ANALYSIS OF LAY-UP OPTIMALITY CONDITIONS FOR BUCKLING OPTIMIZATION OF VAT (STEERED FIBER) COMPOSITE PLATES

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Abstract

In the present paper the analysis of the lay-up optimality conditions (derived earlier) for buckling optimization of VAT composite plates is performed. The plates have a symmetric lay-up and loaded along their contour by the in-plane forces. The consideration employs the von Karman approach. The lay-up optimality conditions correspond to the maximization of a single-mode (lowest) buckling eigenvalue. The order of magnitude of the items of the conditions is assessed and compared. The performed analysis gives the theoretical explanation of the known numerical results of other authors on re-distributing the pre-buckling stresses to supported regions for improving the buckling behavior.

Keywords: composite plate, buckling, lay-up, steered fibers, optimality

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1. Introduction

The buckling optimality of composite plates is studied in the last two-three decades by many authors (see, e.g., the reviews of Ghiasi et al. 2009, 2010 and the book of Falzon and Aliabadi 2008). The works in the field consider the problem mainly from the point of view of numerical solutions. The papers of Wu et al (2012) and Groh and Weaver (2015) and others referenced in them may be indicated as ones among the big number of the numerically targeted papers.

In the paper of Selyugin (2019) the lay-up optimality conditions for buckling optimization of the VAT (steered fiber) composite plates have been derived. The present paper is devoted to the theoretical analysis of the conditions.

Section 2 contains some theoretical background.

In Section 3 the analysis of the optimality conditions is performed.

In Section 4 the results of the analysis are discussed.

Section 5 contains the conclusions.

2. Theoretical background

We consider the laminated composite plate with the symmetric lay-up. The lay-up is composed of the orthotropic fiber-reinforced plies of the same thickness h_{ply} and various point-wise orientation angles. The total number of plies is even and equal to *2K*, the generalization to the odd ply number is straightforward and is not considered in the present paper. The flat plate is located in *XY* plane, it has a piecewise smooth contour *C* with external normal ν and tangent direction s; the directions ν, l and Z create a right-hand triplet. The coordinate system XYZ is a Cartesian one.

The plate is loaded by the in-plane forces p_x, p_y, acting at the part C_1 of the contour C. The remaining part C_2 of the contour the plate is not moving in X and Y. The plate is simply supported, i.e., the z-displacement and the normal bending moment M_ν acting perpendicular to the contour C are equal to zero. The clamping boundary conditions are not considered in the present paper, the change to the case is straightforward.

The X, Y, Z displacements are denoted as u, v, w, respectively.

The signs of the loads are the following. The flows N_x, N_y (along X and along Y) are positive in tension, the shear flow N_{xy} is positive when it decreases the 90° angle between the orthogonal lines at the plate surface. The flows are in equilibrium. The Composite Lamination Plate Theory (CLPT) is used for description of plate deflections (see the book of Gibson 1994).

The Classical Lamination Plate Theory (CLPT), based on linear elastic relations for every layer and Bernoulli's hypotheses, is used for obtaining the equation for the deflections w (see the books of Gibson 1994). The equation is written as follows:

$$\begin{aligned} & D_{11}\frac{\partial^4 w}{\partial x^4} + 2(D_{12}+2D_{66})\frac{\partial^4 w}{\partial x^2 \partial y^2} + D_{22}\frac{\partial^4 w}{\partial y^4} + 4D_{16}\frac{\partial^4 w}{\partial x^3 \partial y} + 4D_{26}\frac{\partial^4 w}{\partial x \partial y^3} - \\ & - N_x \frac{\partial^2 w}{\partial x^2} - N_y \frac{\partial^2 w}{\partial y^2} - 2N_{xy}\frac{\partial^2 w}{\partial x \partial y} = 0 \end{aligned} \tag{1}$$

where $D_{ij}, i = 1,2,6; j = 1,2,6,$ are the elements of the bending stiffness matrix D, coupling the bending/twisting moments and various second derivatives of the deflection w with respect to x and y. The coupling is written as follows:

$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{bmatrix} -\dfrac{\partial^2 w}{\partial x^2} \\ -\dfrac{\partial^2 w}{\partial y^2} \\ -2\dfrac{\partial^2 w}{\partial x \partial y} \end{bmatrix} \tag{2}$$

We denote the left-hand side of (2) as a vector-column $\vec{M}$, and as $\vec{k}$ the vector-column at the right-hand side (which is multiplied to D matrix). Also we use the notations for the corresponding curvatures:

$$k_x = -\frac{\partial^2 w}{\partial x^2}; k_y = -\frac{\partial^2 w}{\partial y^2}; k_{xy} = -\frac{\partial^2 w}{\partial x \partial y} \tag{3}$$

Then (2) is rewritten in the matrix-vector form as:

$$\vec{M} = D\vec{k} \tag{4}$$

With the boundary conditions indicated above, the boundary-value problem for deflections is completed.

The distribution of the in-plane stress flows is given by the relations:

$$\vec{N} = A\begin{bmatrix} u_{,x} \\ v_{,y} \\ u_{,y} + v_{,x} \end{bmatrix} \tag{5}$$

where A is the in-plane stiffness matrix, and the column-vector

$$\vec{N} = \left(N_x, N_y, N_{xy}\right)^T \tag{6}$$

T means transposing, and index after comma means the differentiation w.r.t. the corresponding coordinate.

The components of the $\vec{N}$ must satisfy the equilibrium equations, and the in-plane displacements must satisfy the corresponding boundary conditions.

It is supposed that the buckling eigenvalue is a single-modal one. The optimization goal is to design a lay-up maximizing the first (lowest) eigenvalue.

As it is shown in Selyugin (2019), the first order necessary local optimality conditions are written as follows, m=1,…,K:

$$
\begin{aligned}
&\sin 2(\theta_m-\psi)\left[\frac{U_2}{4U_3}\left(k_1^2-k_2^2\right)+\left(k_1-k_2\right)^2\cos 2(\theta_m-\psi)\right]\frac{1}{3}\left(z_m^2+z_m z_{m-1}+z_{m-1}^2\right)+\\
&+\lambda\left\{w_{,x}^2\sin 2(\theta_m-\varphi)\left[\left(-\frac{U_2}{4U_3}-\cos 2(\theta_m-\varphi)\right)\varepsilon_1+\cos 2(\theta_m-\varphi)\varepsilon_2\right]+\right.\\
&+w_{,y}^2\sin 2(\theta_m-\varphi)\left[\cos 2(\theta_m-\varphi)\varepsilon_1+\left(\frac{U_2}{4U_3}-\cos 2(\theta_m-\varphi)\right)\varepsilon_2\right]+\\
&\left.+w_{,x}w_{,y}\left[\begin{array}{l}\left(\dfrac{U_2}{4U_3}\cos 2(\theta_m-\varphi)+\cos 4(\theta_m-\varphi)\right)\varepsilon_1+\\ +\left(\dfrac{U_2}{4U_3}\cos 2(\theta_m-\varphi)-\cos 4(\theta_m-\varphi)\right)\varepsilon_2\end{array}\right]\right\}=0
\end{aligned}
\tag{7}
$$

where all angles are measured wrt X axis, $\theta_m,\psi,\varphi,U_2,U_3,k_1,k_2,\varepsilon_1,\varepsilon_2,\lambda$ respectively are the ply orientation angle, the direction angle of the largest principal curvature k_1, the direction angle of the largest in-plane strain ε_1, two elastic constant of the ply, first and second principal curvatures, first and second principal in-plane strains, the buckling eigenvalue. The subscript after comma means the differentiation wrt the corresponding coordinate. The value of z_0 is equal to zero.

The first item in (7) corresponds to the out-of-plane deflection, the second item in figure brackets (multiplied to λ) corresponds to the 2D pre-buckling behavior.

3. Analysis of the optimality conditions

The typical deflection buckling shape for the 2D uniformly compressed simply supported composite plate is shown in Figure 1.

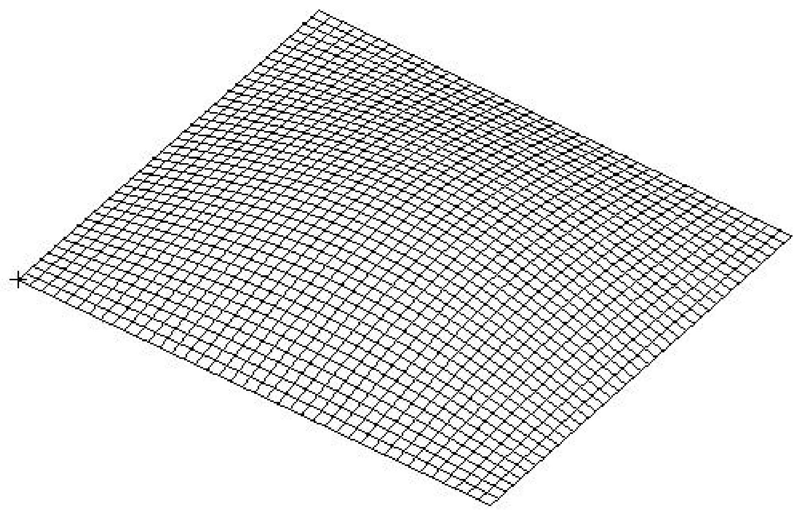

Fig. 1. Typical buckling shape of the 2D compressed composite plate.

Now we analyze the relation (7) in details. Estimating the order of magnitude of the derivatives involved in (7), we may write:

$$w_{,x} \sim \frac{w_{\max}}{a}\ ;\ w_{,y} \sim \frac{w_{\max}}{a}\ ;\ w_{,xx} \sim \frac{w_{\max}}{a^2}\ ;\ w_{,yy} \sim \frac{w_{\max}}{a^2}\ ;\ w_{,xy} \sim \frac{w_{\max}}{a^2} \tag{8}$$

where $w_{\max}, a$ respectively are the maximal deflection and the characteristic linear size like the plate width.

Supposing the ply elastic constants U_2, U_3 as having the same order of magnitude (this is true for the ordinary composite materials) and substituting (8) into (7), we obtain the estimation of the lhs of (7) as:

$$\frac{w_{\max}^2}{a^4} z_m^2 + \lambda \frac{w_{\max}^2}{a^2} \varepsilon_1 \tag{9}$$

Supposing that λ is about 1. (it is possible to do that by the proper pre-buckling level) and dividing by $\frac{w_{\max}^2}{a^2}$, we have from (9):

$$\frac{z_m^2}{a^2} + \varepsilon_1 \tag{10}$$

It follows from (10) that for the innermost plies (supposing that K~10) we may write approximately

$$\frac{(0.1h)^2}{a^2} + \varepsilon_1 \tag{11}$$

where h is the plate thickness.

In the practical buckling cases (see Kassapoglou 2010) the value of the largest principal 2D pre-buckling strain is between 0.0001 and 0.001. The h/a ratio is between 0.01 and 0.1.

It follows from the consideration that for the innermost plies of the heavily loaded wide plates the second (2D-strain) item in the optimality conditions dominates. In fact, the item determines the orientations of the innermost plies.

Now we consider the case of the outermost plies and write (10) in the form:

$$\frac{h^2}{a^2} + \varepsilon_1 \tag{12}$$

Analyzing (12) and keeping in mind the above-mentioned typical values, we may say that for the outermost plies of the heavily loaded wide plates the second (2D-strain) item in the optimality conditions may also dominate or may have the same order of magnitude as the first item related to the out-of-plane deflections.

The above analysis is a very approximate one for the central area of the plate of Figure 1. In the area the curvatures become greater and the slopes of the surface become lower as compared to the areas close to the boundaries. As we see from Figure 1, near the lateral sides the shape is nearly flat. Hence, in the central plate area the distribution of the outermost ply orientation angles is determined by the first item in (7). As to the area close to the boundaries, the optimal ply orientations are mainly determined by the second item in (7).

4. Discussion

As it is shown above, the 2D-strain item of the optimality conditions plays an important role in determination of the optimal ply orientation angles. The above consideration gives the theoretical basis, explains and confirms the indications of Wu et al 2012 ("the load redistribution is a main driver for buckling improvements of VAT plates") and Groh and Weaver 2015 ("Numerous works have shown that tailoring the in-plane stiffness of a plate allows pre-buckling stresses to be re-distributed to supported regions, thereby improving the buckling behavior"). The indications were based on the numerical experience.

5. Conclusions

- The known numerical results are theoretically justified

- The theoretical analysis of the optimality conditions confirms the important general role of the 2D strain distribution
- Near the boundaries of the plate the 2D-strain distribution mainly determines the optimal ply angle distribution
- For the innermost plies of the heavily loaded wide plates the 2D-strain distribution mainly determines the ply orientations
- Near the maximal deflection point(s) of the plate the curvature distribution mainly determines the optimal ply angle distribution

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