ANALYSIS OF LAY-UP OPTIMALITY CONDITIONS FOR BUCKLING OPTIMIZATION OF VAT (STEERED FIBER) COMPOSITE PLATES

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Abstract

In the present paper the analysis of the lay-up optimality conditions (derived earlier) for buckling optimization of VAT composite plates is performed. The plates have a symmetric lay-up and loaded along their contour by the in-plane forces. The consideration employs the von Karman approach. The lay-up optimality conditions correspond to the maximization of a single-mode (lowest) buckling eigenvalue. The conditions are comprehensively analyzed. The conclusions of the analysis are presented. The key conclusion is that the optimality conditions with and without account of variation of the in-plane forces are the same.

Keywords: composite plate, buckling, lay-up, steered fibers, optimality

1. Introduction

The buckling optimality of composite plates is studied in the last two-three decades by many authors (see, e.g., the reviews of Ghiasi et al. 2009, 2010 and the book of Falzon and Aliabadi 2008). The works in the field consider the problem mainly from the point of view of numerical solutions.

In the paper of Selyugin (2019) the lay-up optimality conditions for buckling optimization of the VAT (steered fiber) composite plates have been derived. The present paper is devoted to the theoretical analysis of the conditions.

Section 2 contains some theoretical background.

In Section 3 the analysis of the optimality conditions is performed.

Section 4 contains the conclusions.

2. Theoretical background

We consider the laminated composite plate with the symmetric lay-up. The lay-up is composed of the orthotropic fiber-reinforced plies of the same thickness h_{ply} and various point-wise orientation angles. The total number of plies is even and equal to $2K$, the generalization to the odd ply number is straightforward and is not considered in the present paper. Fig. 1 illustrate the plate and the notations.

Fig. 1

The mid-plane of the flat plate is located in the XY plane. The plate mid-plane is restricted by the piecewise smooth contour C with external normal n (having the components m , l) and tangent direction s ; the directions n , l and Z create a right-hand triplet. The coordinate system XYZ is a Cartesian one.

The plate is loaded by the in-plane forces, acting at the part C_1 of the contour C . The remaining part C_2 of the contour the plate is not moving in X and Y . The plate is simply supported, i.e., the z -displacement and the normal bending moment M_n acting perpendicular to the contour C are equal to zero. The clamping boundary conditions are not considered in the present paper, the change to the case is straightforward.

The X , Y , Z displacements are denoted as u , v , w , respectively.

The signs of the loads are the following. The force flows N_x, N_y (along X and along Y) are positive in tension, the shear force flow N_{xy} is positive when it decreases the 90° angle between the orthogonal lines at the plate surface. The flows are in equilibrium. The Composite Lamination Plate Theory (CLPT) is used for description of plate deflections (see the book of Gibson 1994).

The Classical Lamination Plate Theory (CLPT), based on linear-elastic relations for every layer and Bernoulli's hypotheses, is used for obtaining the equation for the deflections w (see the book of Gibson 1994). The equation is written as follows:

$$D_{11} \frac{\partial^4 w}{\partial x^4} + 2(D_{12} + 2D_{66}) \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_{22} \frac{\partial^4 w}{\partial y^4} + 4D_{16} \frac{\partial^4 w}{\partial x^3 \partial y} + 4D_{26} \frac{\partial^4 w}{\partial x \partial y^3} - N_x \frac{\partial^2 w}{\partial x^2} - N_y \frac{\partial^2 w}{\partial y^2} - 2N_{xy} \frac{\partial^2 w}{\partial x \partial y} = 0 \quad (1)$$

where $D_{ij}, i = 1, 2, 6; j = 1, 2, 6$, are the elements of the bending stiffness matrix D , coupling the bending/twisting moments and various second derivatives of the deflection w with respect to x and y . The coupling is written as follows:

$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{bmatrix} -\frac{\partial^2 w}{\partial x^2} \\ -\frac{\partial^2 w}{\partial y^2} \\ -2\frac{\partial^2 w}{\partial x \partial y} \end{bmatrix} \quad (2)$$

We denote the left-hand side of (2) as a vector-column $\vec{M}$, and as $\vec{k}$ the vector-column at the right-hand side (which is multiplied to D matrix). Also we use the notations for the corresponding curvatures:

$$k_x = -\frac{\partial^2 w}{\partial x^2}; k_y = -\frac{\partial^2 w}{\partial y^2}; k_{xy} = -\frac{\partial^2 w}{\partial x \partial y} \quad (3)$$

Then (2) is rewritten in the matrix-vector form as:

$$\vec{M} = D\vec{k} \quad (4)$$

With the boundary conditions indicated above, the boundary-value problem for deflections is completed.

The distribution of the in-plane stress flows is given by the relations:

$$\vec{N} = A \begin{bmatrix} u_{,x} \\ v_{,y} \\ u_{,y} + v_{,x} \end{bmatrix} \quad (5)$$

where A is the in-plane stiffness matrix, and the column-vector

$$\vec{N} = (N_x, N_y, N_{xy})^T \quad (6)$$

the superscript T means transposing, and index after comma means the differentiation w.r.t. the corresponding coordinate.

The components of the $\vec{N}$ must satisfy the equilibrium equations, and the in-plane displacements must satisfy the corresponding boundary conditions.

It is supposed that the buckling eigenvalue is a single-modal one. The optimization goal is to design a lay-up maximizing the first (lowest) eigenvalue.

As it is shown in Selyugin (2019), the first order necessary local optimality conditions for the ply orientation angle θ_i are written as follows, $i, \dots, K$:

$$\sin 2(\theta_i - \psi) \left[\frac{U_2}{4U_3} (k_1^2 - k_2^2) + (k_1 - k_2)^2 \cos 2(\theta_i - \psi) \right] \frac{1}{6} (z_i^3 - z_{i-1}^3) = 0, \quad i=1, \dots, K \quad (7)$$

where all angles are measured relatively to the X axis, $k_1, k_2, \psi, z_i, z_{i-1}$ respectively are the largest and the lowest principal curvatures, the angle between the direction k_1 and the X axis, the top and the bottom z coordinates of the i -th layer, $i=1, \dots, K$. The value of z_0 is equal to zero. The relation (7) was obtained in the more simplified case in (Selyugin, 2013) using (Gibson, 1994) background on composites.

3. Analysis of the optimality conditions

The first conclusion coming from (Selyugin, 2019) is that the lay-up optimality conditions in the case of varied in-plane forces of the plate are the same as (7). This means that the in-plane force variation doesn't influence the optimality conditions.

It may be also noticed from the optimality condition derivation (Selyugin, 2019) that the parameters α_1, α_2 there (they are some Lagrange multipliers) are some generalized in-plane displacements.

Further we look at the typical deflection buckling shape for the 2D uniformly compressed simply supported composite plate shown in Figure 2. The Figure will support us in our analysis.

Fig. 2. Typical buckling shape of the 2D compressed composite plate.

Now we analyze the relation (7) in details. Rewrite (7) without the mutual multiplier as

$$\sin 2(\theta_i - \psi) \left[\frac{U_2}{4U_3} (k_1^2 - k_2^2) + (k_1 - k_2)^2 \cos 2(\theta_i - \psi) \right] = 0, \quad i=1, \dots, K \quad (8)$$

The umbilical points (where $k_1 = k_2$) are not discussed below.

First we consider the term within the square brackets. When it is equal to zero, we have

$$\cos 2(\theta_i - \psi) = -\frac{U_2(k_1 + k_2)}{4U_3(k_1 - k_2)} \quad (9)$$

We remember that the value of the ratio $\frac{U_2}{4U_3}$ is about 1.0 (see Selyugin, 2013). When the both principal curvatures k_1, k_2 are of the same sign (see Fig. 1) and rather close to each other in values (e.g., 5.0 and 4.0), there is no solution θ_i of (9). But in principle, for some combination of the principal curvature values, there are some solutions of (9).

The equation

$$\sin 2(\theta_i - \psi) = 0 \quad (10)$$

gives the solutions

$$\theta_i = \psi \quad \text{or} \quad \theta_i = \psi \pm \pi/2 \quad (11)$$

The solutions mean that the ply material direction coincide the first or the second principal curvature direction.

It was shown in (Selyugin, 2013) that the sum of (8) for $i=1, \dots, K$ leads to a relation

$$M_{xy}^{pr.cur.}(k_1 - k_2) = 0 \quad (12)$$

where $M_{xy}^{pr.cur.}$ is the twisting moment measured in the principal curvature axes. The relation (12) means that the moment tensor and the tensor of curvatures are co-axial. In fact, the optimal plate is a locally orthotropic plate.

4. Conclusions

- Possible solutions of the optimality conditions are discussed;
- The variation of the in-plane forces does not influence the optimality conditions obtained without account of such variation;
- The plate with optimal lay-up is a locally orthotropic one.

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