

# SILO-Radar: Self-Injection Locked Oscillator based Radar

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**Abstract**—A compact Self-Injection Locked Oscillator based Doppler Radar (SILO-Radar) at 5 GHz has been studied, analytically and numerically. The SILO-Radar consists of just a cross-coupled oscillator and a Schottky diode baseband detector, with the help of the Hilbert transform at the baseband to extract the Doppler phase information. Both analytical analysis and PSpice simulation have been performed to verify the validity of the SILO-Radar. The compact, low-power and low-cost SILO-Radar has many potential consumer applications such as auto-driving radars, Unmanned Aerial Vehicle (UAV) navigation radars, remote heartbeat/respiration healthcare biomedical radars and so on.

**Index Terms**—Self-Injection Locked Oscillator (SILO), Doppler radar, cross-coupled oscillator.

## I. INTRODUCTION

Making use of the remote capability of the electromagnetic waves from the Radio-Frequency (RF) wave to the high-frequency optics wave [1]- [63], Doppler sensors such as RF/microwave/millimeter-wave radars [1]- [10] and high-frequency Lidars have been widely used in many important areas such as police car-speeding radars, auto-driving radars, Unmanned Aerial Vehicles (UAVs) navigation radars and remote heartbeat/respiration healthcare biomedical radars [7]- [10] *etc.* In particular, the long wavelength of the RF wave makes it ideal for long-range Doppler radar applications.

Conventional Doppler radars are either based on broadband pulse time-domain technique or Frequency Modulated Continuous Wave (FMCW) frequency-domain technique, which makes the radar architectures very complex and expensive. So it is desirable to develop a compact cost-effective consumer Doppler radars. Self-Injection Locked Oscillator based radars (SILO-Radar) are such Doppler radar architectures [2]. In this paper, we propose a compact SILO-Radar architecture at 5 GHz that consists

of only a cross-coupled oscillator and a Schottky diode baseband detector, via the help of Hilbert transform.

## II. THE SILO-RADAR

Fig. 1 shows the architecture of the SILO-Radar: it consists of a Voltage Controlled Oscillator a cross-coupled oscillator, which launches an outgoing wave through a patch antenna towards the Doppler object (a car here); then the outgoing wave is reflected by the Doppler object and is injected to the cross-coupled oscillator, changing the phase of the cross-couple oscillator  $\phi(s = j\omega)$  as follows [2],

$$\phi(s) = \frac{\omega_{LR} \cos(\omega_{osc}\tau_D)}{s + \omega_{LR} \cos(\omega_{osc}\tau_D) (1 - e^{-s\tau_D})} \phi_D(s), \quad (1)$$

where  $\phi_D(s)$  is the Laplace transform of the Doppler phase;  $\tau_D$  is the round trip delay time between the antenna and the Doppler object;  $\omega_{osc}$  is the angular frequency of the free running cross coupled oscillator; and  $\omega_{LR}$  is the locking range of cross coupled oscillator.

The real part of the Doppler modulated phase signal  $r(t)$  is then detected by a time-delay ( $\tau$ ) differential operator with a Schottky diode,

$$r(t) = \cos\{\omega_{osc}\tau + \phi(t + \tau) - \phi(t)\}, \quad (2)$$

Then the detected real part signal  $r(t)$  is sampled by an Analog-to-Digital Converter (ADC); and finally, a digital processor (e.g., an FPGA) is used to perform the Hilbert transform [64] to obtain the imaginary part of the Doppler modulated phase signal,

$$i(t) = \frac{1}{\pi t} \otimes r(t), \quad (3)$$

where  $\otimes$  denotes the convolution.

Now the Doppler modulated phase can be obtained from Eq. (2) and Eq. (3),

$$\omega_{osc}\tau + \phi(t) - \phi(t - \tau) = \arctan\left\{\frac{i(t)}{r(t)}\right\}, \quad (4)$$

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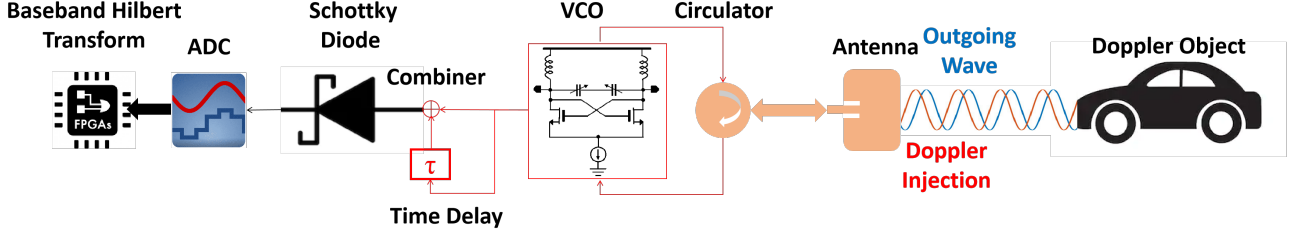


Fig. 1. Schematic of the SILO-Radar.

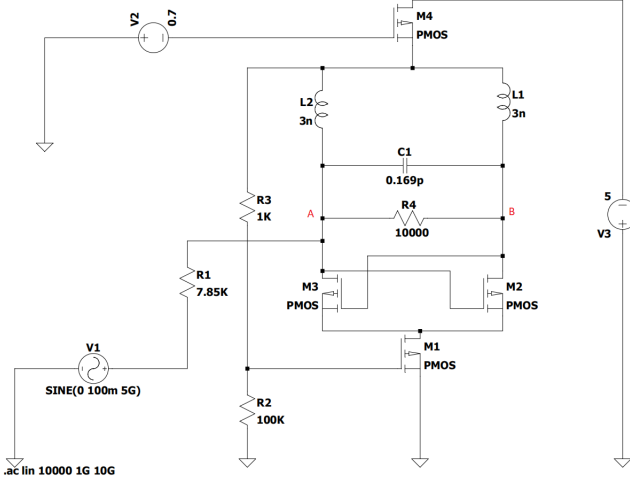


Fig. 2. The SILO circuit under study based on a cross coupled oscillator.

which can be solved through Laplace transform as follows,

$$\phi(s) = \frac{\mathcal{L} \left\{ \arctan \left\{ \frac{i(t)}{r(t)} \right\} - \omega_{osc} \tau \right\}}{1 - e^{-s\tau}}. \quad (5)$$

Finally, the Doppler phase can be obtained by substituting Eq. (5) into Eq. (1),

$$\phi_D(s) = H(s) \mathcal{L} \left\{ \arctan \left\{ \frac{i(t)}{r(t)} \right\} - \omega_{osc} \tau \right\}, \quad (6)$$

$$H(s) = \frac{s + \omega_{LR} \cos(\omega_{osc} \tau_D) (1 - e^{-s\tau_D})}{\omega_{LR} \cos(\omega_{osc} \tau_D) (1 - e^{-s\tau})}.$$

### III. ANALYSIS AND SIMULATION OF THE SILO-RADAR

In this Section, we show the theoretical analysis of the cross coupled oscillator and the PSpice simulation of the SILO, whose circuit is shown in Fig. 2.

#### A. The Cross-Coupled Oscillator

The transfer function of the cross-coupled oscillator is given by,

$$H_{osc}(s) = \frac{sg_m L}{1 + s \frac{L(1-g_m R_H)}{R_H} + s^2 LC}, \quad (7)$$

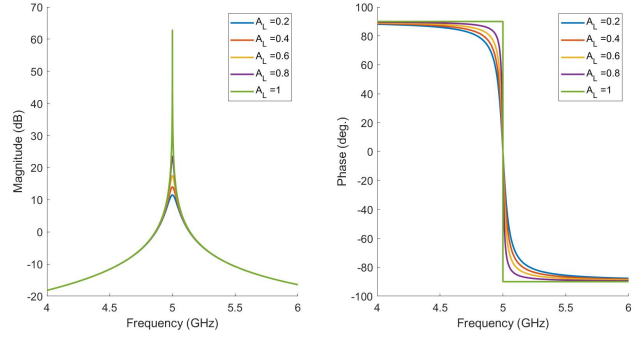


Fig. 3. The analytical transfer function of the cross-coupled oscillator: left) magnitude (dB); and right) phase (degree).

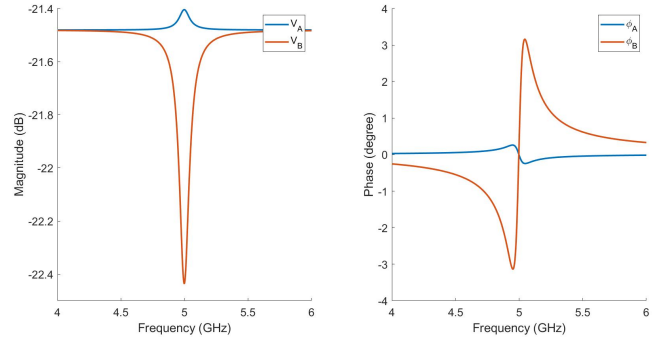


Fig. 4. The PSpice simulation of the transfer function of the cross-coupled oscillator at points A and B of Fig. 2: left) magnitude (dB); and right) phase (degree).

where  $g_m$  is the trans-conductance of the transistor of the cross-coupled oscillator;  $L$ ,  $C$  and  $R_H$  are the inductance, the capacitance and the resistance of the half circuit.

The two poles of the transfer function  $H_{osc}$  of Eq. (7) are given by,

$$s_{1,2} = -\frac{1 - g_m R_H}{2R_H C} \pm j \sqrt{\frac{1}{LC} - \left( \frac{1 - g_m R_H}{2R_H C} \right)^2}. \quad (8)$$

From Eq. (8), it can be seen that the start-up condition for the cross coupled oscillator is given by,

$$g_m > \frac{\alpha}{R_H}, \quad (9)$$

where  $\alpha \geq 1$  is required and usually  $\alpha \geq 3$  is used to ensure the start-up of the cross coupled oscillator.

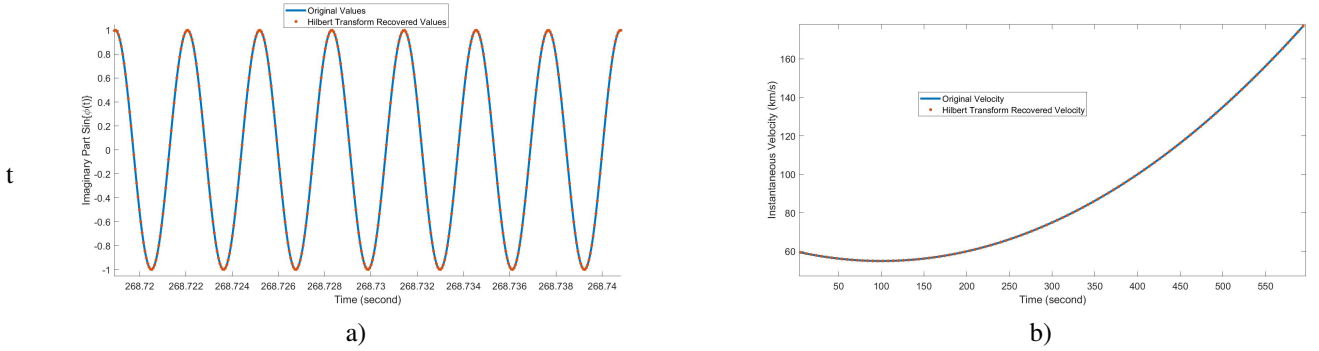


Fig. 5. Doppler information recovered by the Hilbert transform: a) the recovered imaginary part  $\sin\{\phi(t)\}$ ; and b) the recovered Doppler velocity.

In particular, when  $\alpha = 1$ , the resonant frequency is calculated as follows according to Eq. (8),

$$\omega_{1,2} = \omega_{osc} = \sqrt{\frac{1}{LC}}. \quad (10)$$

On Fig. 3, we have plotted the magnitude and the phase of the transfer function given in Eq. (7), for the parameters of  $L_1 = L_2 = L = 3$  nH,  $C_1 = C/2$  0.169 pF,  $R_4 = 2R_H = 10000$  Ohm, and  $g_m = 0.6$  mS. From Fig. 3, it can be seen that when the feedback amplitude  $A_L = fg_m R_H$  approaches 1, *i.e.*, the feedback coefficient  $f$  approaches  $1/(g_m R_H)$ , Full Width at Half Maximum (FWHM) the resonant peak becomes narrower, meaning higher Q factor, which is given by  $Q = 5$  GHz/FWHM.

### B. PSpice Simulation of the SILO

We also performed the PSpice simulation of the SILO circuit shown in Fig. 2. The magnitude and phase plots for the output points of A and B of Fig. 2 have been shown in Fig. 4. Compared with the analytical transfer function of the cross coupled oscillator shown on Fig. 3, it can be seen that both the magnitude and phase behave similarly around the design frequency of 5 GHz.

### C. Doppler Phase Recovered by the Hilbert Transform

To show the validity of Doppler phase recovery by the Hilbert transform, we have performed Hilbert transform for a car moving at the following velocity (in unit of km/hour),

$$v(t) = 60 + \frac{t^2}{20} + \frac{t^3}{6000}. \quad (11)$$

Fig. 5a) shows the imaginary part of the Doppler phase  $\sin\{\phi(t)\}$  obtained from the Hilbert transform according to Eq. (3), and Fig. 5b) shows the recovered velocity. It can be seen that both the imaginary part of the Doppler phase and the recovered velocity agree well with their given original values, showing the feasibility of the Hilbert transform for Doppler information recovery.

## IV. CONCLUSION

In this paper, we have studied a compact 5-GHz SILO-Radar that consists of only a cross-coupled oscillator and a Schottky diode baseband detector. The Doppler phase is extracted through the Hilbert transform at the baseband. Both analytical analysis and PSpice simulation have been performed to show the feasibility of the SILO-Radar. The SILO-Radar has the benefits of compactness, low-power consumption and low cost, enabling its use in many important consumer applications such as autonomous cars, UAVs, and remote heartbeat/respiration remote healthcare etc.

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