

Inverse Modeling of Zonal Transmission Network

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Abstract

Presently, many ideas have been proposed for constructing zonal models of the European transmission network. Most of them are based on transmission network reduction and bus clustering, which requires the ex-ante knowledge on the details (impedance, interconnections, etc.) of the network's elements. Generally, the information is not publicly disclosed. This paper describes a method for serving the purpose, which only requires the ex-post (i.e., published) inter-zonal power flow data as input. It performs inverse modeling on the ex-post data to reveal the virtual reactance of the interconnectors in the zonal model. The reactance values are then used for computing the zonal Power Transfer Distribution Factors (PTDFs) of the model, which is the best representation of the loop flows. A large number of such zonal PTDFs could exist in one zonal model because of its dependency on the Generation Shift Keys (GSKs). Theoretically, the GSKs could exist as an infinite set of unique combinations but only a finite number of them applied in real system operations. As a solution, the authors propose a heuristic approach for grouping the ex-post power flow cases into clusters, and find the zonal PTDF for each of them; in order to effectively reduce the domain of the zonal PTDFs and enhance the tractability of the solution. The identified zonal PTDFs meaningfully represent the zonal model. Additionally, they can also be used for studying the best and worst case scenarios of cross-border security-constrained economic dispatch.

Keywords: Zonal model, transmission network modeling, inverse problem, test case clustering, clustering heuristics

I Introduction

The last decade has seen several exciting developments in the European single market in electricity [1]. Among others, the development of a common market design through system couplings remain active [2]. As a result, the Flow-Based Market Coupling (FBMC) mechanism is established in the Central-Western Europe (CWE) in May 2015 [3]. Regulators in other regions too (for example, the Nordic Regional Security Coordinator) had begun to study the feasibility of FBMC in their respective region. Eventually, the model is expected to be extended to the entire Europe [3].

The FBMC model entails many contributions from the stakeholders (mainly the Transmission Systems Operators (TSOs)) to establish a common set of rules for the sharing of generation and transfer capacity between the coupled market zones. Its most important design is the use of a multi-zonal common grid model [4] (henceforth called as the zonal model) to properly model the cross-border loop flow of electric power. This is important for the market and operational planning involving Security Constrained Economic Dispatch (SCED).

For example, the 9-bus transmission network model shown in Figure 1 (a) can be represented by a 3-bus zonal model (Figure 1 (b)). The SCED could be calculated on the zonal models to derive the least-cost dispatch for serving the demand of the coupled markets. The calculation requires the knowledge of the aggregated Power Transfer Distribution Factors (PTDFs), $\beta_{z \rightarrow z_0, m}^A$ which is the magnitude of change in power transferred at a cross-border interconnection m (comprise of all lines that connects buses which belongs to two market zones) contributed by the transfer of one unit of power at a zone, (zone z) to a reference zone (zone z_0); and the allocated cross-border transfer capacity. The transfer capacity can be allocated more precisely compared to the existing market coupling (the existing method is based on the available transfer capacity approach), because the loop flow pattern in the entire zonal model is well defined by the $\beta_{z \rightarrow z_0, m}^A$.

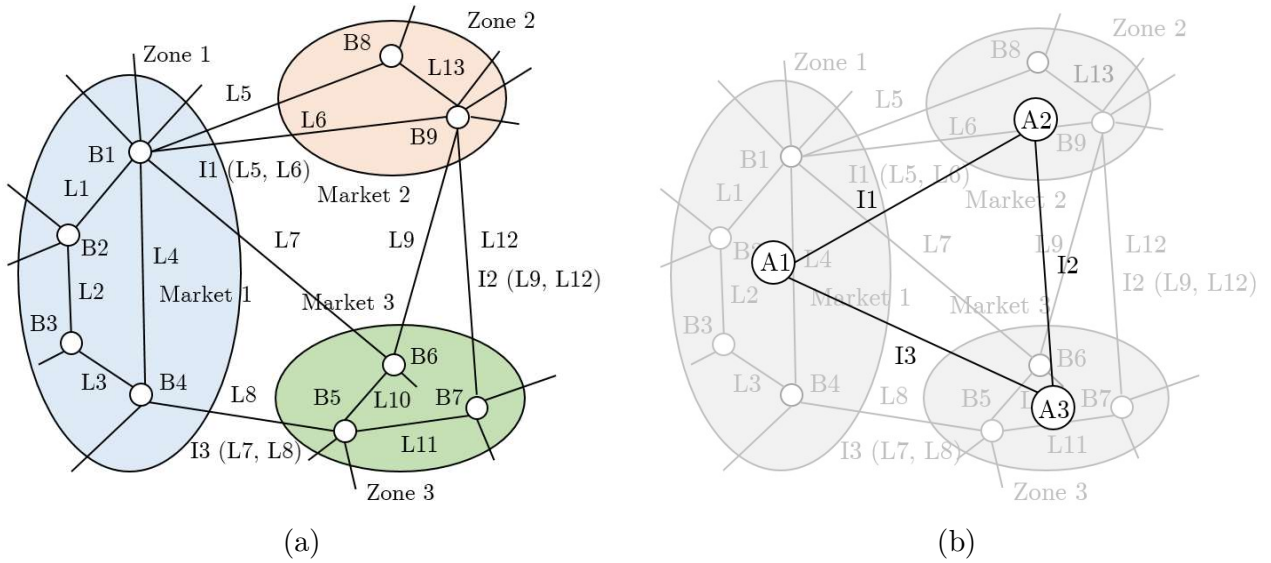


Figure 1. (a) A 9-bus transmission network model, (b) the 3-bus zonal model of the transmission network

II The Formulations

The greatest advantage of FBMC is its ability to remove the impediments to competition by means of promoting the non-discriminatory access to the external power generation and transmission services through proper modeling of the cross-border loop flows. This is the key to the running of a successful regional electricity market [5]. Nonetheless, its implementation is not straightforward. Especially, an infinite number of unique zonal PTDF matrix Γ^A (a two-dimension matrix containing all $\beta_{z \rightarrow z_0, m}^A$) could exist in the zonal model. Each of them is associated with a Generation Shift Keys (GSKs) matrix, Π_z which comprise the commitment of the generators in zone z to the production of one unit of power. The Γ^A is derived based on the nodal (bus-level) PTDFs matrix, Γ comprises all $\beta_{i \rightarrow i_0, k}$ (the magnitude of change in power transfer at transmission line k corresponding to the transfer of a one unit of power at a bus (bus i) to a reference bus (bus i_0)). It is calculated as follows [9]

$$\Gamma = B_f B_r^{-1} \quad (1)$$

where, B is a matrix defined based on the susceptance of the transmission lines, $b_{ij} = 1/x_{ij}$ (where x_{ij} is the reactance of a transmission line k which connects bus i and j), such that its non-diagonal elements are, $B_{ij} = B_{ji} = -b_{ij}$; $\forall i \neq j$ and its diagonal elements are, $B_{ii} = \sum_{j \neq i}^N b_{ij}$. The B_r^{-1} is the inverse of the reduced B matrix, B_r (the row and column in the B matrix associated with an arbitrarily selected reference bus is removed to form the B_r). The B_f is a two-dimension matrix where each row is associated with transmission line k ; in each row, the value is b_{ij} at column i , and $-b_{ij}$ at column j . The column associated with the reference bus is as well removed. The derived Γ is a two-dimension matrix where the row index correspond to transmission line k and each column corresponds to bus i .

The Γ^A cannot be directly calculated by (1), since the reactance of its cross-border interconnectors (interconnector m), x_m is not known. Also, the x_m is not a simple sum of x_{ij} of all of its constituting transmission lines. Instead, it has to be derived using Γ and the GSKs of generators in each zone, $\Pi_z = \{\pi_{1,z}, \dots, \pi_{G,z}\}$, where $\sum_{g=1}^G \pi_{g,z} = 1$. For example, $\Pi_1 = \{0, 0.4, 0.5, 0.1\}$ indicates the first generator in zone 1 is not involved in the dispatch of the next unit of power, the second generator will dispatch 40% of it, the third generator will dispatch 50% of it and the forth generator will dispatch 10% of it. The $\beta_{z \rightarrow z_0, m}^A$ is calculated as follows

$$\beta_{z \rightarrow z_0, m}^A = \sum_g \left(\sum_k \pi_{g,z} \beta_{i^* \rightarrow i_0, k} \right), \forall k \in K_m, z \in \{z_0, z\}, z \neq z_0 \quad (2)$$

where, i^* is the index of the bus which is connected with generator g , z_0 is the index of the reference zone, and K_m is the set of indices of all the constituting transmission lines of interconnector m .

Table I shows a sample calculation of $\beta_{2 \rightarrow 1, 1}^A$ of the zonal model in Figure 1 (b), which is the changes in power flow of interconnector I1 associated to the supply of 1 unit of marginal power in zone A2 to the reference zone, A1 (the $\pi_{g,z}$ of generators in zone 1 is of negative sign). The result shows a 1 MW change in A2 will produce 0.6825 MW power flow on I1 in reverse direction (indicated by the negative sign).

Table I. An example of calculation of $\beta_{2 \rightarrow 1, 1}^A$ of the zonal model of Figure 1 (b)

Zone	g	Bus	$\pi_{g,z}$	Line	$\beta_{i^* \rightarrow i_0, k}$	$\pi_{g,z} \beta_{i^* \rightarrow i_0, k}$
A1	1	B1	0	L5	0	0
A1	1	B1	0	L6	0	0
A1	2	B2	-0.4	L5	-0.0090	0.0036
A1	2	B2	-0.4	L6	-0.0381	0.0152
A1	3	B3	-0.5	L5	-0.0186	0.0093
A1	3	B3	-0.5	L6	-0.0785	0.0393
A1	4	B4	-0.1	L5	-0.0312	0.0031
A1	4	B4	-0.1	L6	-0.1321	0.0132
A2	1	B8	0.7	L5	-0.5612	-0.3928
A2	1	B8	0.7	L6	-0.2590	-0.1813
A2	2	B9	0.3	L5	-0.1224	-0.0367
A2	2	B9	0.3	L6	-0.5180	-0.1554
$\beta_{2 \rightarrow 1, 1}^A$						-0.6825

The entire Γ^A could be fully computed by repeating the calculation for all of the zones and interconnectors. Once the Γ^A is computed, the power flow at the interconnector m , F_m can be computed as follows

$$F_m = \sum_{z=1}^Z P_z^A \beta_{z \rightarrow z_0, m}^A \quad (3)$$

where, P_z^A is the sum of power generation in zone z .

As shown in (2), the Γ^A is a function of Π_z because the $\beta_{i^* \rightarrow i_0, k}$ is constant in value. This is a known shortcoming of FBMC [6] [7]. Generally, there could exist an infinite number of possible Π_z (by changing the $\pi_{g,z}$) which yield an infinite set of Γ^A . As a different combination of $\pi_{g,z}$ values (in a unique Π_z) would yield a completely different power flow at the interconnectors, the domain of Γ^A of a zonal model is non-linear. In fact, a change in power dispatched by a generator located at a different position relative to an interconnector would result in a different power flow (in magnitude and direction) on it. Table II shows an example on the relationship between the $\beta_{2 \rightarrow 1, 1}^A$ and Π_2 of the 3-bus model (Figure 1 (b)). The value of $\beta_{2 \rightarrow 1, 1}^A$ is different for a different Π_2 .

Table II. A example of the non-linear relationship between Γ^A and Π_2

Bus	$\pi_{g,z}^1$	$\pi_{g,z}^2$	$\pi_{g,z}^3$
B8	0.7	0.9	0.5
B9	0.3	0.1	0.5
$\beta_{2 \rightarrow 1, 1}^A$	-0.68	-0.72	-0.65

Table III. Three cases of Π_z of the 3-bus zonal model (Figure 1 (b))

Zone	Bus	$\pi_{g,z}^{\text{ref}}$	$\pi_{g,z}^1$	$\Delta \pi_{g,z}^1$	$\pi_{g,z}^2$	$\Delta \pi_{g,z}^2$
A1	B1	0	0	-	0	-
A1	B2	-0.4	-0.400	-	-0.400	-
A1	B3	-0.5	-0.525	5 %	-0.525	5 %
A1	B4	-0.1	-0.075	-25 %	-0.075	-25 %
A3	B5	0.5	0.500	-	0.500	-
A3	B6	0.4	0.420	5 %	0.420	5 %
A3	B7	0.1	0.080	-20 %	0.080	-20 %
A2	B8	0.7	0.700	-	0.100	-86 %
A2	B9	0.3	0.300	-	0.900	200 %

III The non-linearity of the zonal PTDFs

Nonetheless, the changes in the Γ^A is less sensitive to minor changes in Π_z . An example is shown in Table III, which comprises a reference case with $\pi_{g,z}^{\text{ref}}$ values, a case (case 1) with $\pi_{g,z}^1$ being slightly different from the $\pi_{g,z}^{\text{ref}}$ (up to 25 %), and a case (case 2) with $\pi_{g,z}^2$ being substantially different than $\pi_{g,z}^{\text{ref}}$ (up to 200%). Table IV shows the difference between $\beta_{z \rightarrow z_0, m}^A$ of case 1 and 2 and the reference case. As shown, case 1 only entails a 0.61 % maximum shift in $\beta_{z \rightarrow z_0, m}^A$. Meanwhile, case

2 entails a greater shift in $\beta_{z \rightarrow z_0, m}^A$ for up to 33.49 %. Thus, the Γ^A of cases with low variance in Π_z are approximately similar. For example, the Γ^A of the reference case could be applied to approximate the F_m without a great compromise in its accuracy.

Table IV. The $\beta_{z \rightarrow z_0, m}^A$ value for each case given in Table III

Zone	Interface	$\beta_{z \rightarrow z_0, m}^{A, \text{ref}}$	$\beta_{z \rightarrow z_0, m}^{A, 1}$	$\Delta\beta_{z \rightarrow z_0, m}^{A, 1}$	$\beta_{z \rightarrow z_0, m}^{A, 2}$	$\Delta\beta_{z \rightarrow z_0, m}^{A, 2}$
A1	I1	0	0	0	0	0
A1	I2	0	0	0	0	0
A1	I3	0	0	0	0	0
A2	I1	-0.6825	-0.6842	-0.25 %	-0.5763	15.56%
A2	I2	0.3174	0.3157	-0.54 %	0.4237	33.49%
A2	I3	-0.3174	-0.3157	0.54 %	-0.4237	-33.49%
A3	I1	-0.2118	-0.2105	0.61 %	-0.2105	0.61%
A3	I2	-0.2118	-0.2105	0.61 %	-0.2105	0.61%
A3	I3	-0.78881	-0.7894	-0.07 %	-0.7894	-0.16%

Hence, a few clusters of cases which share an approximately identical Γ^A exist in a zonal model. The cases can be identified and grouped based on their proximity in Π_z . Each of them possesses an approximate $\Gamma^{A,*}$, that could be used for approximating their actual F_m with minor errors. This greatly reduces the (otherwise infinite) number of Γ^A in the zonal model, making it (and the FBMC) a more tractable problem. The concept also resembles the actual network operations where there are preferably less diversity of Π_z in used.

IV The inverse modeling of transmission network

The grouping of the cases into clusters would effectively reduce the non-linear domain of zonal model. Nevertheless, the zonal modeling still could not be done as the details of the transmission network (i.e., the x_k and Π_z) are not known. The authors applied the inverse modeling [4] to construct the zonal model and identify all of its $\Gamma^{A,*}$ based on the ex-post inter-zonal power flow data.

4.1 The method

The inverse modeling of a transmission network is introduced in [4]. It is derived based on the linear power flow theory [8]. As shown in (1), knowing the x_k is the most important step for the computation of the Γ . Similarly, knowing the x_m allows us to compute the Γ^A . The authors applied the inverse modeling and the clustering approach to compute the list of all $\Gamma^{A,*}$ in a zonal model.

The linear power flow theory relates the voltage angle to the net power generated at the buses of a transmission network as follows

$$P = B\Theta \quad (4)$$

where, $P = [P_1, \dots, P_N]^T$ contains the net active power at each bus, and $\Theta = [\theta_1, \dots, \theta_N]^T$ contains the voltage angle at each bus. The knowledge of Θ is needed for calculating the power flow on each transmission line, which is computed as follows

$$\Theta = B^{-1}P \quad (5)$$

Then, the power flow on the transmission line k , F_k which connects buses i and j can be calculated as follows

$$F_k = B_{ij}(\theta_i - \theta_j) \quad (6)$$

The inverse modeling intends to find an approximated value of all x_k in the network, or $X^* = [x_1^*, \dots, x_K^*]$, that will meet the following objective:

$$\phi = \sum_{k=1}^K |F_k^* - F_k| \approx 0 \quad (7)$$

The F_k is the ex-post power flow, and F_k^* is the power flow calculated using the X^* based on (5) and (6). The approximation is successful when an X^* that yield F_k^* value closest possible to the known F_k for all of the transmission lines is found. There are two stages of calculation involved. Firstly, the approximated x_k has to be determined and the B matrix (in (5)) has to be calculated. Secondly, the Θ and F_k^* has to be calculated. The problem has to be solved iteratively. The calculation of the approximated Γ , Γ^* is performed using (1) after an optimal X^* is found.

The approach is directly applicable to zonal modeling, by designating the zones as buses, the cross-border interconnectors as the transmission lines, and the cross-border power flow as the F_k . The X^* derived is used to compute the approximated Γ^A , $\Gamma^{A,*}$. The authors applied the Genetic Algorithm (GA) as the solver. The details of the GA's implementation is described in [4].

During implementation, it is possible to find several unique X^* that would meet the ϕ (in (7)). The solution space of X^* is thus unbounded. It is difficult to ensure the uniqueness of the solution by only inverting one set of the ex-post power flow data. In such case, the GA finds different sub-optimal X^* in different repetitions, while only one of them is globally optimal. This problem could be solved by adopting multiple ex-post power flow data (henceforth denoted by index s) in ϕ to evaluate the quality of X^* :

$$\phi = \sum_s \sum_{k=1}^K |F_k^{*,s} - F_k^s| \approx 0 \quad (8)$$

Generally, a globally optimal X^* could be found by adopting 3 cases in (8). Interestingly, the concept of the cross-border flow clustering matches well with (8) as the clusters comprises a group of ex-post zonal flow cases with approximately identical X^* and $\Gamma^{A,*}$. The main advantage of the inverse modeling is its capability in identifying all $\Gamma^{A,*}$ in the zonal network model without requiring the ex-ante knowledge of Π_z . The method is especially useful for correctly identifying all of the $\Gamma^{A,*}$ of a zonal model. For transmission network model reduction (where the transmission network details are fully known), it would be sufficient to use (2) to calculate the $\Gamma^{A,*}$ of the zonal model. Then, a zonal model based SCED could be meaningfully implemented on the $\Gamma^{A,*}$.

4.2 The clustering of ex-post power flow cases

The clustering of the cases in the ex-post data of a zonal model could be done using a simple heuristic method. The cases (denoted by notion s) in a cluster (that has the same $\Gamma^{A,*}$) would exhibit the following characteristics:

- a) Each of the zones in the cases will have an approximately identical value in power generation ratio, w_z^P compared to their counterpart in the other cases of the same cluster. For each case,

the w_z^P is calculated by dividing the aggregated total generation at zone z , P_z^A by a weighting factor, W which is the largest absolute value of the P_z^A in the case. For a case denoted with s , the W^s is calculated as follows

$$W^s = \max(|P_z^{A,s}|); \forall z \in \{1, \dots, Z\} \quad (9)$$

Hence, the $w_z^{P,s}$ is defined as:

$$w_z^{P,s} = \frac{P_z^{A,s}}{W^s} \quad (10)$$

The cases could potentially be grouped in the same cluster if they meet the following criteria:

$$w_z^{P,s} \approx w_z^{P,s_{\text{ref}}}; \forall s \in \{1, \dots, S\}; \forall z \in \{1, \dots, Z\} \quad (11)$$

where, s_{ref} is an arbitrarily selected case.

- b) The cases which met the criteria of (11) could be grouped in the same cluster if they have an approximately identical value in their interconnector power flow ratio, $w_m^{F,s}$ compared with their counterpart in the other cases. The ratio is calculated by dividing the F_m^s by the W^s in (9):

$$w_m^{F,s} = \frac{F_m^s}{W^s} \quad (12)$$

The cases from the same cluster (possessing the same $\Gamma^{A,*}$) would meet the following criteria:

$$w_m^{F,s} \approx w_m^{F,s_{\text{ref}}}; \forall s \in \{1, \dots, S\}; \forall z \in \{1, \dots, Z\} \quad (13)$$

For instance, the w_m^F of I1, w_1^F of Figure 1 (b) of two arbitrary cases are formulated as follows

$$\begin{aligned} w_1^{F,1} &= \frac{F_1^1}{W^1} \\ &= \frac{P_2^{A,1} \beta_{2 \rightarrow 1,1}^{A,1} + P_3^{A,1} \beta_{3 \rightarrow 1,1}^{A,1}}{W^1} \\ &= w_2^{P,1} \beta_{2 \rightarrow 1,1}^{A,1} + w_3^{P,1} \beta_{3 \rightarrow 1,1}^{A,1} \end{aligned} \quad (14)$$

and,

$$\begin{aligned} w_1^{F,2} &= \frac{F_1^2}{W^2} \\ &= \frac{P_2^{A,2} \beta_{2 \rightarrow 1,1}^{A,2} + P_3^{A,2} \beta_{3 \rightarrow 1,1}^{A,2}}{W^2} \\ &= w_2^{P,2} \beta_{2 \rightarrow 1,1}^{A,2} + w_3^{P,2} \beta_{3 \rightarrow 1,1}^{A,2} \end{aligned} \quad (15)$$

In (14) and (15), if $w_1^{F,1} \approx w_1^{F,2}$ and $[w_2^{P,1}, w_3^{P,1}]^\top \approx [w_2^{P,2}, w_3^{P,2}]^\top$, then $[\beta_{2 \rightarrow 1,1}^{A,1}, \beta_{3 \rightarrow 1,1}^{A,1}]^\top = [\beta_{2 \rightarrow 1,1}^{A,2}, \beta_{3 \rightarrow 1,1}^{A,2}]^\top$. Both case 1 and 2 would have the same $\Gamma^{A,*}$.

Table V shows an example of three ex-post power flow cases of the same cluster, where three of them have approximately identical value of $w_z^{P,s}$ and $w_m^{F,s}$. A $\Gamma^{A,*}$ could be found for the cluster. Table VI, and VII respectively shows the $\Gamma^{A,*}$ found by GA (with $\phi = 0.0013$) and the precision of the resulting power flow at the interconnectors computed using the $\Gamma^{A,*}$.

Basically, this is a hard clustering approach because the cases in each cluster must strictly fulfil (11) and (13). In reality, the $\Gamma^{A,*}$ of some of the distinct clusters are identical. The cases are identified as the members of different cluster because they have different pattern of P_z^A . This issue can be handled by testing a new case with $\Gamma^{A,*}$ of identified clusters. The cases belong to the same cluster if the identified $\Gamma^{A,*}$ could reproduce the F_m of all the interconnectors in the new case.

Table V. An example of three ex-post power flow cases

Zone	$P_z^{A,1}$	$w_z^{P,1}$	$P_z^{A,2}$	$w_z^{P,2}$	$P_z^{A,3}$	$w_z^{P,3}$
A1	12	1.00	5.0	1.00	45	1.00
A2	-3	-0.25	-1.3	-0.26	-11	0.24
A3	-9	-0.75	-3.7	-0.74	-34	-0.76
Interconnector	F_m^1	$w_m^{F,1}$	F_m^2	$w_m^{F,2}$	F_m^3	$w_m^{F,3}$
I1	3.9537	0.3295	1.6709	0.3342	14.7087	0.3269
I2	0.9540	0.0795	0.3710	0.0742	3.7098	0.0824
I3	8.0451	0.6704	3.3286	0.6657	30.2868	0.6730

Table VI. The $\Gamma^{A,*}$ of the test cases (in Table V)

Zone	Interconnector	$\beta_{z \rightarrow z_0,m}^A$
A1	I1	0
A1	I2	0
A1	I3	0
A2	I1	-0.684672
A2	I2	0.315328
A2	I3	-0.315328
A3	I1	-0.211141
A3	I2	-0.211141
A3	I3	-0.788859

Table VII. The precision of F_m computed using the $\Gamma^{A,*}$ (in Table VI)

Case	Interconnector	F_m^*	F_m	$ F_m^* - F_m $
1	I1	3.9539	3.9537	0.0002
1	I2	0.9542	0.9540	0.0002
1	I3	8.0449	8.0451	0.0002
2	I1	1.6711	1.6709	0.0002
2	I2	0.3713	0.3710	0.0003
2	I3	3.3284	3.3286	0.0002
3	I1	14.7807	14.7087	0
3	I2	3.7098	3.7098	0
3	I3	30.2868	30.2868	0
ϕ				0.0013

V Discussions and conclusions

The paper described a method to construct the zonal network model based on ex-post power flow data. It described the concept of Γ^A , which is a function of Π_z and $\beta_{i^* \rightarrow i_0,k}$. It also pointed out that the domain of Γ^A is large and non-linear, because of its dependency of Π_z which exist in unique combinations. Then it demonstrates that the Γ^A is insensitive to reasonably small changes in Π_z .

This allows the inverse modeling of cases with close proximity in Γ^A to derive the $\Gamma^{A,*}$ through the iterative search of the X^* . The authors introduced a simple heuristic approach to identify cases with identical $\Gamma^{A,*}$. The clustering effectively reduce the domain of the inverse modeling problem, thus enhancing its tractability.

The method is developed for serving market coupling studies under the FBMC initiatives. Its major strength is its independence on the knowledge of the physical transmission network. It only requires the TSO's historical cross-border power flow data as input in its implementation. Besides the FBMC studies, the method can also be used for long-term (multi-stage) simulation of SCED (i.e., hydroelectric power scheduling). It has substantially better representation of the physical flow, compare to the point-to-point simulation approach. The multiple $\Gamma^{A,*}$ of the zonal network model derived could be used for studying the best and worst case grid operation scenarios.

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