

A Ping-Pong Algorithm for Computational Electromagnetics of 2D Antennas/Metasurfaces

Lu Ou

College of Computer Science and Electronic Engineering
Hunan University
Changsha, Hunan, China
oulu9676@gmail.com

Shaolin Liao*

Department of Electrical and Computer Engineering
Illinois Institute of Technology
Chicago, IL, USA
sliao5@iit.edu; ORCID: 0000-0002-4432-3448

Abstract—An efficient iterative Ping-Pong Computational Electromagnetics (CEM) algorithm has been developed to simulate the 2D planar reflector antennas and metasurfaces of arbitrary shapes. Without the storage and inverse of the impedance matrix required by the Method of Moments (MoM), the Ping-Pong algorithm updates the surface current and the scattering electric field iteratively. In particular, the first iteration gives the surface current for the reflector antennas or metasurfaces on the substrates of prefect absorption. Numerical simulation has been carried out for an ellipse reflector antenna and a cross-shaped metasurface. It has been shown that convergence occurs within 10 iterations and good agreement between the Ping-Pong algorithm and the MoM has been achieved.

Index Terms—Computational Electromagnetics (CEM), Ping-Pong algorithm, Method of Moments (MoM), planar reflector antennas, metasurfaces.

I. INTRODUCTION

Efficient Computational Electromagnetics (CEM) algorithms are important for design of 2D planar reflector antennas [1] and metasurfaces [2]. However, fast and accurate simulation is challenging when the wavelength is comparable to the dimensions of the reflector antennas or metasurfaces because resonance phenomenon could be involved. Even worse, for electrically large structures for applications from microwave and millimeter wave to optics [3]–[64], highly efficient CEM algorithms have to be developed to save the memory and computation time. Method of Moments (MoM) has been a popular CEM algorithm for PEC structure simulation [65] because only the unknown surface current coefficients are needed to be stored and solved, greatly saving the memory and computation time.

However, MoM still needs to store the impedance matrix and perform the inverse of the impedance matrix to obtain the solution of the surface current. In this paper, we present an iterative Ping-Pong CEM algorithm to remove all of these limitations met in the MoM, *i.e.*, the storage and inverse of the impedance matrix. What's

more, the Ping-Pong algorithm works for 2D planar reflector antennas and metasurfaces of arbitrary shapes.

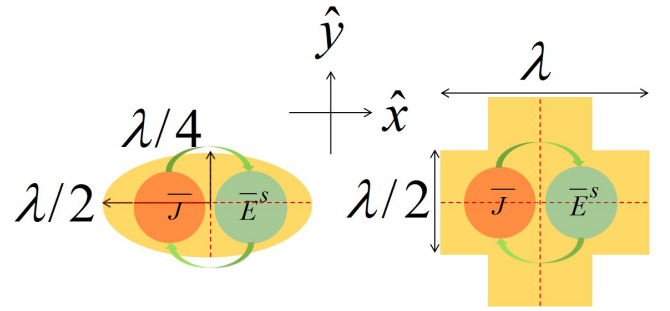


Fig. 1. Planar 2D structures under study: left) an ellipse antenna; and 2) a cross-shaped metasurface. Also shown is the Ping-Pong updating process of the surface current $\bar{J}$ and the scattering electric field $\bar{E}^s$.

II. PROBLEM FORMULATION

The electromagnetic scattering problem is shown in Fig. 1. The electromagnetic source field is incident upon the 2D planar PEC reflector antenna (the ellipse antenna here) or the metasurface (cross-shaped metasurface here) and we want to solve for the unknown surface current. In this paper, the plane wave incident field polarized in $\hat{y}$ direction is used.

A. The PEC Boundary Condition

According to the PEC boundary condition, the total tangential electric field vanishes on the PEC surface,

$$\hat{n} \times [\bar{E}^s(\bar{r}) + \bar{E}^i(\bar{r})] \Omega(\bar{r}) = 0, \quad (1)$$

where $\bar{r}$ is the observation point; Ω is the binary mask that defines the geometry area of the reflector antenna; the superscripts s and i denote the scattering and incident electric field, respectively; finally, $\hat{n}$ is the unit normal vector of the surface.

B. Forward Update: Surface Current to Scattering Field

During the forward update process, the scattering electric field $\bar{E}^s$ can be obtained through the convolution

*Shaolin Liao is the corresponding author.

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of the surface current $\mathbf{J}$ and the electric dyadic Green's function $\bar{\bar{G}}_e$,

$$\bar{E}^s(\bar{r}) = \mathcal{L}\{\bar{J}\} = -j\omega\mu \iint_{S' \in \Omega} \bar{\bar{G}}_e(R) \bar{J}(\bar{r}') dS', \quad (2)$$

where $\mathcal{L}$ is the forward update operator; $R = |\bar{r}| - |\bar{r}'|$ is the distance from the source point $\bar{r}'$ to the observation point $\bar{r}$; ω is the angular frequency; μ is the permeability; and the electric dyadic Green's function is given as Eq (3),

$$\bar{\bar{G}}_e(\bar{r}) = g(\bar{r})\bar{I} + \frac{1}{k^2} \nabla \nabla g(\bar{r}), \quad (3)$$

and $\bar{I}$ is the identity matrix; k is the magnitude of the wave vector $\bar{k}$; and $g(\mathbf{r})$ is the scalar Green's function,

$$g(r) = \frac{e^{-jkr}}{4\pi r}, \quad k = |\bar{k}| = \omega\sqrt{\mu\epsilon},$$

with ϵ being the permittivity.

C. The Electric Field Integral Equation (EFIE)

According to the boundary condition of Eq. (1) and with the help of the scattering electric field of Eq. (2), the EFIE is obtained as follows,

$$\hat{n} \times \left[\iint_{-\infty}^{\infty} \iint_{-\infty}^{\infty} \bar{\bar{G}}_e(R) \bar{J}(\bar{r}') dS' + \bar{E}^i(\bar{r}) \right] \Omega(\bar{r}) = 0. \quad (4)$$

D. Backward Update: Scattering Field to Surface Current

In the spectral domain, the Fourier spectrum of the surface current of Eq. (4) can be solved from the Fourier spectrum of the scattering electric field [4],

$$\begin{aligned} \mathcal{J}_x(k_x, k_y) &= \frac{-2}{\omega\mu} \left[\frac{k^2 - k_y^2}{k_z} \mathcal{E}_x^s(k_x, k_y) + \frac{k_x k_y}{k_z} \mathcal{E}_y^s(k_x, k_y) \right], \\ \mathcal{J}_y(k_x, k_y) &= \frac{-2}{\omega\mu} \left[\frac{k^2 - k_x^2}{k_z} \mathcal{E}_y^s(k_x, k_y) + \frac{k_x k_y}{k_z} \mathcal{E}_x^s(k_x, k_y) \right], \end{aligned}$$

whose inverse Fourier transform gives the surface current in the spatial domain,

$$\begin{aligned} \bar{J}(x, y) &= \mathcal{L}^{-1} \left\{ \bar{E}_{//}^s \right\} \\ &= \frac{4\omega\epsilon}{j} \left\{ \bar{I} + \frac{1}{k^2} \begin{bmatrix} \frac{\partial^2}{\partial y^2} & -\frac{\partial^2}{\partial x \partial y} \\ -\frac{\partial^2}{\partial x \partial y} & \frac{\partial^2}{\partial x^2} \end{bmatrix} \right\} \left[g(x, y) \otimes \bar{E}_{//}^s(x, y) \right], \end{aligned} \quad (5)$$

where $\mathcal{L}^{-1}$ is the backward update operator and the subscript $//$ denotes the tangential component.

III. THE ITERATIVE PING-PONG ALGORITHM

With the forward update from the surface current to the scattering field of Eq. (2) and the backward update from the scattering field to the surface current of Eq. (5), the surface current can be updated iteratively like a Ping-Pong process,

$$\bar{J}_{n+1} = \mathcal{L}^{-1} \left\{ \bar{\Omega} \bar{E}_{//,n}^s - \Omega \bar{E}_{//}^i \right\}, \quad (6)$$

$$\bar{E}_{n+1}^s = \mathcal{L} \left\{ \Omega \bar{J}_{n+1} \right\}, \quad (7)$$

where $\bar{\Omega} = 1 - \Omega$ is the complementary area of the reflector antenna or metasurface.

Also, the initial solution of the surface current is given by,

$$\bar{J}_1 = \mathcal{L}^{-1} \left\{ -\Omega \bar{E}_{//}^i \right\}, \quad (8)$$

which corresponds to the solution for the reflector antennas or metasurfaces on substrates of perfect absorption because the scattering electric field vanishes outside of the PEC surface.

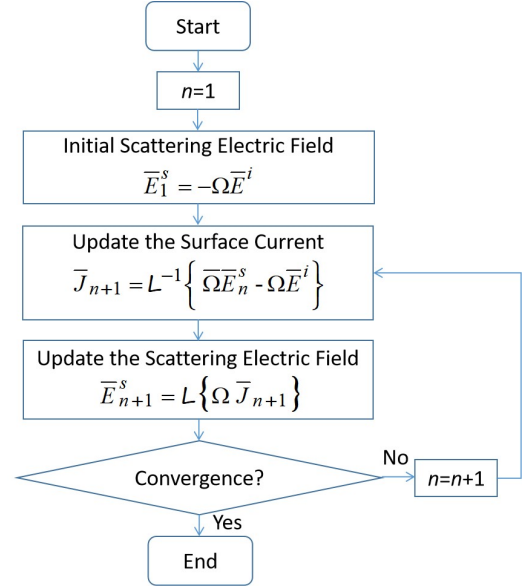


Fig. 2. The Ping-Pong algorithm.

IV. THE PING-PONG ALGORITHM

Fig. 2 shows the Ping-Pong algorithm: it starts with the initial surface current of Eq. (8); then it perform ping-pong process to update the scattering field and surface current, as shown in Fig. 1 — the forward update of the scattering field from the surface current is done according to Eq. (7); followed by the backward update of the surface current from the scattering field according to Eq. (6); finally, this Ping-Pong process repeats until the convergence condition is met.

V. NUMERICAL VALIDATION

Two typical 2D PEC structures are used to show the efficiency of the Ping-Pong algorithm, as shown in Fig. 1: 1) an ellipse antenna with semi-major width of $\lambda/2$ and semi-minor height of $\lambda/4$; and 2) a cross-shaped metasurface with length of 1λ and width of $\lambda/2$. Also, plane wave incidence of $\hat{y}$ polarization is used.

Fig. 3 shows the surface current distribution (normalized by the wave impedance $\eta = \sqrt{\mu/\epsilon}$) for the ellipse antenna with 4 iterations and Fig. 4 shows the comparison

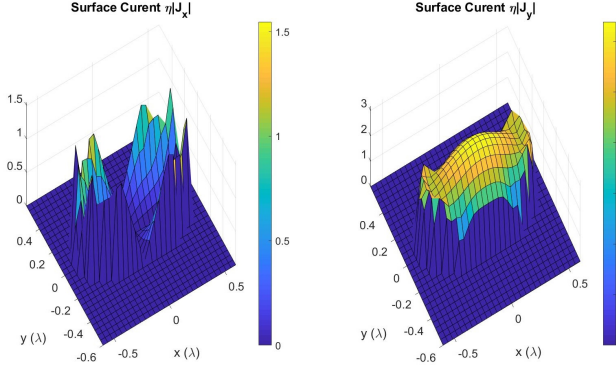


Fig. 3. Surface current distribution of the ellipse antenna: left) $\hat{x}$ component $\eta|J_x|$; and right) $\hat{y}$ component $\eta|J_y|$.

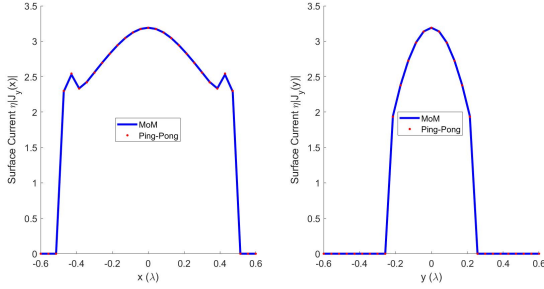


Fig. 4. Comparison of the ellipse antenna's surface current ($\hat{y}$ component $\eta|J_y|$) between our Ping-Pong algorithm and those of the : left) along the $\hat{x}$ direction; and right) along the and $\hat{y}$ direction.

of the surface currents between the Ping-Pong algorithm (red dots) and the MoM (blue lines), along $\hat{x}$ and $\hat{y}$ directions, as shown by the red dashed lines in Fig. 1. It can be seen that good agreement is achieved between the Ping-Pong algorithm and the MoM.

Similarly, Fig. 5 and Fig. 6 show the surface current distribution of the cross-shaped metasurface with 6 iterations and its comparison between the Ping-Pong algorithm (red dots) and the MoM (blue lines), from which it can be seen that good agreement has been achieved again.

VI. CONCLUSION

An efficient iterative Ping-Pong algorithm has been developed to simulate the 2D planar reflector antennas and metasurfaces of arbitrary shapes. Without the storage and inverse of the impedance matrix required by the MoM, the Ping-Pong algorithm iteratively updates the surface current and the scattering electric field until they converge. In particular, the first iteration gives the solution for the 2D planar reflector antennas or metasurfaces on substrates of perfect absorption. What's more, it usually takes less than 10 iterations for the Ping-Pong algorithm to converge, dramatically increasing the simulation speed. At last, it is straightforward to apply the Ping-Pong algorithm to simulate the 2D arrays of reflector antennas and metasurfaces.

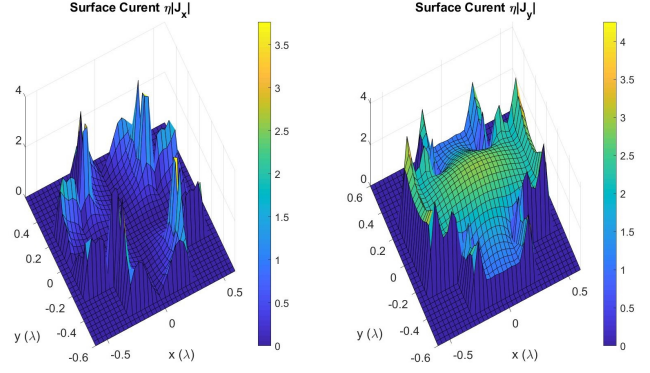


Fig. 5. Surface current distribution of the cross-shaped metasurface: left) $\hat{x}$ component $\eta|J_x|$; and right) $\hat{y}$ component $\eta|J_y|$.

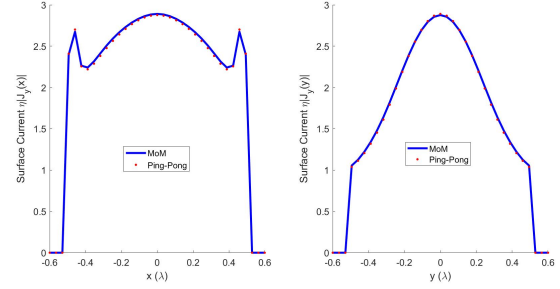


Fig. 6. Comparison of the cross-shaped metasurface's surface current ($\hat{y}$ component $\eta|J_y|$) between our Ping-Pong algorithm and those of the : left) along the $\hat{x}$ direction; and right) along the and $\hat{y}$ direction.

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