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Blast actions from high explosives. Studies on their simulation and effects

Candidate
Filippo Masi

Advisor
Prof. Paolo Maria Mariano

Co-advisors
Prof. Bruno Facchini
Prof. Paolo Vannucci

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To my parents

*Illud in his rebus vereor, ne forte rearis
impia te rationis inire elementa viamque
indugredi sceleris. Quod contra saepius illa
religio peperit scelerosa atque impia facta.
Aulide quo pacto Triviai virginis aram
Iphianassai turparunt sanguine foede
ductores Danaum delecti, prima virorum.
Cui simul infula virgineos circum data comptus
ex utraque pari malarum parte profusast,
et maestum simul ante aras adstare parentem
sensit et hunc propter ferrum celare ministros
aspectuque suo lacrimas effundere civis,
muta metu terram genibus summissa petebat.
Nec miserae prodesse in tali tempore quibat,
quod patrio princeps donarat nomine regem.
Nam sublata virum manibus tremibundaque ad aras
deductast, non ut sollemni more sacrorum
perfecto posset claro comitari Hymenaeo,
sed casta incestu nubendi tempore in ipso
hostia concideret mactatu maesta parentis,
exitus ut classi felix faustusque daretur.
Tantum religio potuit suadere malorum.*

Tito Lucrezio Caro,
De Rerum Natura, Liber I (vv. 80-101)

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Abstract

Bombing attacks are one of the most destructive means of the terrorist threat in terms of both human lives and cultural heritage. The mechanical problem of the effects of a blast on structures, in order to assess their vulnerability and design functioning protections for saving human lives and preserving historical monuments, is a crucial issue for the present international situation.

Here, we introduce a strategy suited for the calculation of the blast actions on structures comparing three different numerical schemes: JWL, CONWEP and TM5-1300. A procedure based upon precise interpolations of the experimental data allows comparison of the above models on simple cases.

We simulate first a blast inside Pantheon in Rome, taking into account for the complex pattern of multiple reflected blast waves and the nonlinearities of no-tension materials. Then, we consider the response to an explosion inside a fuselage made of aluminium alloys.

In this last case we discuss the opportunity given by cost-effective numerical approaches in designing protections through the development of a new concept of a passive protective device for aircrafts cabins.

Keywords: blast actions, fast dynamics, JWL, CEL approach, nonlinear FEA, abrupt damage of monuments and aircrafts.

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Introduction

Terrorist threats have become, unfortunately, a frequent occurrence in 21st Century, although terrorist attacks significantly afflict the world since the second half of the 20th Century. Bombing attacks, in particular, have been the most destructive means in terms of both human lives and cultural heritage. Well-known examples are the Metrojet passenger flight 9268 in 2015, the destructions at Palmyra in 2015 and 2016 and Saint Petersburg metro bombing on April 3rd 2017.

The present international situation has brought to the attention of people and governments the threat on highly representative monuments, that can be today the target of destructive bomb attacks, e.g., the bombings of two Coptic Christian churches in Tanta and Alexandria, Egypt, on April 9th 2017, and the destruction of Great Mosque of al-Nouri, on June 21th 2017.

Bombing attacks in the aviation field take their roots from the first attack on a Boeing 247D in 1933 and had strongly changed embark procedures and laws in international flights. Indeed, in this context, the damage of an aircraft due to an explosion does not consist in a loss of a cultural heritage but in a large amount of human lives. This is why, particular attention in preventing terrorist threats has been paid in the aviation field, starting from the establishment of the Commission on Aviation Security in 1989 charged for determining the explosive limits of current airplanes and proposing measures to prevent disasters similar to that of Pan Am Flight 103 that cost 270 human lives.

Hence, it is of prominent interest to consider the effects of a blast on structures, in order to assess their vulnerability and design functioning protections for saving human lives and preserving historical monuments.

The aim of this thesis consists on the investigation of numerical strategies able to simulate blast actions and their effects, in particular on monuments and aircraft structures. In Chapter 1 we discuss a strategy suited for the calculation of the blast actions on structures by three different schemes: JWL, CONWEP and TM5-1300. Precise interpolations of experimental data allow comparisons on case studies of a barrel vault referring to a typical monumental structure.

Once a numerical procedure has been evaluated, the effects of explosions can be studied in rather heterogeneous fields, i.e., the preservation of monumental buildings and the definition of new designs in protecting fuselage in the aviation field.

Chapter 2 presents an investigation on the effects of an explosion on the structure of the Pantheon in Rome by taking into account the pattern of multiple reflected blast waves in a complex geometry and the nonlinearities of no-tension materials.

Finally, in Chapter 3 we simulate the effects to an explosion inside the fuselage of an aircraft and propose a new concept of protections from bomb attacks.

Chapter 1

State of the art in blast actions modeling

1.1 Introduction

Common investigations on the effects of a blast on a civil structure usually consider simple geometries, which do not represent the actual complexity of the real ones, see e.g. [Remennikov, 2003], [Ngo et al., 2007], [Kocczaz et al., 2008], [Draganic and Sigmund, 2012], [Birhane, 2009]. The same problem stands in the case of mechanical structures such as aircrafts, as it will be discussed in Ch. 3.

In this chapter, we consider a simple structure typical of some monuments, like churches or palace galleries: parallel walls covered by a barrel vault. The chapter is subdivided into six Sections: the first one, Sect. 1.2, is devoted to a general description of the mechanical effects of a blast (we do not consider neither thermal effects nor the projection of fragments). We introduce the JWL model [Jones and Miller, 1948], [Wilkins, 1964], [Lee et al., 1968] in Section 1.3, while Section 1.4 is devoted to the empirical models for the calculation of the blast overpressure. In Section 1.5 we propose a procedure for the evaluation of the blast overpressure, using precise interpolating functions of the experimental data contained in the report [USACE, 1990], while in Section 1.6 we give a brief account of the model CONWEP [USACE, 1986]. Finally, a comparison of the three models is given in Section 1.7, in a case study representing schematically the interior of a possible monument. Section 1.8 collects final remarks.

1.2 The mechanical effects of a blast

An explosion is actually an extremely rapid and exothermal chemical reaction that lasts just few milliseconds. During detonation, hot gases, produced by this chemical reaction, expand quickly and, for the hot temperatures produced instantaneously, the air around the blast expands too. The result is a blast shock wave, characterized by a thin zone of air propagating spherically much faster than the sound speed, through which pressure is discontinuous.

The shock-wave, travelling along a solid surface, produces an almost instantaneous increase of the air pressure on the surface, that decreases very quickly to the ambient pressure; this

is the so-called positive phase of the blast. Then, the pressure decreases further, below the ambient pressure and increases again to the ambient pressure, but in a longer time; this is the negative phase of the blast, see Fig. 1.1. The shock wave is the main mechanical effect of a blast on a structure, but not the only one: the hot gases, expanding, produce the so-called dynamic pressure, least in value with respect to the shock wave and propagating at a lesser speed, while the impinging shock wave can be reflected by solid surfaces and act again on other surfaces as reflected shock wave.

To better understand all the mechanics of a blast, let us first introduce some quantities, used in the following:

- W : explosive mass;
- $R = |q - o|$: distance of a point q from the detonation point o ;
- P_o : ambient pressure;
- P_s : overpressure due to the blast; it is the pressure in the air relative to P_o ;
- P_{so} : side-on overpressure peak: the shock-wave peak pressure, relative to P_o , measured in the air at q ;
- P_r : reflected overpressure: the pressure, relative to P_o , acting at a point q of a solid surface when hit orthogonally by a shock-wave;
- P_{ro} : peak value of P_r ;
- $P_{r\alpha}$: peak value of the reflected overpressure at a point q of a solid surface at tilt of the angle α on the direction of $q - o$;
- t_A : arrival time, i.e., the instant at which the shock-wave peak arrives at q , taking as $t = 0$ the instant of detonation; of course, t_A increases with R , but experimental evidence has shown that it decreases with W , i.e., the velocity of the shock-wave increases with the quantity of explosive;
- t_o : positive phase duration; this is the duration of the time interval, starting from t_A , during which $P_s \geq 0$; t_o increases with R ;
- t_{o-} : negative phase duration; this is the duration of the time interval, starting from $t_A + t_o$, during which $P_s < 0$;
- Z : Hopkinson-Cranz scaled distance [Karlos and Solomos, 2013] defined as,

$$Z = \frac{R}{W^{\frac{1}{3}}}; \quad (1.1)$$

- i_s : impulse of the shock-wave positive overpressure, defined as

$$i_s = \int_{t_A}^{t_A+t_o} P_s(q, t) dt; \quad (1.2)$$

- i_r : impulse of the shock-wave positive reflected overpressure, defined as

$$i_r = \int_{t_A}^{t_A+t_o} P_r(q, t) dt; \quad (1.3)$$

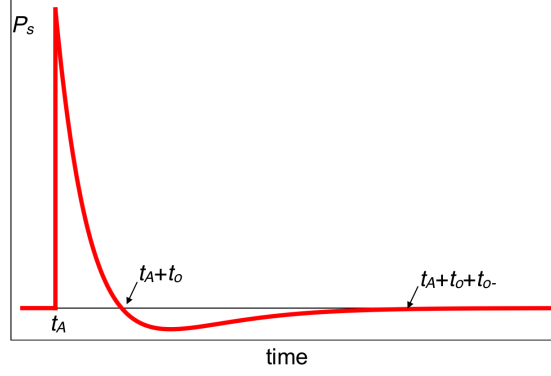


Figure 1.1: Scheme of the time variation of the pressure due to a blast.

- t_{rf} : fictitious duration of the positive phase of the blast, defined as

$$t_{rf} = \frac{2 i_r}{P_{ro}}; \quad (1.4)$$

- t_{Aw} , t_{ow} , t_{ow-} : scaled durations, obtained dividing the respective durations by $W^{\frac{1}{3}}$;
- i_{sw} , i_{rw} : scaled impulses, obtained dividing the respective impulses by $W^{\frac{1}{3}}$.

The overpressure P_s at a point q decreases with both time $t > t_A$ and R . Generally, the time rate decrease is much greater than the space rate decrease: the blast overpressure is really like a very localized pressure wave that propagates at high speed and whose intensity decreases, like for any other wave, with the travelled distance.

Figure 1.1 represents an ideal profile of the overpressure $P_s(q, t)$. When the shock wave arrives at q , after t_A from detonation, the pressure instantaneously increases, from the ambient pressure P_o to a peak P_{so} , which is a strong discontinuity.

For $t > t_A$ the overpressure decreases with exponential rate until time $t_A + t_o$, the end of the positive phase. After $t_A + t_o$ we have the negative phase: the pressure decreases with respect to P_o and then it returns to P_o after a time $t_{o-} > t_o$. Anyway, during the negative phase the decrease of the pressure is much lower, in absolute value, than the peak pressure of the positive phase, so usually the negative phase can be neglected for structural analyses, though it can be important in some particular cases, due to its duration, always much longer than the positive phase.

Generally, the activation time t_A decreases, for the same distance R , when the amount of explosive W increases, i.e., the velocity of the shock-wave increases. The peak value P_{so} , on its side, increases with W and decreases with R , while the time duration t_o does exactly the opposite, see Figure 1.2.

The decrease of the pressure wave, i.e. the function $P_s(q, t)$, is an extremely rapid phenomenon; it can be modeled by the Friedlander's equation, [Karlos and Solomos, 2013]:

$$P_s(q, t) = P_{so}(q) \left(1 - \frac{t - t_A}{t_o} \right) e^{-b \frac{t - t_A}{t_o}}. \quad (1.5)$$

With this model, the impulse can be calculated analytically:

$$i_s(q, t) = P_{so}(q) \frac{t_o}{b^2} (b - 1 + e^{-b}). \quad (1.6)$$

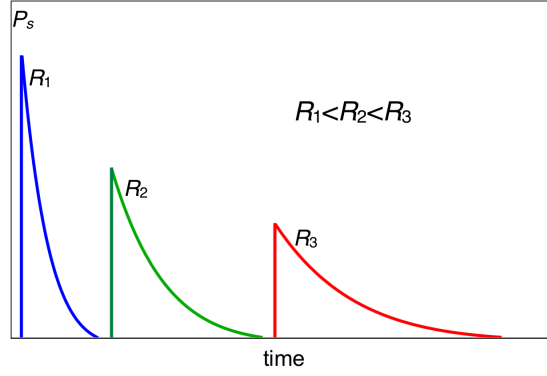


Figure 1.2: Influence of the distance R on the positive phase of a blast.

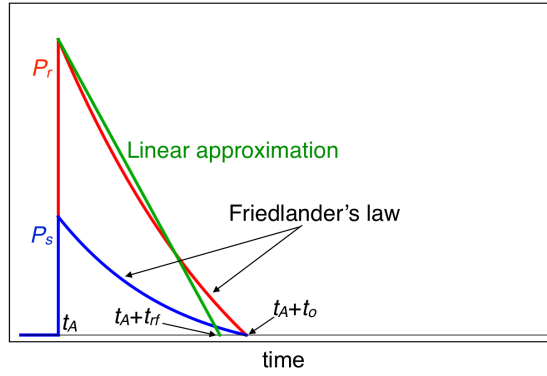


Figure 1.3: The Friedlander law.

This relation is useful for determining the coefficient b if i_s is known, e.g., from experimental data or with the method proposed below. The same rate decrease (1.5) is used also for the reflected overpressure, P_r .

P_r is the pressure that acts on a surface impinged by the incident overpressure P_s . The peak P_{ro} of P_r is normally much greater than P_{so} measured at the same point in the absence of any surface.

In fact, if we consider air as an ideal linear-elastic fluid, the air particles should bounce back freely from the surface, this giving a P_r equal to the double of the incident pressure. But normally, $P_{ro}/P_{so} > 2$ because a blast is actually a nonlinear shock phenomenon, where the reflection of the particles is hindered by subsequent air particles, with consequently a far higher reflected pressure.

A formula relating the values of P_{ro} and P_{so} for normal shocks is

$$P_{ro} = 2P_{so} \frac{4P_{so} + 7P_o}{P_{so} + 7P_o}. \quad (1.7)$$

The above equation indicates, on one side, that the ratio P_{ro}/P_{so} is not constant but depends upon P_{so} and, on the other side, that this ratio can vary between 2 and 8 or more. Of course, it is the value of P_{ro} to be used for structural design. The Friedlander's law is used also for describing the decrease of P_r .

Sometimes, the Friedlander's law is replaced by a simpler linear approximation, see Fig. 1.3. The fictitious positive duration time t_{rf} is then calculated imposing to preserve the same P_{ro} and i_r , which gives equation (1.4).

The reflected pressure becomes, on its turn, an incident pressure for other surfaces: different incident waves can hit a surface besides the first one originated directly by detonation, all reflected by other surfaces, so giving a time history of the overpressure at a point that can have several successive peaks. However, normally the first peak is the highest one.

Another phenomenon produced by a blast is the dynamic pressure; the air behind the front of the blast wave moves like a wind, but with a smaller velocity. This wind causes a *dynamic* or *drag* pressure Q , loading a surface for the whole duration of the wind produced by the blast. Its peak value Q_o is less than P_{so} and it is delayed with respect to t_A but Q has a much longer duration (up to $2 \div 3$ s) than t_o (some ms).

The blasts can be of different types: in *free-air bursts* the detonation occurs in the air and the blast waves propagate spherically outwards and impinge first and directly onto the structure, without prior interaction with other obstacles or the ground. In *air bursts* the detonation occurs still in the air but the overpressure wave arrives at the structure after having first interacted with the ground; a *Mach wave front* is created. *Surface bursts* are the explosions where detonation occurs almost at ground surface: the blast waves immediately interact locally with the ground and they propagate next hemispherically outwards and impinge onto the structure. Finally, in *internal blasts* the detonation occurs inside a structure: blast waves propagate and interact with the different walls, reflected waves are generated and the effects of dynamic pressure due to gas expansion are amplified by the surrounding space.

In the case of explosions on monuments, we are concerned with surface blasts, i.e., with hemispherical overpressure waves. Surface blasts result in much greater shock overpressure than air-blasts, because of the ground effect, that reflects and amplifies the overpressure wave.

The case of internal blasts is also important in the study of monuments; nevertheless, in several cases an internal blast can be considered as an external one. This happens for buildings like churches or great halls, where the internal volume is so great to limit the effects of dynamic pressure and reflected waves.

The simulation of a blast can be conducted using different approaches, the most widely used being three: JWL, CONWEP and TM5-1300 models.

1.3 JWL model

JWL stands for Jones, Wilkins, and Lee [Jones and Miller, 1948], [Wilkins, 1964], [Lee et al., 1968]. Basically, JWL is a physically based equation of state (EOS) using the laws of thermodynamics to recover the physics of a chemical blast.

This model allows, in principle, to obtain a complete description of a blast phenomenon, i.e., including not only the propagation of the shock-wave in a medium, e.g. air, but also its reflection on solid surfaces and the expansion of the hot gases, i.e., the dynamic pressure. JWL model is implemented in different commercial codes and its use needs the meshing not only of the structure but also of air and charge volumes involved in the blast.

JWL model gives the overpressure P_s inside the explosive material as function of different parameters:

$$P_s = A \left(1 - \frac{\omega \rho}{R_1 \rho_0} \right) \exp \left(-R_1 \frac{\rho_0}{\rho} \right) + B \left(1 - \frac{\omega \rho}{R_2 \rho_0} \right) \exp \left(-R_2 \frac{\rho_0}{\rho} \right) + \omega \rho E_m. \quad (1.8)$$

In the above constitutive phenomenological equation, A , B , R_1 , R_2 and ω are parameters depending upon the explosive, along with ρ_0 , its density, while ρ is the density of the detonation products and E_m is the internal energy per unit mass. In addition, detonation velocity v_D need to be specified. All the parameters are calibrated by fitting results of the cylinder expansion test, see Tab. 1.1.

JWL EOS is based upon a first order expansion in energy of the principal isentrope for the detonation products, i.e.,

$$P_s = A \exp\left(-R_1 \frac{\rho_0}{\rho}\right) + B \exp\left(-R_2 \frac{\rho_0}{\rho}\right) + C \left(\frac{\rho_0}{\rho}\right)^{-(\omega+1)}. \quad (1.9)$$

The principal isentrope, (1.9), consists of three different contributes, each one describing the propagation of a detonation wave inside an explosive material: (i) the first exponential term describes the high pressure regime and dominates when the explosive is compressed so that $\rho_0/\rho < 1$, (ii) the second one represents the high-intermediate pressure range, i.e. when $1 < \rho_0/\rho < 3 \div 4$, and (iii) the last term is the equation of state of an ideal gas that ensures a correct behaviour at large expansion, $\rho_0/\rho > 4$, see Fig. 1.4.

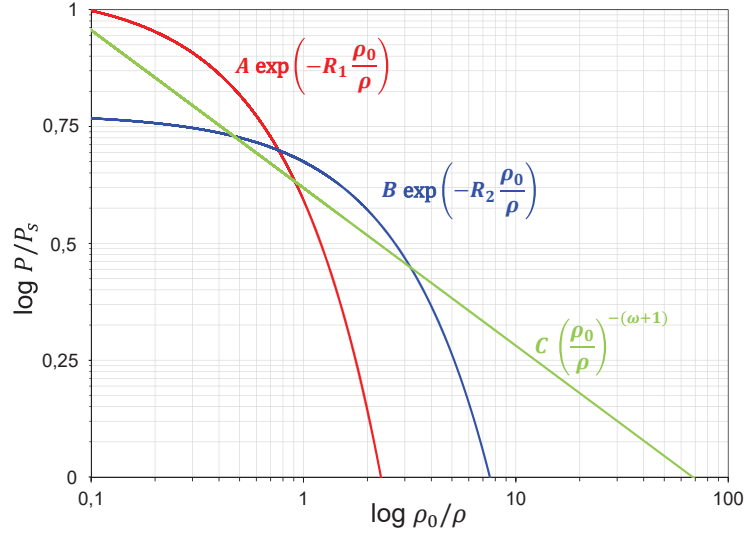


Figure 1.4: Overpressure inside an explosive material, trinitrotoluene, at varying of the density of the detonation products.

The use of JWL model allows a rather precise and complete simulation of the blast phenomenon, but its drawback is the need of discretizing, finely, the charge and the fluid domain, the latter can be very large, besides the structure for the coupled structural analysis.

Air is modeled as an ideal gas with total pressure given by

$$p + p_A = \rho \bar{R}(T - T^z), \quad (1.10)$$

where p_A and p are, respectively, ambient and current (fluid) pressures; $\bar{R}$ is the universal gas constant, i.e., the product between Boltzmann and Avogadro constants; T is the current temperature; and T^z is the absolute zero temperature, namely $T^z = 0$ K. The model permits to compute the specific energy, E_m , as a function of the temperature alone:

$$E_m = E_{m0} + \int_{T_0 - T^z}^{T - T^z} c_v(\theta) d\theta, \quad (1.11)$$

where E_{m0} is the initial specific energy at the initial temperature T_0 ; c_v is the specific heat at constant volume (we assume it remains constant at varying of temperature); and $d\theta$ is the differential of the temperature θ .

We can easily assess that multi-physics transient problems, with a strong coupling between fluid and structure dynamics, lead to numerical simulations that can be very heavy, from a computational viewpoint.

Table 1.1: Physical parameters for the state equations for air and explosive, after [Rogers and Mayhew [1988]] and [Lee et al. [1973]].

AIR	ρ [kg/m ³] 1.225	Temperature [K] 288.2	Gas constant [J/(kg K)] 287.058	Specific heat [W/(m K)] 717.6				
TNT	ρ_0 [kg/m ³] 1630	v_D [m/s] 6930	A [MPa] 373770	B [MPa] 3747.1	ω 0.35	R_1 4.15	R_2 0.9	E_m [kJ/kg] 3680

1.4 Empirical models

Because of the drawbacks of the model JWL detailed hereon, more empirical methods are often used in the calculations. They offer a good balance between computing cost and precision. These models consider the effects of the blast, namely the pressure field, and rely on data fitting.

With such models, the characteristic parameters of the explosive serve to calculate the overpressure shock-wave and its propagation speed. The wave propagates spherically from the detonation point o to the elements of the structure. The distance of o from any impact point q on a surface of the structure and the inclination of the perpendicular to the surface with respect to the vector $q - o$ are the only geometric parameters needed by the models.

In fact, these models consider just the incident wave, not the reflected ones, nor the dynamic pressure, that can be anyway calculated afterward. So, what is mainly lost with empirical models is the possibility of taking into account for the set of reflected waves that impinge again the wall surfaces.

Nevertheless, the effect of the reflection of the shock-wave by the ground, in the case of a ground-explosion, is taken into account by specific laws, different from those modeling a free-air burst: the models for hemispherical blasts are not those for the spherical ones, the first ones giving higher values of the overpressure to take into account for the ground reflection and the formation of the so-called Mach stem.

This phenomenon is due to the reflection of the wave pressure by a surface. In general, the overpressure shock-wave due to an air detonation is produced by an incident wave, emanating from the explosive charge, and by a wave reflected, at least, by the ground. For small incident angles, up to about 40°, the incident wave is ahead of the reflected wave produced by the surface and typical reflection occurs. However, for larger angles, coalescence between the incident and the reflected wave takes place, creating a Mach stem.

In ground explosions, the interaction between the ground and the blast wave takes place since the beginning, due to the closeness of the detonation point to the ground surface. Instead of the creation of a Mach front at a certain distance from the detonation point, the

incident wave is reflected immediately by the ground. This wave coalescence can give much greater pressure values than the normal reflection.

If the ground were a rigid surface, the reflected pressure P_r would be twice that of a free-air burst. In practice, a part of the energy is absorbed by the creation of a crater, so P_r is less than the double of a free-air burst pressure, say $1.7 \div 1.8$ times.

The two most commonly used empirical models are based upon different but related studies of the U.S. Army Corps of Engineers (USACE): the document [USACE, 1986], containing the model CONWEP, and the Technical Manual TM5-1300, [USACE, 1990], completed by successive documents, [USACE, 2008]. The Joint Research Center of the European Union has produced in 2013 a Technical Report, [Karlos and Solomos, 2013], substantially referring to these two last documents and to another Technical Report of the U.S. Army, [Kingery and Bulmash, 1984]. In [Karlos and Solomos, 2013] all the empirical laws of [USACE, 1990] are reproduced using S. I. units.

It is worth recalling that [USACE, 1990] gives different rules for the calculation of the effects of a blast inside a building; all of them lead to an increment of the overpressure, but they all refer to some simple geometrical situations, that hardly can represent, for dimensions and geometry, a typical internal volume of a monument, like a cathedral or a palace. That is why the use of these rules in such a context is quite problematic.

We give below a short description of these two models, but before we need to introduce the empirical laws used to calculate the overpressure peak P_{so} and other technical parameters.

1.4.1 Empirical laws for P_{so}

In the literature, there are several empirical laws for the prediction of P_{so} , all of them expressing it as function of the scaled distance Z , eq. (1.1). The most well known are listed below (R in m, W in kg, P_{so} in MPa for all the formulas). The *Kinney-Graham formula*

$$P_{so} = P_o \frac{808 \left[1 + \left(\frac{Z}{4.5} \right)^2 \right]}{\sqrt{\left[1 + \left(\frac{Z}{0.048} \right)^2 \right] \left[1 + \left(\frac{Z}{0.32} \right)^2 \right] \left[1 + \left(\frac{Z}{1.35} \right)^2 \right]}}, \quad (1.12)$$

is valid for spherical explosions [Kinney and Graham, 1985], as the *Brode formula* [Brode, 1955],

$$P_{so} = \begin{cases} \frac{0.67}{Z^3} + 0.1, & \text{for } P_{so} > 1 \text{ MPa,} \\ \frac{0.0975}{Z} + \frac{0.1455}{Z^2} + \frac{0.585}{Z^3} - 0.0019, & \text{for } P_{so} < 1 \text{ MPa,} \end{cases} \quad (1.13)$$

and the *Mills equation* [Mills, 1987],

$$P_{so} = \frac{1.772}{Z^3} - \frac{0.114}{Z^2} + \frac{0.108}{Z}, \quad (1.14)$$

while the *Newark-Hansen formula* serves to model hemispherical blasts [Newmark and Hansen, 1961]:

$$P_{so} = \frac{678400}{Z^3} + \frac{2941}{Z^{\frac{3}{2}}}. \quad (1.15)$$

However, the most widely used method for blast parameters evaluation is based upon the

experimental data of Kingery and Bulmash [Kingery and Bulmash, 1984]. Such data concern both the cases of spherical and hemispherical explosions and most important they provide the data not only for the incident pressure peak P_{so} , but also for the reflected one, P_{ro} , which is by far most interesting for design purposes.

In Figure 1.5 we show the value of P_{so} as predicted by the above models and by the Kingery's and Bulmash's experiments, with data fitted for both the cases of spherical and hemispherical blasts.

One can observe that the hemispherical case gives always higher values of P_{so} than the spherical one, for the two curves provided by Kingery and Bulmash. Also, at small scaled distances, the predictions of the models by Brode, Newmark and Mills substantially deviates from Kingery's and Bulmash's. This is mainly due to the fact that the three models have been proposed for predicting the effects of nuclear blasts, not those of conventional explosives. On its side, the Kinney curve gives rather satisfactory predictions, i.e., similar to Kingery's and Bulmash's, for all the scaled distance range.

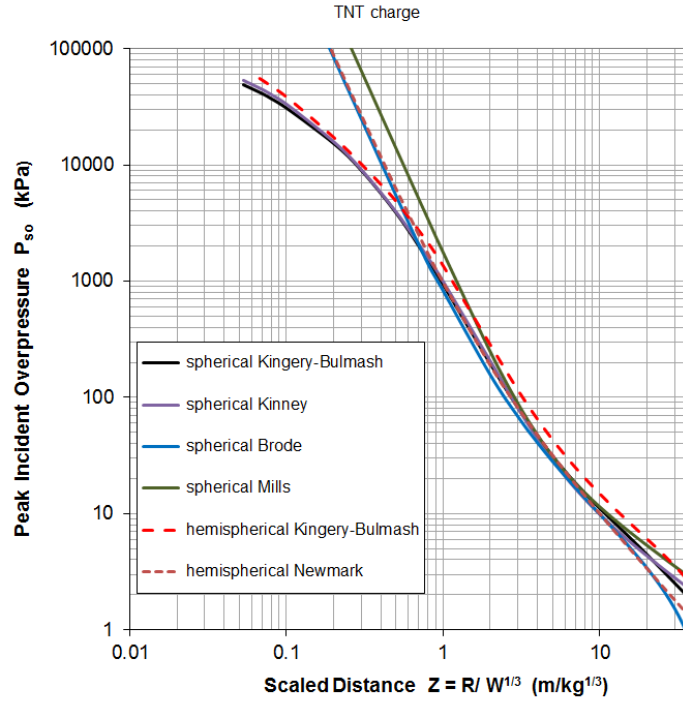


Figure 1.5: Comparison of P_{so} for different blast models (from [Karlos and Solomos, 2013]).

The experimental data by Kingery and Bulmash are represented in Figure 1.6, uniquely for the positive phase of a blast, the most important one; such curves have been obtained by data fitting. The diagrams are referred to the explosion of 1 kg of TNT and concern a distance range from 0.5 to 40 m. As they are given as functions of Z , it is easy to adapt such data also to other cases, by multiplying by $W^{\frac{1}{3}}$ the value of the scaled parameters, except for pressure and velocity.

1.4.2 Interpolation of the Kingery-Bulmash data

In order to have precise analytical expressions of the diagrams in Figure 1.6, we interpolated them and we give below the results for the different blast parameters of interest. The results are referred to the case of a hemispherical blast, the one of concern in our case. For

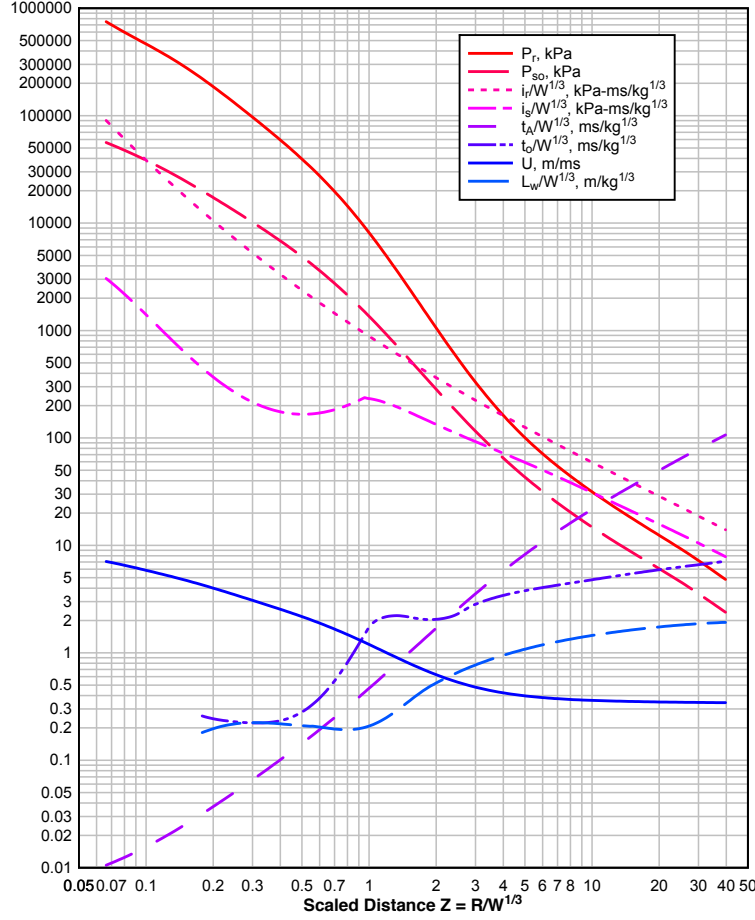


Figure 1.6: Blast parameters for the positive phase of a hemispherical blast, according to Kingery and Bulmash (from [Karlos and Solomos, 2013]).

each parameter, we give the analytical interpolating formula and a diagram comparing the experimental data, in blue, and the interpolating curve, in thin red. Among the parameters in Figure 1.6, we have not interpolated the incident impulse i_s , the shock wave speed, U , and the blast wavelength, L_w , because they are not necessary in the following. The interpolating functions that we have used are listed below. For the incident pressure P_{so} , we accept the expression

$$P_{so} = \exp(0.14 - 1.49 \ln Z - 0.08 \ln^2 Z - 0.62 \sin(\ln Z)) \left(1 + \frac{1}{2e^{10Z}}\right), \quad (1.16)$$

and, for the reflected pressure P_{ro} ,

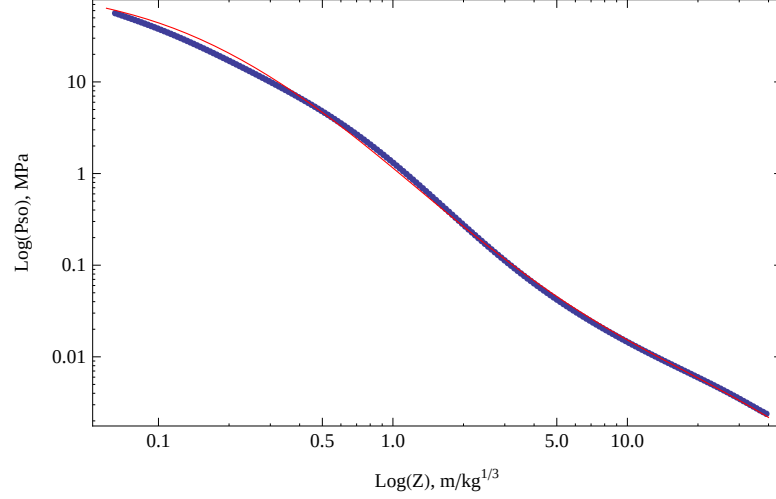
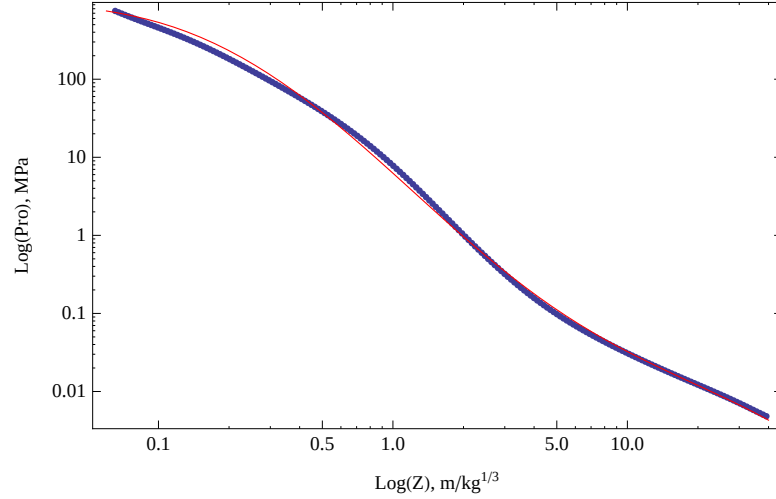
$$P_{ro} = \exp(1.83 - 1.77 \ln Z - 0.1 \ln^2 Z - 0.94 \sin(\ln Z)) \left(1 + \frac{1}{2e^{10Z}}\right). \quad (1.17)$$

The function interpolating the scaled reflected impulse i_{rw} is

$$i_{rw} = \exp(-0.11 - 1.41 \ln Z + 0.085 \ln^2 Z), \quad (1.18)$$

the one for the scaled activation time t_{Aw} is

$$t_{Aw} = \exp(-0.685 + 1.429 \ln Z + 0.029 \ln^2 Z + 0.411 \sin(\ln Z)), \quad (1.19)$$

Figure 1.7: Comparison of experimental and fitted values of P_{so} .Figure 1.8: Comparison of experimental and fitted values of P_{ro} .

while that for the scaled duration time t_{ow} is

$$t_{ow} = \exp(-0.846 + 1.041Z + 0.408 \ln Z - 1.105 \ln^2 Z - 0.295 \ln^3 Z + 0.143 \ln^4 Z - 0.054 \ln^5 Z). \quad (1.20)$$

1.4.3 Influence of the type of explosive

The experimental data and formulae for the blasts are always referred to TNT, used as reference explosive. To assess the effects of a blast produced by another explosive, an equivalent weight W of TNT is computed according to the following formula, that links the weight W_e of a chosen explosive to W using the ratio of the heat produced during detonation, [Karlos and Solomos, 2013]:

$$W = W_e \frac{H_e}{H_{TNT}}, \quad (1.21)$$

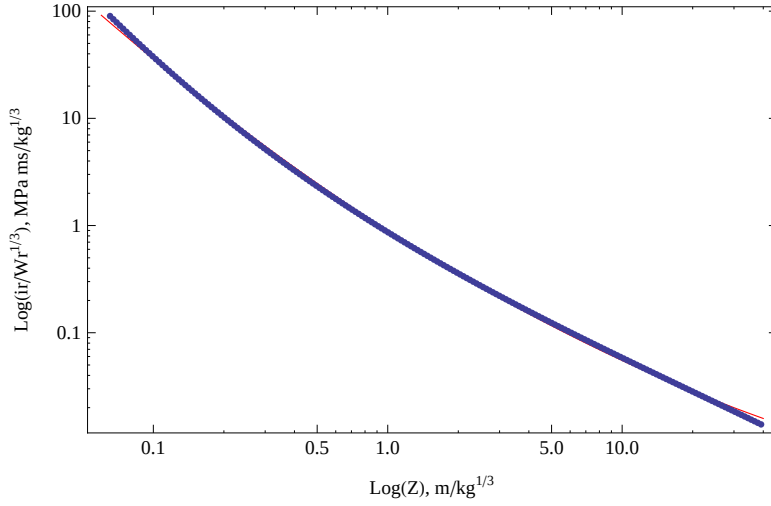


Figure 1.9: Comparison of experimental and fitted values of i_{rw} .

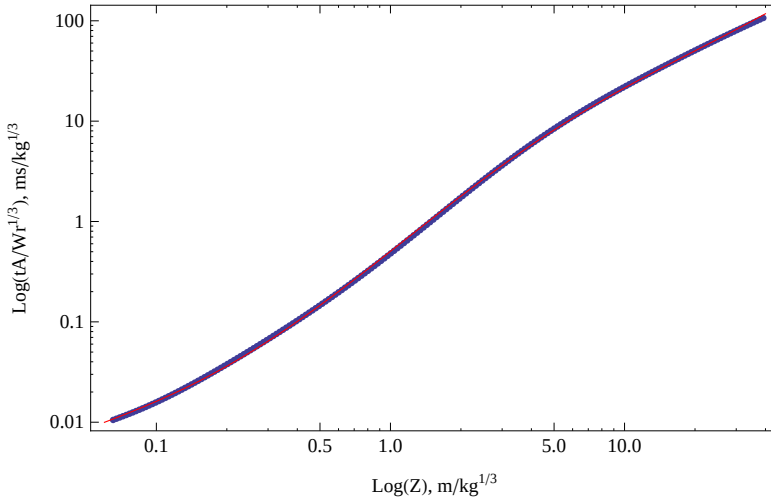


Figure 1.10: Comparison of experimental and fitted values of t_{Aw} .

where H_e is the heat of detonation of the explosive and H_{TNT} is that of TNT. The values of the heat of detonation of some explosives are given in Tab. 1.2.

1.5 Calculation of P_r on a surface using TM5-1300

1.5.1 Influence of the direction

The values of the incident and reflected pressures and impulses are intended for a normal shock, i.e., when the vector $q - o$ is orthogonal to the impinged surface in q . In such a case, the reflected pressure takes its maximum local value, that decreases when the shock is not orthogonal, i.e. when the vector $q - o$ forms an angle $\alpha > 0$ with the inward normal to the impinged surface.

The effect of the lack of orthogonality in the shock is taken into account introducing the

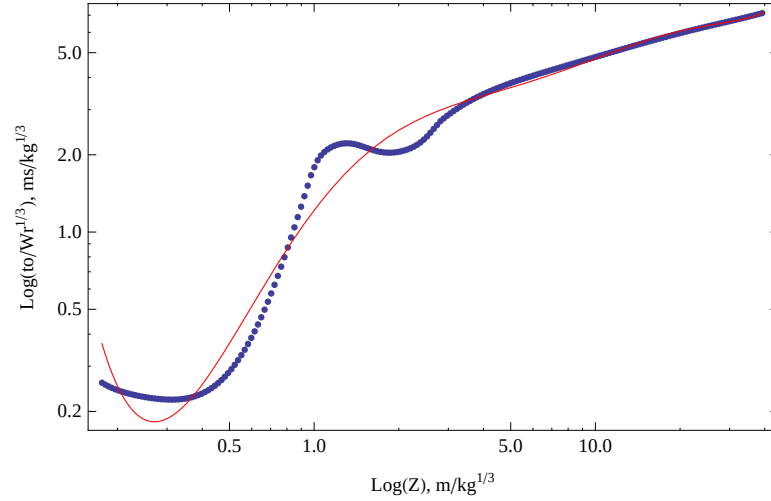
Figure 1.11: Comparison of experimental and fitted values of t_{ow} .

Table 1.2: Heat of detonation of different explosives (from [Karlos and Solomos, 2013]).

Type of explosive	Heat of detonation
	[MJ/kg]
TNT	4.10 ÷ 4.55
C4	5.86
RDX	5.13 ÷ 6.19
PETN	6.69
Pentolite 50/50	5.86
Nitroglycerin	6.30
Nitrocellulose	10.60

reflection coefficient $c_{r\alpha}$, defined as

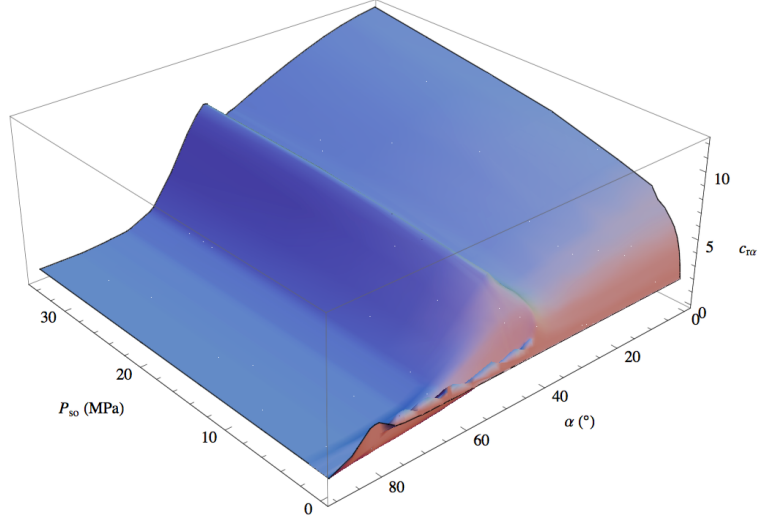
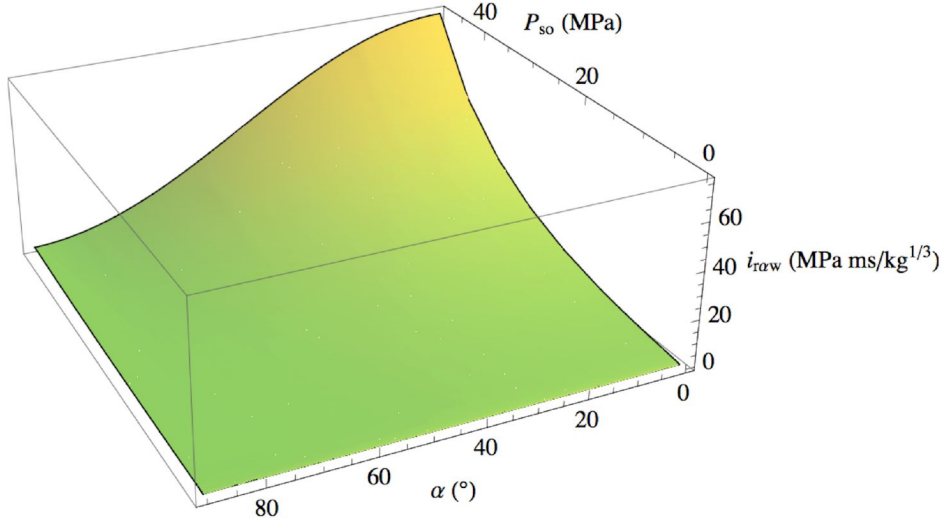
$$c_{r\alpha} = \frac{P_{r\alpha}}{P_{so}}, \quad (1.22)$$

where $P_{r\alpha}$ is the peak of the reflected pressure in q for a surface inclined of the angle α onto the direction of $q - o$. It is worth noticing that the reflection coefficient is defined as the ratio of the reflected inclined pressure $P_{r\alpha}$ with the incident pressure P_{so} , not P_{ro} .

The value of $c_{r\alpha}$ has been evaluated experimentally, and the results are shown in Fig. 1.12; this figure has been obtained from the parametrized curves given in [Karlos and Solomos, 2013]. To remark that $c_{r\alpha}$ depends not only upon α , but also upon P_{so} .

In Fig. 1.13 we show also the interpolation of the empirical values of $i_{r\alpha w}$, still obtained from [Karlos and Solomos, 2013].

Looking at Fig. 1.12, one can see that $c_{r\alpha}$ is not a decreasing function of α , as it could be expected. Actually, for $\alpha > \sim 40^\circ$, $c_{r\alpha}$ increases reaching a maximum and then it decreases again. This fact is attributed to the creation of the Mach stem cited above. For small values of P_{so} the behavior is more complicated, but this presumably could be the effect of imperfections in the experimental data.

Figure 1.12: Surface representing $c_{r\alpha}(\alpha, P_{so})$.Figure 1.13: Surface representing $i_{r\alpha w}(\alpha, P_{so})$.

1.5.2 Computational procedure

We detail in the following the sequence of the calculations to obtain, for a given blast, the time history of the reflected pressure $P_r(q, t)$ at the time t on a point q of a surface placed at a distance R from the detonation point o with inward normal forming an angle α with the vector $q - o$.

We make the following assumptions:

- i. the blast occurs at point o and is produced by a mass W_e of a given explosive;
- ii. the blast is hemispherical;
- iii. the time rate decrease is of the same type for P_s and P_r and it is ruled by the Friedlander's equation;
- iv. the calculation of the time history of P_s and P_r , as well as the impulse, is made pointwise;

- v. only the positive duration phase is considered (this assumption, usually done, is justified because of the much larger values of the positive pressures in comparison with the negative ones);
- vi. reflected waves and dynamic pressure are ignored.

Then, the algorithm goes as follows:

- a time duration t_{max} is fixed;
- t_{max} is subdivided into time intervals dt ;
- the impinged surface is discretized into regular patches, whose centroids are points q ;
- the equivalent mass of TNT W is calculated using eq. (1.21);
- then, for each point q we calculate:

- the distance from the blast point: $R = |q - o|$;
- the inward normal $\mathbf{n}$ to the wall;
- the angle α between $q - o$ and $\mathbf{n}$:

$$\cos \alpha = \frac{q - o}{R} \cdot \mathbf{n}; \quad (1.23)$$

- the scaled distance Z , eq. (1.1);
- the normal incident pressure peak P_{so} using the fitted curve of Kingery-Bulmash for hemispherical blasts, eq. (1.16), Fig. 1.7;
- the normal reflected pressure peak P_{ro} using the fitted curve of Kingery-Bulmash for hemispherical blasts, eq. (1.17), Fig. 1.8;

- the ratio

$$c = \frac{P_{ro}}{P_{so}}; \quad (1.24)$$

- the angular coefficient $c_{r\alpha}$, eq. (1.22), by a linear interpolation of the experimental data represented by the surface in Figure 1.12;

- the ratio

$$c_{r0} = \frac{P_{r0}}{P_{so}}, \quad (1.25)$$

i.e. $P_{r\alpha}$ for $\alpha = 0^\circ$, still linearly interpolating the data of the surface in Fig. 1.12;

- the corrected value of $P_{r\alpha}$ as

$$P_{r\alpha} = c \frac{c_{r\alpha}}{c_{r0}} P_{so} = \frac{c_{r\alpha}}{c_{r0}} P_{ro}; \quad (1.26)$$

this correction is done to adapt the data of the interpolating surface in Fig. 1.12 to those of the Kingery-Bulmash fitted curves, Fig. 1.6, more conservative;

- the reduction coefficient

$$c_{red} = \frac{P_{r\alpha}}{P_{ro}} = \frac{c_{r\alpha}}{c_{r0}}; \quad (1.27)$$

- the positive normal scaled reflected impulse i_{rw} using the fitted curve of Kingery-Bulmash for hemispherical blasts, eq. (1.18), Fig. 1.9;
- the effective angular reflected impulse $i_{r\alpha}$ as

$$i_{r\alpha} = c_{red} i_{rw} W^{\frac{1}{3}}; \quad (1.28)$$

the value of $i_{r\alpha}$ is voluntarily *not* calculated interpolating the surface of experimental data, Fig. 1.13, like for $c_{r\alpha}$. Indeed, we have observed, numerically, that the value of $i_{r\alpha}$ from the fitted surface can present underestimations, for some α , if compared to the one obtained by computing the ratio between the effective angular reflected and the positive normal reflected impulses as the value of the reduction coefficient, namely $i_{r\alpha}/(i_{rw}W^{1/3}) = c_{red}$. Therefore, the selected procedure, eq. (1.28), is conservative;

- the scaled activation time t_{Aw} using Kingery-Bulmash's fitted curve for hemispherical blasts, eq. (1.19), Fig. 1.10;
- the activation time t_A as

$$t_A = t_{Aw} W^{\frac{1}{3}}; \quad (1.29)$$

- the scaled positive duration time t_{ow} using Kingery-Bulmash's fitted curve for hemispherical blasts, eq. (1.20), Fig. 1.11;
- the positive duration time t_o as

$$t_o = t_{ow} W^{\frac{1}{3}}; \quad (1.30)$$

- the fictitious positive duration time t_{rf} , eq. (1.4), as

$$t_{rf} = 2 \frac{i_{r\alpha}}{P_{r\alpha}}; \quad (1.31)$$

- check on t_o : if $t_o < t_{rf}$ then put $t_o = 1.1 t_{rf}$; this is done to avoid pathological situations such as a negative, thus unphysical, value of the exponential decay coefficient b in Friedlander's equation, eq. (1.5), due to the fact that the Kingery-Bulmash curve for t_o does not cover low ranges of Z , see Fig. 1.6 and 1.11;

- solve numerically the equation

$$\frac{b-1+e^{-b}}{b^2} P_{r\alpha} t_o = i_{r\alpha} \quad (1.32)$$

to determine the coefficient b of the Friedlander's equation.

- the time history of the pressure wave can now be calculated:

- $t = n \, dt$;
- $\forall q$:
 - * if $t < t_A$ or $t > t_A + t_o$ then $P_r(q, t) = 0$;
 - * else, use the Friedlander equation (1.5) with $P_{r\alpha}$ in place of P_{so} to evaluate $P_r(q, t)$;

- iterate on t until $t > t_{max}$.

This sequence has been implemented in a program for the formal code *Mathematica* and applied to the case study presented in Section 1.7.

1.6 The CONWEP model

CONWEP is the acronym of CONventional Weapons Effects Programme, a study made by USACE, [USACE, 1986], for the simulation of the effects of a blast produced by conventional, i.e. not nuclear, explosives. The report [USACE, 1986] uses in many parts the same experimental results shown above, but not completely; in particular, a noticeable difference is the way in which the reflected pressure is calculated; in place of using the data represented in Figure 1.13, and integrating the effect of the Mach stem, CONWEP makes use of the following law, using circular functions:

$$P_{r\alpha} = \begin{cases} P_{so}(1 + \cos \alpha - 2 \cos^2 \alpha) + P_{ro} \cos^2 \alpha & \text{if } \cos \alpha \geq 0, \\ P_{so} & \text{if } \cos \alpha < 0. \end{cases} \quad (1.33)$$

The algorithm CONWEP is today implemented in different finite elements commercial codes, e.g. in *LS-DYNA*, *AUTODYN* or *ABAQUS*. In particular, in *ABAQUS*, the code we used for numerical simulations with CONWEP, the following assumptions are made:

- for each point q of the impinged walls, $P_{r\alpha}$ is calculated according to eq. (1.33);
- reflected waves are ignored;
- the dynamic pressure is neglected;
- the negative phase is taken into account;
- hemispherical blasts can be modeled.

So, the assumptions made by CONWEP are similar to those introduced in Sec. 1.5.2, except for the calculation of $P_{r\alpha}$ and the fact that in CONWEP the negative phase is considered too. That is why a comparison of the two methods is interesting, see the next Sec. 1.7.

In CONWEP, reflected waves and dynamic pressure are ignored, it means that there is no difference between internal and external blasts: the only geometric parameters that matter are R and α , regardless of the surrounding geometry. For numerical simulations, it allows for modeling only a part of the building, the closest one to the detonation point, and to reasonably neglect the effects, at least in terms of applied pressures, of more distant parts. Therefore, only a part of the building can be modeled, saving in this way computing time.

1.7 Comparison of the models on a case study

Even though CONWEP and JWL models are implemented in some commercial finite element codes, this is not the same for TM5-1300. This is probably due to the fact that it is easier to implement the dependence on the direction described by eq. (1.33) than the more accurate of TM5-1300, described in Figure 1.12.

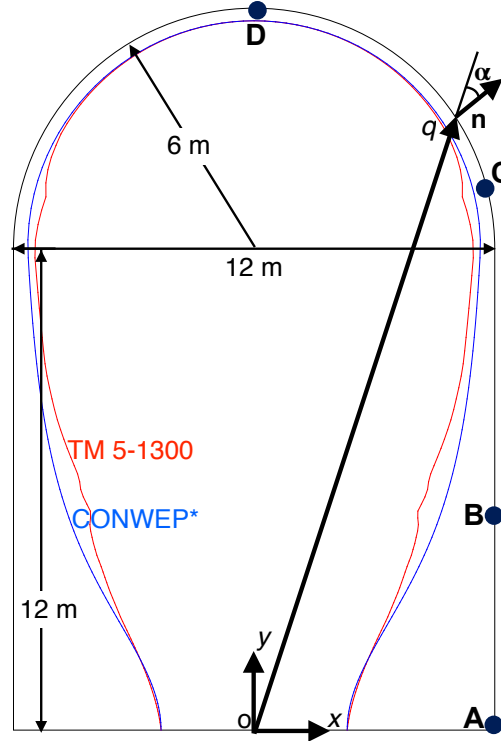


Figure 1.14: Case study: layout and $P_{r\alpha}$ distribution for the model TM5-1300 and CONWEP*.

As said above, in the case of large buildings, like monuments, to make a complete nonlinear fluid-solid simulation using the model JWL can be computationally expensive and unnecessary from an engineering point of view. This is why CONWEP is generally preferred.

The following comparison is hence made with the objective of evaluating the performance of CONWEP, in comparison with both JWL and TM5-1300, and finally to calibrate its use in numerical simulations.

To evaluate the differences in the prediction of the blast parameters, and namely of the reflected pressure, between the three models, we consider an explosion in the interior of a building covered by a barrel vault, see Fig. 1.14. This is intended to simulate the typical structure of a monumental building, like a church or a palace gallery and so on, or also a domed structure. The dimensions used in this case are listed below:

- width: 12 m,
- height of the walls: 12 m,
- radius of the vault: 6 m,
- total height: 18 m.

The explosive charge is composed by $W = 20$ kg of TNT, and detonates at the instant $t = 0$ at point $o = (0,0)$. The analysis is planar, except for the JWL model, where we use a 3D approach.

We evaluate $P_{r\alpha}$ in four different ways. In the first one, we use JWL model, as implemented in *ANSYS Autodyn*; the results are indicated by JWL. Then, we consider the data of TM5-1300 and following modifications, as integrated into [Karlos and Solomos, 2013], according

to the calculation sequence presented in Sec. 1.5.2; the results are indicated by TM5-1300. The third way is the use of the data of TM5-1300 and following modifications, as integrated into [Karlos and Solomos, 2013], but evaluating the pressure $P_{r\alpha}$ according to equation (1.33); the results are indicated by CONWEP*. Finally, we apply the model CONWEP as implemented in *ABAQUS* to the above structure; the results are indicated as CONWEP.

TM5-1300 and CONWEP* results have been obtained by using a program we have done in *Mathematica*.

1.7.1 The results from JWL

The analyses with JWL model run with *AUTODYN Hydrocode*, a code allowing good facilities in modeling the set of explosive, air domain and structure, where the equations of mass, momentum and energy conservation for inviscid flows are coupled with the dynamics of simple solids. A *Coupled Eulerian-Lagrangian* (CEL) method is expedient: explosive and surrounding air, as fluids, are modeled with an Eulerian frame, while solid walls are identified in a Lagrangian reference.

The 3D geometrical model, taking advantage of symmetries, consists of a quarter of the entire domain. A volume having the transversal section like in Figure 1.14 and a depth of 2 m hosts the mesh. Ground and planes of symmetry are modeled as *reflecting planes* to prevent flow of material through them. The ending transversal surfaces are modeled as *transmitting planes*, i.e., boundary surfaces whereupon the gradients of velocity and stress vanish. This approach is used to simulate a far field solution at the boundary, it is only exact for outflow velocities higher than the speed of sound and is just approximated for lower velocities.

A preliminary analysis of the solution sensitivity to the cell size has been made studying the peak of the reflected pressure at point A of Figure 1.14 for five different meshes. For each one of them, the dimensions of the volume elements discretizing the explosive charge and of the surface elements modeling the explosive-air and air-solid interfaces are decreased each time by a factor 1.5, starting from the coarsest mesh.

In Table 1.3 and in Figure 1.15 we show the relative percent error, computed with respect to the finest discretization; it remains very small in all cases. Attention needs to be paid when dealing with a CEL approach: more precisely, an almost equal size of Eulerian and Lagrangian elements and an average grid size smaller than the thickness of the solid walls must be guaranteed. The simulations have been made with the mesh M2.

Table 1.3: Convergence study for different meshes.

Meshes	M1	M2	M3	M4	M5
Body sizing EXPLOSIVE [m]	0.0033	0.005	0.0075	0.01	0.015
Face sizing EXPLOSIVE-AIR / AIR-SOLID [m]	0.013	0.02	0.03	0.04	0.06
Relative error on P_{ro} at point A [%]	0.	0.018	0.059	0.106	0.274

The results of the numerical simulation appear in Figure 1.16.

We may see the formation not only of the principal, hemispheric shock-wave, but also of the reflected ones, which produce the Mach stem, well visible in the photogram at $t = 14$ ms, at the bottom of the waves reflected by the vertical walls. At $t \sim 48$ ms, the incident and reflected waves focus just at the top of the vault, giving rise to a very high localized pressure. Then, the reflected waves propagate downwards and upwards and decrease in

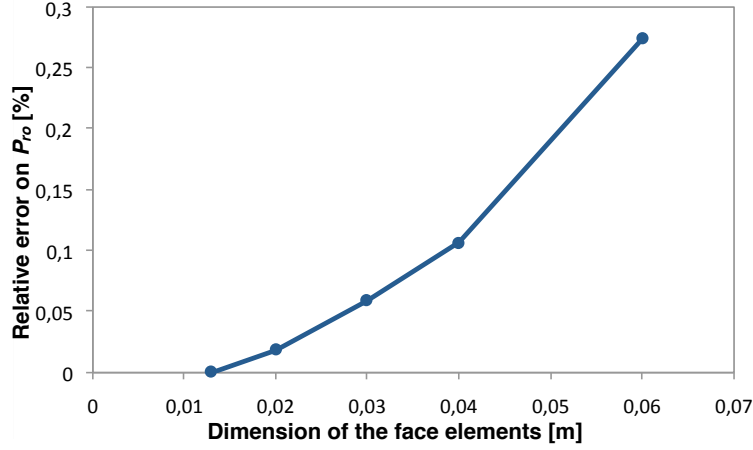


Figure 1.15: Convergence study for different meshes.

intensity. Though not well visible in the figures, also the dynamic pressure is taken into account in this analysis.

The focusing of waves in the vault is not a surprising result and it is very similar to what observed by Rayleigh for acoustic waves in the study of the *whispering galleries*, [Strutton–Lord Rayleigh, 1910], [Strutton–Lord Rayleigh, 1914]: due to its geometry, the vault behaves like a concave (or converging) mirror for the shock waves, which has the tendency to collect the blast energy. As far as it concerns blasts, we can hence say that barrel vaults and domes have a dissatisfactory behavior.

Clear is the true complexity of the pressure dynamics, which cannot be predicted in advance and, a crucial thing, is strongly dependent on the geometry and dimensions of the structure. Namely, the focalization described hereon takes place just because of the geometry of the vault and of its dimensions. To this purpose, we have also performed a simulation where the lowest part of the vertical walls, for a height of 7 m, are replaced by transmitting planes, to simulate the presence of openings, like in the case of an aisle. In such simulation, the peak at the vault's key, produced by the focusing of the reflected waves, gives a sensibly smaller peak of the reflected pressure, reduced by a factor 2.72 with respect to that calculated in the above simulation, see Fig. 1.17, passing from 0.299 to 0.109 MPa.

It is not possible to recover such a complete description of the pressure history everywhere in the fluid domain, and on the walls, using the other empirical models. So it is interesting, for comparison, to have the time-history of the pressure at a given point of the solid boundary. To this purpose, we give in Figure 1.18 the diagram of the time variation of the reflected pressure P_r at four points of the boundary, indicated in Figure 1.14 as points A, at the base of the vertical vault ($Z = 2.21 \text{ m/kg}^{1/3}$), B, at 5.63 m from the base of the vertical wall ($Z = 3.03 \text{ m/kg}^{1/3}$), C, at the springing of the vault ($Z = 5.40 \text{ m/kg}^{1/3}$), and D, the key of the vault ($Z = 6.63 \text{ m/kg}^{1/3}$).

The resulting decay phase is not exactly as predicted by Friedlander's law, and this because of the reflected waves, clearly visible on each diagram as secondary peaks of the curve. It is interesting to notice that the successive peaks are not necessarily decreasing, which confirms the complexity of the interactions and dynamics of the reflected waves.

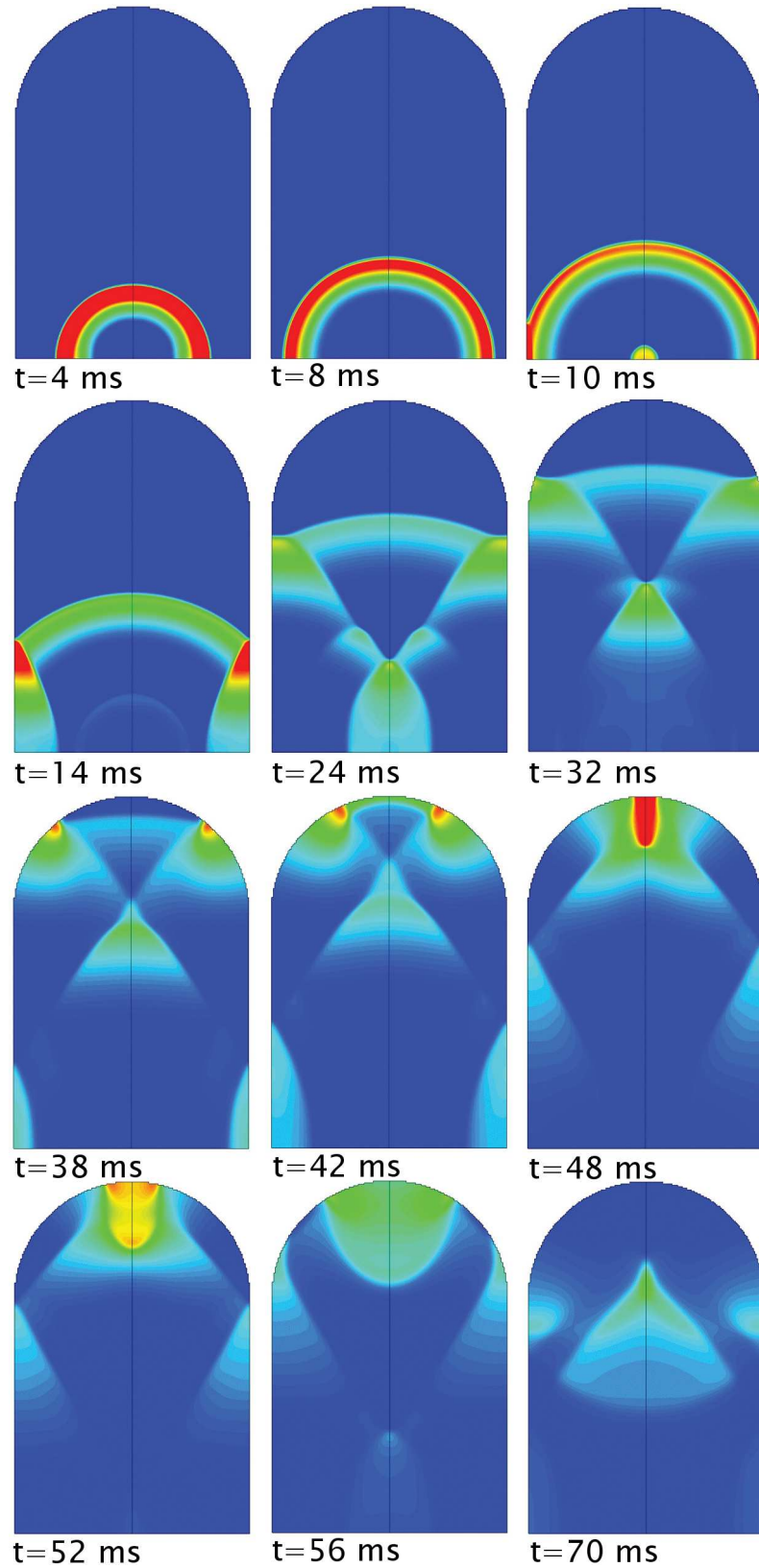


Figure 1.16: Evolution of the shock-wave as predicted by JWL; the color scale represents the overpressure and runs from blue for the minimum value, 0 kPa, to red for the maximum one, 300 kPa

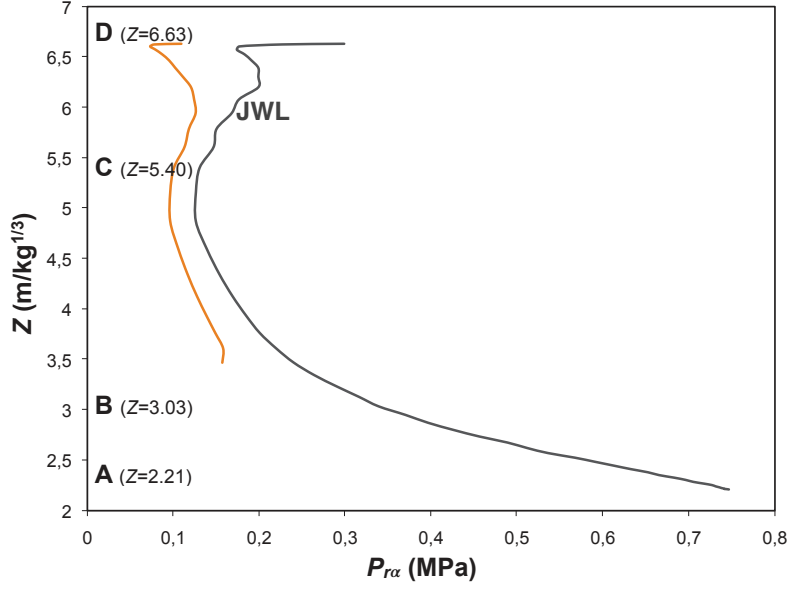


Figure 1.17: Comparison of $P_{r\alpha}$ as obtained by JWL for the two cases of complete vertical walls (black curve) and of openings until the height of 7 m (orange curve).

Table 1.4: Simulation results for models TM5-1300 ($P_{r\alpha 1}$) and CONWEP* ($P_{r\alpha 2}$).

#	x [m]	y [m]	R [m]	Z [m/kg ^{1/3}]	α [°]	φ [°]	$P_{r\alpha 1}$ [MPa]	$P_{r\alpha 2}$ [MPa]	t_A [ms]	t_o [ms]	$t_A + t_o$ [ms]	b
1.	6.	0.	6.	2.21042	0.	90.	0.735886	0.735886	5.80172	7.20288	13.0046	5.08752
2.	6.	0.188558	6.00296	2.21151	1.8	90.	0.730865	0.73454	5.80677	7.20494	13.0117	5.08511
3.	6.	0.377488	6.01186	2.21479	3.6	90.	0.724034	0.730518	5.82196	7.2111	13.0331	5.07787
4.	6.	0.567167	6.02675	2.22027	5.4	90.	0.715496	0.723861	5.84739	7.22137	13.0688	5.06578
5.	6.	0.757976	6.04769	2.22799	7.2	90.	0.705346	0.714641	5.88323	7.23574	13.119	5.04882
6.	6.	0.950307	6.07479	2.23797	9.	90.	0.693531	0.702954	5.92974	7.25421	13.1839	5.02696
7.	6.	1.14456	6.10819	2.25028	10.8	90.	0.680578	0.688923	5.98724	7.27677	13.264	5.00017
8.	6.	1.34116	6.14807	2.26497	12.6	90.	0.666244	0.672692	6.05613	7.30342	13.3595	4.96838
9.	6.	1.54054	6.19462	2.28212	14.4	90.	0.650548	0.654428	6.1369	7.33415	13.4711	4.93155
10.	6.	1.74316	6.24809	2.30182	16.2	90.	0.633576	0.634314	6.23016	7.36895	13.5991	4.88962
11.	6.	1.94952	6.30877	2.32417	18.	90.	0.615423	0.612547	6.33659	7.40781	13.7444	4.84252
12.	6.	2.16013	6.377	2.34931	19.8	90.	0.596186	0.589338	6.45699	7.45072	13.9077	4.79019
13.	6.	2.37557	6.45316	2.37737	21.6	90.	0.575971	0.564906	6.59232	7.49766	14.09	4.73255
14.	6.	2.59643	6.5377	2.40851	23.4	90.	0.554887	0.539474	6.74364	7.54862	14.2923	4.66955
15.	6.	2.82339	6.6311	2.44292	25.2	90.	0.533405	0.513267	6.9122	7.60359	14.5158	4.60111
16.	6.	3.05715	6.73396	2.48081	27.	90.	0.514042	0.486509	7.09944	7.66255	14.762	4.52717
17.	6.	3.29853	6.84692	2.52243	28.8	90.	0.493955	0.459421	7.30699	7.7255	15.0325	4.44768
18.	6.	3.54839	6.97073	2.56804	30.6	90.	0.474135	0.432213	7.53675	7.79243	15.3292	4.36261
19.	6.	3.80772	7.10624	2.61796	32.4	90.	0.455844	0.405088	7.79088	7.86333	15.6542	4.27191
20.	6.	4.0776	7.25443	2.67256	34.2	90.	0.437248	0.378234	8.07188	7.93822	16.0101	4.17558
21.	6.	4.35926	7.41641	2.73223	36.	90.	0.41569	0.351825	8.38265	8.01711	16.3998	4.07363
22.	6.	4.65408	7.59345	2.79745	37.8	90.	0.398883	0.32602	8.72652	8.10004	16.8266	3.96608
23.	6.	4.96363	7.78702	2.86876	39.6	90.	0.384757	0.30096	9.10737	8.18708	17.2944	3.85299
24.	6.	5.28971	7.99882	2.94679	41.4	90.	0.383963	0.276765	9.52971	8.27832	17.808	3.73447
25.	6.	5.63438	8.23081	3.03226	43.2	90.	0.365806	0.253538	9.99883	8.3739	18.3727	3.61063
26.	6.	6.	8.48528	3.126	45.	90.	0.335087	0.231364	10.5209	8.47402	18.995	3.48166
27.	6.	6.38935	8.76492	3.22902	46.8	90.	0.319545	0.210307	11.1033	8.57895	19.6823	3.34776
28.	6.	6.80566	9.07288	3.34248	48.6	90.	0.285542	0.190412	11.7547	8.68909	20.4438	3.20923
29.	6.	7.25275	9.41289	3.46774	50.4	90.	0.251119	0.17171	12.4855	8.80492	21.2904	3.06641
30.	6.	7.73515	9.78941	3.60645	52.2	90.	0.218771	0.154212	13.3081	8.92713	22.2352	2.91969
31.	6.	8.25829	10.2078	3.76059	54.	90.	0.191973	0.137918	14.2375	9.05663	23.2942	2.76959
32.	6.	8.82873	10.6746	3.93255	55.8	90.	0.167226	0.122812	15.2922	9.19459	24.4868	2.61669
33.	6.	9.45449	11.1976	4.12525	57.6	90.	0.149989	0.108867	16.4948	9.34257	25.8374	2.46169
34.	6.	10.1454	11.7869	4.34232	59.4	90.	0.136025	0.096049	17.8733	9.50263	27.376	2.3054
35.	6.	10.914	12.4545	4.58828	61.2	90.	0.12172	0.0843127	19.463	9.67744	29.1405	2.1488
36.	6.	11.7757	13.2161	4.86887	63.	90.	0.106993	0.0736089	21.3087	9.87051	31.1792	1.99303
37.	5.96239	12.6707	14.0035	5.15893	58.3816	83.5816	0.115654	0.0703287	23.2485	10.0649	33.3134	1.85383
38.	5.82149	13.4527	14.6582	5.40014	52.5888	75.9888	0.129849	0.0692648	24.8837	10.2236	35.1073	1.75251
39.	5.60197	14.1489	15.2176	5.6062	47.4129	69.0129	0.0984861	0.0682742	26.2947	10.3576	36.6523	1.67499
40.	5.31949	14.7754	15.7038	5.78534	42.6467	62.4467	0.0854331	0.067366	27.5311	10.4731	38.0042	1.61363
41.	4.98432	15.3401	16.1296	5.94219	38.1727	56.1727	0.0772309	0.0665434	28.6206	10.5735	39.1941	1.56408
42.	4.60409	15.8474	16.5026	6.07963	33.9164	50.1164	0.0739119	0.0658065	29.5801	10.6611	40.2412	1.5236
43.	4.185	16.2995	16.8282	6.19956	29.8268	44.2268	0.0712106	0.0651542	30.4211	10.7372	41.1583	1.49037
44.	3.7324	16.6978	17.1098	6.30332	25.8672	38.4672	0.0690077	0.0645845	31.1512	10.8029	41.9541	1.46308
45.	3.2511	17.0429	17.3502	6.39186	22.0096	32.8096	0.0672195	0.0640955	31.776	10.8588	42.6347	1.44083
46.	2.74559	17.335	17.551	6.46586	18.2322	27.2322	0.0657855	0.0636853	32.2994	10.9054	43.2048	1.42292
47.	2.22013	17.5741	17.7138	6.52583	14.517	21.717	0.0646619	0.0633522	32.7244	10.9431	43.6675	1.40885
48.	1.67885	17.7603	17.8395	6.57213	10.8487	16.2487	0.0638169	0.0630946	33.053	10.9721	44.0252	1.39824
49.	1.12576	17.8934	17.9288	6.60503	7.21431	10.8143	0.063228	0.0629115	33.2868	10.9928	44.2796	1.39085
50.	0.564835	17.9734	17.9822	6.62471	3.60178	5.40178	0.0628804	0.0628019	33.4267	11.0051	44.4318	1.38648
51.	0.	18.	18.	6.63126	0.	0.	0.0627655	0.0627655	33.4732	11.0092	44.4824	1.38503

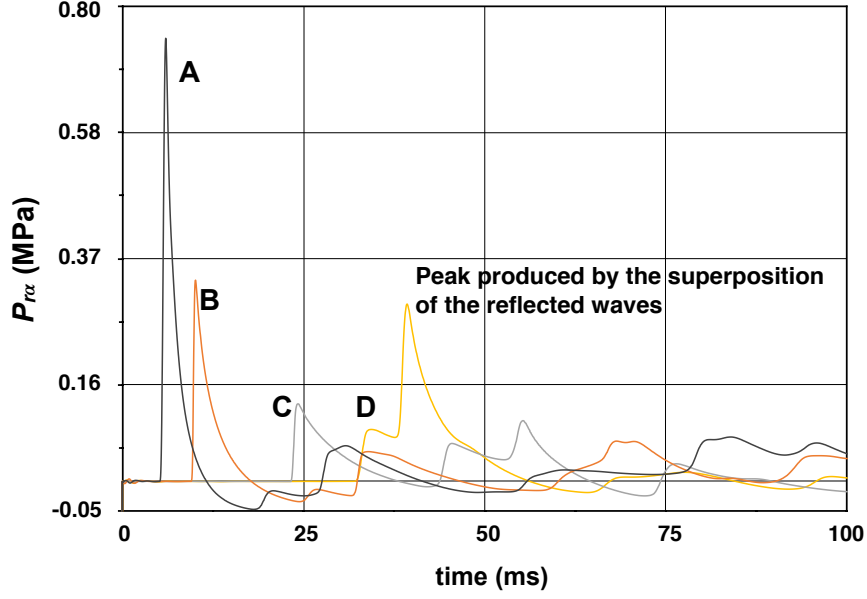


Figure 1.18: Time variation of $P_{r\alpha}$ at four points of the solid boundary, by JWL.

1.7.2 The results from TM5-1300 and CONWEP*

The spatial distributions of the maximum value of $P_{r\alpha}$, as given by TM5-1300 and CONWEP* and calculated as specified above and in Sect. 1.5.2, are shown in Figure 1.14. Such distributions are similar, though some differences exist: first of all, CONWEP* gives maximum values of $P_{r\alpha}$ that are almost always less or equal of those given by TM5-1300. Then, the greatest differences appear for $0^\circ < \alpha < 90^\circ$; this can be explained by the fact that the angular variation taken by CONWEP does not take into account the formation of the Mach stem.

Such occurrence happens really for $\alpha \simeq 40^\circ$ and is clearly indicated by the local increase of $P_{r\alpha}$ in the diagram of TM5-1300, which shows two humps: at midway of the vertical wall and at the springing of the vault where $\alpha \simeq 40^\circ$ in both the cases; this fact can be essential for vaulted structures, because an increase of $P_{r\alpha}$ in the zone between 0° and 30° can be very dangerous for the stability of the vault, that normally has on its back a filling with a material like rubble or gravel to improve the stability of the structure.

The numerical data of the simulations TM5-1300 and CONWEP* are shown in Tab. 1.4; φ is the angle formed by the normal $\mathbf{n}$ with the axis y , while b is the coefficient appearing in the Friedlander's law, eq. (1.5).

Observing the results concerning $P_{r\alpha}$ and t_o , we see clearly that the peak of the shock wave decreases with the distance R , passing from a maximum of 0.736 MPa for $R = 6$ m, to a minimum of 0.063 MPa for $R = 18$ m, while its time duration increases, passing from 7.2 ms to 11 ms.

In Figure 1.19 we show the same curves of Fig. 1.18 but now obtained with the models TM5-1300, red curves, and CONWEP*, green curves. The red and green curves are distinct only for $\alpha \neq 90^\circ$, for the way the values of CONWEP* are calculated.

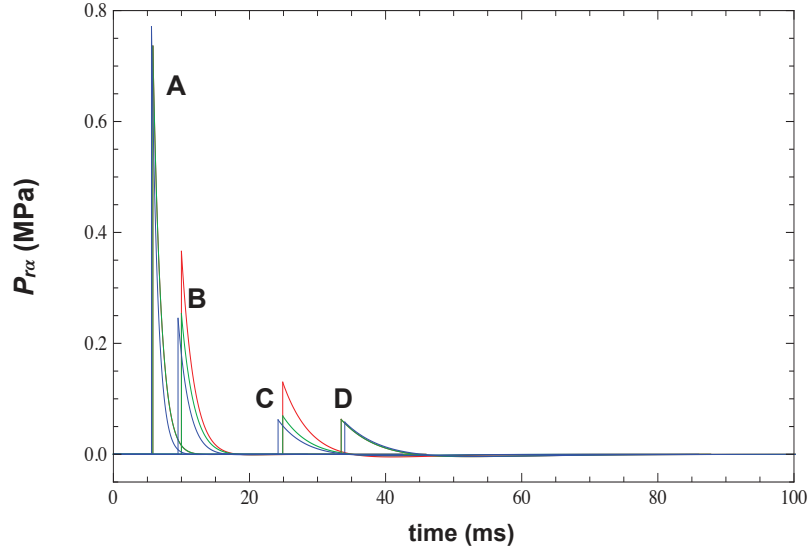


Figure 1.19: Time variation of $P_{r\alpha}$ at four points of the solid boundary, as predicted by TM5-1300 (red curves), CONWEP (blue curves) and CONWEP* (green curves).

Table 1.5: Simulation results for the CONWEP model.

#	x [m]	y [m]	R [m]	Z [m/kg ^{1/3}]	α [°]	φ [°]	$P_{r\alpha}$ [MPa]	t_A [MPa]	t_o [ms]	$t_A + t_o$ [ms]
1.	6.	.001	6.	2.21042	.0096	90.	.771063	5.5634	5.7978	11.3612
2.	6.	.005	6.	2.21042	.0478	90.	.740335	5.3609	5.7303	11.3612
3.	6.	.504	6.02113	2.21820	4.8040	90.	.686422	5.7658	5.7975	11.5633
4.	6.	1.3	6.13922	2.26171	12.2313	90.	.68598	5.8333	5.7974	11.6307
5.	6.	1.7	6.23618	2.29743	15.8272	90.	.628755	6.0357	5.8645	11.9002
6.	6.	2.3	6.42573	2.36726	20.9841	90.	.588982	6.2381	5.999	12.237
7.	6.	2.9	6.66408	2.45507	25.8091	90.	.536047	6.5079	5.9986	12.5065
8.	6.	3.3	6.84763	2.52269	28.8254	90.	.475994	6.8451	6.2004	13.0455
9.	6.	3.7	7.04911	2.59692	31.6768	90.	.414128	7.2497	6.1999	13.4496
10.	6.	4.1	7.26705	2.6772	34.3635	90.	.369243	7.6543	6.4016	14.0559
11.	6.	4.5	7.5	2.76302	36.8886	90.	.335817	8.0588	6.6707	14.7295
12.	6.	4.9	7.74661	2.85388	39.2573	90.	.311071	8.4633	7.0744	15.5377
13.	6.	5.3	8.00562	2.9493	41.4763	90.	.273256	9.0026	7.2087	16.2112
14.	6.	5.7	8.27587	3.04886	43.5533	90.	.244744	9.5418	7.4777	17.0194
15.	6.	6.1	8.55628	3.15216	45.4966	90.	.216588	10.1483	7.6119	17.7602
16.	6.	6.5	8.8459	3.25886	47.3146	90.	.194903	10.7548	7.8809	18.6357
17.	6.	6.7	8.99389	3.31338	48.1793	90.	.188719	11.0243	8.0155	19.0398
18.	6.	6.9	9.14385	3.36862	49.0158	90.	.178286	11.3612	8.0826	19.4438
19.	6.	7.1	9.2957	3.42456	49.8251	90.	.173488	11.6307	8.2172	19.8479
20.	6.	7.3	9.44934	3.48117	50.6083	90.	.160889	12.0349	8.217	20.2519
21.	6.	7.5	9.60469	3.53840	51.3662	90.	.157606	12.3044	8.3516	20.656
22.	6.	7.7	9.76166	3.59623	52.1	90.	.150868	12.6413	8.4187	21.06
23.	6.	7.9	9.92018	3.65463	52.8103	90.	.141494	13.0455	8.4859	21.5314
24.	6.	8.1	10.08018	3.71357	53.4983	90.	.133298	13.4496	8.5532	22.0028
25.	6.	8.3	10.24158	3.77303	54.1646	90.	.131731	13.7191	8.6877	22.4068
26.	6.	8.5	10.40433	3.83299	54.8102	90.	.124746	14.1232	8.8223	22.9455
27.	6.	8.7	10.56835	3.89341	55.4358	90.	.121109	14.46	8.8895	23.3495
28.	6.	8.9	10.73359	3.95429	56.0422	90.	.115390	14.8642	8.8893	23.7535
29.	6.	9.1	10.9	4.01559	56.6302	90.	.107966	15.3357	8.8892	24.2249
30.	6.	9.3	11.06752	4.07731	57.2005	90.	.107626	15.6051	9.1585	24.7636
31.	6.	9.5	11.2361	4.13942	57.7536	90.	.103298	16.0092	9.2257	25.2349
32.	6.	9.7	11.4057	4.2019	58.2904	90.	.099298	16.4133	9.2256	25.6389
33.	6.	9.9	11.57627	4.26473	58.8114	90.	.095704	16.8174	9.2929	26.1103
34.	6.	10.1	11.74777	4.32791	59.3172	90.	.092426	17.2215	9.4275	26.649
35.	6.	10.3	11.92015	4.39142	59.8085	90.	.086272	17.7602	9.2928	27.053
36.	6.	10.5	12.09339	4.45524	60.2857	90.	.083664	18.1643	9.36	27.5243
37.	6.	10.7	12.26744	4.51936	60.7494	90.	.082638	18.501	9.562	28.063
38.	6.	10.9	12.44227	4.58377	61.2001	90.	.080329	18.9051	9.6292	28.5343
39.	6.	11.1	12.61784	4.64845	61.6382	90.	.075713	19.4438	9.5618	29.0056
40.	6.	11.3	12.79414	4.7134	62.0643	90.	.073825	19.8479	9.6964	29.5443
41.	6.	11.5	12.97112	4.7786	62.4789	90.	.072095	20.2519	9.7637	30.0156
42.	6.	11.7	13.14876	4.84405	62.8822	90.	.070498	20.656	9.8983	30.5543
43.	6.	11.9	13.32704	4.90972	63.2748	90.	.067946	21.1273	9.8983	31.0256
44.	5.99832	12.10025	13.5054	4.97543	63.6638	90.	.064633	21.6661	9.8981	31.5642
45.	5.99163	12.30062	13.68228	5.0406	62.1427	88.0809	.064329	22.0028	10.0328	32.0356
46.	5.95817	12.70015	14.02831	5.16807	59.1296	84.2299	.062510	22.9455	10.1	33.0455
47.	5.85816	12.79302	14.07051	5.38362	63.5136	88.0843	.062294	24.2249	10.2346	34.4595
48.	5.69932	13.8729	14.99799	5.52531	50.2785	72.5782	.062214	25.4369	10.5038	35.9407
49.	5.48322	14.43397	15.44038	5.68828	45.9365	66.7023	.061377	26.6490	10.5036	37.1526
50.	5.21205	14.97060	15.85195	5.83991	41.6396	60.7994	.059801	27.8610	10.5708	38.4318
51.	4.88854	15.47738	16.23105	5.97957	37.3838	54.8758	.060474	28.8036	10.8402	39.6438
52.	4.09793	16.38143	16.88621	6.22093	28.9900	42.9962	.059756	30.6216	10.9073	41.5289
53.	3.47738	16.88854	17.24282	6.35231	23.4836	35.0786	.059435	31.6316	11.1092	42.7408
54.	2.43397	17.48322	17.65183	6.50299	15.3680	23.2520	.058859	32.8435	11.1765	44.0200
55.	1.29302	17.85816	17.90491	6.59623	7.4347	11.5324	.058495	33.6515	11.3111	44.9626
56.	.50299	17.93144	17.93850	6.6086	1.53009	.0318	.059572	33.7188	11.4458	45.1646
57.	.001	17.99832	17.99832	6.63064	.00319	.0456	.058515	34.0555	11.3784	45.4339

1.7.3 The results from CONWEP

The same case study has finally been implemented in *ABAQUS* in order to allow comparison with results given by CONWEP. In this way we understand the quality of the predictions given by CONWEP and may calibrate its use.

The problem is still considered planar and the mesh consists of linear quadrilateral elements with an average size of 0.2 m and a thickness of 0.1 m.

The results obtained with this simulation are reported in Table 1.5, while the diagrams of the time variation of the reflected pressure P_r at the four points A, B, C and D of the boundary are still shown in Figure 1.19.

Table 1.6: Comparison of the characteristic parameters evaluated at points A, B, C and D of Fig. 1.14 as calculated by JWL, TM5-1300, CONWEP* and CONWEP; $R_1 = JWL/TM5-1300$, $R_2 = JWL/CONWEP$, $R_3 = TM5-1300/CONWEP$.

	JWL	TM5-1300	CONWEP*	CONWEP	R_1	R_2	R_3
	$P_{r\alpha}$ [MPa]						
A	0.746	0.736	0.736	0.771	1.01	0.97	0.95
B	0.339	0.366	0.253	0.245	0.93	0.72	1.49
C	0.131	0.130	0.070	0.062	1.01	2.11	2.10
D	0.299	0.063	0.063	0.058	4.75	5.15	1.09
	t_A [ms]						
A	5.9	5.8	5.8	5.6	1.02	1.05	1.03
B	10.1	10.0	10.0	9.5	1.01	1.06	1.05
C	24.3	24.9	24.9	24.2	0.97	1.00	1.03
D	39.3	33.5	33.5	34.0	1.17	1.15	0.98
	t_o [ms]						
A	5.7	7.2	7.2	5.8	0.81	0.98	1.24
B	7.7	8.4	8.4	7.5	0.92	1.03	1.12
C	13.9	10.2	10.2	10.2	1.36	1.36	1.00
D	16.9	11.0	11.0	11.4	1.53	1.48	0.96

1.7.4 Comparisons

In order to compare the results of the above simulations, something must be mentioned about the results from JWL. On one hand, the multiple reflections of the shock-wave, visible in Figure 1.16, completely alter the time variation of P_r after the positive phase, that in some cases can be even difficult to be determined. On the other hand, the multiple reflections add together and can considerably increase the value of P_r . Nevertheless, it is still possible, at least for a part of the boundary surface, to focus on just the characteristic elements of the positive phase, namely $P_{r\alpha}$, t_A and t_o . Such a comparison, of course, is possible only if the peak $P_{r\alpha}$ has not yet been affected by other reflected shock-waves and if a positive phase is still clearly distinguishable, which depends again on the reflected waves.

To assess the results, we have chosen to consider the relevant physical parameters of the blast: $P_{r\alpha}$, t_A and t_o . We have compared them as evaluated at the four points A, B, C and D of Fig. 1.14; these values are summarized in Tab. 1.6.

In Figure 1.20 we show a comparison of the results for the values of $P_{r\alpha}$ obtained with JWL, TM5-1300, CONWEP* and CONWEP. The latter ones, as already pointed out, use the empirical data for computing P_r for $\alpha = 0$, but a different angular dependence from α , eq. (1.33).

It can be observed that the values given by CONWEP and the code *ABAQUS* are practically coincident with those calculated using CONWEP*, except some oscillations, to be imputed to the finite element approximation. Hence, on one side the computation of P_{ro} is practically the same for the three models, but what changes is the way the effect of the inclination α is taken into account. In particular, except the value $\alpha = 0$, where the three models give the same value of P_{ro} , for all the other values of α , CONWEP and CONWEP* underestimate the value of $P_{r\alpha}$ with respect to TM5-1300.

TM5-1300 and JWL give values that are comparable until point C; here, JWL diverges and gives values of $P_{r\alpha}$ that can be considerably greater. This is actually the effect of converging the reflected waves, that increases significantly the value of the overpressure. It is a local phenomenon, essentially depending upon the geometry and dimensions of the structure, as shown above. This same phenomenon is noticed also on the time variation of $P_{r\alpha}$ for point D in Fig. 1.18, where the peak due to the reflected waves is clearly visible.

It is worth noticing that, besides the convergence of the reflected waves starting from point C, JWL and TM5-1300 give not only comparable values of $P_{r\alpha}$, but also of its time variation. In Fig. 1.21 we show the time diagrams of JWL, reduced to the positive phases, presented together with those of TM5-1300. The curves are in a rather good agreement, apart from the peak on the curve of point D, due to reflected waves.

As far as it concerns the time durations, represented in Fig. 1.22, TM5-1300 and CONWEP* give, of course, the same values, while CONWEP underestimates slightly the durations for small values of Z . Nevertheless, these discrepancies are not significant. The diagrams in Figs 1.20 and 1.22 explain the aforementioned slight differences, namely in the value of t_A , appearing in Fig. 1.19 between the curves obtained with CONWEP and those relative to TM5-1300 and CONWEP*.

For what concerns JWL, the curve of t_A is in a very good agreement with those of TM5-1300 and CONWEP until point C, where it diverges, once more due to wave reflection. The curve of $t_A + t_o$ has been obtained by interpolating the values estimated for points A, B, C and D for the duration t_o because, as mentioned above, it is not easy to determine the exact duration of the positive phase for all the points, due to the interaction of the reflected waves. Globally, the four models give values that are comparable, apart the zone of the wave reflections, where the values given by JWL diverge from those of the other models.

1.8 Conclusion

The results of the simple example treated above clearly show that the distribution of the pressures given by TM5-1300 is sensibly similar to that given by the more exact JWL model, apart from those zones where the interaction of the reflected waves alters the overpressure distribution. This phenomenon, as already explained, strongly depends upon the geometry and the dimensions of the structure and cannot be predicted *a priori*.

Simulations done with the model JWL are hence the best way to predict the effects of an explosion on a monument. However, the use of JWL can be impractical, if not impossible, in real problems of large structures. The need of discretizing finely not only the structure but, even more, the explosive charge and the volume of air is an almost insurmountable obstacle for numerical simulations. Due to the great volume of the air and of the structure to be finely discretized, the size of the numerical problem becomes very heavy and impossible to treat in reasonable time.

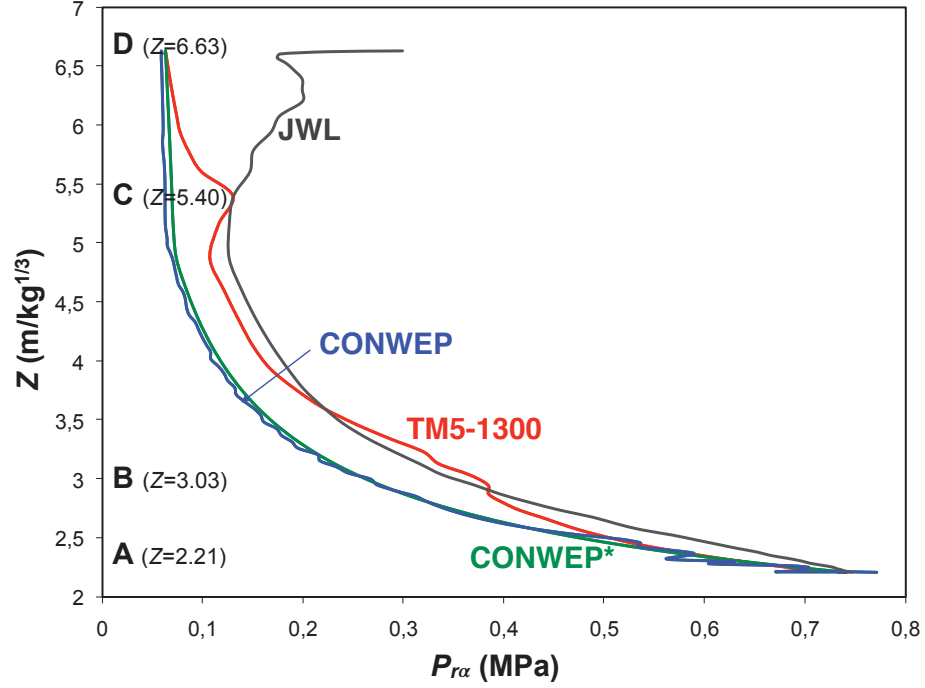


Figure 1.20: Comparison of $P_{r\alpha}$ as obtained by the four models.

That is why the use of empirical models, like TM5-1300 and CONWEP, can be very useful. If abstraction is made concerning the effect of reflected waves, the results of TM5-1300, though obtained neglecting the dynamic pressure, are close to those of JWL, incorporating it, which confirms that the dynamic pressure can be neglected in simulations.

The comparison made of CONWEP, TM5-1300 and JWL confirms what usually said in the literature: TM5-1300 is more accurate than CONWEP. Nevertheless, as mentioned above, it is this last model that normally is implemented in commercial finite element codes. That is why its use must be accurately considered, adopting a multiplying factor $p_f > 1$. Such a coefficient, affecting only the reflected pressure $P_{r\alpha}$ and not the other blast parameters, namely t_A and t_o , is needed to obtain values that are similar to those predicted by TM 5-1300 and JWL; it represents hence, on the average, the ratio between the value of $P_{r\alpha}$ given by JWL and that given by CONWEP. This coefficient must be chosen in function of the problem at hand, namely considering the characteristic dimensions of the structure: as apparent from Fig. 1.20, p_f depends upon Z , hence upon R , and upon the angle α . For structures with a simple geometry, we can suggest a value in the interval $1.5 \div 2.5$, in order to take into account, though indirectly, of the effect of reflected waves. This choice is also suitable for a first investigation on the effect of an explosion inside structures with complex geometry, but, as it will be discussed below, the theoretical model JWL allows a deeper understanding of the physics behind such an event.

In short, the evaluation of blast actions is case-dependent and the choice of the modelling strategy is not-trivial.

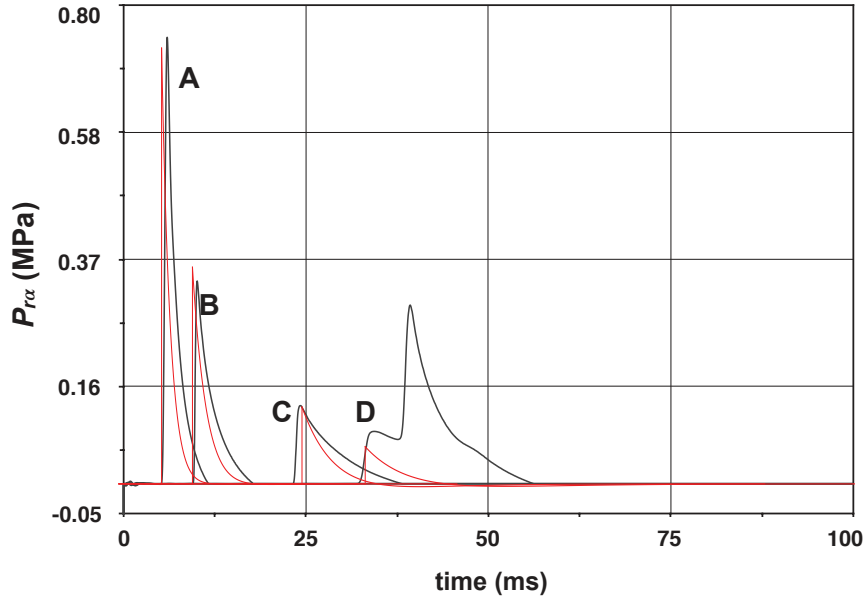


Figure 1.21: Comparison of the positive phases for points A, B, C and D as given by JWL (black) and TM5-1300 (red).

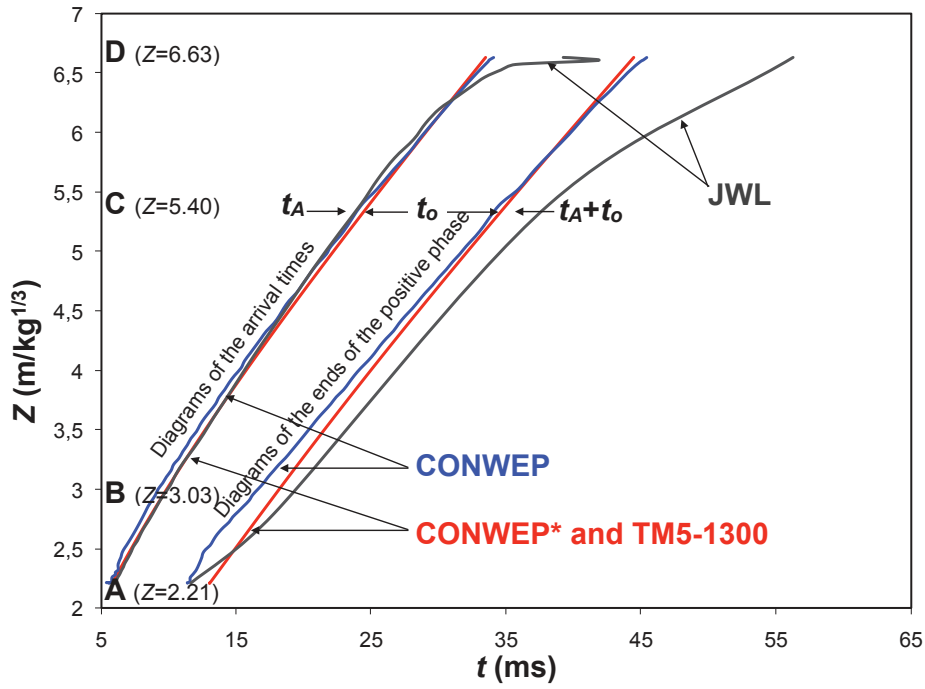


Figure 1.22: Comparison of the arrival time t_A and of the positive phase duration t_o as obtained by the four models.

Chapter 2

The effects of an explosion on a monument structure: the case of Pantheon in Rome

Highly representative monuments are for their proper symbolic nature too often the objective of iconoclastic damage attacks, even destructions: Parthenon in 1687, the Cathedral of Reims in 1914, Buddha's statues of Bamyan in 2001 and the more recent destructions at Palmyra and of Great Mosque of al-Nouri, respectively, in 2015 and 2016 and in 2017.

Research on the effects of an explosion on monumental architectures are hence interesting for assessing the potential effects of a blast onto a monument structure and also for helping in designing reinforcements or any other possible passive protection device aiming at reducing the consequences of an explosion on the structure of a monument.

This domain is still almost unexplored; in fact, the most part of papers concerning the effects of an explosion on a civil structure regard modern reinforced concrete or steel structures with simple geometries, normally squared buildings [Remennikov, 2003], [Ngo et al., 2007], [Koccaz et al., 2008], [Draganic and Sigmund, 2012].

This is not the case of monumental structures that have often a complex geometry, sometimes very articulated, which renders the assessment of the blast loads strongly case dependent and affects the type of simulation. This point is tackled in Sect. 2.2, where a short account of the state of the art for what concerns the simulation of blast loads is given and, on its base, the choice of the method used for the case of Pantheon is justified.

On the other hand, monumental structures are either masonry-like or timber structures, or both of them at the same time. In particular, monuments constituted by masonry-like materials have a structural response strongly affected by the nearly no-tension behavior of the material. This is a key point, considered in Sect. 2.3.

Hence, a special attention must be paid to the way we perform numerical simulations, that must account, on one hand, for the high-velocity of the phenomenon and, on the other hand, for the peculiar constitutive law of the material, that must be able to describe non-linear damage, i.e., cracks propagating into the structure as a consequence of the blast actions. Section 2.4 describes the procedure used for the numerical simulations, which are presented in Sect. 2.5. All results are normalized.

But let us start by briefly introducing, in Sect. 2.1, the object of this chapter, the Pantheon in Rome.



Figure 2.1: The destructions at Palmira.

2.1 The Pantheon in Rome

The Pantheon is one of the most admired and studied monuments ever. The actual building is the fourth Pantheon, built upon the rests of previous temples of classical rectangular form, by the Emperor Hadrian, since A.D. 118 to about 128, or later, perhaps until 140, under Emperor Antoninus Pius.



Figure 2.2: The Pantheon of Rome (from [Pulvirenti, 2014]).

The Pantheon is probably the joint work of Hadrian and of the Nabatean great architect Apollodorus of Damascus, at least until 121, when seemingly he was first exiled by Hadrian and then executed in 125, according to Dione Cassius. Apollodorus was the architect of many great constructions during the reign of the Emperor Trajan, namely of the great bridge over the Danube, of the Trajan's Forum and Markets, of the artificial harbour of Porto and so on.

The Pantheon has inspired number of architects during the centuries, especially during the Neo-Classical period, e.g., Pietro Bianchi in the church San Francesco di Paola in Naples (1816-46), Ferdinando Bonsignore in the church Gran Madre di Dio in Turin (1818-31) or Thomas Jefferson in The Rotunda of the University of Virginia (1822-26).

The main body of the Pantheon, the so-called *Rotonda* recalls, probably intentionally, the celebrated discovery of Archimedes of the volume of a sphere inscribed in a cylinder [Boyer, 1968]. In fact (see Fig. 2.3) the *Rotonda* is composed by a cylinder whose inscribed sphere is coincident, for its upper part, with the dome, while its bottom touches the ground. The touch of Archimedes is present also with regard to another aspect: the coffering that is sculpted in the intrados of the dome is subdivided into 28 parts, and it was Archimedes

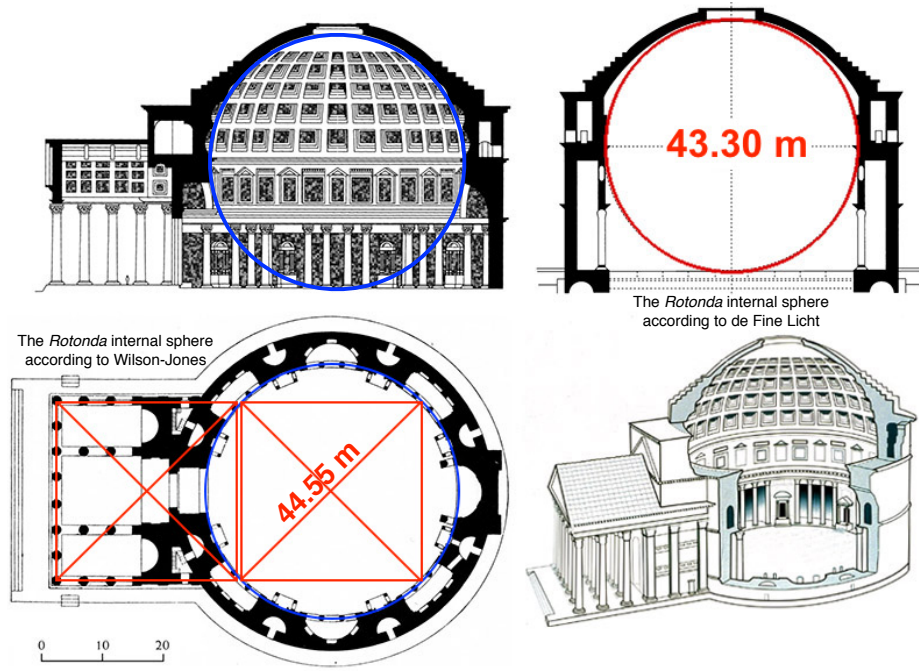


Figure 2.3: Scheme and dimensions of the Pantheon (from [Pulvirenti, 2014]).

that studied the related problem of partitioning a circle into 7 parts. The coffering, besides its aesthetic role, serves also to reduce dome's weight.

The diameter of the dome is of 43.30 m, according to the measurements of de Fine Licht [de Fine Licht, 1968], [Mark, 1990]. However, according to Wilson-Jones [Wilson-Jones, 2000], [Como, 2013], the right measure of the diameter is 44.55 m and it corresponds to the circle passing through the axes of the columns in the interior of the *Rotonda*. This measure equals 150 Roman feet and is exactly identical to $\sqrt{2}$ times the width of the *pronaos*, the entrance *portico* of the monument: in the interpretation of Wilson-Jones, the *Rotonda* perfectly circumscribes a square equal to that of the *pronaos*, see Fig. 2.3.

Pantheon's dome is, still today, the largest dome in the world, apart the modern realizations in reinforced concrete. In fact, it is larger than the dome of Saint Peter in Vatican, whose diameter is of 42 m, and also of the octagonal dome of Filippo Brunelleschi, in Santa Maria del Fiore at Florence, whose base is circumscribed to a circle of 41.57 m. However, unlike these two famous domes, and also of other ones, made by bricks, Pantheon's one is made of concrete, a technique already mastered by Apollodorus in other previous works (e.g. the vault of the Great Hall in Trajan's markets in Rome, see [Perucchio and Brune, 2008], [Perucchio and Brune, 2009]).

If the intrados of the dome is sculpted by a coffering, the external lowest part of the dome is modeled by stepped rings. The function of these stepped rings, and more generally the structural behavior of the *Rotonda*, has been studied by Mark and Hutchinson [Mark and Hutchinson, 1986], [Mark, 1990]. For their study, the first attempt to apply finite element analysis to Roman structures, they used a rather simplified bi-dimensional model of the *Rotonda*, exploiting axe-symmetric conditions. The results are rather troubling: according to Mark and Hutchinson, and contrarily to what commonly thought, the step-rings do not contrast the hoop tension in the lowest part of the dome, because actually the dome is spread of cracks that have an almost meridional direction and that stop at $\sim 57^\circ$ above the equatorial plane passing through the dome's base. In this zone, the hoop stress begins to be compressive.

The distribution of these cracks, Fig. 2.4, was detailed in 1934 by A. Terenzio, the Superintendent of the Monuments of Latium, who had carried on a series of inspections on the dome of the *Rotonda* after that some fragments had fallen down [Terenzio, 1934].

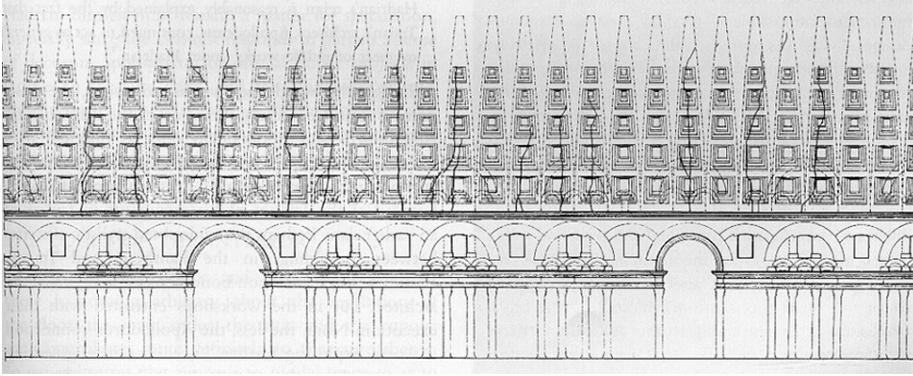


Figure 2.4: Meridional cracks in the *Rotonda*'s dome (after [Terenzio, 1934]).

We cannot establish here what is the true origin of these cracks, but certainly they are to be modeled to have a correct response of the structure to both gravity and blast. In fact, on one hand, the cracks undoubtedly change the stress regime of the *Rotonda*, on the other hand, they considerably affect the response of the structure to a blast by a special local mechanism, as we will see in Sect. 2.5.2.

2.2 Modeling the blast actions

In the case of explosions on monuments, we are concerned with surface blasts, i.e., with detonation points close to the ground surface, which give hemispherical overpressure waves. The closeness of the blast to the ground results in an augmentation of the shock overpressure, because the ground reflects and amplifies the overpressure wave.

The simulation of a blast can be conducted, as already discussed in Ch. 1 and in [Vannucci et al., 2017a], by using different approaches, the most widely used being three: JWL, CONWEP and TM5-1300 models.

The case of the *Rotonda* is particularly interesting: by its geometry, focalizing effects can happen and be determinant. That is why we have chosen to study the problem of a blast inside the Pantheon using the JWL model, that allows for modeling completely the explosion. This needs the discretization not only of the structure, to perform the coupled structural analysis, but also of the air volume inside and partly outside the Pantheon. This is a step forward with respect to a previous study of the authors, where the CONWEP model was used [Vannucci et al., 2017b]. Finally, we have a very huge numerical model, needing a considerable computational effort; this aspect is detailed in Sect. 2.4.

The material parameters that characterise the equations of state of explosive and air are those in Tab. 1.1.

2.3 Modeling the material behavior

Pantheon is made of different materials: bricks, mortar and concrete, for the cylindrical part of the *Rotonda*, granite for the columns inside and concrete for the dome. The cylindrical

part is actually composed also by pillars and arches, having a structural role and merged in the wall, whose thickness is of about 6 m. Concerning the dome, it is more correct to say that it is composed of *concretes*, because the Romans used different types of concrete, from the heaviest one in the lower part, with thickness ~ 5.9 m, to the lightest one in the upper part of the dome, where the thickness decreases to ~ 1.5 m (see Fig. 2.5).

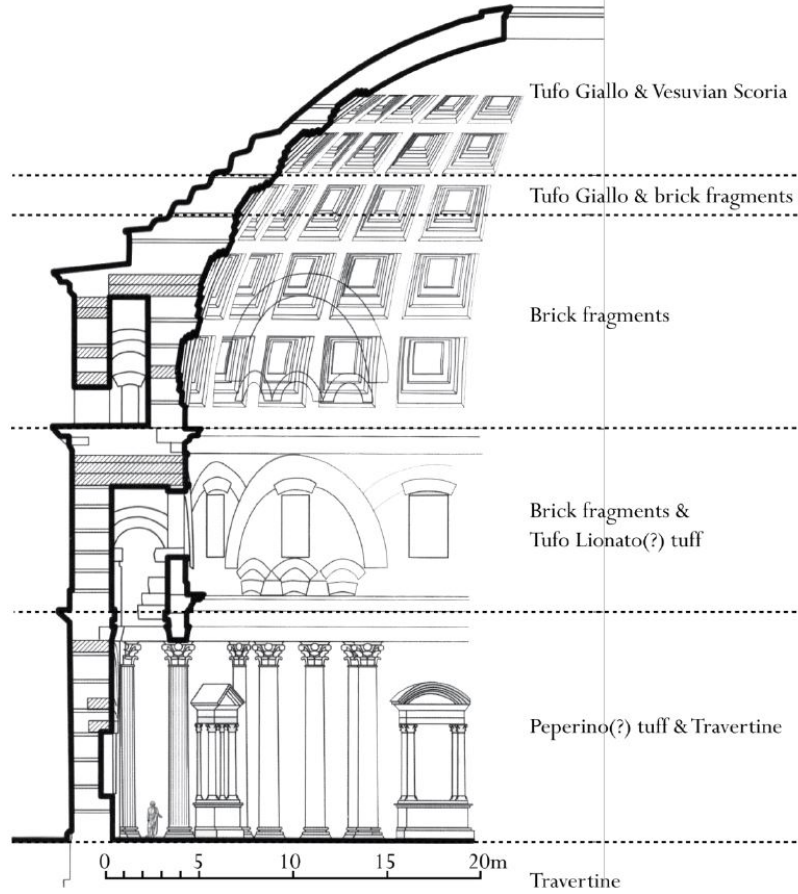


Figure 2.5: Materials composing the *Rotonda* (after [Lancaster, 2005])

2.3.1 The mechanical model of the materials

Masonry and concrete share a peculiarity: they have a low strength to tensile stresses, so small that often they are modeled as no-tension materials. In this study, we have used the same mechanical model for the constitutive law of the types of concrete composing the dome or the foundations, the masonry of the cylindrical part and the granite of the columns. We precise that we have modeled the masonry as an isotropic homogeneous material, as it is almost impossible, for such a large structure, to model precisely the actual disposition of the bricks composing the pillars and numerous arches included in the masonry of the *Rotonda* (see Fig. 2.5).

The constitutive law that we have used considers a linear elastic behavior in compression, with an unlimited strength. This corresponds to the normal compression response of masonry and concrete and to the fact that normally in monumental structures compression remains far below the compressive strength of the material (cf. [Heyman, 1995], [Stefanou et al., 2015]).

Under tension, the material is assumed to be linearly elastic until the maximum principal

stress does not exceed the tensile strength. Actually, a small, but not null, tensile strength f_t is considered for the material. When f_t is exceeded, damage occurs. This is modeled in terms of the nonlinear brittle cracking model proposed in [Hillerborg et al., 1976]. In this scheme, a crack appears when the maximum principal stress over an element exceeds its tensile strength f_t . The crack forms in the plane orthogonal to the direction of the principal stress exceeding the tensile limit (see also [DS, 2016]).

An energy criterion, based on the fracture energy G_f for linear-elastic-brittle solids, is used to model the cracks propagation. This allows to minimize mesh dependency due to the softening behavior of the material and to dissipate adequately the energy. We have used a simplified Hillerborg's law, shown in Figure 2.6, to represent the tensile part of the constitutive law: in tension, the elastic phase is followed by a piecewise linear softening one, modeling the damage of the material.

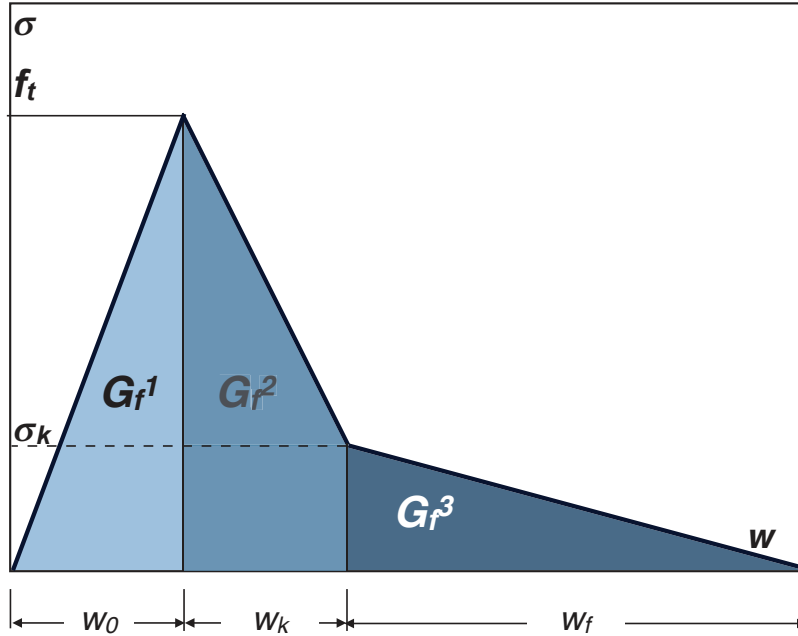


Figure 2.6: Simplified Hillerborg's model used as constitutive law in tension.

Referring to Fig. 2.6, the fracture energy for normal tensile stresses is defined by

$$G_f = G_f^1 + G_f^2 + G_f^3 = \int_0^{w_0} \sigma^m dw + \int_{w_0}^{w_k} \sigma^m dw + \int_{w_k}^{w_f} \sigma^m dw, \quad (2.1)$$

where σ^m is the maximum principal stress; w is the displacement normal to the crack surface, defined as the product of the normal strain ε and a characteristic length h , i.e., $w = \varepsilon h$; w_0 is the normal displacement corresponding to f_t , w_k that relative to the kink point, with stress σ_k , and w_f the one corresponding to the complete loss of strength.

Because the assumed constitutive law is piecewise linear, putting, like in [Brune, 2010],

$$\Psi = \frac{\sigma_k}{f_t}, \quad \zeta = \frac{w_k}{w_f}, \quad (2.2)$$

the three integrals in eq. (2.1) become (E is the Young's modulus)

$$\begin{aligned} G_f^1 &= \int_0^{w_0} \sigma dw = \frac{f_t w_0}{2} = \frac{f_t^2}{2E} h, \\ G_f^2 &= \int_{w_0}^{w_k} \sigma dw = \frac{f_t}{2} (1 + \Psi) (\zeta w_f - \frac{f_t}{E} h), \\ G_f^3 &= \int_{w_k}^{w_f} \sigma dw = \frac{f_t}{2} \Psi (1 - \zeta) w_f, \end{aligned} \quad (2.3)$$

so that

$$G_f = \frac{f_t}{2} \left[(\Psi + \zeta) w_f - \Psi h \frac{f_t}{E} \right]. \quad (2.4)$$

Usually, G_f , E and f_t are known for a given material; fixing Ψ and ζ allows to find the value of w_f :

$$w_f = \frac{1}{\Psi + \zeta} \left(\frac{2G_f}{f_t} + \Psi h \frac{f_t}{E} \right). \quad (2.5)$$

The question is now how to fix the material characteristics; this point is addressed in the next Section.

2.3.2 The choice of the material characteristics

Different material parameters can be fixed for each material: the density ρ , the Young's modulus E , the Poisson's ratio ν , the fracture energy G_f , the tensile strength f_t and the two ratios Ψ and ζ .

We have considered five zones for the materials of the *Rotonda*, corresponding to those indicated in Figure 2.5: foundations, brick-faced concrete of the cylindrical wall, lower, intermediate and upper zone of the dome. In addition, we have considered apart the material of the columns in the interior of the *Rotonda*, the ones made of granite. The materials data are shown in Table 2.1 and their location in the *Rotonda* is shown in Figure 2.7, while their choice is discussed below.

Table 2.1: Materials data

Material	Part	ρ [kg/m ³]	E [GPa]	G_f [J/m ²]	f_t [MPa]	ν	Ψ	ζ
Granite	Columns	2500	55.3	17.5	3.50	0.16	0.25	0.50
Concrete 1	Foundations	2000	13.5	62.1	0.68	0.20	0.35	0.15
Concrete 2	Cylindrical wall	1750	7.0	59.4	0.63	0.20	0.35	0.15
Concrete 3	Lower dome	1600	4.6	57.7	0.60	0.20	0.35	0.15
Concrete 4	Intermediate dome	1500	3.6	56.6	0.58	0.20	0.35	0.15
Concrete 5	Upper dome	1350	3.0	55.0	0.55	0.20	0.35	0.15

The concrete densities have been fixed upon the indications given in [Mark and Hutchinson, 1986]. About the other quantities, the choice is much more problematic. Though we know today rather well the behavior of light concretes, according to historians and researchers in mechanics of ancient monuments, the physical properties of concretes and mortars of the past have quite different characteristics. Moreover, the data that can be found in the literature about ancient concrete are often quite different and most of all fragmentary. The only reliable data about Pantheon were the densities. So we needed a rationale linking in

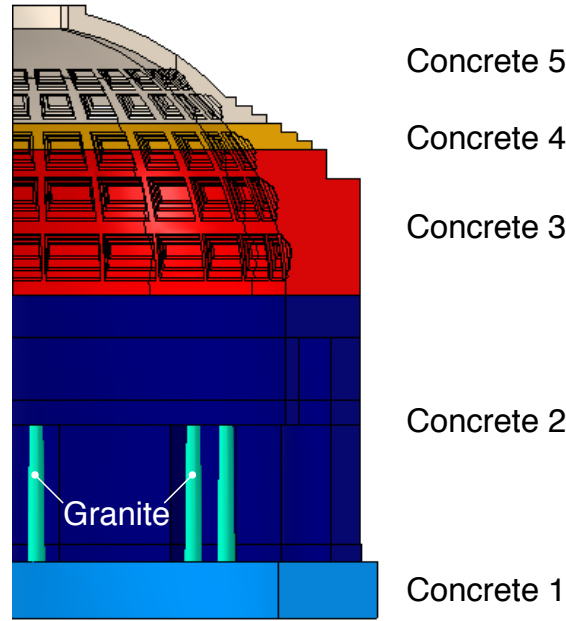


Figure 2.7: Location of the materials in Tab. 2.1.

some way the other material parameters to densities in such a way that, starting from few known data, the other ones could be determined.

First of all, for all the concretes, we have put $\nu = 0.2$, a mean value often used for the concrete Poisson's coefficient in ancient monuments (see e.g. [Perucchio and Brune, 2009]).

About Young's modulus E , the question is more delicate. Tests done on Roman mortars and concretes give very low values for E (see [Brune, 2010], [Jackson et al., 2009], [Jackson et al., 2014]). In his PhD thesis, Brune gives a table reporting, as a function of the curing duration, some typical values of different material parameters, in particular E , f_t , G_f , Ψ and ζ . Such values are reported in Table 2.2.

Table 2.2: Roman concrete properties as function of the set (after [Brune, 2010]).

Set [Days]	E [GPa]	f_t [MPa]	G_f [J/m ²]	Ψ	ζ
28	1.00	0.08	5	0.50	0.10
90	2.90	0.47	45	0.40	0.10
180	3.37	0.55	55	0.35	0.15

Of course, the interesting values for the present analysis are those referring to a set of 180 days. Also comparing the value of E given by Brune with other tests (see e.g. [Jackson et al., 2009], [Jackson et al., 2014], [Brune et al., 2010], [Samuelli-Ferretti, 1996]), it is likely to consider the value $E = 3.37$ GPa as an approximate lower bound for Young's modulus (this value is referred to a concrete whose density is 1540 kg/m³).

Another value that can be extrapolated from the literature is that concerning E for the cylindrical wall; it is composed by concrete and masonry and we have found a reliable value for ancient masonry in [Como, 2013]: $E = 6.6$ GPa. So, rounding the above values, we have put $E = 3$ GPa for concrete 5, the lightest one and $E = 7$ GPa for concrete 2. The other values can be determined upon a rule stating the dependence of E on ρ . We did not find it

in the literature about ancient concretes and the scarcity of data about these materials did not allow us to extrapolate such a rule.

That is why we have considered modern technical recommendations about lightweight concretes, [ACI, 1999], [FIB, 2000]. The following law is proposed in [Sanpaolesi and Formichi, 2009] about the Young's modulus:

$$E = 22 \left(\frac{f_c}{10} \right)^{0.3} \left(\frac{\rho}{2200} \right)^2, \quad (2.6)$$

with E in GPa, ρ in kg/m³ and f_c , the mean compressive strength, in MPa. So, what is apparent from this relation is that E is a quadratic function of ρ . We have hence looked for a law in which E satisfies this rule, giving a lower bound for E for $\rho = 1350$ kg/m³ and the two values given above for concrete 5 and concrete 2. Some algebra gives the relation

$$E = 2.5 \times 10^{-5} \rho^2 - 0.0675 \rho + 48.5625, \quad (2.7)$$

with E in GPa and ρ in kg/m³. This allows to have an estimation of E also for the other concretes.

About the tension strength f_t , the only reliable datum is that of Brune (see Tab. 2.2): $f_t = 0.55$ MPa. We attribute this value to concrete 5, the lightest one. Like for E , we need hence to put in relation f_t with ρ . We still use a relationship proposed for modern lightweight concrete [Sanpaolesi and Formichi, 2009]:

$$f_t = 0.3 f_{ck}^{2/3} \left(0.4 + 0.6 \frac{\rho}{2200} \right), \quad (2.8)$$

where f_{ck} is the characteristic value of the statistical distribution of f_c . It is likely that the compressive strength of the concrete types in Table 2.1 do not change considerably. Then, making the assumption that f_{ck} is practically constant, we get the following relation between the values of f_t for two different types of concrete:

$$\frac{f_{t1}}{f_{t2}} = \frac{0.4 + 0.6 \frac{\rho_1}{2200}}{0.4 + 0.6 \frac{\rho_2}{2200}}. \quad (2.9)$$

So, putting $f_{t1} = 0.55$ MPa and $\rho_1 = 1350$ kg/m³, we get the following linear relation for f_t as function of ρ :

$$f_t = 3.25 \times 10^{-4} (880 + 0.6 \rho), \quad (2.10)$$

with f_t in MPa and ρ in kg/m³. This relation allows us to have an estimation of f_t also for the other types of concrete in the *Rotonda*.

Concerning the fracture energy G_f , once more we have attributed the value of 55 J/m² of Table 2.2 to concrete 5. The data concerning G_f in the literature are even more questionable than those of the other material parameters: experimental or numerical procedure affect them, besides the intrinsic properties of the material, like the size of the aggregates, the curing process and so on. As a consequence, the spread of data is considerable (see for instance [Hillerborg, 1985], [Weerheijm and Vegt, 2010]). Hence, once more we have looked for a relation between G_f and another known material parameter of concrete.

In [Dehn, 2004] we found a linear relationship between G_f and the tensile strength f_t for lightweight concretes with natural sand, which is the case of the concretes of the *Ro-*

tonda:

$$G_f = 24 + 16f_t, \quad (2.11)$$

with G_f in J/m² and f_t in MPa. So, for two different concretes, we get

$$\frac{G_{f1} - 24}{G_{f2} - 24} = \frac{f_{t1}}{f_{t2}}, \quad (2.12)$$

and putting, as indicated above, $G_{f1} = 55$ J/m² and $f_{t1} = 0.55$ MPa, data relative to concrete 5 in Tab. 2.1, we get the linear relation

$$G_f = 24 + 56.36f_t, \quad (2.13)$$

allowing us to obtain an estimation of G_f for all the other concretes. The values of E , f_t and G_f so obtained for all the concretes are indicated in Table 2.1.

For what concerns the ratios Ψ and ζ we have put $\Psi = 0.35$ and $\zeta = 0.15$, as suggested by Brune (see Tab. 2.2).

For the granite of the columns, we have taken the values found in [Buyukozturk, 1993] and we have put $\Psi = 0.25$ and $\zeta = 0.50$.

By using eqs. (2.2) and (2.5), along with the Hooke's law for the value of w_0 , namely

$$w_0 = \frac{f_t}{E}h, \quad (2.14)$$

we get also all the other parameters defining the constitutive law represented in Fig. 2.6. These parameters are shown in Table 2.3. The values of w_0 , w_k and w_f have been calculated for a reference length $h = 100$ mm.

Table 2.3: Materials parameters defining the constitutive law

Material	E [GPa]	G_f [J/m ²]	f_t [MPa]	Ψ	ζ	σ_k [MPa]	w_0 [mm]	w_k [mm]	w_f [mm]
Granite	55.3	17.5	3.50	0.25	0.50	0.88	0.006	0.008	0.015
Concrete 1	13.5	62.1	0.68	0.35	0.15	0.24	0.005	0.006	0.371
Concrete 2	7.0	59.4	0.63	0.35	0.15	0.22	0.009	0.058	0.385
Concrete 3	4.6	57.7	0.60	0.35	0.15	0.21	0.013	0.059	0.395
Concrete 4	3.6	56.6	0.58	0.35	0.15	0.20	0.016	0.060	0.403
Concrete 5	3.0	55.0	0.55	0.35	0.15	0.19	0.018	0.062	0.413

In Figure 2.8 we show the diagrams of the constitutive law $\sigma - w$. The diagram of the granite has been reduced by a factor 4, for graphical reasons.

It is worth noting that the values of the different material parameters are close to experimental data measured on ancient Roman concrete (see [Lamprecht, 1984], [Jackson et al., 2009], [Brune, 2010], [Brune et al., 2010] and [Jackson et al., 2014]). They represent, hence, a plausible variation of such parameters with the concrete density.

The properties of Roman concrete are remarkable when compared to modern concrete: the low value of the Young's modulus, the high tensile strength and the important fracture energy confer to Roman types of concrete good properties of ductility and toughness, which render such materials particularly suited for energy dissipation.

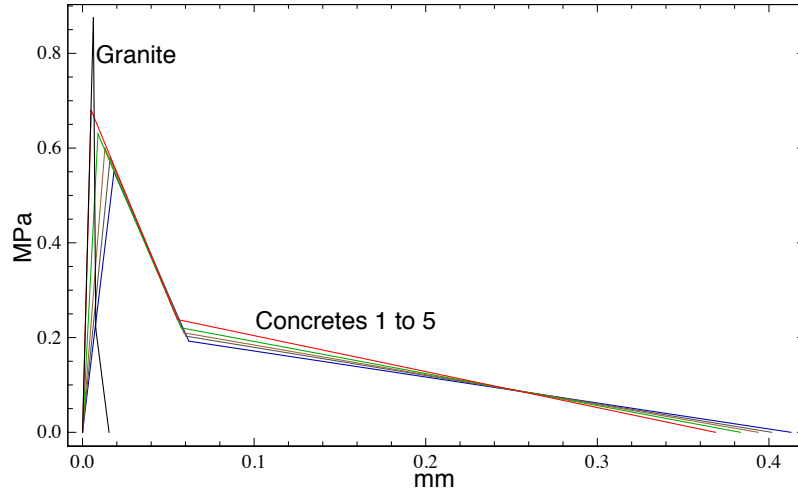


Figure 2.8: Tensile constitutive law of the materials (for $h = 100$ mm).

2.4 The numerical procedure

The use of JWL for the blast simulation needs, as already mentioned, a meshing not only of the Pantheon's structure, but also of the air volume interested by the explosion, in this case the internal volume of the *Rotonda* and also a volume around the two openings, the entrance door and the *oculus*, "Pantheon's eye", the round opening at the dome's top, with a neat diameter of ~ 7 m [de Fine Licht, 1968]. The details on the models of the air volume and of the structure and the procedure used for their numerical validation are given below, separately for the two parts. Using the symmetry of the structure, (see below Fig. 2.13) we have modeled just one half of the structure and of the air domain. This allows, for the same computing effort, to obtain a more detailed model, i.e., a finer discretization. All the computations have been done with the code ABAQUS, [DS, 2016].

2.4.1 Mesh sensitivity of the fluid domain finite volumes model

The model of the air domain is shown in Fig. 2.9. The air domain of half the structure has been decomposed into 2.068×10^6 8-nodes hexahedral volume elements, for 10.34×10^6 degrees of freedom (DOF) on the whole (5 DOF for each volume element: the 3 displacement components and the pressure of the volume element centroid plus the EVF [DS, 2016]).

In Coupled-Eulerian-Lagrangian (CEL) simulations, ABAQUS/Explicit takes into account for the Eulerian fluid domain through the so-called volume-of-fluid method: the material flowing through the mesh is tracked by the definition of an additional variable within each element, the Eulerian volume fraction (EVF). In our case, this allows to compute not only the propagation of the shock waves, but also the diffusion of the explosive material inside the air domain. The coupling between the Lagrangian (solid) domain and the Eulerian (fluid) one, i.e., the fluid-structure interaction, is achieved by a general contact algorithm, with a null interface friction coefficient and a penalty method.

Boundary conditions on the Eulerian domain are of two kind: a reflecting boundary is used for the symmetrical plane, while a nonreflecting outflow condition is applied to the other surfaces. The first one imposes a zero flow across the boundary, i.e. zero normal velocities. The nonreflecting outflow condition consists, instead, of the application of additional normal and shear tractions on the domain boundary that are proportional to normal and shear

components of the velocity at the boundary. In a sense, it consists on using damping constants in order to minimize the reflection of waves back into the fluid domain. However, the need for achieving a condition similar to plane body waves impinging on the boundary surfaces requires to extend the Eulerian domain beyond the solid domain. This is why we have modeled a volume of air as deep as 5 meters above the *oculus* and a volume of 2 meters of air on the lateral sides.

The choice of the fineness of the mesh has been done upon a convergence analysis: four different meshes for the air domain have been considered: M1, with 1.48×10^6 DOF, M2 with 6.3×10^6 DOF, M3, the chosen mesh detailed above, and a reference mesh M4, the finest one, with 16.94×10^6 DOF.

For each mesh, we have computed the blast overpressure in correspondence of three different observation points: P1, P2 and P3, indicated in Figure 2.9, respectively close to the base of the cylindrical wall, to its top and to the *oculus*. The time histories of the overpressure, for the three different points and for each mesh, are plotted in Figure 2.10 (the curves has been normalized with respect to the highest value).

In Figure 2.11 we show, for points P1 to P3, the relative error evaluated for meshes M1 to M3 with respect to mesh M4; such an error, in percent, is computed as

$$err = 100 \left| \frac{p_{Mi} - p_{M4}}{p_{M4}} \right|, \quad (2.15)$$

where p_{Mi} is the peak of the overpressure, in P1, p2 or P3, evaluated for mesh M_i , while p_{M4} is the same for the reference mesh M4.

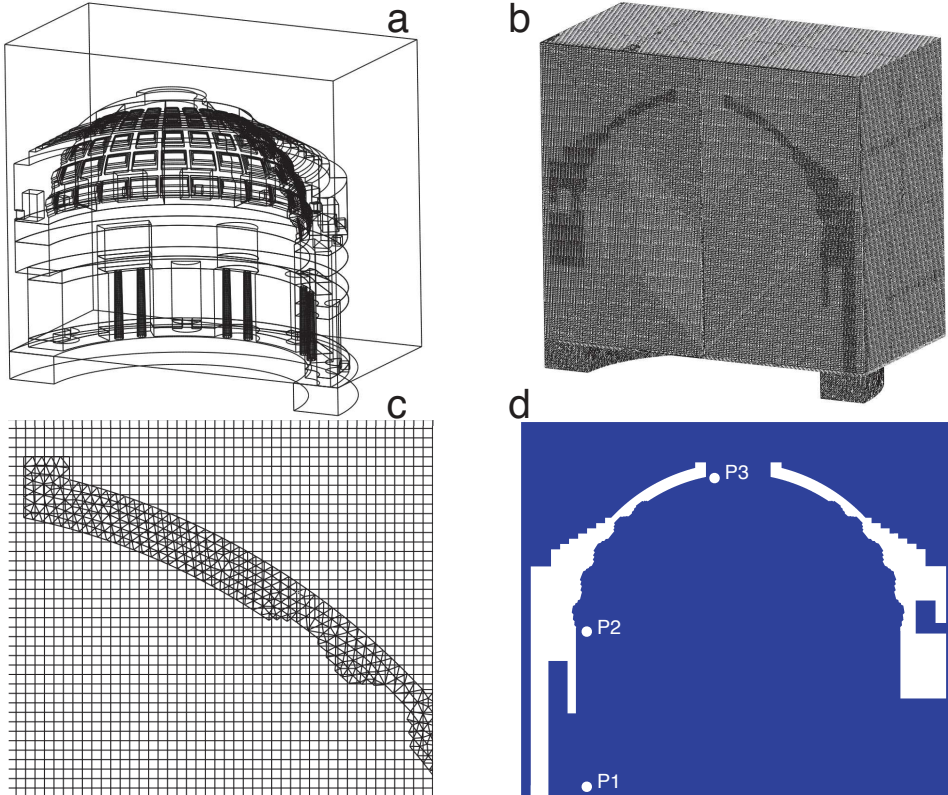


Figure 2.9: The air domain; a) overall view, b) complete mesh, c) detail of the mesh, d) location of the observation points P1 to P3.

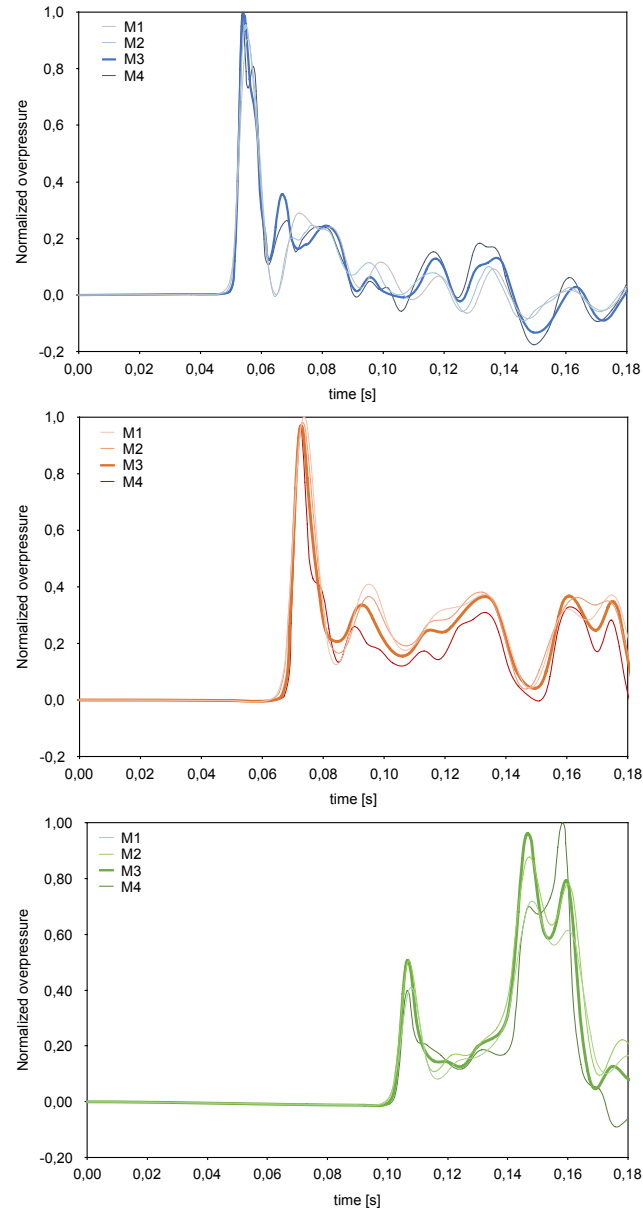


Figure 2.10: Time history of the overpressure for points P1 to P3 (from the top).

Considering the results shown in Fig. 2.10 and 2.11, we can see that the choice of the mesh M3 is a fair compromise: it guarantees a good quality of the result (the maximum relative error is 3.8%, for point P3) with a problem size that is not extreme.

We have also compared the results given by JWLP with those given by CONWEP for the reflected pressure P_r on the solid points corresponding to P1, P2 and P3, see Fig. 2.12 (the curves have been normalized with respect to the highest value). The comparison clearly shows how much the use of CONWEP in a case like this one can lead to erroneous results: the time history of the overpressure is completely different, mainly due to the reflected shock waves.

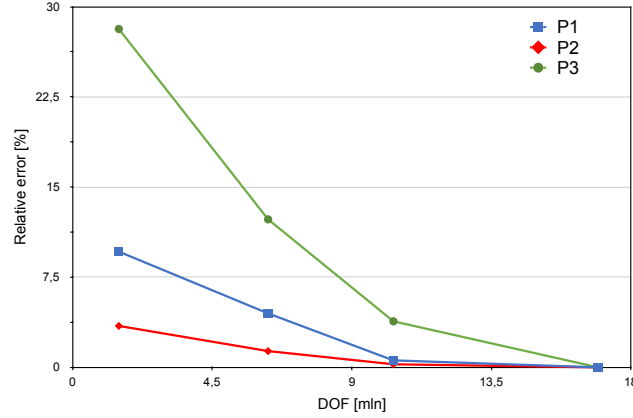


Figure 2.11: Relative error, eq. (2.15), for points P1 to P3 as function of the number of DOF.

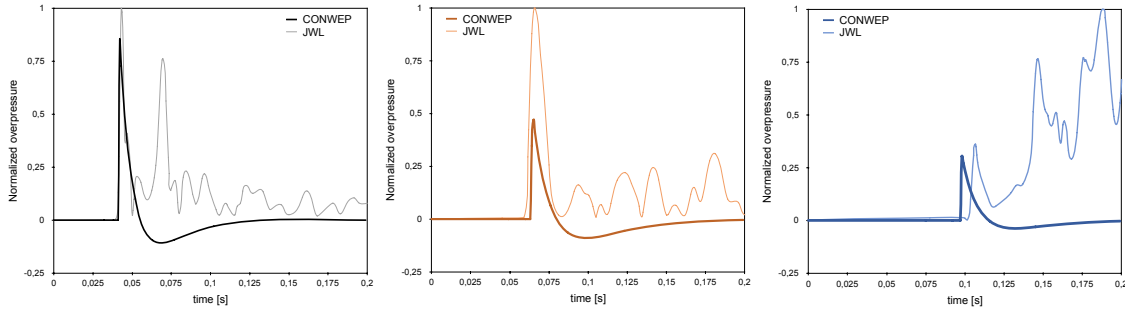


Figure 2.12: Comparisons of P_r time history as evaluated by JWEL and CONWEP; from the left, comparison for points P1, P2, P3.

2.4.2 Mesh sensitivity of the Pantheon finite elements model

The entire structure of the Pantheon has been modeled using CAD (see Fig. 2.13). The model comprehends the coffering of the dome intrados, the step-rings, the interior columns, absides and cavities present in the cylindrical wall and the foundation ring.

The CAD model has been successively meshed to obtain a finite element (FE) model. The FE model comprehends only the *Rotonda*, because the structural role of the *pronaos* is undoubtedly little, for the blast actions of an explosion in the interior of the Pantheon; so, we have neglected it.

Just like for the mesh of the air domain, we have performed a convergence analysis for validating the choice of the *Rotonda* mesh, composed of four-nodes tetrahedral elements, supported by both the standard and explicit solvers of ABAQUS.

Also in this case, the fineness of the mesh has been chosen after a convergence analysis, performed in order to obtain a reliable degree of accuracy with an acceptable size of the whole numerical model (air domain plus structure).

The convergence analysis has been done using the standard (implicit) scheme for eleven different structural meshes MSi, $i=1, \dots, 11$, whose characteristics are given in Tab. 2.4.

The standard analysis made for each mesh MSi is divided into two parts: a modal analysis and a static one, for the only gravity load. We have monitored the eigenfrequencies of the first twenty vibration modes of the structure and, for the static analysis, the vertical

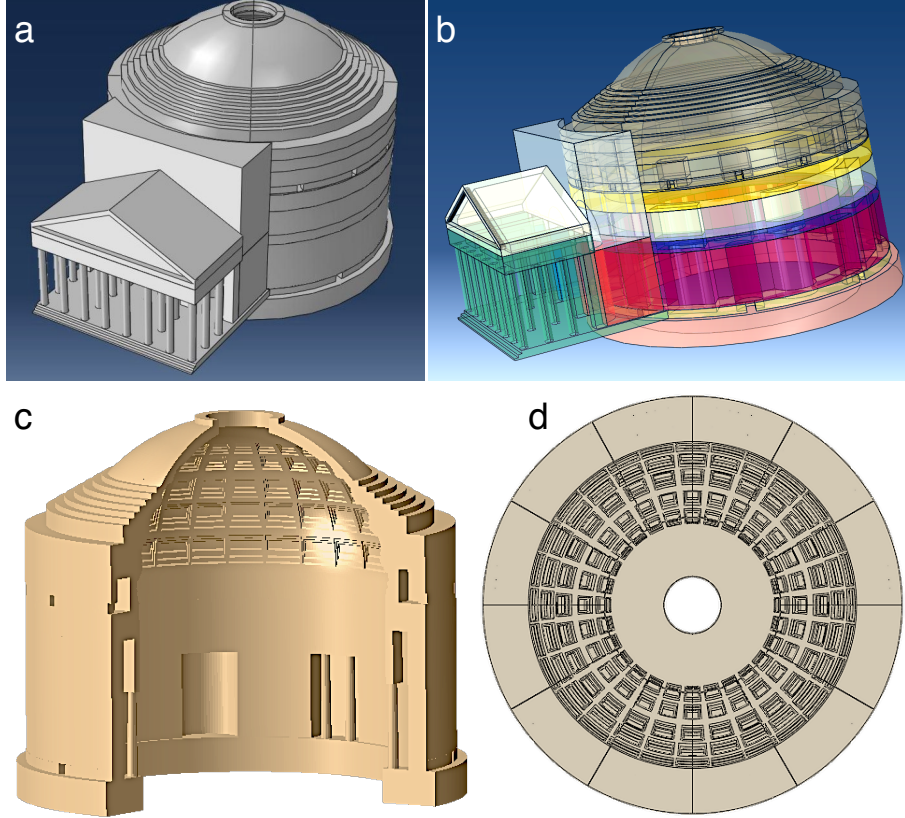


Figure 2.13: The CAD model of the Pantheon; a): overall view, b) transparency view, c) interior view of the *Rotonda*, d) detail of the dome with indicated the twelve meridional cracks inserted in the model.

displacement of point P3 in Fig. 2.10. The twenty eigenfrequencies f_j , $j = 1, 2, \dots, 20$, and the vertical displacement u of the point P3, are calculated for each structural mesh MS. The convergence has been evaluated like for the air domain, calculating for each mesh $MS_i = 1, \dots, 10$ the errors Δf_j of the frequencies f_j and Δu of the displacement u , relatively to the same quantities f_j^r and u^r , calculated for the reference mesh MS11, the finest one:

$$\Delta f_j = \left| \frac{f_j - f_j^r}{f_j^r} \right|, \quad j = 1, 2, \dots, 20, \quad \Delta u = \left| \frac{u - u^r}{u^r} \right|, \quad \forall MS = 1, \dots, 10. \quad (2.16)$$

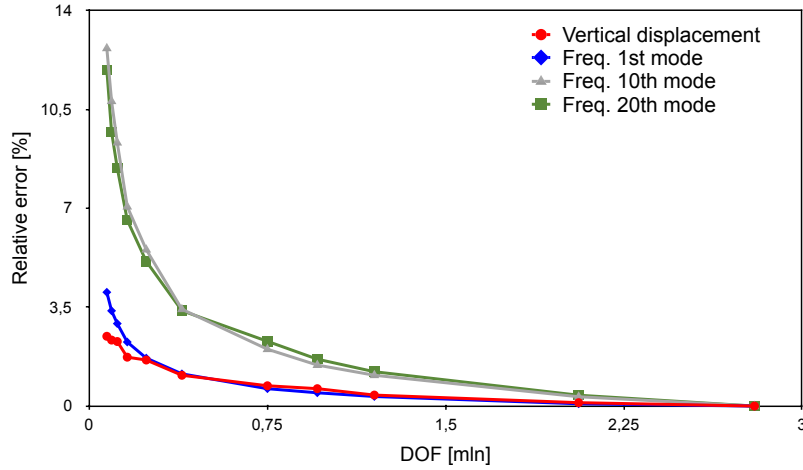
In Figure 2.14 we show the diagrams of Δf_1 , Δf_{10} and Δf_{20} along with Δu as functions of the DOF number.

In Figure 2.15, left, we report the cumulative normalized CPU time necessary for the static and modal analysis as function of the DOF number; for each mesh MS_i , $i = 1, \dots, 11$, this is the ratio between the CPU time corresponding to MS_i and that of MS1. In Fig. 2.15 right, we show the relative errors Δf_1 , Δf_{10} , Δf_{20} and Δu versus the normalized CPU time.

After evaluation of the results in Figs 2.14 and 2.15, we have selected the mesh MS11, the finest one, for the computations. In fact, we have estimated that the convergence has been reached (the diagrams in Figs 2.14 and 2.15 show an asymptotic behavior) while the increase of the computation time can still be considered as tolerable. A detail of the structural mesh MS11 is shown in Figure 2.16.

Table 2.4: Characteristics of the studied structural meshes.

Mesh	Average element size [m]	Number of elements	Number of DOF
MS1	1.80	95000	74000
MS2	1.60	122200	94000
MS3	1.40	159300	118000
MS4	1.20	224000	160000
MS5	1.00	348000	240000
MS6	0.80	590000	390000
MS7	0.60	1200000	750000
MS8	0.55	1500000	960000
MS9	0.50	1980000	1200000
MS10	0.45	3450000	2060000
MS11	0.35	4800000	2800000

Figure 2.14: Relative errors Δf_1 , Δf_{10} , Δf_{20} and Δu as function of the number of DOF.

To take into account for the existing cracks in the dome, Fig. 2.4, we have modeled 12 cracks, propagating along the meridians up to 57° above the plane passing through the sphere center in Figure 2.3 and equally spaced; they are indicated in Fig. 2.13 d.

Parallel computing allowed us to decrease significantly the length of the analyses. All the simulations have been performed using a 24-cores workstation. The entire model is, thus, divided into 24 geometric domains, taking advantage of 24 processors in the machine.

2.5 Numerical simulations

We have simulated the effects of an explosion in the center of the *Rotonda*, taking into account the nonlinearities of large strains formulation; some images representing the simulation at different times are represented in Figs 2.17 to 2.21. The pictures represent the pressure field in the interior of the *Rotonda* and, starting from the instant where the first cracks appear in the dome, the effects on the structure.

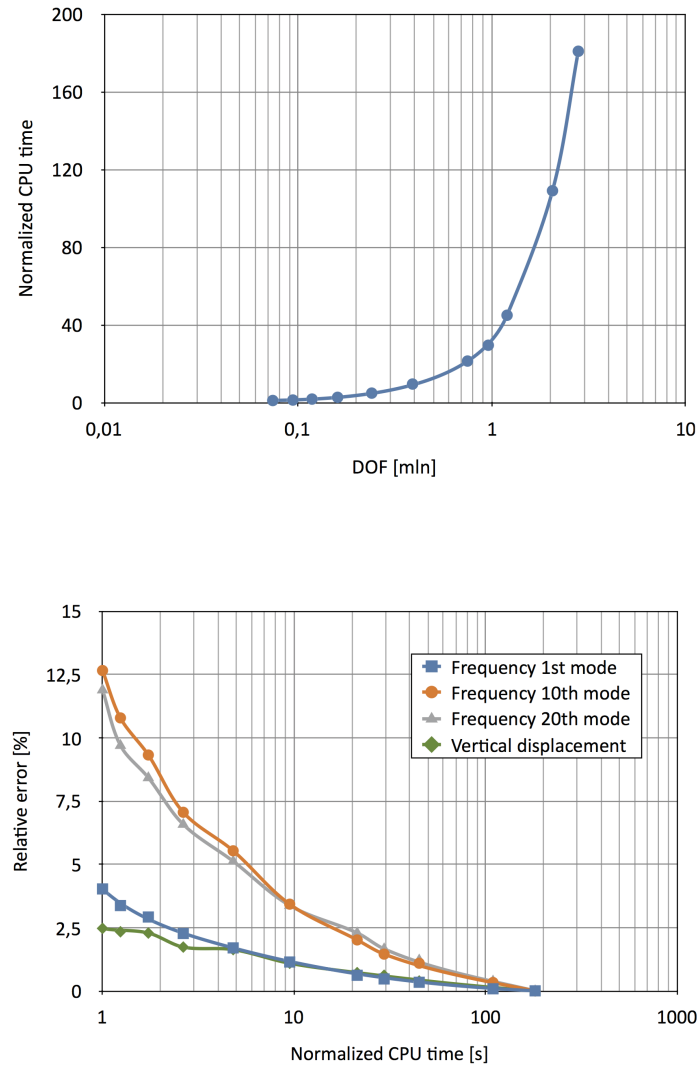


Figure 2.15: Normalized CPU time as function of the DOF number, left, and relative error versus the normalized CPU time, right.

2.5.1 Evolution of the pressure field

The pressure field represented in Figs 2.17 to 2.20 is comprehensive of all the blast phenomena: shock wave, reflected shock waves and dynamic pressure. The first two sketches in Figure 2.17 show the very initial phases of the blast, with the shock wave propagating hemispherically until it touches the cylindrical wall. At this moment, the reflected shock waves add to the principal one, creating a complex pressure field and, though the principal shock wave decreases in intensity, important local concentrations of pressure are possible due to the interaction between the principal shock wave and the reflected ones.

Local concentration of the pressure are clearly visible in the interior of the niches and in the parts of the coffering looking downward, but the most important concentration happens exactly in the central axis of the *Rotonda*, of course as an effect of its cylindrical symmetry. An important zone of high pressure, in the form of a butterfly, forms and progresses towards the *oculus*. This last represents an important escape way for the pressure and the gases, as

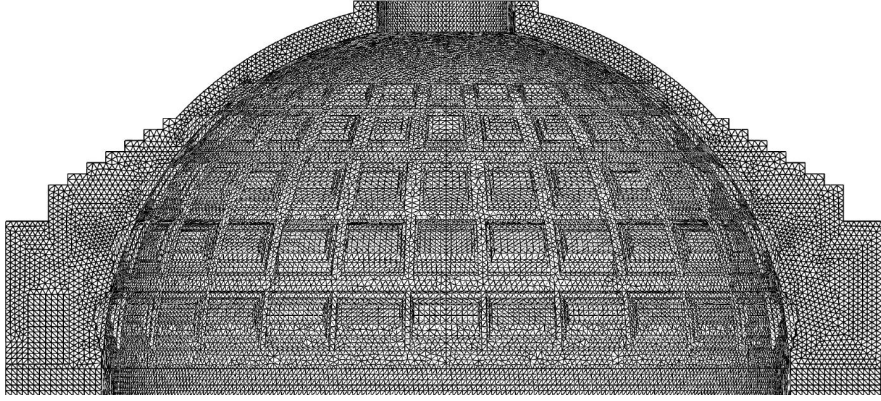


Figure 2.16: Detail of the structural mesh MS11.

clearly visible in the pictures of Figure 2.18. Also the entrance door is an escape way, but apparently it has a least effect of the *oculus*, because the focalized shock waves move toward the dome's top.

Such pressure concentrations can lead to local value of the overpressure far greater than that produced by the original shock wave. This is clearly visible, e.g., in Figure 2.10, where the pressure for point P3, close to the *oculus*, clearly shows secondary peaks far higher than the first one, due to the impinging shock wave. These secondary peaks mark the passage, through the *oculus*, of the "butterfly" pressure wave. Another phenomenon is visible in Figure 2.10: the overpressure lasts much more than the single original shock wave. Of course, this has a strong effect on the structure damage, because the structure itself is much longer exposed to high impact pressures.

Rather surprising is the fact that the coffers acts like dampers of the pressure impulse, in terms of both its module and time. On one hand, the sharp-cornered surfaces of the coffers are nothing but obstacles to the propagation of the internal blast wave: the waves reflected by the coffering array looking downward contrast the rising incident wave through the particular combination of incident and reflection angles, as clearly shown in Figure 2.17. The arrangement of the coffers themselves results in a step-by-step attenuation, mitigating the incident pressure as getting closer to the upper part of the dome and the intense, butterfly-shaped, concentration that rises in the center of the *Rotonda*. On the other hand, this translates in a temporal phasing of the blast wave, well visible in Figure 2.12. Indeed, comparing the pressure time history of the empirical model CONWEP with that from JWJ, we can see that the pressure impulse from the latter one arrives at the *oculus*, i.e., at point P3, delayed with respect to that computed by CONWEP. Therefore, we can assess that the role of the coffering is structurally crucial: it serves not only to reduce the dome's weight but it is able also to mitigate the effects of an internal blast.

2.5.2 Evolution of the structural damage

It is exactly the "butterfly" zone of focalized pressure, result of the interaction of the principal and reflected shock waves, that produces the principal damages to the *Rotonda*, and in particular it is likely to cause the destruction of the dome.

This is apparent looking at Figs 2.19 and 2.20: the first cracks appear in the compression ring of the *oculus* when the focalized pressure approaches the dome and when it passes through the *oculus*, the existing cracks begin to propagate toward the dome's top and

eventually they reach it. The static regime of the dome is completely changed, because a shell behavior is no more possible and, each part of the dome behaving like a wedged cantilever, a circular crack appears in the upper part of the dome as consequence of the bending tensile stresses produced by gravity so that, in the end, the upper part of the dome falls down to ground.

In the same time, the lower part of the *Rotonda*, the cylindrical wall, is interested by cracks, that propagate through the entire wall thickness starting from the existing ones, as apparent in Figs 2.20 and 2.21, while also the granite columns are fractured by the blast.

The failure of the dome is hence the consequence of different causes: the blast, the propagation of the existing cracks and the weight of the dome itself.

The real extent of the existing cracks in the *Rotonda's* dome is uncertain. So, for the sake of completeness, we have performed the same simulation described above and on exactly the same model of the Pantheon but now without the existing cracks. All the other parameters are left unchanged.

The results of this analysis are shown in Figure 2.22, where the two cases of structure with or without cracks are compared at different instants. Looking at these pictures, it is evident that the case without cracks has a different failure mechanism: the upper part of the dome is still destroyed, but to a less extent with respect to the case with cracks.

This is because in the case of the cracks, as mentioned above, the blast effect is a propagation of these toward the top of the dome and the creation of the cantilever effect, that destroy a large part of the dome under the action of the gravity. In the case without cracks, the upper part of the dome is destroyed mainly by the cracks produced at the intrados by the blown of the blast. Moreover, while in the case with cracks these propagates also downward in the cylindrical wall of the *Rotonda*, this last is practically unaffected by the explosion, apart the granite columns. As a conclusion, we can say that also if the existing cracks are ignored in the simulation, the dome is destroyed by the blast, though to a less extent, while the cylindrical wall remains almost intact. The existing cracks constitute hence a weakness of the whole structure of the Pantheon for what concerns the effects of an explosion.

2.5.3 Remarks about numerical simulations

The results presented hereon have been produced by a numerical simulation; though there is always, in such a kind of problems, a certain ineradicable difference between reality and simulation, nevertheless they certainly grasp the main response of the structure to a blast.

For what concerns the simulation of the blast itself, it clearly shows how much it is important to take into account for reflected shock waves, a fundamental aspect of the phenomenon for internal explosions. This means that a study of such phenomena done using empirical models like CONWEP, not taking into account for reflected shock waves, can lead to results really far from reality and principally underestimate the effects of the blast on the structure.

The response of the structure deserves a commentary. The results shown hereon have been obtained for the least quantity of explosive able to destroy, at least in part, the Pantheon. Without entering in details, we precise that such a quantity is really enormous, impossible to be brought in the interior of the *Rotonda* by hands, without some mechanical means.

In other words, the Pantheon is intrinsically safe with respect to inner explosions induced

by a single individual. This is due to different reasons. On one hand, its dimensions: it is a large building and the structure itself is really massive: the cylindrical wall thickness is ~ 6 m, while that of the dome of ~ 1.5 m at its top, as already mentioned. Then, its form: the cylindrical form is by itself more likely to resist to impact pressures than a flat one, more exposed to bending stresses. Finally, and this point is really surprising, the material. As already highlighted in Sec. 2.3.2, Roman concrete has mechanical properties quite different from modern light concretes: its Young's modulus (~ 3 to ~ 18 GPa [Brune, 2010], [Lamprecht, 1984]), good tensile strength, ~ 0.6 MPa, and fracture energy, ~ 60 J/m², confer to Roman concrete very good properties of resilience and shock absorbing. Though, of course, ancient Romans did not conceive this material for absorbing a blast effects, nevertheless it reveals once more to be a key of Pantheon's structural success.

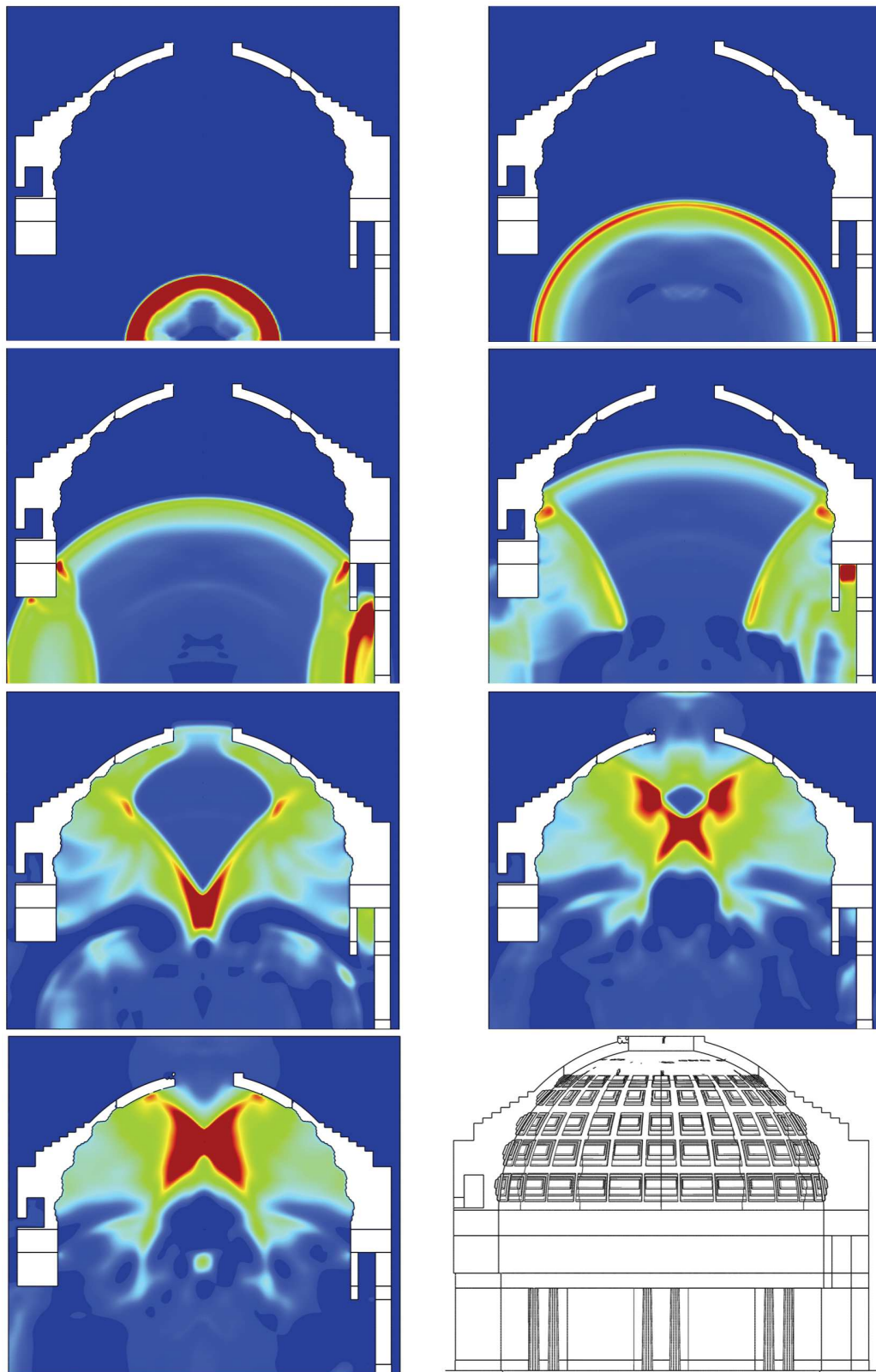


Figure 2.17: The simulation of a blast inside the *Rotonda*.

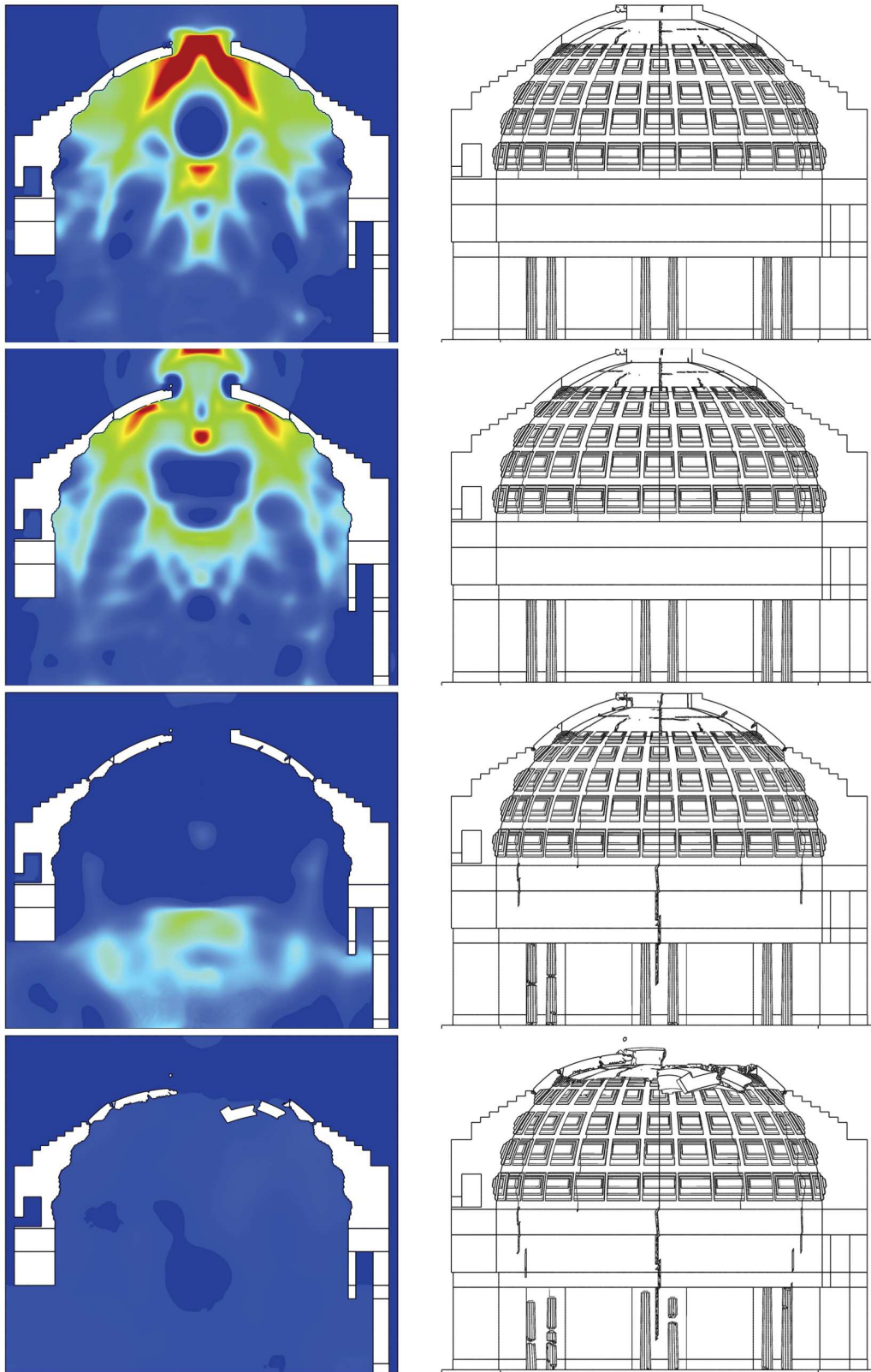


Figure 2.18: The simulation of a blast inside the *Rotonda* (continued).

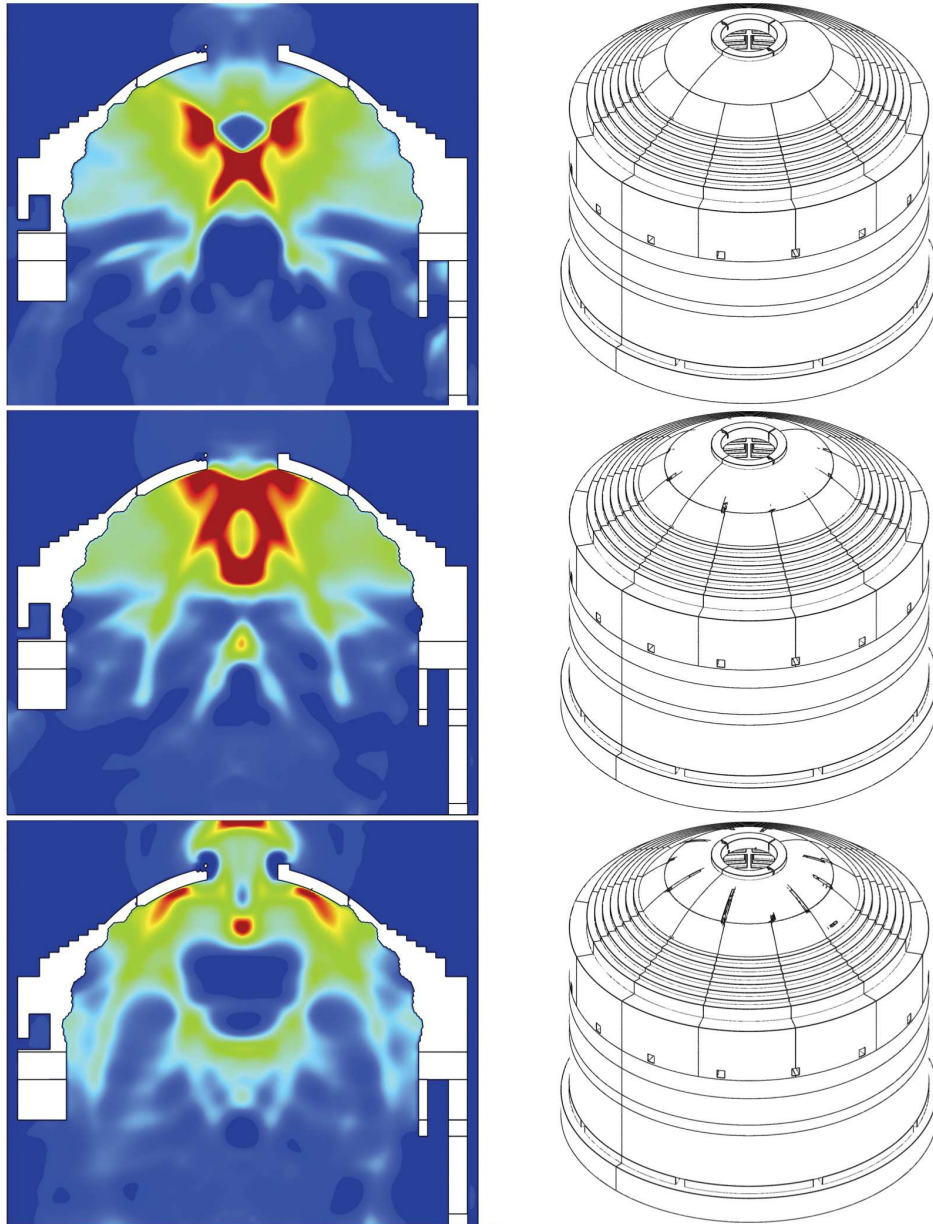


Figure 2.19: The simulation of a blast inside the *Rotonda*; external view.

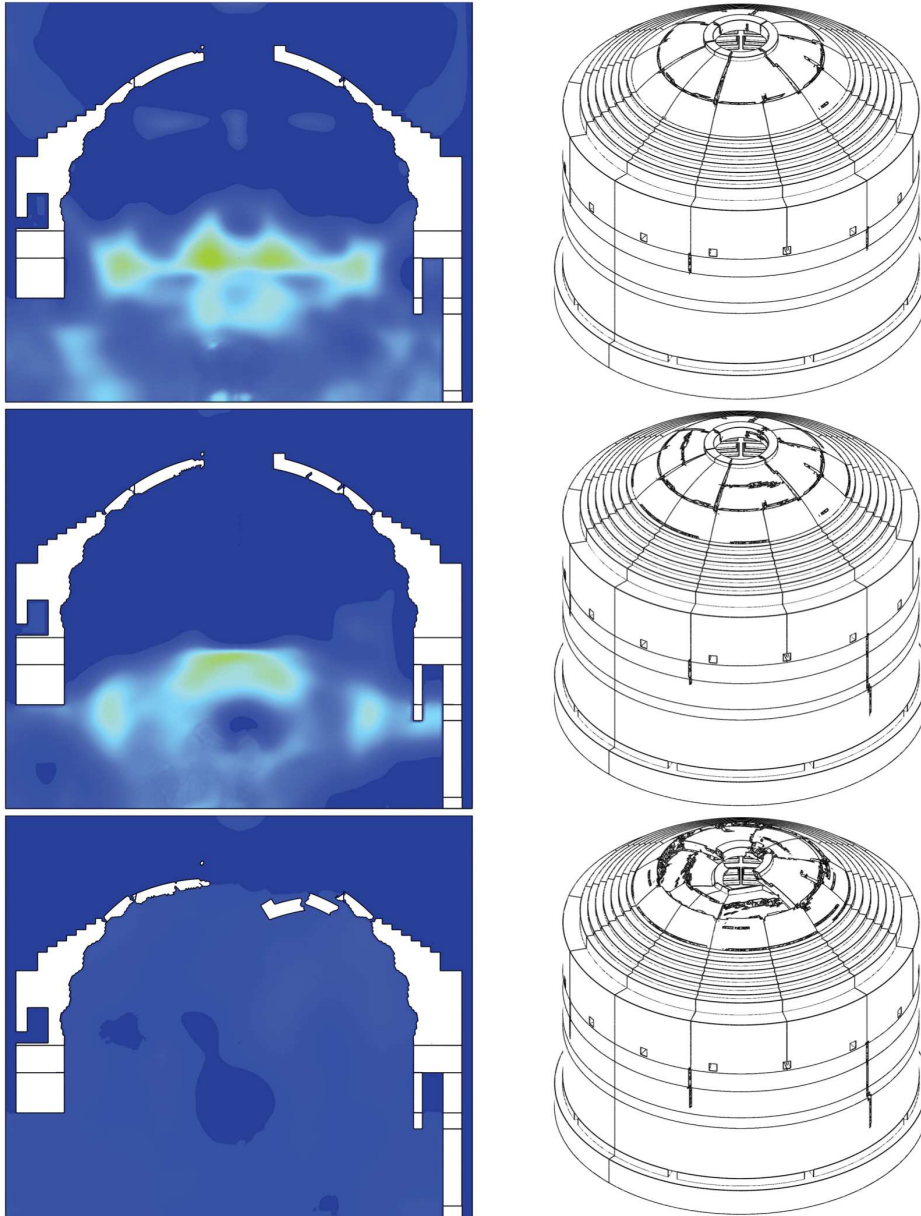


Figure 2.20: The simulation of a blast inside the *Rotonda*; external view (continued).

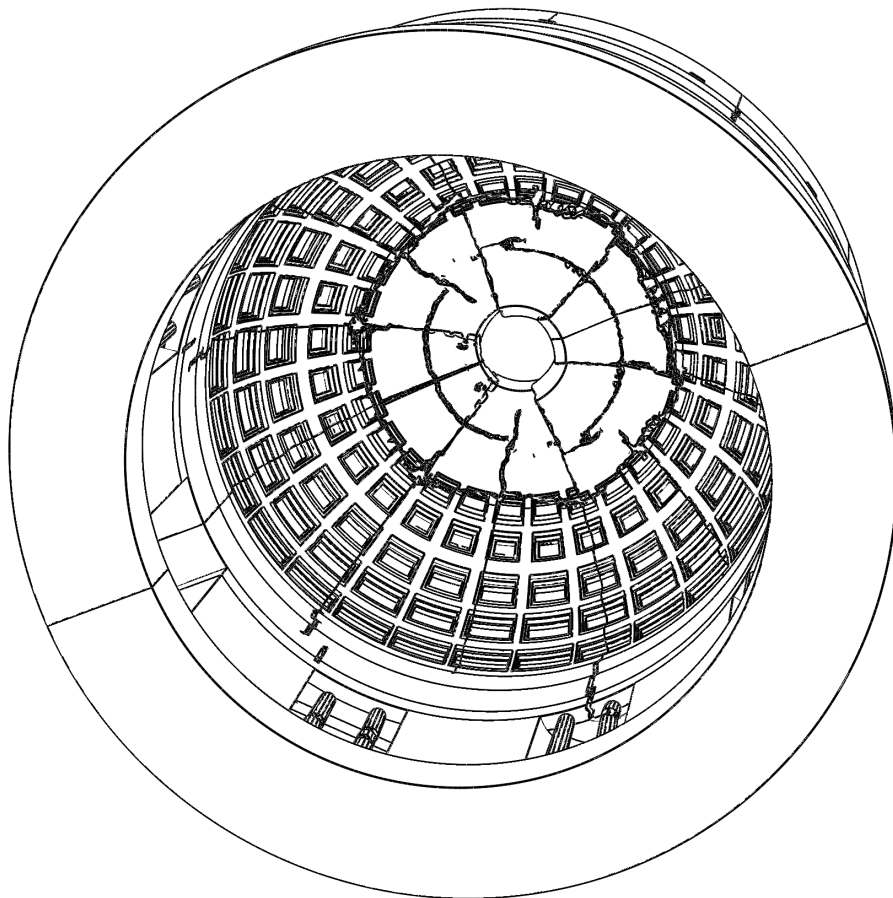


Figure 2.21: The simulation of a blast inside the *Rotonda*; internal view of the final situation.

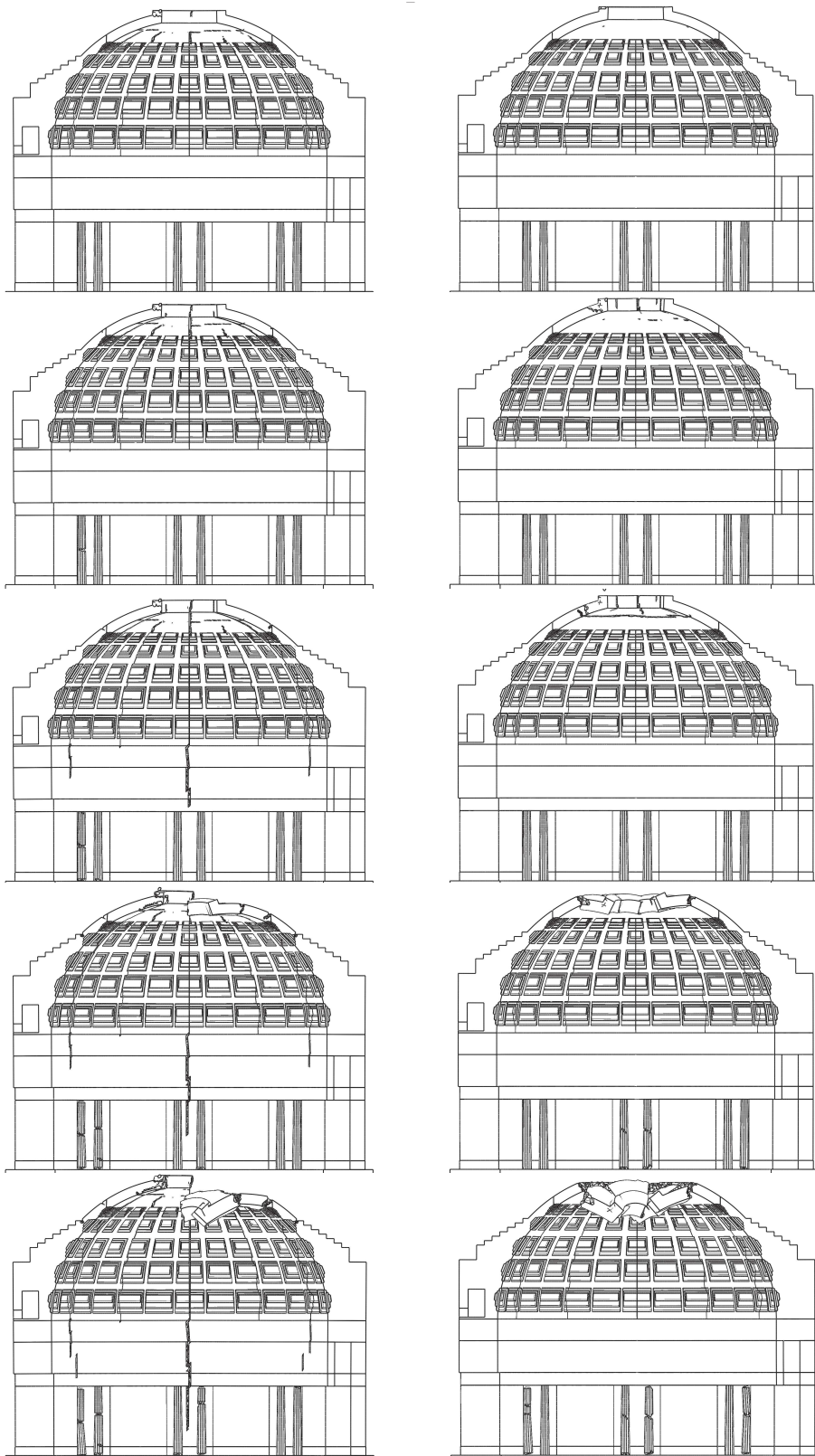


Figure 2.22: Comparison between the case with existing cracks (on the left) and without cracks (on the right).

Chapter 3

Blast actions on an Al-alloy made fuselage and their mitigation

We record airplane bombing attacks afflict the early 20th Century, with the first bomb made of nitroglycerin, detonated in 1933 in the cargo hold of a Boeing 247D. In Table 3.1 we present the development of terrorist attacks from the early 1930s until now.

Until the bombing of a Boeing 707-124 in 1962, all the attacks were focused on very simple explosive devices placed in the baggage compartment. Many of the following disasters were caused by in-cabin bombs, placed mostly under seat cushions.

In 1987, the first liquid explosives were used in a terrorist attack, but they were rapidly replaced by plastic explosives hidden inside shoes, laptop computers and other electronic devices.

The response of airlines and airports to these threats was and is focused on their prevention. A rising aviation security has brought to strict measures to be applied: between 1960s and 1970s a pre-board screening was imposed, from 1990s hold baggage screening was applied, from 2000s the first rules on the restriction of liquids, aerosols and gels appeared, from 2010s fully body scanning procedures were required. Finally on March 21st 2017, electronic and electrical devices are banned, with immediate effect, from cabins in some international flights coming from countries such as Turkey, Lebanon, Egypt, Jordan, Morocco, Qatar, Tunisia, Saudi Arabia and United Arab Emirates.

However, the prevention of bombing attacks guaranteed by these and others security procedures has required enormous amount of investments and characterise, today, an important economic issue for airports, well discussed in [Gillen and Morrison, 2015].

The substantial costs associated with active protections, i.e., prevention, from terrorist threats have been steeply increasing in the last years. Gillen and Morrison report a comparative study of European total expenditures on aviation security: 2.8 billion euros, while in 2011 they have been grown up to 5.7 billion euros.

This leads to the need of investigating functioning passive protections to be integrated in the aircraft design to prevent the subsequent disaster caused by an explosion of a little charge.

Studies on the vulnerability of commercial aircrafts started thirty years ago. The Commission on Aviation Security established in 1989 by the former United States President G. H. W. Bush, in response of the terrorist bombing of a Boeing 747-121 in 1988, had two main objectives: to determine explosive limits of current commercial aircrafts and to pro-



Figure 3.1: February 2016, Daallo Airlines: the one meter sized hole caused by an internal explosion.

pose measures to prevent disasters similar to that of Pan Am Flight 103. In this context, the idea of developing the so-called *unit load devices*, a new design of luggage containers with the aim of absorbing energy from an in-cargo explosion, was developed. A ULD made of composite panels with reinforcing fibres has been designed [Mlakar et al., 1992]; also a bilayer hardened luggage container made of an inner layer of lightweight foam for capturing debris and an outer pressure mitigation layer has been designed [Sanai and Greenfield, 1993].

Protections for fuselages (see e.g. [Fray, 1992] and [Hararat-Tehrani, 1992]) usually consist in blowout panels, that are incorporated into fuselages and designed to be weaker than the surrounding airframe. In a bombing event, blowout panels will fail allowing for pressure releasing and a controlled fuselage failure.

The first important thing in defining new protections is to understand the behaviour of an aircraft fuselage subjected to an internal explosion. The most critical challenge is to describe the complex nature of loads that act on such a structure.

Let us suppose an airplane at cruising altitude. First, we cannot neglect the pressurization and gravity loads; also, although we may suppose the absence of turbulence and wind gusts, we need to take into account the rigid body motion of the system itself.

Standard experimental tests on fuselages are usually employed in achieving a fatigue design and cannot be used in investigating a bombing event.

Experiments on the response of fuselages to a blast usually consist simply in detonating the explosive inside the aircraft, while it is at ground and without any acting load on it, exception made for gravity (see [Settles et al., 2003]).

In [Backman and Gould, 2015] the response of a partially pressurized fuselage is investigated in retired Boeing 727 fuselages. In [Veldam et al., 2008] the response of a plane pre-pressurized panel is analysed, despite the fact that the dynamics of a fuselage is afflicted by large-scale effects due to its size.

A lower-cost branch of methods for investigating the fast-dynamics of a fuselage subjected to blast actions is a numerical approach. However, the lack of literature in this field makes clear the difficult of setting up reliable numerical procedures. Indeed, many articles deal with the modelling strategies of reinforced plates in Al-alloys or glass reinforced aluminium (GLARE), i.e., a fibre metal laminate composed of aluminium and glass fibre layers, under blast actions, but still the real size of the problem is neglected (see e.g. [Soutis et al., 2011]). The only papers that deal with the investigations of blast actions on fuselages are [Dacko and Toczyski, 2011] and [Kotzakolios et al., 2011], where a Coupled-Eulerian-Lagrangian approach is followed. The first investigates the effects of an in-cabin bomb placed inside

Table 3.1: Airliner bombing attacks, from [Foundation].

Date	Flight	Description	Casualties
1933	Boeing 247D, United Air Lines	bomb made of nitroglycerin placed in the baggage compartment	7
1949	Douglas C-47-DL, Canadian Pacific Airlines	bomb made of dynamite placed in the baggage compartment	23
1955	Lockheed L-749A, Air India	explosive device	16
1955	Douglas DC-6B, United Air Lines	bomb made of dynamite	44
1962	Boeing 707-124, Continental Air Lines	explosive device inside passenger cabin	45
1966	Douglas RD4-1, Aden Airlines	explosive device inside passenger cabin	30
1967	DH-106 Comet 4, British European Airways	high explosive device within the cabin under seats	66
1970	Convair CV-990-30A-6 Conorado, Swissair	bomb in the baggage compartment	47
1976	Douglas DC-8-43, Cubana de Aviación	explosive device at the rear of the cabin	73
1976	Boeing 720-023B, Middle East Airlines	bomb in the baggage compartment	81
1982	Boeing 747-121, Pan Am	bomb placed under a seat cushion	1
1983	Boeing 737-2P6, Gulf Air	bomb in the cargo compartment	112
1985	Boeing 747-237B, Air India	high explosive device inside the cargo compartment	329
1986	Boeing 727-231, Trans World Airlines	explosive device in the cabin	4
1986	Boeing 737-270C, Iraqi Airways	two hand grenades in the cabin	63
1987	Boeing 707-3B5C, Korean Air	liquid explosives concealed as liquor bottles	115
1988	Boeing 747-121, Pan Am	high-explosive device in the cabin	270
1989	McDonnell Douglas DC-10-30, Union de Transport Aériens	high explosive device in the cargo hold	170
1989	Boeing 727-21, Avianca Airlines	explosive near the fuel tank	110
2001	Boeing 767, American Airlines	plastic explosive concealed within shoes	0
2004	Tupolev Tu-134-3, Volga-Avia Express Tupolev Tu-154B-2, Siberia Airlines	high explosive devices	90
2015	Airbus A32-231, Metrojet	1 kg of TNT	224
2016	Airbus A321-111, Daallo Airlines (see Fig. 3.1)	explosive device concealed within a laptop computer	1

the luggage compartment for an Al-alloy made fuselage, assumed as a cantilever beam and thus neglecting any possible rigid body motion and limiting its response in terms of strain and stresses, especially when the structural damage becomes significant.

In [Kotzakolios et al., 2011] the authors investigate a fuselage from Airbus A380, where the pressurized load is considered just as a static load acting on the skin, without modelling the different volumes of air, inside and outside the cabin, and therefore neglecting the different velocities of travelling shock waves through the volumes and the changing of pressurization load. Although little quantities of explosives are tested and any damage of the fuselage is recorded, these assumptions may lead to a questionable understanding of the real dynamics, as shown in the Appendix (3.5.3).

In this chapter, we highlight the main issue of a numerical approach in evaluating the response of a narrow-body fuselage subjected to an in-cabin explosion, with the help of JWL model. The main objective is to show numerical approaches able to help in designing and testing new protections.

In Section 3.1 we present the geometry of the fuselage, while in Section 3.2 we show the material model for aluminium alloys. Section 3.3 deals with the development of a new concept of passive protections in Kevlar fabric and polyurethane foam. A numerical strategy capable of recreating stress distributions on fuselage-type structures under gravity and pressurization loads, at cruising altitude, is presented in Sec. 3.4 and it will be used as the

background of the following simulations of an internal blast. Finally, Section 3.5 is divided into a first part where the effects of an explosion are studied and a second one where the reliability of the new design for protections is evaluated.

3.1 Fuselage geometry

A fuselage is the central body of an aircraft and it is designed for holding both passengers and cargo. Fuselages are usually built in two or more sections, depending on the size of the whole aircraft. As already pointed out, these structural parts are demanded to resist to a complex combination of loads that result in a multi-axial stress distribution. The structural integrity of fuselages is a critical issue due to the need of building lightweight airframes to reduce the high cost associated with fuel consumption in civilian flights.

Fuselages are of three different designs: i) truss, ii) monocoque and iii) semimonocoque types.

The truss design, commonly belonging to the first generation of aircrafts, consists of steel tubes, welded together, organized in a framework in order to achieve a high global stiffness. The monocoque type, i.e., *unit shell*, relies mostly on the strength of the skin or covering to carry loads. It consists of formers, frame assemblies and bulkheads to provide shape with the skin carrying the primary stresses. The main problem related to such a design is keeping the weight within allowable limits while assuring enough strength.

The more recent semimonocoque is a modification of the latter design in order to overcome its drawbacks. Once again, it consists of frame assemblies, bulkheads and formers but the additional reinforcements, called longerons, make the structure lightweighted and stiffer. Semimonocoque fuselages are usually made of aluminium alloys, although steel and titanium are used in high temperature regions.

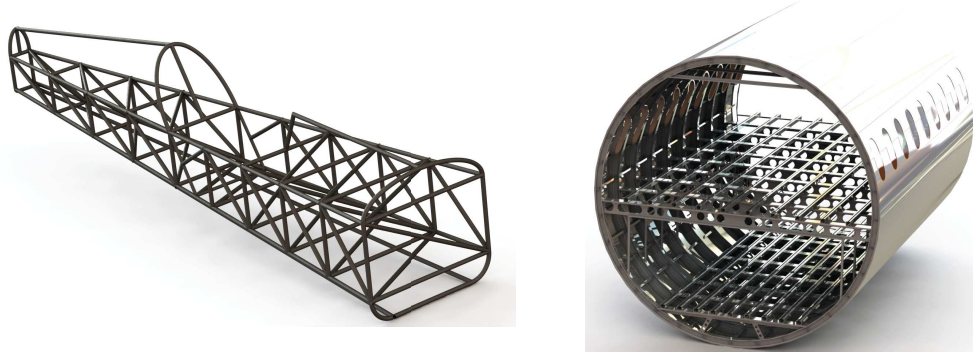


Figure 3.2: Truss design (*left*) and semimonocoque design (*right*).

In the simulations presented here, we adopt a simplified semimonocoque design shown in Figure 3.3. The fuselage is 4 meters long and has a diameter of 3 meters. Longerons and bulkheads are shown in Figure 3.4, together with their section.

The floor consists of plates with a thickness of 0.8 mm, while the skin is assumed to have a 2 mm thickness. The continuity between different parts is assured with the definition of tied contact pairs. Frames along the floor, longerons and bulkheads are made of Al2024-T3, while the skin is made of Al7075-T6.

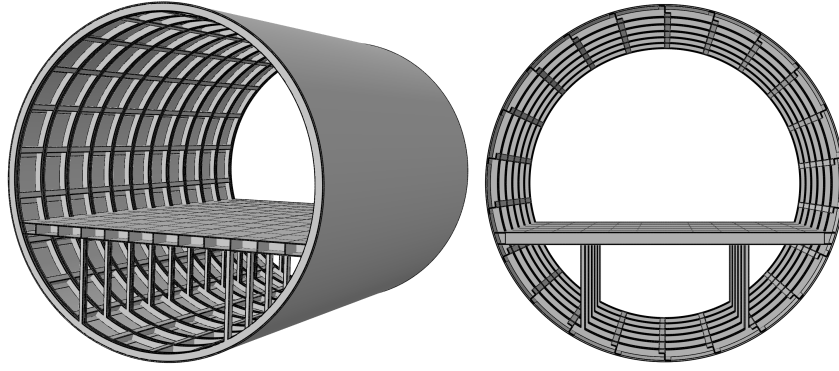


Figure 3.3: The geometrical model.

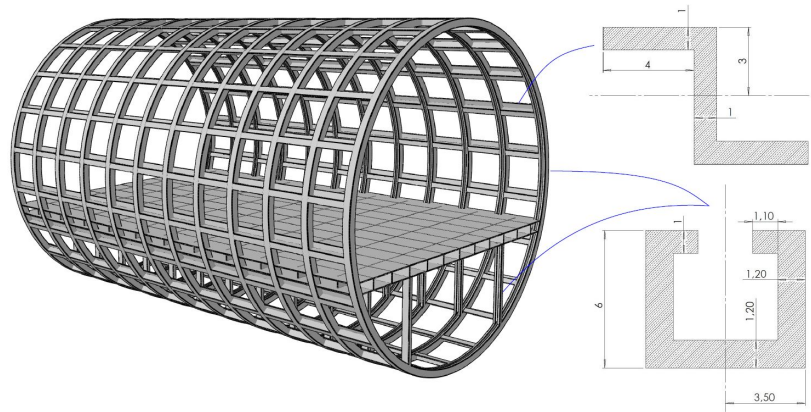


Figure 3.4: Longerons and frame assemblies and related sections (measures are in millimeters).

3.2 Modelling Aluminium alloys behaviour

The small size of the problem of an explosion inside a fuselage, if compared to the previous case study of the Pantheon in Rome, requires a special care. Indeed, the heat released by a blast would affect the behaviour of aluminium alloys, thus the global structural behaviour. The evaluation of this dependence is mandatory and this is why we have considered both the elastic parameters and thermal properties of the material, as shown in Table 3.2.

Table 3.2: Material elastic and thermal parameters.

Material	ρ [kg/m ³]	E [GPa]	ν	Specific heat [J/(kg K)]	Thermal conductivity [W/(m K)]
Al7075-T6	2810	71.7	0.33	848	130
Al2024-T3	2770	73.1	0.33	863	121

Aluminium alloys as ductile materials need to be modeled taking into account for the so-called first-order metal-working effects such as the influence of plastic deformation, strain rate and temperature.

Many theoretical and empirical methods are available in FEM softwares and, among them, the empirical one proposed by Johnson and Cook [Johnson and Cook, 1983] is widely used in the aviation field and capable of considering strain hardening, strain rate and thermal softening effects on material behaviour and fracture.

Johnson-Cook material model is used for modelling the plasticity and damage behaviour while a elastic constitutive law is assumed when plastic strain is zero.

Plasticity behaviour Johnson-Cook plasticity is a particular type of von Mises plasticity model with analytical form of hardening law and rate dependence.

Johnson-Cook hardening. The model expresses the static yield stress, σ^0 , as a function of a nondimensional temperature, θ , and plastic small strain, ε_{pl} , namely as

$$\sigma^0 = [A + B (\varepsilon_{pl})^n] (1 - \theta^m), \quad (3.1)$$

where A is the yield strength, while B and n are, respectively, the hardening constant and an exponent; m is the thermal softening exponent; and θ is defined as

$$\theta = \begin{cases} 0 & \text{if } T < T_{tr} \\ (T - T_{tr}) / (T_m - T_{tr}) & \text{if } T_{tr} \leq T \leq T_m \\ 1 & \text{if } T > T_m \end{cases} \quad (3.2)$$

where T is the actual temperature, T_m is the melting temperature and T_{tr} is the transition temperature defined as the one at which the yield stress is independent of temperature. If $T > T_m$, the definition of θ allows us to have a zero resistance, i.e., $\sigma^0 = 0$, and to remove the hardening memory, imposing $\varepsilon_{pl} = 0$, since the material would be melted.

However, Johnson-Cook plasticity model is not completely appropriate in analyses where temperatures are higher than the material recrystallization temperature, due to the presence of microstructural changes such as a decreasing density of dislocations and an increasing grain size.

Johson-Cook strain rate dependence. The yield stress at non-zero strain is expressed by

$$\sigma = \sigma^0(\varepsilon_{pl}, T) r(\dot{\varepsilon}_{pl}), \quad (3.3)$$

and the equivalent plastic strain rate is

$$\dot{\varepsilon}_{pl} = \dot{\varepsilon}_0 \left(\frac{r - 1}{C} \right), \quad (3.4)$$

where $\dot{\varepsilon}_0$ is the reference strain rate; C is a material parameter; r is the ratio of the yield stress at nonzero strain rate to the static yield stress. Finally, we can evaluate the yield stress as

$$\sigma = [A + B (\varepsilon_{pl})^n] \left[1 + C \ln \left(\frac{\dot{\varepsilon}_{pl}}{\dot{\varepsilon}_0} \right) \right] (1 - \theta^m). \quad (3.5)$$

Damage behaviour An additional dynamic failure model is required when high-strain deformations will cause the failure of the material. It is based on the value of the equivalent plastic strain and it defines a damage parameter δ such as

$$\delta = \sum_{k=0}^{k=K} \frac{\Delta \varepsilon_{pl,k}}{\varepsilon_{pl,f}}, \quad (3.6)$$

where $\Delta\varepsilon_{\text{pl}}$ is an increment of the equivalent plastic strain, $\varepsilon_{\text{pl},f}$ is the strain at failure and K is the total number of increments in the analysis.

The strain at failure is evaluated as

$$\varepsilon_{\text{pl},f} = \left[d_1 + d_2 \exp\left(d_3 \frac{p}{q}\right) \right] \left[1 + d_4 \ln\left(\frac{\dot{\varepsilon}_{\text{pl}}}{\dot{\varepsilon}_0}\right) \right] (1 + d_5 \theta), \quad (3.7)$$

where p is the pressure stress, q is the Mises stress and d_i , $i = 1, \dots, 5$, are the failure parameters experimentally derived.

The usefulness of the empirical Johnson-Cook model relies on numerous experimental investigations of puncturing and, more in general, ballistic problems accomplished in order to evaluate the materials parameters. Tables 3.3 and 3.4 show the parameters of Johnson-Cook model for Al7075-T6 and Al2024-T3.

Table 3.3: Material parameters for Johnson-Cook plasticity [Corona and Orient, 2014] and [Kray, 2003].

Material	A [MPa]	B [MPa]	n	m	C	T_{tr} [K]	T_{m} [K]	$\dot{\varepsilon}_0$ [1/s]
Al7075-T6	517	405	0.41	1.1	0.0075	292.2	750	0.000161
Al2024-T3	265	426	0.34	1.0	0.015	293.2	775	1.0

Table 3.4: Material parameters for Johnson-Cook damage [Corona and Orient, 2014] and [Kray, 2003].

Material	d_1	d_2	d_3	d_4	d_5	Elongation at break [%]
Al7075-T6	0.005	0.34	-1.5	-0.039	8	22
Al2024-T3	0.13	0.13	-1.5	0.011	0	18

3.3 Passive protections. Design and material behaviour

Laminated composites are often used in aerospace and aviation fields due to the fact that are lightweight materials with an excellent in-plane performance in terms of strength. Aramidic fibres are usually adopted in aerospace engineering and ballistic applications thanks to their high degree of toughness. Indeed, differently from glass and carbon fibres, when Kevlar fibres break, they fail by a series of energy absorbing fibril failures and not by a brittle cracking, where fibrils are sub-fibres that compose each aramidic fibre.

Kevlar fabric properties make it an excellent candidate to be used in designing blast-protections in the case of a fuselage. A new concept of cabin load device (CLD) has been designed to mitigate the blast waves that arise from an explosion. The concept relies on the extension of the previous described ULDs: a Kevlar fabric is inserted inside the passenger cabin and tied with longerons and bulkheads, upholstering the interiors. During an explosion, the CLD, going under large deformations, has the ability of protecting the skin, as it will be discussed in Section 3.5. In a sense, the contribution of a Kevlar-made CLD can be regarded as an increment of the viscosity of the medium through which blast waves propagate.

An improvement of the CLD protection can be obtained using foam in cavities of the

fuselage such as the spaces between longerons and the interior of the floor. Foams, as energy absorbing materials, allow to mitigate the stresses that arise during an impact, being characterised, at the same time, by low densities. These properties make a polyurethane foam another good candidate for protecting aircrafts, also thanks to its good insulation properties that make it exploitable for upholstering the skin.

Kevlar Kevlar fabric is assumed to have an orthotropic elastic behaviour up to failure, with a plane stress formulation.

An orthotropic material is a material with mechanical properties differing along three mutually-orthogonal axes of material symmetry. Let us denote the Cauchy stress tensor by σ , the total elastic strain by ε and the fourth-order elasticity tensor with D , therefore the elastic relation between stress and strain is

$$\sigma = D \varepsilon \quad (3.8)$$

in linear elastic setting. Linear elastic relations in an orthotropic material can be explicitly written as functions of Young moduli E_i , Poisson ratios ν_{ij} and shear moduli G_{ij} , where $i, j = 1, 2, 3$ are the principal axes of the material, namely as (we adopt here Voigt's notation with $\gamma_{ij} = 2\varepsilon_{ij}$)

$$\begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \gamma_{23} \\ \gamma_{13} \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} 1/E_1 & -\nu_{21}/E_1 & -\nu_{31}/E_1 & 0 & 0 & 0 \\ -\nu_{12}/E_1 & 1/E_2 & -\nu_{32}/E_2 & 0 & 0 & 0 \\ -\nu_{13}/E_1 & -\nu_{23}/E_2 & 1/E_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/G_{23} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/G_{13} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/G_{12} \end{bmatrix} \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{13} \\ \sigma_{12} \end{Bmatrix} \quad (3.9)$$

Plane stress conditions, having denoted the surface (1,2) as the surface of plane stress, correspond to $\sigma_{13} = \sigma_{23} = \sigma_{33} = 0$. Therefore, eq. (3.9) becomes

$$\begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} 1/E_1 & -\nu_{21}/E_1 & 0 \\ -\nu_{12}/E_1 & 1/E_2 & 0 \\ 0 & 0 & 1/G_{12} \end{bmatrix} \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{Bmatrix} \quad (3.10)$$

and

$$\varepsilon_{33} = -\frac{\nu_{13}}{E_1} \sigma_{11} - \frac{\nu_{23}}{E_2} \sigma_{22} \neq 0 \quad (3.11)$$

In Abaqus an orthotropic plane stress elasticity, the lamina type [DS, 2016] has also the additional shear moduli G_{13} and G_{23} , included for taking into account for transverse shear

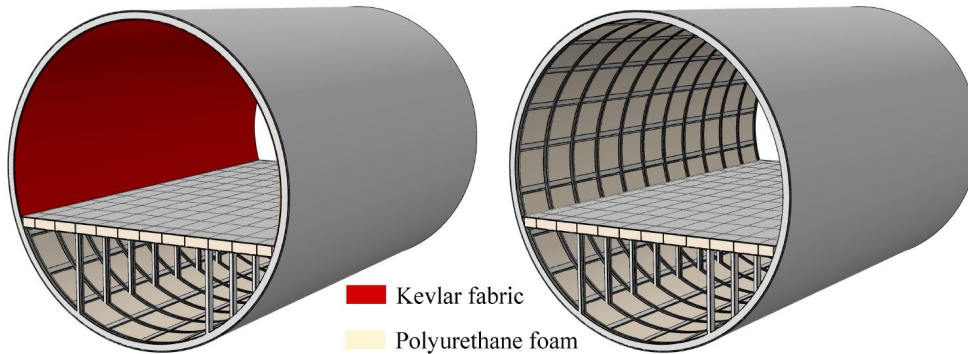


Figure 3.5: CLD and covering in polyurethane foam.

deformations in a shell. The elastic parameters, shown in Table 3.5, are derived from a woven fabric Kevlar-29 with a polyvinyl-butyril-phenolic matrix, with 18% of fibres volume fraction.

Table 3.5: Material parameters for Kevlar fabric [Zhu et al., 1992] and [Cheeseman and Bogetti, 2003].

ρ	E_1	E_2	G_{12}	G_{13}	G_{23}	ν_{12}
[kg/m ³]	[GPa]	[GPa]	[GPa]	[GPa]	[GPa]	
1230	18.5	18.5	0.77	5.43	5.43	0.25

The strength of Kevlar is modeled by the Hashin failure criteria for fibre-reinforced materials [Hashin, 1980]. The model allows to take into account for four different damage mechanisms, namely fibre tension, fibre compression, matrix tension and matrix compression. The initiation criteria relies on a set of damage variables, each one corresponding to a damage mechanism

$$d_f^t = \left(\frac{\widehat{\sigma}_{11}}{X^t} \right)^2 + \left(\frac{\widehat{\sigma}_{12}}{S^l} \right)^2, \quad (3.12)$$

$$d_f^c = \left(\frac{\widehat{\sigma}_{11}}{X^c} \right)^2, \quad (3.13)$$

$$d_m^t = \left(\frac{\widehat{\sigma}_{22}}{Y^t} \right)^2 + \left(\frac{\widehat{\sigma}_{12}}{S^l} \right)^2, \quad (3.14)$$

$$d_m^c = \left(\frac{\widehat{\sigma}_{22}}{2S^t} \right)^2 + \left[\left(\frac{Y^c}{2S^t} \right)^2 - 1 \right] \frac{\widehat{\sigma}_{22}}{Y^c} + \left(\frac{\widehat{\sigma}_{12}}{S^l} \right)^2, \quad (3.15)$$

where X^t and X^c are, respectively, the longitudinal tensile and compressive strengths; Y^t and Y^c the transverse tensile and compressive strengths; S^l and S^t the in-plane and transversal shear strengths; $\widehat{\sigma}_{11}$, $\widehat{\sigma}_{22}$ and $\widehat{\sigma}_{12}$ are the components of the effective stress tensor computed as

$$\widehat{\sigma} = M\sigma, \quad (3.16)$$

where σ is the Cauchy stress tensor and M is the damage operator, that is assumed to be the identity matrix before any damage initiation occurs, computed as

$$M = \begin{bmatrix} 1/(1-d_f) & 0 & 0 \\ 0 & 1/(1-d_m) & 0 \\ 0 & 0 & 1/(1-d_s) \end{bmatrix} \quad (3.17)$$

The parameters d_f , d_m and d_s are the damage variables that characterise fibre, matrix and shear damage modes, namely

$$d_f = \begin{cases} d_f^t & \text{if } \widehat{\sigma}_{11} \geq 0 \\ d_f^c & \text{if } \widehat{\sigma}_{11} < 0 \end{cases} \quad (3.18)$$

$$d_m = \begin{cases} d_m^t & \text{if } \widehat{\sigma}_{22} \geq 0 \\ d_m^c & \text{if } \widehat{\sigma}_{22} < 0 \end{cases} \quad (3.19)$$

$$d_s = 1 - (1-d_f^t)(1-d_f^c)(1-d_m^t)(1-d_m^c) \quad (3.20)$$

In Table 3.6 the parameters of Hashin's failure criterion are shown, together with the fracture energy G_f [Wang et al., 1995], assumed to be equally distributed in the longitudinal,

transverse and shear directions.

Table 3.6: Material parameters for Hashin’s failure criterion [Cheeseman and Bogetti, 2003].

X^t	X^c	Y^t	Y^c	S^t	S^l	G_f
[MPa]	[MPa]	[MPa]	[MPa]	[MPa]	[MPa]	[kJ/m ²]
555	555	34.5	34.5	898	77	1.64

Polyurethane foam Polyurethane foam is modeled as a low density foam material in ABAQUS [DS, 2016] with a highly strain-rate sensitive behaviour. The material is assumed to be visco-hyperelastic, with a null Poisson’s ratio and a density of 240 kg/m³. The mechanical behaviour is characterised by tabular data derived from uniaxial tension and compression tests, at different strain rates, from [Goods et al., 1997]. An additional viscous regularization technique is used in order to avoid unphysical jumps in stress caused by sudden changes in the deformation rate.

Failure in compression is neglected, while a cut-off value of 3.8 MPa of the maximum principal tensile stress is assumed, independently of the strain rate.

3.4 Numerical procedure

Simulations of an in-cabin explosion are based on the same approach defined in Ch. 2: the fuselage is immersed inside a volume of air, the explosion is modeled by JWL model and a Coupled-Eulerian-Lagrangian analysis is run.

In order to recreate the stress state at which a fuselage is subjected at a cruising altitude before a bombing event, two analyses are used.

- First, gravity and a distributed load, acting on the bottom half of the skin, equivalent to lift are applied incrementally through a quasistatic analysis. Figure 3.6 shows the applied symmetry boundary conditions, making clear the possibility of having a rigid body motion as soon as the equilibrium between vertical loads would be broken.
- Then, the results obtained are used as initial state of a subsequent simulation where pressurization load is applied through a gradient of pressure between internal and external volumes of air, using a CEL approach.

The aircraft is assumed to be at an altitude of 10000 m, while the equivalent effective cabin altitude is 2000 m.

Differently from the previous case of the Pantheon in Rome, the evaluation of the thermal sensitive behaviour of Al-alloys is mandatory and it results in the need of running a Coupled-Temperature-Displacement (CTD) analysis. In this way, the equilibrium state achieved at the end of the second analysis is used as the initial state of a CEL-CTD explicit analysis for studying the effects of blast waves on the structure. The state equations for explosive and air are those already shown in Table 1.1.

Once again, mesh sensitivity analyses are required for both the types of analyses and both the fluid and structure domains.

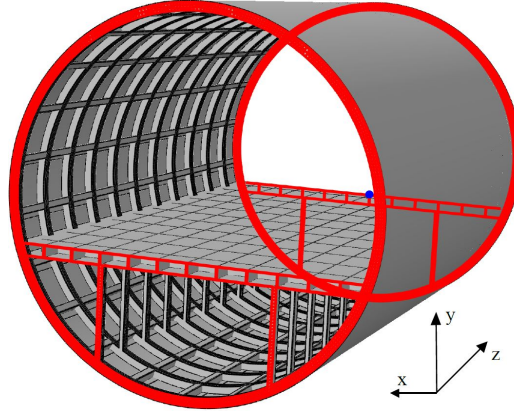


Figure 3.6: Symmetry boundary conditions (*red*) along z axis applied in every analysis and no x -translation boundary condition (*blue*) applied in the CEL-CTD analysis.

3.4.1 Mesh sensitivity of the fluid domain

The model of air domain is shown in Figure 3.7. The domain has been decomposed into 1.156×10^6 8-nodes hexaedral volume elements, for 5.78×10^6 DOF.

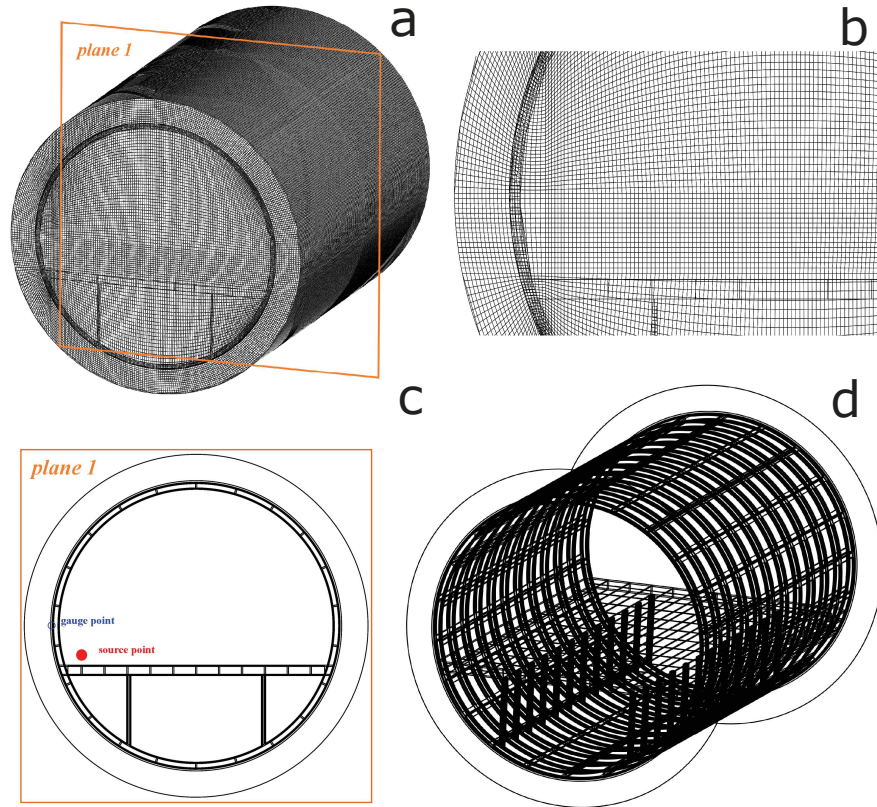


Figure 3.7: The air domain: a) complete mesh M5, b) detail of the mesh M5, c) location of the gauge and source points, d) overall view.

Boundary conditions on the Eulerian domain consist on nonreflecting outflow conditions applied to all the boundary surfaces. The cylindrical air domain is substructured into two different volumes: the one inside the fuselage plus an exterior air volum corresponding to a cylinder with a diameter of 4 m.

The choice of the fineness of the mesh has been done upon a convergence analysis and six different meshes for the air domain have been considered: from the coarsest, M1, with 0.58×10^6 DOF, to the finest one M6, with 10.54×10^6 DOF. For each mesh, we have computed the blast overpressure due to an explosive charge placed in the middle of the fuselage length, identified by the plane 1, in correspondence of the gauge point, highlighted in Figure 3.7.

The time histories of the overpressure for each mesh are plotted in Figures 3.8 and 3.9 (the curves have been normalized with respect to the highest value).

In Figure 3.10 we show the relative error evaluated for meshes M1, ..., M5 with respect to mesh M6; such an error, in percent, is computed as

$$err = 100 \left| \frac{p_{Mi} - p_{M6}}{p_{M6}} \right|, \quad (3.21)$$

where p_{Mi} is the peak overpressure, in the gauge point evaluated for grid Mi, while p_{M6} is the same for the reference mesh M6. After the evaluation of the results, we have selected the mesh M5, with 5.72×10^5 DOF.

3.4.2 Mesh sensitivity of the structural domain

Once the grid for the fluid domain has been selected, we have performed a convergence analysis for corroborating the choice of the fuselage mesh, composed of four-nodes thermally coupled finite shell elements, supported by the explicit solver of ABAQUS.

The convergence analysis, performed in order to obtain a reliable degree of accuracy with an acceptable size of the whole numerical model (air domain plus structure) is carried out by two quasistatic analyses. As already pointed out, firstly gravity and lift loads are considered, secondly the equilibrium state is exported into a second analysis where also the fluid domain is considered, discretized with the selected mesh M5, and the pressurization load is applied.

The convergence analysis has been done using explicit schemes for nine different structural meshes MSi, $i = 1, \dots, 9$, whose characteristic are given in Table 3.7.

Table 3.7: Characteristics of the studied structural meshes.

Mesh	Average element size [cm]	Number of elements	Number of DOF [mln]
MS1	10	9191	7.23×10^{-2}
MS2	8	12520	1.01×10^{-1}
MS3	6	21329	1.73×10^{-1}
MS4	5	27971	2.24×10^{-1}
MS5	4	40813	3.22×10^{-1}
MS6	3	74611	5.87×10^{-1}
MS7	2	159764	1.26
MS8	1.5	266811	2.10
MS9	1	5597159	4.7

We have monitored the values of strain at a point belonging to the skin, as shown in Figure 3.11. The need for evaluating the degree of accuracy of the mesh in terms of strains instead

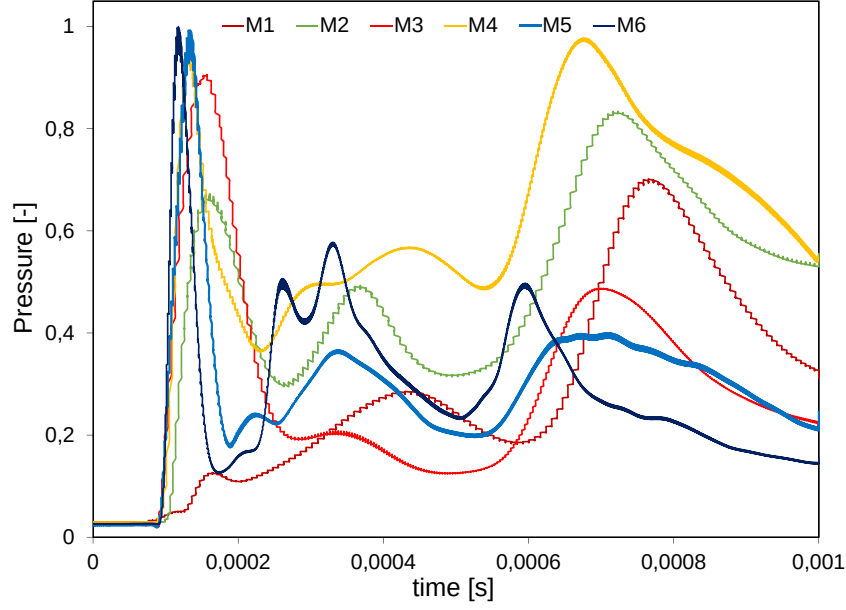


Figure 3.8: Time history of the overpressure for the gauge point (time filtered).

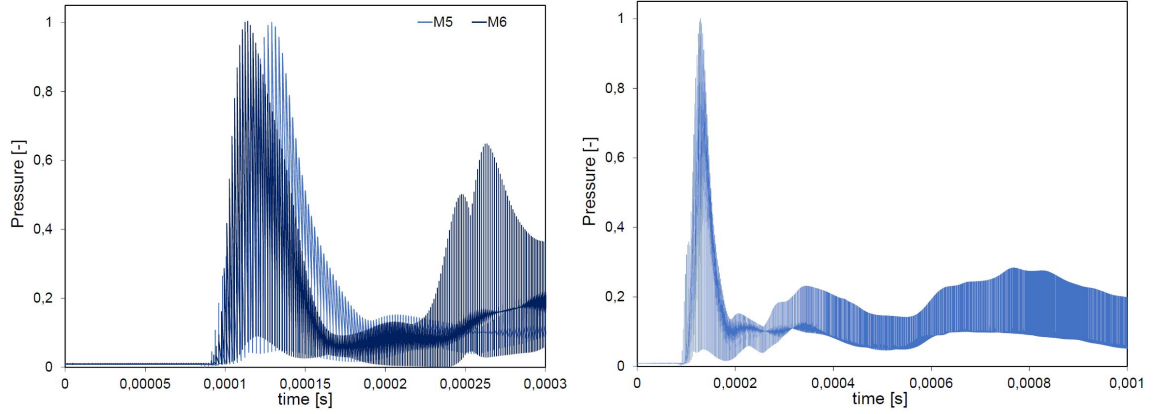


Figure 3.9: Detail of time history of the overpressure, unfiltered, for M5 and M6 (left), and time history of the overpressure for M6, unfiltered (right).

of displacements, as we have done in the case of the Pantheon, is a consequence of rigid body motions for the structure. Indeed, the displacement of any point of the system consists of a part that belongs to the rigid motion and another caused by the strain.

The convergence has been evaluated like for the air domain, calculating for each mesh $MS_i=1, \dots, 8$ the errors $\Delta\varepsilon$ of the strain magnitude ε , relatively to the same quantity ε^r , calculated for the reference mesh MS_9 , the finest one:

$$\Delta\varepsilon = \left| \frac{\varepsilon - \varepsilon^r}{\varepsilon^r} \right|, \quad \forall MS = 1, \dots, 10. \quad (3.22)$$

In Figure 3.11 we show the diagram of $\Delta\varepsilon$ as a function of the DOF. After the evaluation of the results, we have selected the mesh MS_8 , for the computations, since we have estimated that the convergence has been reached. A detail of the structural mesh MS_8 is shown in Fig. 3.12.

Parallel computing is used to decrease significantly the duration of the analyses. All the simulations have been performed using a 24-cores workstation. The entire model is, thus, di-

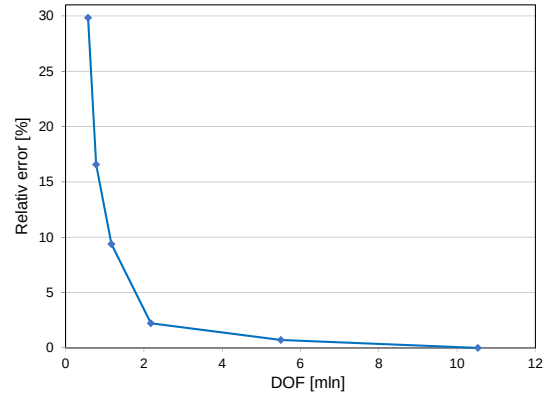


Figure 3.10: Relative error, eq. (3.21), for the gauge point as function of the number of DOF.

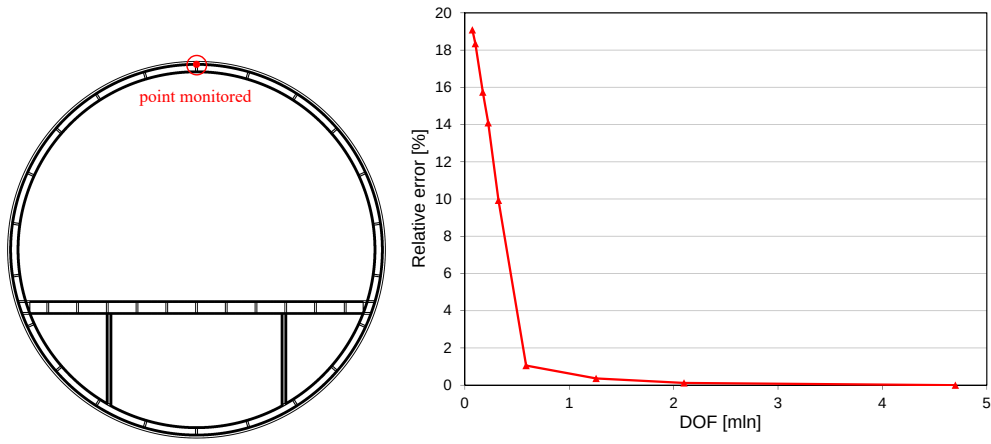


Figure 3.11: Relative error $\Delta\epsilon$ as function of the number of DOF, with respect to a point of the skin.

vided into 24 geometric domains, taking advantage of the 24 processors of the machine.

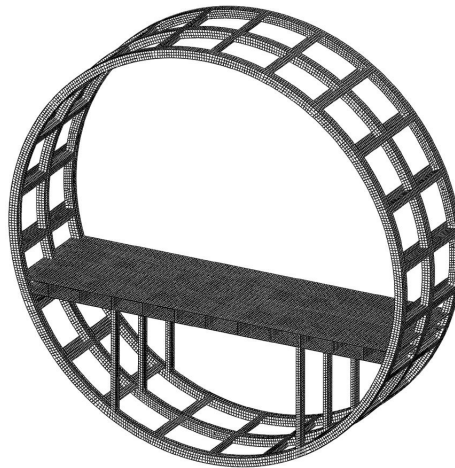


Figure 3.12: Detail of the structural mesh MS8.

3.5 Numerical simulations

The simulation of a blast, produced by the explosion of 1 kg equivalent TNT charge inside the cabin, has been studied through a CEL-CTD analysis, taking into account the nonlinearities of a large strain incremental formulation. From these results a passenger cabin protection, CLD, has been designed, as already shown in Fig. 3.5. Here, we find it reasonable to discuss the complex nature of the following simulations: indeed, the small size of the problems, thus of the elements, together with the complex model of the materials and the coupled approaches result in a long time required for running each simulation. The stable time step of the explicit schemes, mostly afflicted by the small elements size (see [DS, 2016]) varies from 2.9×10^{-7} s to 5.7×10^{-11} s, therefore the computational time required for running a short time period (3.5 milliseconds) analysis is around 35 hours.

3.5.1 In-cabin explosion simulation

Some images of the simulation, at different times, are represented in Figs 3.13 and 3.14. Since we have already pointed out the pattern of strong reflections in circular and dome-like geometries in Ch. 1 and 2, we neglect here the study on the evolution of the pressure inside the fuselage.

Rather more interesting is the evolution of damage upon the structure. The blast event is really faster with respect to that one inside of a large structure and the response of the fuselage can be easily seen in a really short time period, such as 3 milliseconds. Figure 3.13 shows clearly the issue related to an in-cabin explosion of an aircraft: a small amount of charge, capable just of piercing the skin and detonated at an altitude sufficiently high, can be the origin of a disaster. Indeed, as soon as a first hole is produced, the following decompression will result in a continuously acting force that will make the hole become larger, opening the skin and causing the global failure.

Moreover, from Figure 3.14 we can assess that the skin is not the only vulnerable element; indeed, the overpressure, responsible for destroying the skin in the immediate proximity of the charge, punctures the floor, causing another hole in the bottom half of the skin by venting.

It is upon such considerations that the new design of CLD has been done.

3.5.2 Assessing the reliability of the protective CLD

Some images of the simulation with the same quantity of explosive are presented in Fig. 3.15, where the Kevlar fabric is not displayed for a better visualization. The CLD is able to perform an excellent mitigation of blast waves, thanks to both the materials used and the locations in which the protections are inserted.

The results are extremely clear: skin presents a minimum damage from the internal explosion and floor is only partially damaged, avoiding the failure of the bottom half skin due to venting.

The advantages of using a Kevlar fabric are clearly shown in Fig. 3.16. Indeed, such a fabric behaves like a second skin, much less stiff and tougher than the real skin of Al7075-T6, that is capable of going under large deformations reducing the magnitude of the impinging blast waves.

Figure 3.17 shows the comparison between the results of an in-cabin explosion without any protecting devices and those where a CLD is considered. Although the modest puncturing of the skin with CLD, the present results make clear the powerfulness of a numerical approach in evaluating the response of a fuselage subjected to a bombing event and, thus, investigate different types of passive protections.

3.5.3 Remarks about numerical procedure

Aircrafts are characterised by a complex combination of loads and their design usually needs to satisfy conflicting requirements: strength, stiffness and lightness are the key points with which a fuselage is conceived. The protection against terroristic bombing attacks in an aircraft makes the concept of their design even more crucial.

Few works on experimental investigation of the response of fuselages to blast waves can be found in literature. Numerical simulations in this field are usually based on slightly unrealistic hypotheses, such as the application of pressurization load as a distributed load acting on the skin independently from the pressures of the surrounding volumes of air; hypothesis investigated in the Appendix (3.5.3).

The simulations presented here are hence interesting and allow to capture in detail the fast-dynamics of the response of a fuselage to an internal explosion for the first time. The above presented approach can be a rather successful way to be followed in order to minimize the effects of terrorist attacks in aviation. Indeed, a numerical procedure used for evaluating functioning protections, at least in their first design, allows for saving both time and costs. However the existence of a difference between the numerical results of a simulation and reality, as it stands in studying problems so complex and delicate, requires a successive step based upon reliable experimental tests.

Furthermore, the usefulness of a numerical approach of the kind presented can be a support for the definition of new prevention procedures, resting on the evaluation of the vulnerability of existing aircrafts. Indeed, once the limit of a current commercial airplane has been numerically determined, active measures can be applied for preventing explosive devices larger than the minimum estimated weight to be brought inside the cabin.

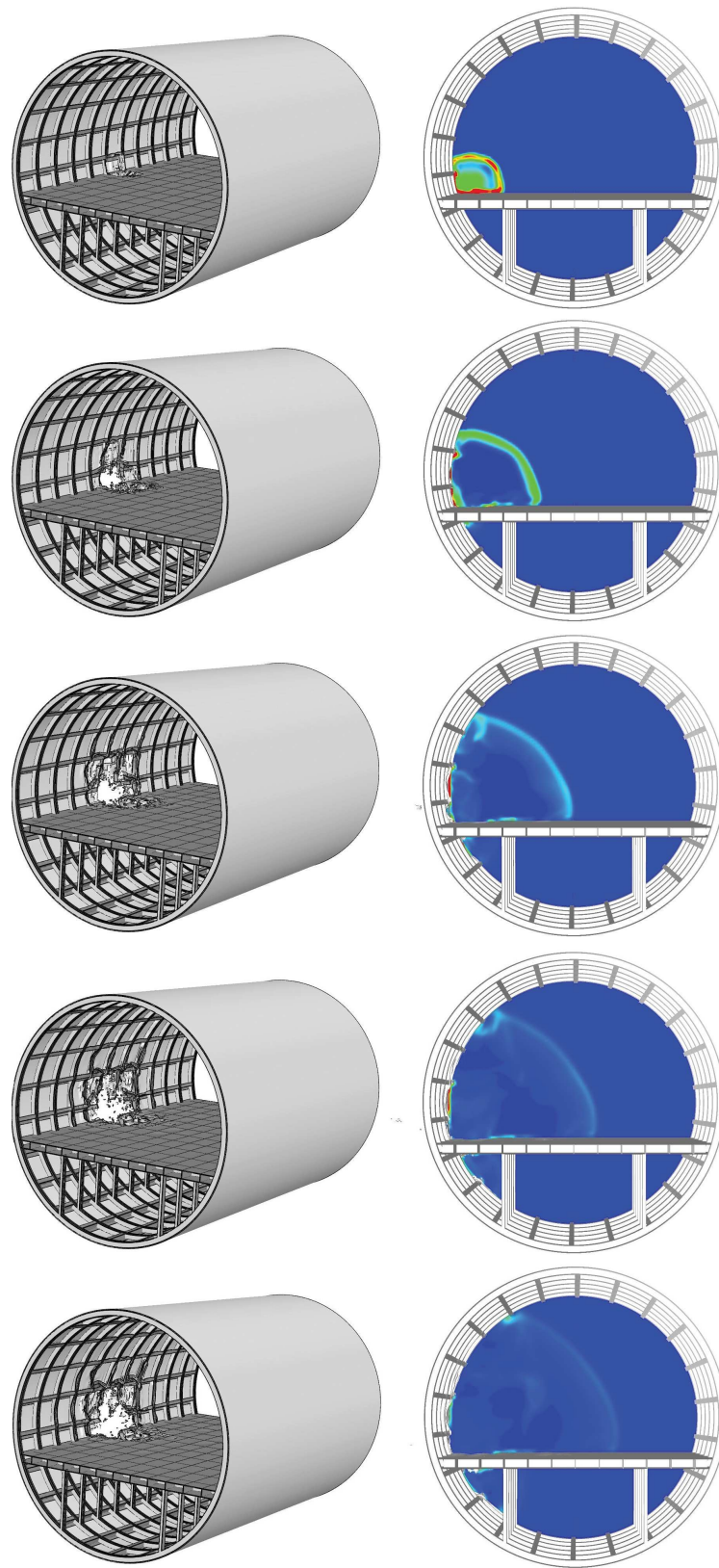


Figure 3.13: Simulation of an in-cabin explosion, global view and blast waves.

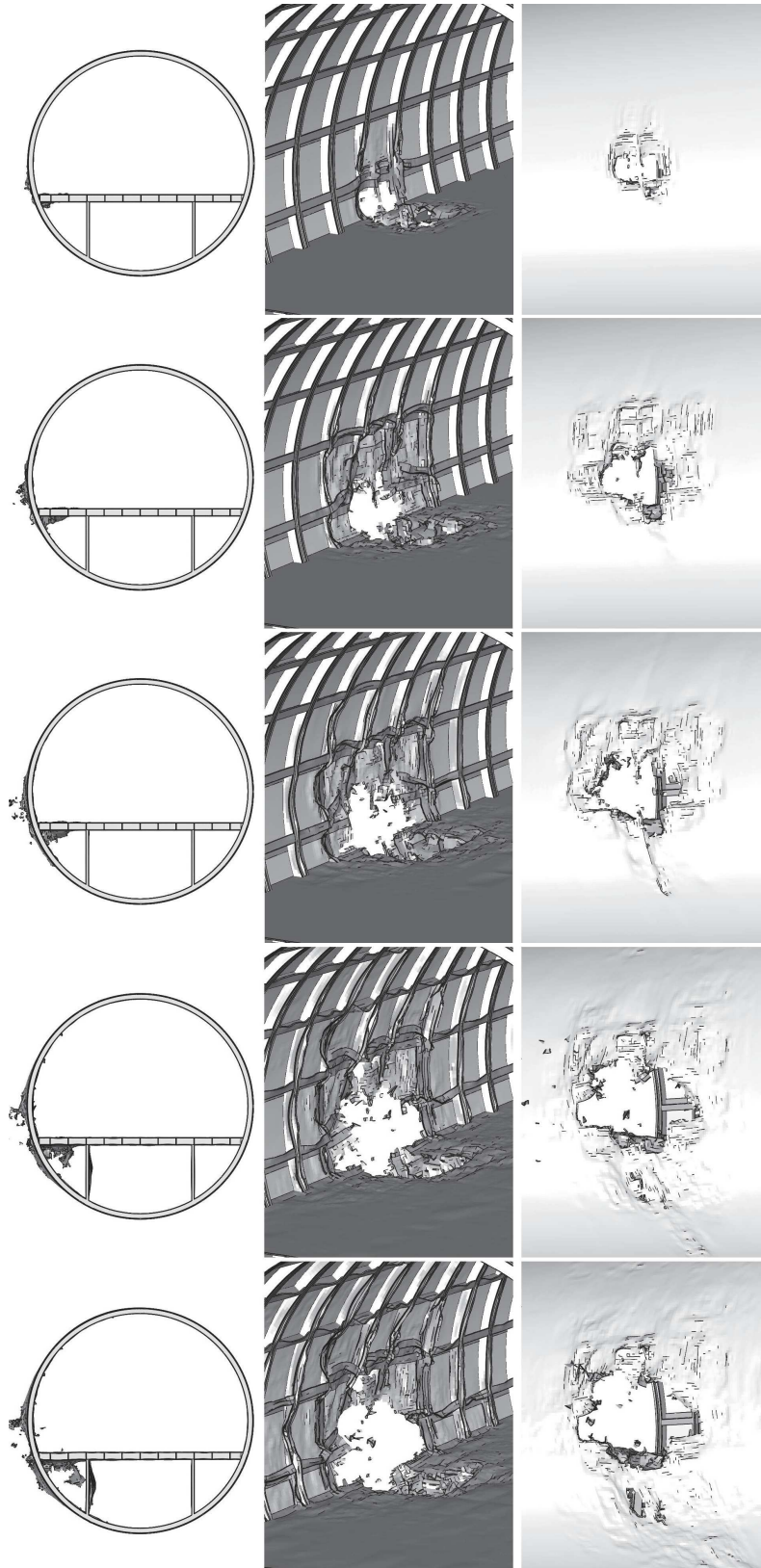


Figure 3.14: Simulation of an in-cabin explosion.

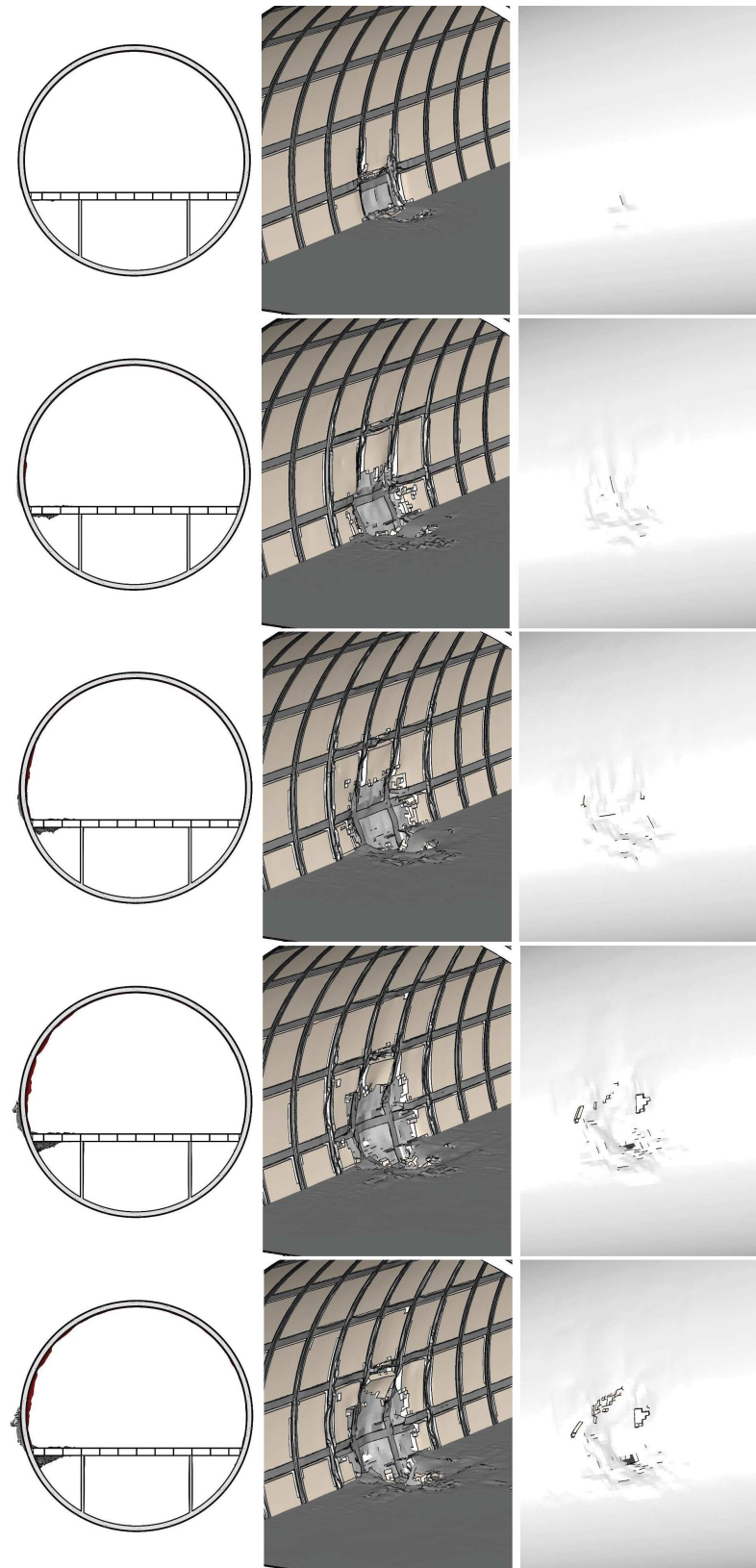


Figure 3.15: Simulation of an in-cabin explosion with CLD.

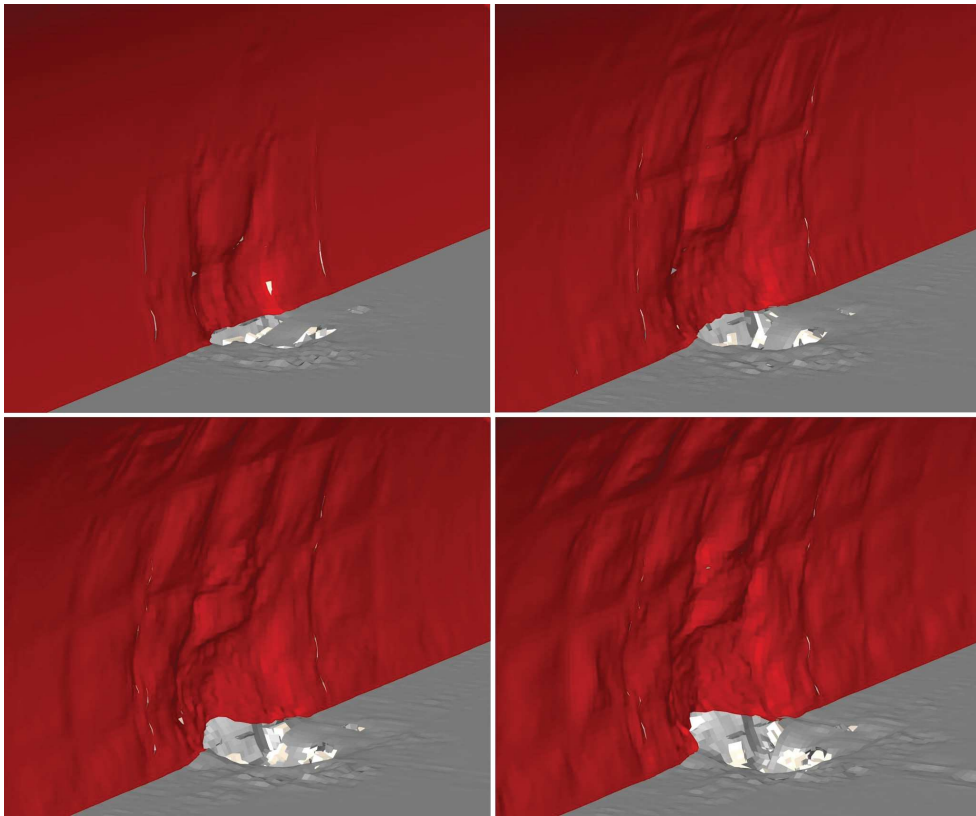


Figure 3.16: Kevlar fabric under large deformations.

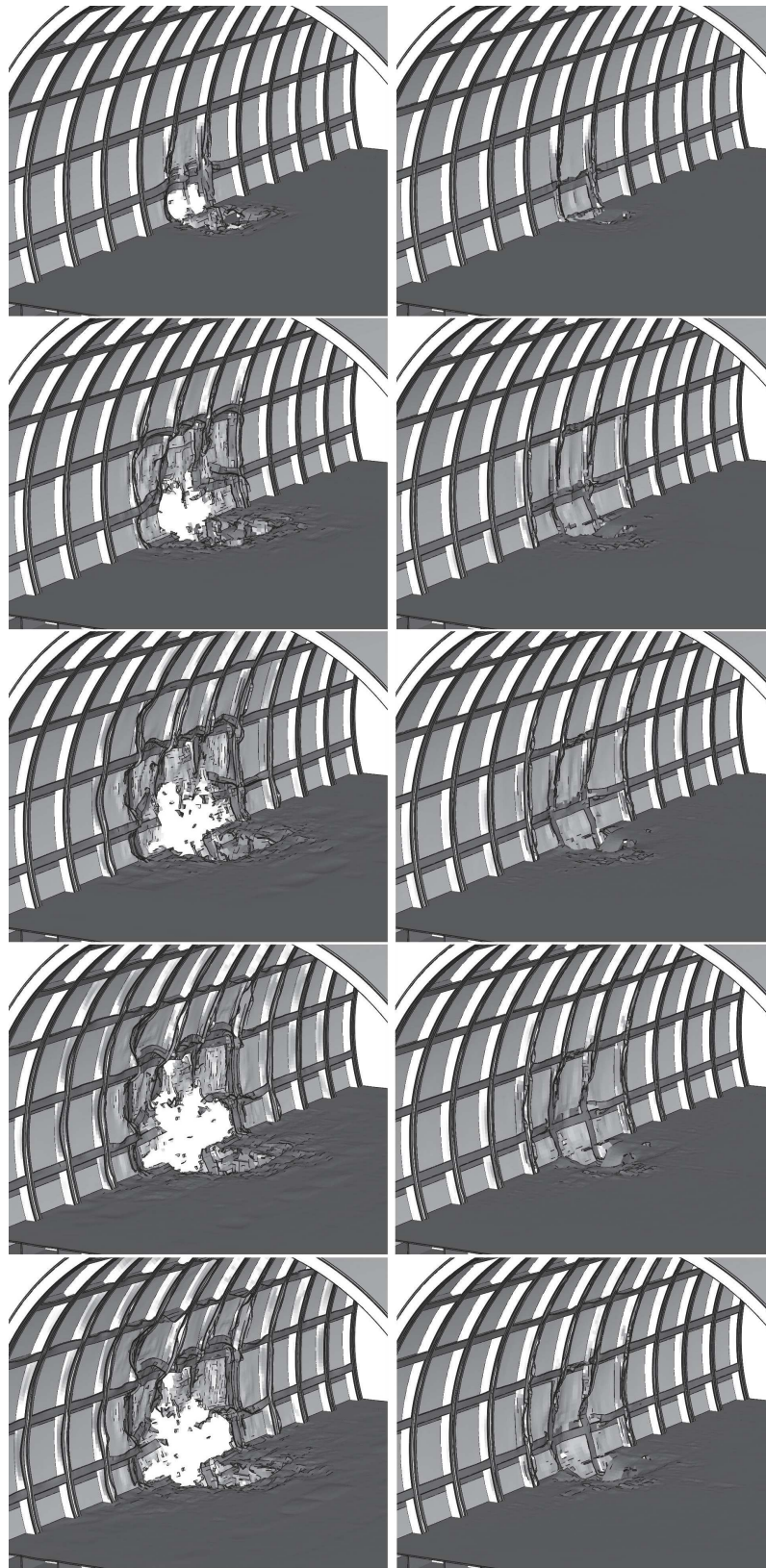


Figure 3.17: Comparison between the reference design (*left*) and the CLD (*right*), foam and Kevlar fabric are not displayed.

Additional remarks

A comparison of three different methods, i.e. JWL, TM5-1300 and CONWEP, used for modeling blast actions has been done. The case study of a barrel vault allows us to deeply understand the differences between them: CONWEP, widely used in the field of simulating the effects of blast waves on buildings, is found to be affected by strong simplifications that make the model itself unsuitable when complex geometries are considered. We propose a feasible correction of the overpressure values estimated by CONWEP although its intrinsic lack of modelling reflected shock waves can lead to a wrong understanding of the real dynamics.

CONWEP can hence be used in a first part of the investigations of the effects of explosions and when extremely large structures are studied, [Vannucci et al., 2017a] and [Vannucci et al., 2017b].

We have used JWL model, the most accurate among the three, for simulating the effects of explosions inside Pantheon in Rome and an aircraft fuselage, [Masi et al., 2017]. The aim is showing the powerfulness of such a numerical approach in studying the fast-dynamics of a blast phenomenon. The method is hence independent from the field in which it is applied and allows for a real understanding of the complex nature of blast actions and especially to derive designs of passive protections that can be used for mitigating a bombing event.

The studies on the Pantheon in Rome and on the fuselage of an aircraft are just two emblematic examples that are presented in order to make the reader aware of the need of changing the old paradigm of designing structures. Terrorist threats force engineers to find a new way of conceiving buildings, aircrafts, etc.: it is no more sufficient to design structures with the objective of making them able to carry only primary loads.

The studies presented in this work are just a contribution in this direction. Applying for the first time JWL method in so different real world contexts, it shows clearly that it is possible to perform appropriate numerical simulations that are a valid support for designers faced with the problem of conceiving blast resistant structures and determining explosive limits of current designs.

Moreover, this work differentiates from others available in literature by presenting simulations done taking into account for the complexity of JWL in modeling blast waves on large structures and the integrated methodology used in evaluating a reliable procedure in recreating the stress state of a fuselage.

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Appendix

The case study of a blast inside an aircraft requires to consider a correct modeling in terms of boundary conditions. In particular, the issue related to the pressurization load is addressed here and a comparison between two different models is studied. [Kotzakolios et al., 2011] compute the stresses distribution due to pressurization applying a distributed load, acting perpendicularly on the internal side of the skin, equivalent to the difference of pressure between inside and outside the cabin.

Differently, the investigations presented in Ch. 3 rely on the evaluation of pressurization by *physically* applying two different values of pressure to the internal and surrounding volumes of air. The two different ways of applying pressurization are depicted in Fig. 3.18.

Figure 3.18: Different pressurization loads; (a) same value of pressure applied in the all air domain plus a continuous load equivalent to the difference between internal and external atmospheric pressures (*left*), (b) distinction between the internal and external values of the pressure (*right*).

Modeling pressurization through a uniformly distributed dead load, case (a), is clearly unphysical as soon as the puncturing of the skin happens. Indeed, when damage occurs, the cabin will be subjected to a decompression and hence the gradient of pressure will rapidly decrease to zero. On the other hand, case (b), i.e. apply pressurization by distinguishing the pressure values in the internal and external volumes of air, allows to take into account for the complete modeling of the decompression phenomenon.

In order to evaluate the difference between the two ways of modeling pressurization, a numerical simulation with a charge of 0.5 kg is run for both the cases. The comparison is depicted in Figs. 3.19 and 3.20.

It is apparent that case (a) leads to a more severe damage of the fuselage, due to the constant pressure acting on it. The more realistic case (b), taking into account for the real pressure distribution inside the cabin, do not lead to the skin perforation.

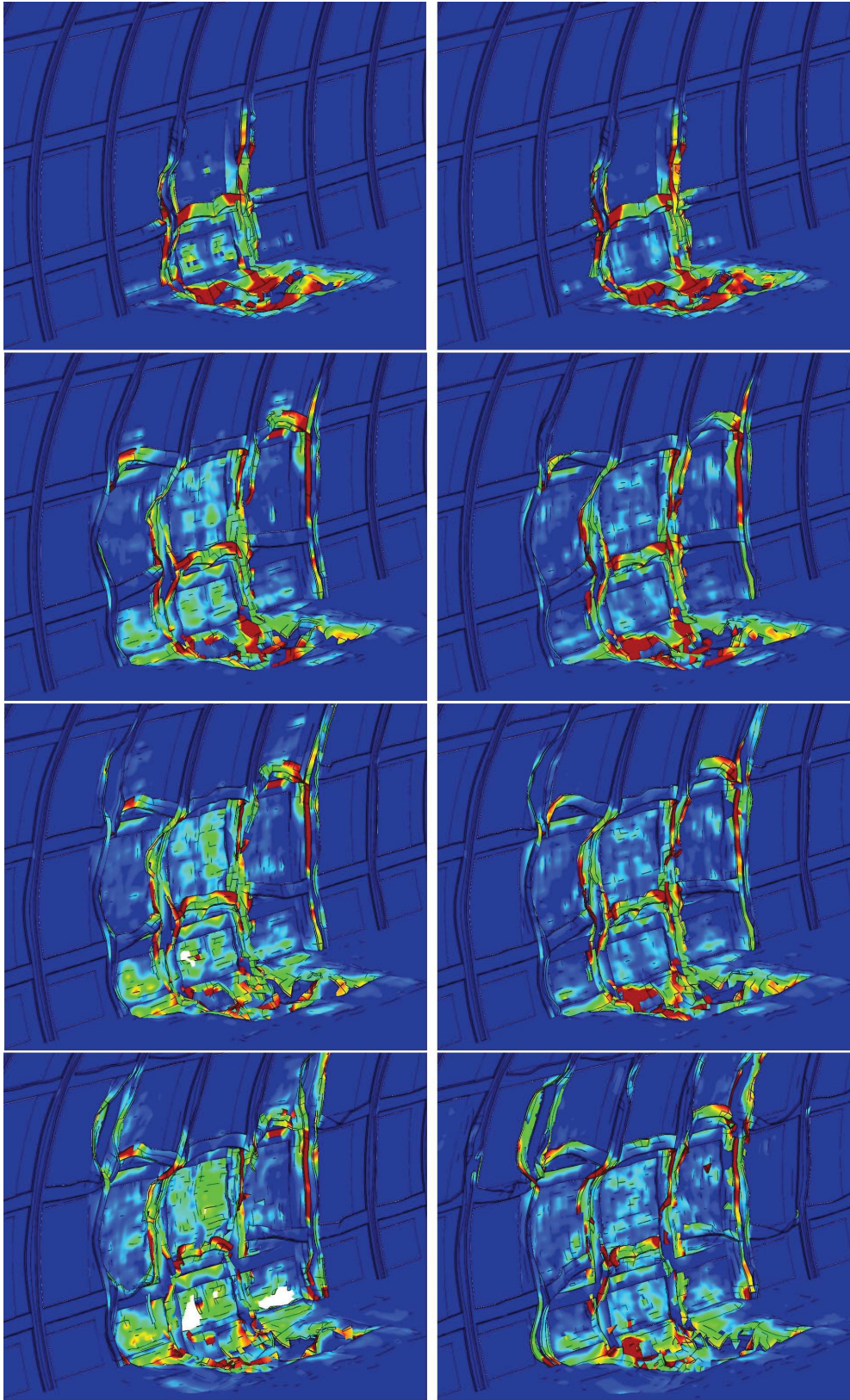


Figure 3.19: Equivalent plastic strain: case (a) (*left*) and case (b) (*right*).

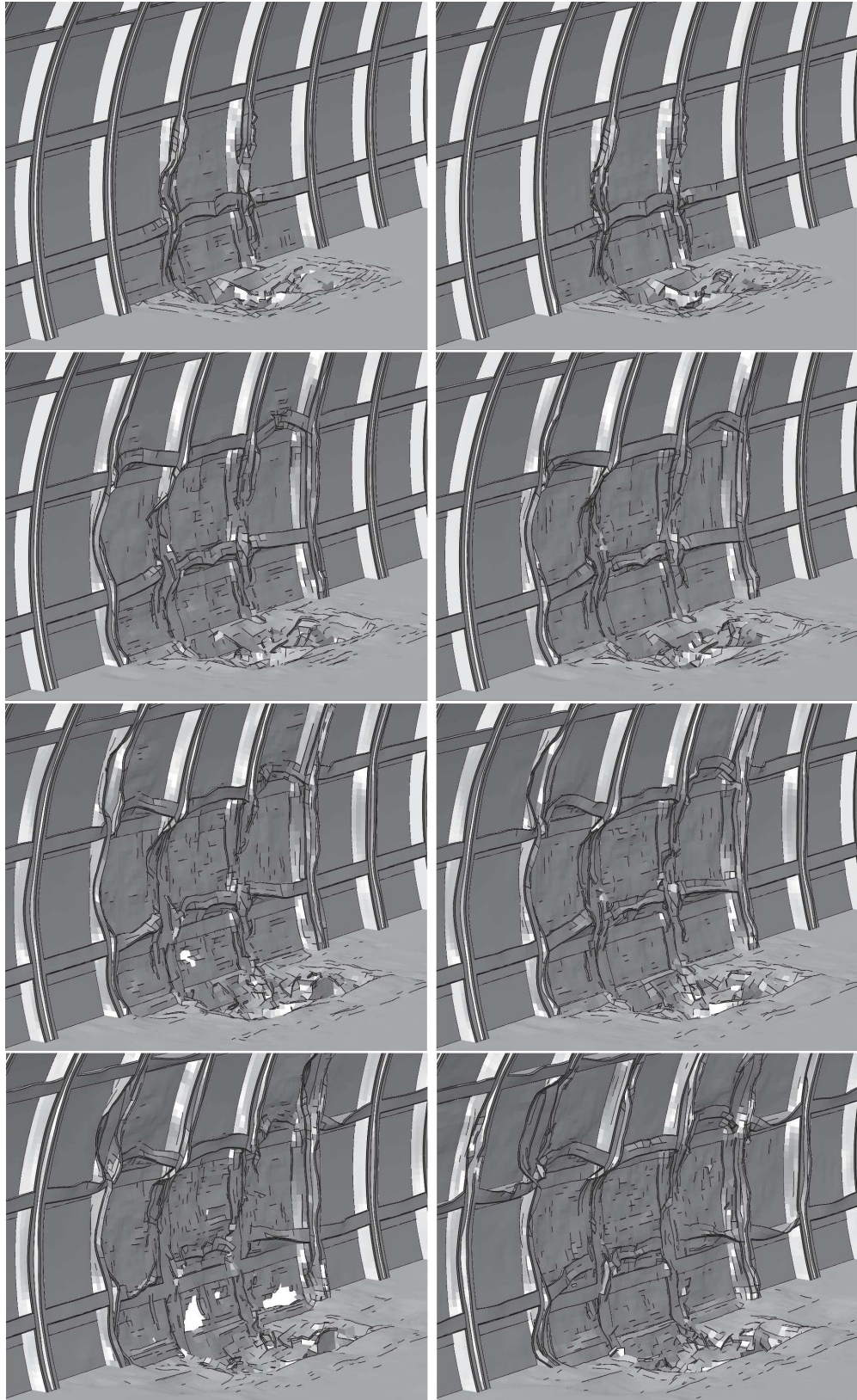


Figure 3.20: Damage of the skin: case (a) (*left*) and case (b) (*right*).