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REDUCING STRUCTURAL ERROR IN FUNCTION GENERATING MECHANISMS VIA THE ADDITION OF LARGE NUMBERS OF FOUR-BAR MECHANISMS

Hessein Ali* **Andrew P. Murray** **David H. Myszka**
Design of Innovative Machines Laboratory
Department of Mechanical & Aerospace Engineering
University of Dayton
Dayton, Ohio 45469
ashourh1@udayton.edu

ABSTRACT

This paper presents a methodology for synthesizing planar linkages to approximate any prescribed periodic function. The mechanisms selected for this task are the slider-crank and the geared five-bar with connecting rod and sliding output (GFBS), where any number of drag-link (or double crank) four-bars are used as drivers. A slider-crank mechanism, when comparing the input crank rotation to the output slider displacement, produces a sinusoid-like function. Instead of directly driving the input crank, a drag-link four-bar may be added that drives the crank from its output via a rigid connection between the two. Driving the input of the added four-bar results in a function that is less sinusoid-like. This process can be continued through the addition of more drag-link mechanisms to the device, slowly altering the curve toward any periodic function with a single maximum. For periodic functions with multiple maxima, a GFBS is used as the terminal linkage added to the chain of drag-link mechanisms. The synthesis process starts by analyzing one period of the function to design either the terminal slider-crank or terminal GFBS. A randomized local search is then conducted as the four-bars are added to minimize the structural error between the desired function and the input-output function of the mechanism. Mechanisms have been "grown" in this fashion to dozens of links that are capable of closely producing functions with a variety of intriguing features.

*Address all correspondence to this author.

1 Introduction

Erdman et al. [1] define function generation as the correlation of an input motion with an output motion of a mechanism. The fundamentals of function generation, particularly its use in designing slider-crank and four-bar linkages, are presented in many machine theory texts [2–4]. A slider-crank is not capable of error-free generation of an arbitrary function as it is able to achieve, at most, five precision points. As solutions to the five precision point problem typically divide between circuits or lack a fully rotating input, Murray and Stumph [5] introduced a defect-free approach to achieve four precision points. For more than five points, structural error is a certainty [6]. Structural error is defined as the difference between the desired function and the actual function generated by the mechanism [7]. Freudenstein developed a technique for locating more than five precision points while minimizing structural error in four-bar mechanisms [8].

More precision points are achievable through the introduction of mechanical adjustments and, hence, additional design parameters in the mechanism. Naik and Amarnath [9] synthesized adjustable four-bar function generators utilizing five-bar loop closure equations. Sandor [10] synthesized adjustable linkages for function generation by modifying the fixed pivot locations. Soong [11] presented a methodology to synthesize four-bar function generation mechanisms to achieve any number of precision points utilizing variable length driving links. One approach to achieve more precision points is to use a mechanism with more links providing additional design parameters. Subbian and Flugrad [12] used a continuation method to synthesize eight-bars to produce six precision points. McLarnan [13] used an iterative solution technique to design Watt and Stephenson linkages to achieve eight precision points. Dhingra et al. [14] employed continuation with m-homogenization to reduce the number of tracked solutions to design the same. Dhingra and Mani [15] applied symbolic computing to synthesize Watt and Stephenson mechanisms, deriving closed-form solutions and handling as many as 11 finitely separated precision points. Links may also be added to a mechanism via the introduction of a drag-link four-bar. A drag-link is the special case of four-bar mechanisms with the ground link being the shortest and the other links dimensioned such that both the input and output link fully rotate. A variety of uses including altering the outputs of cam-follower, quick-return and dwell mechanisms is presented by Al-Dwairi and the references therein [16].

A second approach to introduce design parameters is through the use of gears within the mechanism. Geared five-bar mechanisms (GFBMs) were investigated for their use in function generation by Oleksa and Tesar [17] and Erdman and Sandor [18]. Subbian and Flugrad [12] synthesized geared five-bars to produce seven precision points. Sultan and Kalim [19] synthesized geared five-bar slider-crank mechanisms in which the gears are mounted on the moving pivots. For the kinematic properties of GFBMs with arbitrary gear ratios, see Freudenstein [20] and Primrose [21].

The nonlinear design challenge associated with function generating mechanisms also makes them an ideal testbed for optimization algorithms. Several methods can already be seen in the citations above. In addition, Southerland and Roth [22] utilized an improved least-squares method, Chen and Chan [23] applied Marquardt's compromise technique, Sarganachari [24] a variable topology approach, Norouzi [25] used a gradient-based SQP method, and Akcali and Dittrick used Gelerkin's Method [26].

This paper presents a technique for reducing the structural error in function generating mechanisms via the addition of large numbers of four-bars. The two terminal mechanisms in the chains under consideration are the slider-crank and the geared five-bar with connecting rod and sliding output (GFBS). The remainder of the paper is organized as follows. Section 2 details the design of fully rotatable slider-crank mechanisms to provide an initial mechanism that approximates any single maximum periodic function. Section 3 presents the design of the GFBS to match the number of maxima in any desired periodic function. Section 4 reviews drag-link mechanisms. A randomized local search technique used for adding and optimizing drag-links to either of the terminal mechanisms is presented in Section 5. Section 6 includes a variety of example synthesis problems and Section 7 concludes this paper.

2 The Terminal Slider-Crank Mechanism

Given any periodic function with a single maximum, a slider-crank can be designed to provide an initial approximation to the function. The vector loop of the slider-crank shown in Fig. 1 is

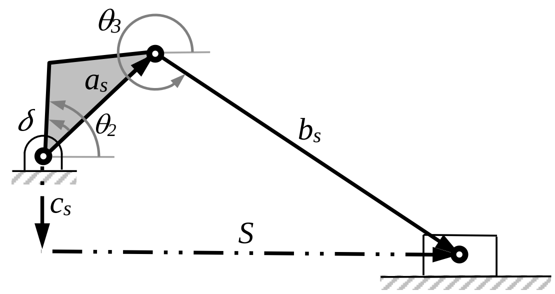

FIGURE 1. The vector loop of the offset slider-crank mechanism.

$$a_s \begin{Bmatrix} \cos(\theta_2 - \delta) \\ \sin(\theta_2 - \delta) \end{Bmatrix} + b_s \begin{Bmatrix} \cos\theta_3 \\ \sin\theta_3 \end{Bmatrix} + \begin{Bmatrix} -S \\ c_s \end{Bmatrix} = 0, \quad (1)$$

where a_s and b_s are link lengths, and c_s, the distance between the fixed crank pivot and the horizontal axis of sliding, can be positively valued (downward, as shown) or negative (upward). Additionally, S is the slider position (positive to the right), θ_2 and θ_3 are joint variables, and δ is an offset angle that shifts the input-output function left or right. The function associated with the slider-crank is then S versus θ_2.

Given a (set of physical parameters for a) slider-crank and θ_2, the unknowns are the angle θ_3 and the slider displacement S. The slider displacement is

$$S = -D_s \pm \sqrt{{D_s}^2 - F_s} \quad (2)$$

where $D_s = -a_s \cos(\theta_2 - \delta)$ and $F_s = {D_s}^2 + (c_s + a_s \sin(\theta_2 - \delta))^2 - {b_s}^2$. Note from Eq. (2) that solutions for S occur in real or complex pairs.

A curve may be generated using Eq. (2) indicating the value of S as a function of θ_2. In the case that $b_s > a_s + |c_s|$ and $a_s < b_s - |c_s|$, the input link fully rotates. For this case, S in Eq. (2) is real with one value being positive and one negative. This work only examines the positive value. Thus, driving θ_2 over the range of 2π produces one period of a repeating curve with a single maximum per period.

Given a desired function with a period of 2π and a single maximum, $(\overline{\theta}_2, \overline{S})$ denotes the values corresponding to the location of the period's maximum. Likewise, $(\underline{\theta}_2, \underline{S})$ corresponds to the period's minimum. Should the period have a horizontal line segment as a maximum or minimum, the midpoint may be used for either of the values. Given a slider-crank with a fully rotating input, $\overline{S}$ is achieved when $\theta_2 - \delta = \theta_3$ and $\underline{S}$ when $\theta_3 = \theta_2 - \delta + \pi$. Substituting these conditions into Eq. (1),

$$a_s \begin{Bmatrix} \cos\left(\overline{\theta}_2 - \delta\right) \\ \sin\left(\overline{\theta}_2 - \delta\right) \end{Bmatrix} + b_s \begin{Bmatrix} \cos\left(\overline{\theta}_2 - \delta\right) \\ \sin\left(\overline{\theta}_2 - \delta\right) \end{Bmatrix} + \begin{Bmatrix} -\overline{S} \\ c_s \end{Bmatrix} = 0, \quad (3)$$
$$a_s \begin{Bmatrix} \cos\left(\underline{\theta}_2 - \delta\right) \\ \sin\left(\underline{\theta}_2 - \delta\right) \end{Bmatrix} + b_s \begin{Bmatrix} \cos\left(\underline{\theta}_2 - \delta + \pi\right) \\ \sin\left(\underline{\theta}_2 - \delta + \pi\right) \end{Bmatrix} + \begin{Bmatrix} -\underline{S} \\ c_s \end{Bmatrix} = 0.$$

Solving Eq. (3),

$$\delta = \tan^{-1}\left(\frac{-B_s \pm \sqrt{{B_s}^2 - 4A_sC_s}}{2C_s}\right), \quad (4)$$

where

$$\begin{aligned} A_s &= \overline{S}\sin\overline{\theta}_2\cos\underline{\theta}_2 - \underline{S}\sin\underline{\theta}_2\cos\overline{\theta}_2, \\ B_s &= \left(\overline{S} - \underline{S}\right)\left(\sin\overline{\theta}_2\sin\underline{\theta}_2 - \cos\overline{\theta}_2\cos\underline{\theta}_2\right), \\ C_s &= \underline{S}\cos\underline{\theta}_2\sin\overline{\theta}_2 - \overline{S}\cos\overline{\theta}_2\sin\underline{\theta}_2. \end{aligned} \quad (5)$$

Having δ, the remaining values are

$$\begin{aligned} c_s &= -\frac{\overline{S}\sin\left(\overline{\theta}_2 - \delta\right)}{\cos\left(\overline{\theta}_2 - \delta\right)} = -\frac{\underline{S}\sin\left(\underline{\theta}_2 - \delta\right)}{\cos\left(\underline{\theta}_2 - \delta\right)}, \\ a_s &= \frac{1}{2}\left(\frac{\overline{S}}{\cos\left(\overline{\theta}_2 - \delta\right)} + \frac{\underline{S}}{\cos\left(\underline{\theta}_2 - \delta\right)}\right), \\ b_s &= \frac{1}{2}\left(\frac{\overline{S}}{\cos\left(\overline{\theta}_2 - \delta\right)} - \frac{\underline{S}}{\cos\left(\underline{\theta}_2 - \delta\right)}\right). \end{aligned} \quad (6)$$

Given that the value of ${B_s}^2 - 4A_sC_s \geq 0$ in Eq. (4), δ has two potential values. Both are checked and the one resulting in the mechanism that matches $\overline{\theta}_2$, $\overline{S}$, $\underline{\theta}_2$, and $\underline{S}$ is selected. The value ${B_s}^2 - 4A_sC_s$ can be negative when the difference between $\overline{\theta}_2$ and $\underline{\theta}_2$ is small. If ${B_s}^2 - 4A_sC_s < 0$, Eqs. (4) and (6) produce complex values. In this case, $\overline{\theta}_2$ and $\underline{\theta}_2$ are shifted apart until the value becomes real and the resulting mechanism used. Another consideration is that the values of a_s and b_s determined by Eq. (6) may be negative. The mechanism is still physically realizable nothing that the line segment between the joints is 180° different from the the joint values in the equation. The values for c_s, a_s, and b_s determined always define a fully rotating input link due to the assumptions resulting in Eq. (3).

3 The Terminal Geared Five-Bar with Connecting Rod and Sliding Output

Multiple maxima per period cannot be achieved with a slider-crank, in which case the geared five-bar with connecting rod and sliding output (GFBS) is used. A GFBS is shown in Fig. 2 and can create a single period with any number of maxima due to the gears being able to have any rational number ratio.

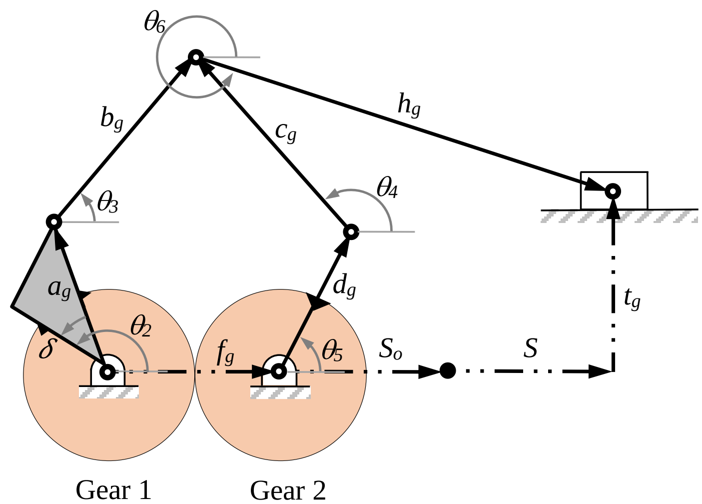

FIGURE 2. The vector loop of the geared five-bar mechanism with a connecting rod and a sliding output.

The two vector loops needed to analyze the GFBS are

$$a_g \begin{Bmatrix} \cos\left(\theta_2 - \delta\right) \\ \sin\left(\theta_2 - \delta\right) \end{Bmatrix} + b_g \begin{Bmatrix} \cos\theta_3 \\ \sin\theta_3 \end{Bmatrix} - c_g \begin{Bmatrix} \cos\theta_4 \\ \sin\theta_4 \end{Bmatrix} - d_g \begin{Bmatrix} \cos\theta_5 \\ \sin\theta_5 \end{Bmatrix} - \begin{Bmatrix} f_g \\ 0 \end{Bmatrix} = 0, \quad (7)$$

and

$$a_g \begin{Bmatrix} \cos\left(\theta_2 - \delta\right) \\ \sin\left(\theta_2 - \delta\right) \end{Bmatrix} + b_g \begin{Bmatrix} \cos\theta_3 \\ \sin\theta_3 \end{Bmatrix} + h_g \begin{Bmatrix} \cos\theta_6 \\ \sin\theta_6 \end{Bmatrix} - \begin{Bmatrix} f_g + S + S_o \\ t_g \end{Bmatrix} = 0, \quad (8)$$

where a_g, b_g, c_g, d_g, and f_g are positively valued link lengths and t_g, the distance between the fixed crank pivot and the horizontal axis of sliding, can be positively valued (upward, as shown) or negative (downward). Additionally, S is the slider position (positive to the right), and is measured from an arbitrary location given by S_0. Note that the value of S_0 can be manipulated to shift the output displacement values of the mechanism vertically on a plot of S as a function of θ_2. A similar strategy (involving an S_0 term) is not needed in the case of the slider-crank because the terminal mechanism is sized to directly match the extrema of the desired period. The angles θ_2, θ_3, θ_4, θ_5 and θ_6 are joint variables, and δ is an offset angle that shifts the input-output function left or right.

Because gears correlate their motions, links a_g and d_g obey an additional relationship,

$$\theta_5 = \lambda\left(\theta_2 - \delta\right) + \phi \tag{9}$$

where ϕ is an initial angular offset of link d_g and λ is the gear ratio which can be any positive or negative rational number.

Given a (set of physical parameters for a) GFBS and θ_2, the unknowns are the angles θ_3, θ_4, θ_5, θ_6, and the slider displacement S. The value of angle θ_3 is

$$\theta_3 = 2\tan^{-1}\left(\frac{-M_g \pm \sqrt{{M_g}^2 - 4L_gN_g}}{2L_g}\right), \tag{10}$$

where

$$L_g = K_g - G_g, M_g = 2H_g, N_g = G_g + K_g, \tag{11}$$

and

$$\begin{aligned} G_g &= 2b_g\left(a_g\cos\left(\theta_2-\delta\right) - d_g\cos\theta_5 - f_g\right), \\ H_g &= 2b_g\left(a_g\sin\left(\theta_2-\delta\right) - d_g\sin\theta_5\right), \\ K_g &= {a_g}^2 + {b_g}^2 - {c_g}^2 + {d_g}^2 + {f_g}^2 - 2a_gf_g\cos\left(\theta_2-\delta\right) \\ &\quad - 2d_g\left(a\cos\left(\theta_2-\delta\right) - f_g\right)\cos\theta_5 - 2a_gd_g\sin\left(\theta_2-\delta\right)\sin\theta_5. \end{aligned} \tag{12}$$

With θ_3, Eq. (8) has the two unknowns θ_6 and the quantity $S+S_0$. Solving for $S+S_0$,

$$S + S_0 = D_g \pm \sqrt{{D_g}^2 - C_g} \tag{13}$$

where $D_g = a_g\cos\left(\theta_2-\delta\right) + b_g\cos\theta_3 - f_g$ and $C_g = {D_g}^2 + \left(a_g\sin\left(\theta_2-\delta\right) + b_g\sin\theta_3 - t_g\right)^2 - {h_g}^2$.

To ensure full rotation of link a_g, a sufficiency condition presented by Ting [27] is used. The link lengths a_g, b_g, c_g, d_g, and f_g are arranged in an ascending order. The ordered link lengths are then labeled L_1, L_2, L_3, L_4 and L_5. The shortest two links, a_g and d_g by design in this case, make a complete revolution if

$$L_1 + L_2 + L_5 < L_3 + L_4. \tag{14}$$

A curve may be generated using Eq. (13) indicating the value of S as a function of θ_2. In the case of Eq. (14) being satisfied, the input link fully rotates. For this case, S in Eq. (13) has two real values, and the larger one corresponding to the slider being to the right (as shown in Fig. 2) is selected. The gear ratio λ plays a fundamental role in the number of maxima per period and the number of rotations needed by gear 1 to create a single period.

The gear ratio of the GFBS is $\lambda = N_a/N_d$ where N_a and N_d are the numbers of teeth on gears 1 and 2, respectively. The ratio can be either negative via a direct meshing of the gears or positive via the use of an idler. Selection of λ dictates the largest number of maxima for a single period. To determine the number of maxima, reduce the fraction N_a/N_d by eliminating common factors. For example, $32/14$ reduces to $16/7$ and this is labeled $\tilde{N}_a/\tilde{N}_d$. The resulting period will have $\max\{\tilde{N}_a, \tilde{N}_d\}$ as its number of maxima. Moreover, the input needs to be driven $2\pi\tilde{N}_d$ to create one period. As an example, if a single period of the desired curve has four maxima, the potential values of $\tilde{N}_a : \tilde{N}_d$ include 4 : 1, 4 : 3, 3 : 4, 1 : 4 and all of their negatives. Note that 4 : 2 and 2 : 4 are not included because these reduce to 2 : 1 and 1 : 2, respectively. Finally, in the case of 4 : 3 (or −4 : 3), gear 1 needs to be driven 6π radians to produce a single period.

Also of note is that the number of maxima is not always produced in a single period due to the presence of inflection points. In all cases tested, changing the other parameters of the mechanism has resulted in converting inflection points into true maxima.

4 The Additional Drag-Link Mechanisms

The vector loop of the drag-link mechanism shown in Fig. 3 is

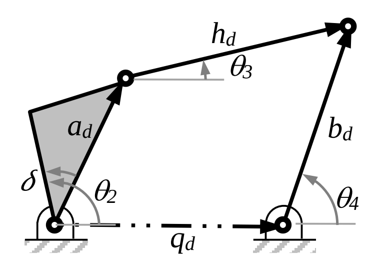

FIGURE 3. The vector loop of the drag-link mechanism.

$$\begin{aligned} &a_d\begin{Bmatrix}\cos\left(\theta_2-\delta\right)\\ \sin\left(\theta_2-\delta\right)\end{Bmatrix} + h_d\begin{Bmatrix}\cos\theta_3\\ \sin\theta_3\end{Bmatrix} \\ &-\begin{Bmatrix}q_d\\ 0\end{Bmatrix} - b_d\begin{Bmatrix}\cos\theta_4\\ \sin\theta_4\end{Bmatrix} = 0, \end{aligned} \tag{15}$$

where a_d, h_d, q_d, and b_d are positively valued link lengths. Additionally, θ_2, θ_3 and θ_4 are joint variables, and δ is an offset angle that shifts the input-output function left or right.

Given a (set of physical parameters for a) drag-link and θ_2, the unknowns are the angles θ_3 and θ_4. The output angle is

$$\theta_4 = \tan^{-1}\left(\frac{B_d}{A_d}\right) \pm \cos^{-1}\left(\frac{-C_d}{\sqrt{{A_d}^2 + {B_d}^2}}\right) \tag{16}$$

where

$$\begin{aligned} &D_d = q_d - a_d \cos(\theta_2 - \delta), E_d = -a_d \sin(\theta_2 - \delta), \\ &C_d = {D_d}^2 + {E_d}^2 + {h_d}^2 - {b_d}^2, \\ &A_d = 2D_d h_d, B_d = 2E_d h_d. \end{aligned} \tag{17}$$

Either the "+" or "-" must be used over the entire rotation of θ_2 in Eq. (16) to address a single assembly. As only drag-links are desired, note that $q_d - a_d + h_d - b_d < 0$, $q_d - a_d - h_d + b_d < 0$ and $-q_d - a_d + h_d + b_d > 0$ [28].

5 Synthesis Methodology

The methodology for reducing structural error in function generating mechanisms via the addition of large numbers of four-bar mechanisms is now detailed.

1. In the case of the desired periodic function having a single maximum per period, a slider-crank may be used as the terminal mechanism, as shown in Fig 4.
   If a period includes two or more maxima, a GFBS is used, noting it can be implemented in the case of a single maximum too. The function generating mechanism that terminates with a GFBS is shown in Fig. 5
   (a) For the slider-crank, the terminal mechanism is synthesized via Eqs. (4) through (6).
   (b) For the GFBS, the terminal mechanism is synthesized by selecting the physical parameters paying particular attention to the possible λ values. The mechanism is then optimized using a randomized local search, discussed at the end of this section.
2. With a terminal mechanism synthesized, a dyad is added between the ground and input link. The dyad is sized such that it operates as an additional drag-link mechanism. The link lengths of the first drag-link are chosen arbitrarily with $q_1 = 1$ being fixed due to the fact that only the ratio of the link lengths matter. While the output stays the same, S, the input to the mechanism is now θ_{21}. Note the overlap between θ_{41} and the input θ_2 to either terminal mechanism. That is, Eqs. (16) and (17) are used to generate θ_{41} given the rotation of θ_{21}. The values of θ_{41} are now used as the θ_2 inputs in Eq. (2) for the slider-crank, and in Eq. (13) for the GFBS. The initial value of the offset angle δ_1 is selected by considering 360 values in the range from 0 to 2π. The choice of δ_1 that minimizes the structural error is selected.
3. A randomized local search is then applied for all physical parameters in the drag-link driven mechanism, including those in the terminal mechanism. Note that as the slider-crank, GFBS or drag-link is altered, all requirements for full rotations are checked where applicable. The combination of the physical parameters that minimizes the structural error is utilized in the next step.
4. A second drag-link mechanism is added. The input to the mechanism is now θ_{22} with its output θ_{42}. This output now drives θ_{21} directly via the offset δ_1. The 360 possible offsets are considered for δ_2. A randomized local search is then applied for all physical parameters in the drag-link driven mechanism, including those in the previous step.
5. Drag-links may continue to be added until a threshold on structural error is met or until the structural error ceases decreasing.

A randomized local search is a brute force and easily implemented optimization technique that simply improves a guess by adjusting it toward the local extrema. Starting with an initial guess for the parameters, say $x_0 = \{x_{1_0}, x_{2_0}, ...\}$, the objective function is calculated, say $f(x_0)$. Here, the parameters are all of the physical parameters of the mechanism and the objective function is the resulting structural error. A random change, much smaller in magnitude than the original values in x_0, is generated. Let the random change be $\alpha_1 = \{\alpha_{1_1}, \alpha_{2_1}, ...\}$. A new set of input values is generated as $x_1 = x_0 + \alpha_1$. The objective function is calculated for x_1. If $f(x_1) < f(x_0)$, x_1 is used in the next iteration, otherwise $x_1 = x_0$. The process is seen to continue where $x_i = x_{i-1} + \alpha_i$, and $x_i = x_{i-1}$ if $f(x_i) > f(x_{i-1})$. A randomized local search is well suited to this application due to the frequent change in the number of variables. No claim is made that an optimal solution is achieved.

6 Synthesis Examples

Several examples are now presented. The examples endeavor to display the successes and potential challenges of the method presented.

6.1 A Piecewise-Linear Function with a Single Maximum per Period – Slider-Crank Terminal Chain

A chain of drag-link mechanisms driving a terminal slider-crank is synthesized to generate the desired piecewise linear curve shown in Fig. 6. The function defining the curve is $S_{des} = \frac{2}{75}\theta_2 + 3$ for $0° \le \theta_2 \le 75°$, $S_{des} = -\frac{2}{105}\theta_2 + \frac{45}{7}$ for $75° \le \theta_2 \le 285°$ and $S_{des} = \frac{2}{75}\theta_2 - \frac{33}{5}$ for $285° \le \theta_2 \le 360°$. The

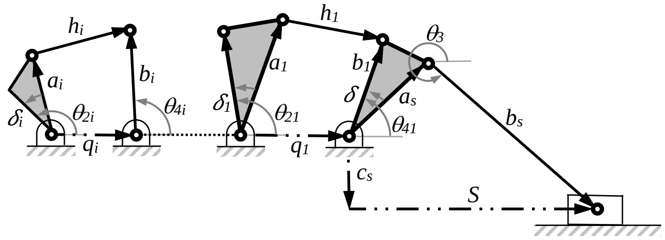

FIGURE 4. The chain of drag-link mechanisms added to a terminal slider-crank mechanism.

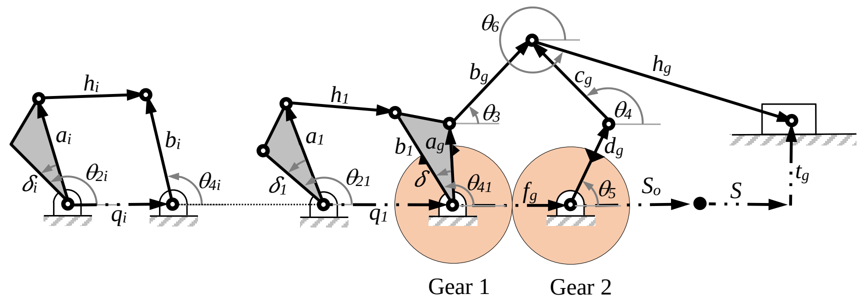

FIGURE 5. The chain of drag-link mechanisms added to a GFBS.

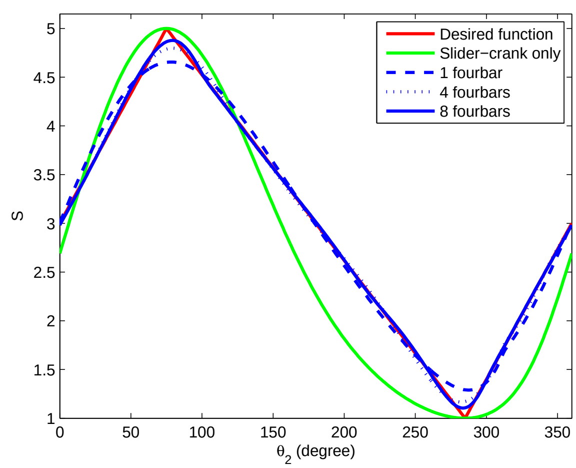

FIGURE 6. The piecewise-linear single-maximum period and the improvement in structural error as the number of four-bars driving the slider-crank increases.

"Slider-crank only" curve shows the match of the non-optimized terminal slider-crank using Eqs. (4) through (6). The associated structural error is 79.92, found by summing the distances squared between the output curve of the mechanism and the desired curve at each degree increment over the range of motion of the input link needed to create one period from the output. The structural error is measured in this fashion throughout this section. The structural error decreased through the addition of eight drag-link mechanisms. Figure 6 also shows the corresponding decreases in structural error over just the slider-crank after one and four drag-link mechanisms are added. The structural error associated with the eight mechanisms is 0.25. To verify the results, a CAD model of the full mechanism was assembled and is shown in Fig. 7. The

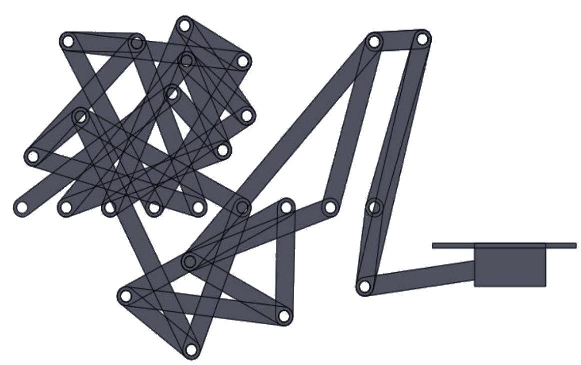

FIGURE 7. A slider-crank driven by eight drag-link mechanisms to produce the desired function shown in Fig. 6.

results of the animation of the eight drag-link driven slider-crank are shown in Fig. 8. The curve shows the relationship between the input angle of the device, measured at the fixed pivot farthest to the left, and the output location of the slide shown on the right. Finally, Table 1 shows the lengths of all the links in the

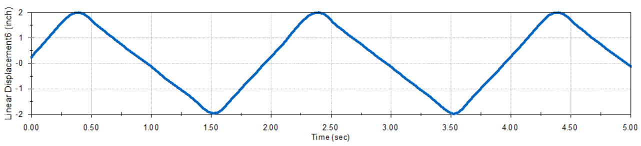

FIGURE 8. The CAD model of the eight drag-link driven slider-crank mechanism is shown to produce a period very close to the desired piecewise-liner curve.

system, including the dimensions of all eight four-bars and the slider-crank mechanism.

Slider-Srank Mechanism				
	c_s	a_s	b_s	δ (rad)
	1.2156	1.6921	3.3343	2.9756
Drag-Link Mechanisms				
	a_d	h_d	b_d	δ (rad)
1	3.6519	1.0952	3.733	2.1929
2	2.306	2.4297	3.388	-1.0722
3	3.2326	1.8728	4.0991	-1.6874
4	3.9044	1.3709	4.1476	-0.9107
5	4.0281	1.5581	3.7591	-1.0819
6	4.2118	1.5213	3.6918	-0.5972
7	4.1404	1.7846	3.661	-0.5198
8	4.1779	1.6606	3.7583	-0.5506

TABLE 1. The link dimensions of the slider-crank and eight drag-link mechanisms that best matches the desired function shown in Fig. 6.

6.2 A Piecewise-Linear Function with a Single Maximum per Period – GFBS Terminal Chain

The same desired function as in the previous example is attempted with a GFBS and a chain of drag-link four-bars. Due to the single maximum, the gear ratios considered were $1:1$ and $-1:1$. The ratio $-1:1$ was selected due to the better initial match found between the GFBS and the desired function. An initial mechanism, in this case the "Non-optimized GFBS" is included in Fig. 9. An initial mechanism is included in many of the examples to indicate the improvement attributable to the optimization. Note that optimizing the GFBS alone results in a structural error of 1.88 and obtains a shape quite close to the desired function. Nevertheless, the addition of drag-links continues to improve the error gradually through the addition of 14 four-bars. The lowest structural error for the GFBS driven by a chain of four-bars was found to be 0.51. Note that the structural error using the slider-crank as the terminal chain driven by eight four-bars in the previous example is 0.25, less than the structural error using the GFBS and fourteen four-bars. Table 2 shows the values of all the physical parameters in the three GFBS mechanisms and Table 3 show the dimensions of the 14 four-bars for the mechanism with the lowest structural error.

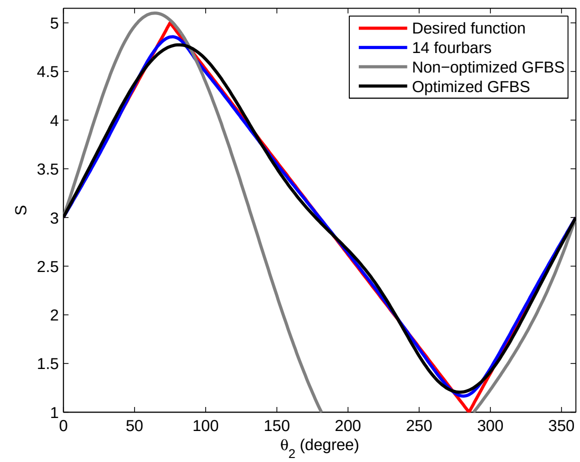

FIGURE 9. The desired function and the input-output functions generated for several GFBS cases.

GFBS			
	Non- optimized	Optimized	Optimized with four-bars
a_g	2.5	1.4034	1.6803
b_g	6	5.0831	7.0185
c_g	10	2.2171	5.6788
d_g	1	1.7998	1.0762
f_g	8	5.4898	8.5243
h_g	3	3.3718	1.8238
t_g	15	9.7971	15.9568
δ (rad)	0	1.2715	1.3094
ϕ (rad)	0	-0.4278	-1.2774
S_0	12.5704	10.8424	18.2785

TABLE 2. The link dimensions of the three GFBS mechanisms whose outputs are shown in Fig. 9.

6.3 A Second Function with a Single Maximum per Period

The desired function in this example is shown in Fig. 10. The function defining the curve is $S_{des} = 0.1221\theta_2 + 1.0$ for $0° \leq \theta_2 \leq 51.56°$, $S_{des} = 0.1219\theta_2 + 1.0102$ for $51.56° < \theta_2 \leq 57.29°$ and $S_{des} = (\pi/180)\,\theta_2 \cos(\theta_2/2) + 7$ for $57.29° < \theta_2 \leq 360°$. Again, a slider-crank is sized using Eqs. (4) through (6) and produces a structural error of 211.18. The addition of ten drag-links generates the match shown, with a structural error of 0.50.

To generate the same target function using a chain with a terminal GFBS, a $-1:1$ gear ratio is chosen. Figure 11 shows the

Drag-Link Mechanisms				
	a_d	h_d	b_d	δ (rad)
1	4.395	5.0712	5.9436	1.2435
2	4.2135	7.6087	9.2994	0.1647
3	2.4555	5.0765	5.6126	-2.3944
4	4.298	4.6333	2.9562	2.9869
5	6.3969	8.3688	10.7886	-0.7530
6	8.2836	5.1974	4.4506	-1.9696
7	3.3327	2.8689	3.7549	-3.1411
8	2.3918	4.7581	5.5921	1.1732
9	3.4946	6.3347	5.6245	0.7206
10	2.5727	2.4156	3.619	-0.2462
11	3.2273	4.689	3.8033	1.3142
12	3.4855	2.0998	4.5822	1.786
13	3.0491	2.7429	2.9417	0.1226
14	2.9981	1.3443	3.2501	-1.1563

TABLE 3. The link dimensions of the 14 added drag-link mechanisms producing the best fit shown in Fig. 9.

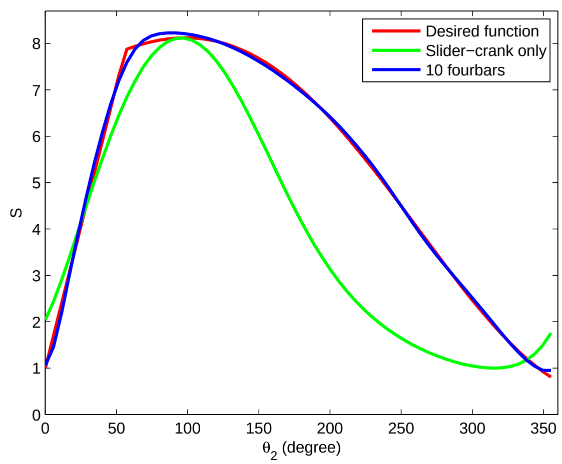

FIGURE 10. The desired function, the slider-crank ouput, and the curve generated using the proposed method resulting in the addition of 10 drag-link mechanisms.

output produced by the initial design of only the GFBS device as well as a chain that has nine drag-links in addition to the GFBS. The structural error obtained with the nine drag-links is 0.38, less than the slider-crank with ten drag-link four-bars added.

6.4 A Periodic Function with Three Maxima

The curve shown in Fig. 12 is the desired periodic function for this example. The function defining the curve is $S_{des} = 3\sin(t) + \sin\left(\frac{1}{2}t + 20\right) + 2\sin(3t - 60)$ where $t = 0.0061\theta_2 + 3.04434$ and $0° \leq \theta_2 \leq 1080°$. Due to a single period having

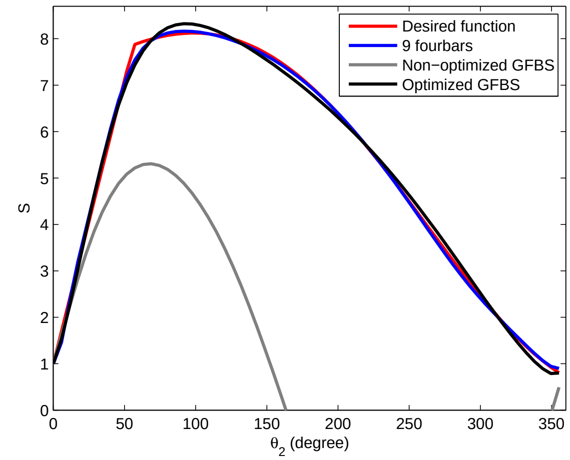

FIGURE 11. The desired function and the input-output functions using an initial guess at the GFBS and the GFBS optimized with nine drag-link mechanisms.

three maxima, the slider-crank is not used for this design. The choices under consideration for the gear ratio are $\pm 1:3$, $\pm 2:3$, $\pm 3:1$ and $\pm 3:2$. The ratio of $1:3$ was found to produce an acceptable initial guess. Because of this choice of ratio, a single period is produced by three full rotations of the input of the mechanism. The structural error after optimizing the GFBS alone was 71.82. After the addition of six drag-links, the structural error was reduced to 34.36. Note that these structural errors cannot be compared to those of previous examples. The structural error in this case is the sum of of the $3 * 360 = 1080$ values found in the range of motion of the input link to create one period of output from the mechanism.

6.5 A Periodic Function with Ten Maxima

The curve shown in Fig. 13 is the desired periodic function for this example. The function defining the curve is $S_{des} = 2\cos(10\pi t)\cos(200\pi t)$ where $t = 0.00003\theta_2 + 0.0525$ and $0° \leq \theta_2 \leq 3240°$. Due to a single period having ten maxima, the slider-crank is not used for this design. The choices under consideration for the gear ratio are $\pm 1:10$, $\pm 3:10$, $\pm 7:10$, $\pm 9:10$, $\pm 10:1$, $\pm 10:3$, $\pm 10:7$ and $\pm 10:9$. The ratio of $10:9$ was found to produce an acceptable initial guess. Because of this choice of ratio, a single period is produced by nine full rotations of the input of the mechanism. The structural error after optimizing the GFBS alone was 155.14. After the addition of 11 drag-links, the structural error was reduced to 93.37. Again, the structural error here is the sum of $9 * 360 = 3240$ values.

6.6 A Challenging Periodic Function

In all examples thus far, the GFBS has produced a visually satisfactory match. In fact, the improvements offered by the additional drag-links are modest when compared to the stand-alone GFBS. This example provides a case in which the GFBS matching was deemed unsatisfactory. The curve shown in Fig. 14 is the desired periodic function. The function defining the curve is $S_{des} = \sin(\theta_2) + \sin(2\theta_2) + \sin(4\theta_2)$ for $0^\circ \leq \theta_2 \leq 360^\circ$. The choices under consideration for the gear ratio are $\pm 1:4$, $\pm 3:4$, $\pm 4:1$ and $\pm 4:3$. The ratio of $-4:1$ was found to produce the best initial guess. Because of this choice of ratio, a single period is produced by one rotation of the input of the mechanism. The structural error after optimizing the GFBS alone was 97.98.

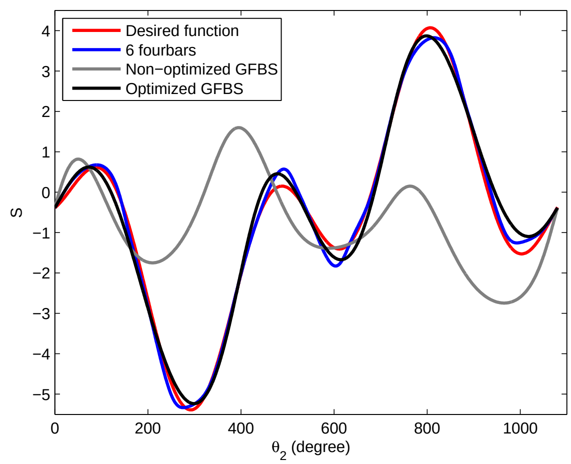

FIGURE 12. The desired function having three maxima and the input-output functions using a non-optimized GFBS and an optimized GFBS with a chain of drag-links.

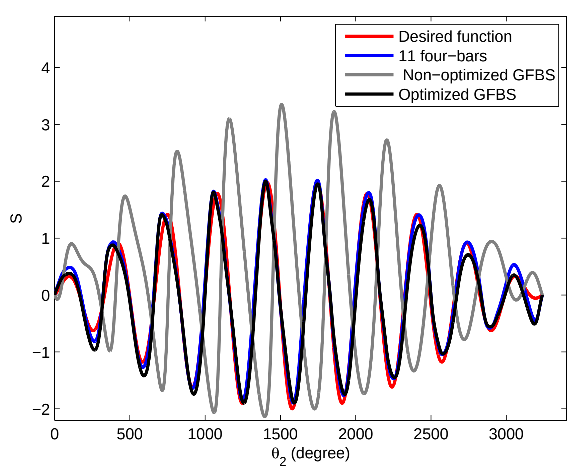

FIGURE 13. The desired function having ten maxima and the input-output functions using a non-optimized GFBS and an optimized GFBS with a chain of drag-links.

After the addition of seven drag-links, the structural error was reduced to 47.35. Note for the previous cases in which the range of motion of the input link was one rotation, structural errors for optimized mechanisms are found to be less than 1.

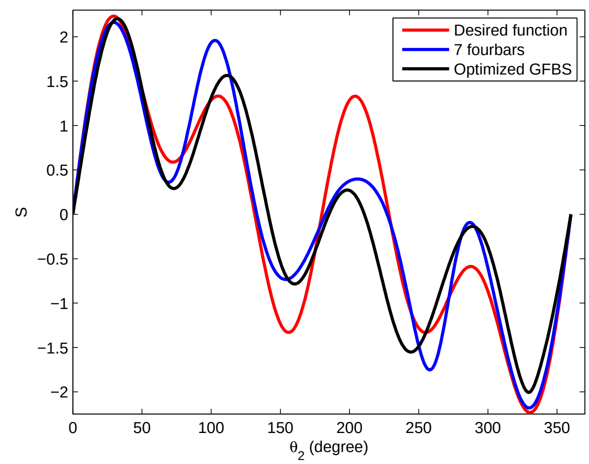

FIGURE 14. The desired function having four maxima proved difficult to match using the proposed technique.

7 Conclusions

Function generating mechanisms are synthesized with the goal of reducing structural error versus a desired output. This paper addresses the challenge of minimizing the structural error with respect to any desired periodic function. The synthesized mechanisms are allowed to be of any number of links. Two basic architectures are considered. The first architecture is any number of drag-link four-bars driving each other in succession concluding with a slider-crank. The second is any number of drag-links driving a geared five-bar with connecting rod and sliding output (GFBS). The slider-crank as a terminal linkage is only used to address functions with a single maximum in the period, whereas the GFBS can be used to create one or more maxima. As expected, an increase in the number of links is seen to yield an improvement in matching the periodic function. Conditions are implemented throughout to ensure fully rotating inputs connect to fully rotating outputs and then to fully rotating inputs again. This guarantees the periodic output aspects of the architectures. Finally, a variety of examples are included to display the methodology presented, including periods composed of piecewise linear segments and a period for which no acceptable solution was found.

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