

Uncertainty analysis of mechanical behavior of natural fiber composites

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Abstract

The natural origin of the fibers combined with random production flaws results in significant uncertainties in the properties of natural fiber reinforced composites. A probabilistic assessment can help to characterize the uncertainties and evaluate the reliability of natural fiber composites, enabling their use in engineering designs. Toward this end, this study aims to quantify the uncertainties in the tensile strength and frequency response of a unidirectional flax/epoxy composite due to the variability of various input parameters, including the fiber material properties and manufacturing flaws. Based on the available data in the literature, the non-deterministic input variables were divided into normal and uniform variables using a statistical test. A computationally efficient response surface approach based on the polynomial chaos expansion was adopted to conduct the uncertainty analysis with multi-type uncertain variables. Moreover, the results were validated by the direct Monte Carlo simulation to demonstrate the accuracy and efficiency of the surrogate model.

Keywords: uncertainty analysis, natural fiber composites, polynomial chaos expansion, sampling techniques, flax fibers

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1. Introduction

The use of natural plant fiber reinforcements in polymer composite materials has recently gained attraction due to their promising properties such as high specific stiffness, good acoustic insulation, and vibration damping, and lower environmental impacts [1,2]. These advantages of natural fibers make them suitable for replacing synthetic glass fiber reinforced composites in many nonstructural and semi-structural applications, and such substitutions based on short natural fiber preforms are becoming increasingly common in the transportation and sport goods industries [3]. Besides, with the aim of utilizing the maximum load-bearing potential of natural fibers, an increasing number of investigations are now beginning to research into continuous long natural fiber composites [4–6].

However, there are still important issues that limit natural fiber's future use in engineering designs, including the difficulties in predicting structural performance as a result of the natural variability in fiber characteristics [7]. The variations in plant origin, quality of the crop, planting condition, harvesting, extraction and refinement method, and fiber characterization are considered as primary aleatoric uncertainty sources in the fiber properties. The uncertainties in fiber physical and mechanical properties generally lead to significant dispersion in the mechanical and structural performance of the natural fiber reinforced composites. Due to the dependency on a large number of stochastic variables, broad analysis of uncertainty propagation in the material behavior and structural response of composites are quite necessary when natural fiber composites are intended to be used in engineering applications.

Uncertainty analysis approaches are useful tools in order to probabilistically assess the behavior, evaluate the reliability, and quantify the contributions of specific sources of uncertainty to the overall uncertainty of composite structures. So far, the uncertainty analysis of synthetic fiber composites has been investigated by a number of researchers, and various methods have been developed. Probabilistic uncertainty analysis methods, such as traditional Mont Carlo simulation (MCS) methods, stochastic finite element, and response surface methodology (RSM) have been widely applied in the analysis and prediction of uncertain response of composite structures [8–14].

The direct MCS is conventionally a robust uncertainty propagation method to evaluate the performance variability of composite structures when there are uncertainties in the input parameters. Despite the superiority of the MCS in terms of precision, it is practically impossible to use it in computationally intensive problems such as high-fidelity models or nonlinear problems with a large number of uncertain parameters. Instead, surrogate model approaches,

which are constructed based on a limited set of actual input/output data points, are a suitable method when dealing with such complex problems. Several methods of application of surrogate models have been reported in the literature for evaluation of uncertainties in composite laminates, such as Kriging method [15,16], radial basis function [17], polynomial chaos expansion (PCE) [18–20], and artificial neural network (ANN) [21,22]. State-of-the-art reviews on the surrogate models for evaluating the uncertainty in structural responses of composite laminates can be found in [23]. The response surface methods (RSMs) are computationally efficient surrogate models that can approximate the response of a system based on the actual response at a limited set of algorithmically chosen points from uncertain inputs. The RSM is ideal for uncertainty propagation of computationally expensive models, such as uncertainties in the structural response of composites, and evaluating their reliability [18].

It has been shown that the application of the non-intrusive polynomial chaos expansion method is more efficient for dynamic analysis of stochastic systems compared to other RSMs [23,24]. Dey et al. [25] considered fuzzy membership functions to represent the parameter uncertainties and successfully applied the non-intrusive polynomial chaos expansion method to uncertainty propagation in the dynamic characteristics of composite structures. Jacquelin et al. [26] extended this method to an uncertain system with mixed uncertainties of random and fuzzy variables and proposed the product of Hermite and Legendre polynomials as a PCE to deal with both types of uncertainties.

Recently, Chen et al. [18] proposed a PCE based approach for uncertainty analysis of composite structures considering mixed input uncertainties, including normal random and interval variables. In the authors' work, the type of uncertain variables is determined based on the available data for input parameters. In other words, an uncertain parameter is treated as a normal random variable if there exists sufficient data to fit a normal distribution; otherwise, it is considered as an interval variable. A similar approach was also employed in constructing an efficient surrogate model for reliability evaluation of composite structures by considering the uncertain parameters as random or interval variables [27]. A more general mixed uncertainties related to uncertain structural, material and geometric parameters with multiple uncertainty types were taken into account for uncertainty quantification of composite laminates by Peng et al. [20]. They divided the uncertain input parameters into different variable types based on the data available for each parameter and applied the PCE method to analyze the uncertainties. It can be concluded from the previous studies the PCE with multi-type uncertainties is an efficient method for uncertainty evaluation of composites. Nevertheless, the validity and

effectiveness of this approach when dealing with a large number of input uncertainties from different types have not been fully studied. As described earlier, unlike the synthetic fibre reinforced composites, there are a larger number of uncertainty sources in the constituents and manufacturing process of natural fiber composites. However, there are few studies dedicated to a systematic assessment of the uncertainty in the properties of natural fiber composites. As one of the few studies in this regard, Blanchard et al. [28] conducted a comparative study on the reliability of a flax and E-glass fiber composite structure using the direct MCS. It was concluded that the structure made of the flax fiber composite needs to be heavier than the E-glass structure to achieve equivalent reliability.

Considering the advantages of the PCE approach, this study aims to provide a computationally efficient framework to quantify the uncertainties in the structural properties of natural fiber composites. Based on the mechanical properties reported in the literature for one type of natural fibers, namely flax fibers, and the statistical tests, the uncertain input parameters are divided into two types, strong statistical and interval variables. The rest of this paper is organized as follows. Section 2 describes the numerical procedure for constructing the response surface model. Then, the uncertainty analysis problems of the strength and natural frequencies of the unidirectional (UD) flax/epoxy laminate are discussed in Section 3. Finally, the conclusions are presented in Section 4.

2. Polynomial chaos expansion method for uncertainty analysis with multi-type uncertain parameters

According to the concept of the polynomial chaos expansion (PCE) introduced by Wiener, the combination of orthogonal polynomial bases is a suitable method for approximation of unknown or complex correlations [29]. The PCE is, therefore, a proper choice for estimating the variability of uncertain systems. Based on the simplified form of the PCE, the uncertain response of a system can be expressed as a function of normally distributed random variables (ξ_i) using the following convergent series [30]

$$\begin{aligned}
X(\xi_1, \xi_2, \dots, \xi_m) &= c_0 + \sum_{i_1=1}^m c_{i_1} H_1(\xi_{i_1}) \\
&+ \sum_{i_1=1}^m \sum_{i_2=1}^{i_1} c_{i_1 i_2} H_2(\xi_{i_1}, \xi_{i_2}) \\
&+ \sum_{i_1=1}^m \sum_{i_2=1}^{i_1} \sum_{i_3=1}^{i_2} c_{i_1 i_2 i_3} H_3(\xi_{i_1}, \xi_{i_2}, \xi_{i_3}) + \dots
\end{aligned} \tag{1}$$

where $H_p(\xi_{i_1}, \xi_{i_2}, \dots, \xi_{i_p})$, $i_1, i_2, \dots, i_p \in \{1, 2, \dots, m\}$ denotes the Hermite-chaos of order p , which is a set of multidimensional Hermite polynomials, $\xi_j, j \in \{1, 2, \dots, m\}$, are m independent standard normal random input variables, and $c_{i_1}, c_{i_2}, \dots, c_{i_p}$ are deterministic constants. The variable ξ_i is determined by means of the following change-of-variable

$$\xi_i = \frac{x_i - \mu_{x_i}}{f \sigma_{x_i}} \tag{2}$$

where x_i is normal variables, μ_{x_i} and σ_{x_i} are the corresponding mean and standard deviation, respectively, and f is a constant coefficient which is determined based on the confidence level. Here, the constant f is set equal to 3 according to the three-sigma rule of the normal distribution. The multidimensional Hermite polynomials can be obtained by

$$H_p(\xi_{i_1}, \xi_{i_2}, \dots, \xi_{i_p}) = (-1)^p e^{\frac{1}{2}\xi^T \xi} \frac{\partial^p e^{-\frac{1}{2}\xi^T \xi}}{\partial \xi_{i_1} \partial \xi_{i_2} \dots \partial \xi_{i_p}} \tag{3}$$

where ξ denotes the p -dimensional vector of standard normal random variables $(\xi_{i_1}, \xi_{i_2}, \dots, \xi_{i_p})$, $i_1, i_2, \dots, i_p \in \{1, 2, \dots, m\}$. The coefficients c_j in Eq.(1) are learnable parameters or weights which can be determined using N sample points of normal random input variables, i.e. $\xi^i, i \in \{1, 2, \dots, N\}$, and the corresponding actual responses $X(\xi^i), i = 1, 2, \dots, N$. A suitable sampling strategy, such as the sparse collocation method, can be employed to determine the N sample points [31,32]. The response surface model of Eq.(1) can be expressed in the matrix form as

$$\mathbf{X} = \mathbf{H}\mathbf{C} \tag{4}$$

Here, $\mathbf{X} = \mathbf{X}(X_1, X_2, \dots, X_N)^T$ is the vector of actual responses for N sample points, in which $X_i, i \in \{1, 2, \dots, N\}$ is an actual outcome obtained through deterministic analysis of the problem with respect to a sample point of uncertain variables. In Eq.(4), $\mathbf{C}$ is the polynomial coefficient vector and $\mathbf{H}$ is the matrix of all Hermite polynomials for N sample points which is defined as

$$\mathbf{H} = \begin{bmatrix} 1 & H_1(\xi_1^1) & \cdots & H_2(\xi_{i_1}^1, \xi_{i_2}^1) & \cdots & H_p(\xi_{i_1}^1, \xi_{i_2}^1, \dots, \xi_{i_p}^1) & \cdots \\ 1 & H_1(\xi_1^2) & \cdots & H_2(\xi_{i_1}^2, \xi_{i_2}^2) & \cdots & H_p(\xi_{i_1}^2, \xi_{i_2}^2, \dots, \xi_{i_p}^2) & \cdots \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots \\ 1 & H_1(\xi_1^N) & \cdots & H_2(\xi_{i_1}^N, \xi_{i_2}^N) & \cdots & H_p(\xi_{i_1}^N, \xi_{i_2}^N, \dots, \xi_{i_p}^N) & \cdots \end{bmatrix} \quad (5)$$

Thus, the polynomial coefficient vector can be determined based on the N input sample points using

$$\mathbf{C} = (\mathbf{H}^T \mathbf{H})^{-1} \mathbf{H}^T \mathbf{X} \quad (6)$$

One can make good use of sampling techniques corresponding to the probability distribution of the input variables and numerical matrix inversion to reduce the computational cost of Eq.(6). Besides, for simplicity, Eq.(1) can be written as

$$X(\xi) = \sum_{j=1}^M \hat{c}_j \Psi_j(\xi) \quad (7)$$

where $\hat{c}_j$ is identical to coefficients $c_0, c_{i_1}, c_{i_2}, \dots, c_{i_p}$ and there is a one-to-one correspondence between the polynomial functions $\Psi_j(\xi)$ and $H_p(\xi_{i_1}, \xi_{i_2}, \dots, \xi_{i_p})$. M is the total number of undetermined polynomial coefficients and can be determined, based on the highest order of polynomials (p) and number of uncertain variables (m), by $\frac{(m+p)!}{m!p!}$.

So far, only normal random variables have been accounted for in the approximation of the structural response X since the random input of Hermite-chaos polynomials of Eq.(1) is limited to normal random variables. The reason is that the Hermite polynomials are uncorrelated when random input (ξ) is normal. This assures the optimal exponential convergence rate of the series of Eq.(7) [30].

However, based on the available data for the uncertain input parameters, there is usually more than one type of random inputs. Here, the random inputs are categorized into two types, normal random variables and uniform variables. The classification of input variables is carried out based on the Anderson-Darling (AD) goodness-of-fit test results. An input is considered as a normal random variable if available data for that parameter fits a normal distribution; otherwise, it is considered as a uniform variable. According to the idea of the multi-type random variables PCE [18,26], the uniform random variables can be incorporated in the polynomial chaos coefficients by expressing them as a function of the orthogonal Legendre polynomials. This is because the orthogonality weighting functions of the Legendre polynomials provide a good match with the probability density function of the uniform random variables [30]. Considering the Legendre polynomials of order 2, the coefficients $\hat{c}_j, j \in \{1, 2, \dots, M\}$ in Eq.(7) take the form

$$\hat{c}_j = a_{j0} + \frac{1}{4} \sum_{l=1}^n b_{jl} \gamma_l + \frac{1}{2} \sum_{l=1}^n d_{jl} (3\gamma_l^2 - 1) \quad (8)$$

where a_{j0}, b_{jl}, d_{jl} are the polynomial coefficients and $\gamma_l, l \in \{1, 2, \dots, n\}$ are the n independent standard uniform random variable, which can be obtained by

$$\gamma_l = \frac{y_l - y_l^c}{y_l^r} \quad (9)$$

where y_l^c and y_l^r are the mean value and the interval radius of the uniform variable, respectively. By substituting Eq.(8) into Eq.(7), the latter can be expressed as a function of two types of random variables as

$$\begin{aligned} X(\xi_1, \xi_2, \dots, \xi_m, \gamma_1, \gamma_2, \dots, \gamma_n) \\ = \sum_{j=1}^M \left(a_{j0} + \frac{1}{4} \sum_{l=1}^n b_{jl} \gamma_l \right. \\ \left. + \frac{1}{2} \sum_{l=1}^n d_{jl} (3\gamma_l^2 - 1) \right) \Psi_j(\xi_1, \xi_2, \dots, \xi_m) \end{aligned} \quad (10)$$

To construct the response surface, the unknown coefficients a_{j0}, b_{jl} and d_{jl} can be determined by solving Eq.(10) by means of an adequate number of sample points. The Latin Hypercube Sampling (LHS) method [29] and the sparse Cleanshaw-Curtis (SCC) collocation strategy [31] can be employed to determine the sample points for the standard uniform and normal random variables, respectively. The sample points should be determined in a two-level sampling strategy. In the inner level, N sample points are determined for the normal random variables by means of the SCC collocation strategy as $(\xi_1^v, \xi_2^v, \dots, \xi_m^v), v \in \{1, 2, \dots, N\}$. In the outer level, depending on the accuracy of the polynomial chaos expansion, K sample points are determined for the uniform variables appeared in the polynomial coefficients using the LHS method as $(\gamma_1^w, \gamma_2^w, \dots, \gamma_n^w), w \in \{1, 2, \dots, K\}$.

Considering Eq.(4)-(6) and applying the sample points of the uniform variables, the coefficients of the Hermite-chaos polynomials in Eq.(10) can be written as the following matrix form

$$\mathbf{\Gamma} \mathbf{E}_j = \mathbf{C}_j, \quad j \in \{1, 2, \dots, M\} \quad (11)$$

where

$$\mathbf{\Gamma} = \begin{bmatrix} 1 & \left(\frac{\gamma_1}{4}\right)^1 & \dots & \left(\frac{\gamma_n}{4}\right)^1 & \left(\frac{3\gamma_1^2 - 1}{2}\right)^1 & \dots & \left(\frac{3\gamma_n^2 - 1}{2}\right)^1 \\ 1 & \left(\frac{\gamma_1}{4}\right)^2 & \dots & \left(\frac{\gamma_n}{4}\right)^2 & \left(\frac{3\gamma_1^2 - 1}{2}\right)^2 & \dots & \left(\frac{3\gamma_n^2 - 1}{2}\right)^2 \\ \vdots & \vdots & \ddots & \vdots & \ddots & \ddots & \ddots \\ 1 & \left(\frac{\gamma_1}{4}\right)^K & \dots & \left(\frac{\gamma_n}{4}\right)^K & \left(\frac{3\gamma_1^2 - 1}{2}\right)^K & \dots & \left(\frac{3\gamma_n^2 - 1}{2}\right)^K \end{bmatrix} \quad (12)$$

is the matrix of Legendre polynomials of order 2 for K sample points of uniform variables and

$$\mathbf{E}_j = [a_{j0} \quad b_{j1} \quad \dots \quad b_{jn} \quad d_{j1} \quad \dots \quad d_{jn}]^T \quad (13)$$

$$\mathbf{C}_j = [\hat{c}_j^1 \quad \hat{c}_j^2 \quad \dots \quad \hat{c}_j^K]^T \quad (14)$$

Using Eq.(11)-(14), the unknown coefficients a_{j0} , b_{ji} and d_{ji} of Eq.(10) are calculated by

$$\mathbf{E}_j = (\mathbf{\Gamma}^T \mathbf{\Gamma})^{-1} \mathbf{\Gamma}^T \mathbf{C}_j \quad (15)$$

Consequently, the response surface model of Eq.(10) is obtained. Here, the number of sample points in LHS and the collocation level in the SCC can be determined based on the accuracy of the developed RSM compared to the actual response of the composite structure. One can make good use of the root mean square errors (RMSE) method to evaluate the precision and the convergence of the surrogate model at any given number of sample points for uniform variables. The RMSE is given by

$$RMSE = \left[\frac{1}{N} \sum_{i=1}^N (X_i - X_{id})^2 \right]^{\frac{1}{2}} \quad (16)$$

where X_i is the predicted response by the RSM and X_{id} is the actual response for an identical input. Clearly, the smaller the value of RMSE, the higher precision of the RSM prediction. The detailed procedure of solving Eq.(10) can be found in [18].

3. Uncertainty analysis of mechanical properties of flax fiber composites by means of PCE-Legendre RSM

Flax fiber is one of the most desirable natural fibers to replace synthetic glass fibers in many applications [33–36]. This is mainly due to the promising characteristics of flax fibers like excellent mechanical properties, lower density, and higher aspect ratio. As a result, the mechanical properties of flax fibers and their composites have been extensively reported in the literature. However, their properties have a relatively large variation that arises from their natural variabilities, such as origin, variety, and maturity of the plant, as well as processing variabilities, such as harvesting, extraction, and refinement of fibers. The uncertainty effects of

these factors are generally hard or impossible to control due to their stochastic nature. Thus, it is necessary to consider the uncertainties of properties when modeling and predicting the behavior of flax fiber reinforced composites.

The uncertainty information of the elastic properties and density for flax fibers based on the available experimental data from the literature are summarized in *Table 1*. One can apply a distributional test to the experimental data obtained from different sources to evaluate data distribution if their probability distributions are not specified. Here, the Anderson-Darling test [37], with a confidence level of 95%, is used to test the data for normality. The uncertain parameter is considered as a normal random variable (ξ) if the calculated observation significance level was greater than 0.05; otherwise, it is treated as a uniformly distributed variable (γ). Besides, when there is insufficient data for an uncertain parameter, that parameter is assumed as a uniformly distributed variable (between its upper and lower limit).

Table 1. Statistical information of flax fiber properties [7,38–52].

Variable	Type	Mean	SD	Lower limit	Upper limit
Young's modulus, E_f (GPa)	Normal	52.01	17.12		
Shear modulus, G_f (GPa)		2.5	0.2	-	-
Density, ρ_f (g.cm ⁻³)		1.47	0.24		
Poisson's ratio, ν_f (%)	Uniform	-	-	27	31

The variables related to matrix properties are assumed to be deterministic. Since thermoset resin systems are widely used for flax fiber composites, the average properties of a common epoxy resin with $E_m = 3.70$ GPa, $G_m = 1.37$ GPa, $\nu_m = 0.35$, and $\rho_m = 1.15$ g.cm⁻³ are considered for the analysis [44].

Considering the uncertainties in the elastic properties and density of flax fibers, the effective elastic properties of the composite can be estimated as a function of the non-deterministic fiber parameters through the rule of mixtures as

$$\begin{aligned}
E_1 &= V_f E_f + (1 - V_f) E_m \\
E_2 &= E_m \frac{1 + 0.5\delta V_f}{1 - \delta V_f}, \\
G_{12} &= G_m \frac{(1 + V_f)G_f + (1 - V_f)G_m}{(1 - V_f)G_f + (1 + V_f)G_m} \\
\nu_{12} &= V_f \nu_f + (1 - V_f) \nu_m
\end{aligned} \tag{17}$$

where E_1 and E_2 are the effective elastic modulus in the fiber and transverse direction, respectively, V_f is the fiber volume fraction, G_{12} is the in-plane shear modulus, and ν_{12} is the in-plane Poisson's ratio. The coefficient δ is defined as

$$\delta = \frac{E_f - E_m}{E_f + 0.5E_m} \quad (18)$$

Eq.(17) is based on the assumption that both the materials are linear elastic and the fiber-matrix bonding is perfect [53,54]. Besides, the variability in the local fiber orientation is considered by treating the fiber volume fraction (V_f) as a random variable. It is assumed that its distribution is normal, with a mean of 40% and a standard deviation of 11%.

In addition to the elastic properties of the fiber, the strength parameters of the UD flax/epoxy laminate are assumed to be nondeterministic and treated as random variables. The statistical information of the strength variables of UD flax/epoxy composites obtained from the literature is given in Table 2.

Table 2. Statistical information of strength parameters for the UD flax/epoxy composite laminate [7,38,39,43–48,50–52,55–64].

Variable	Type	Mean	SD	Lower limit	Upper limit
Longitudinal tensile, X_t (MPa)	Normal	291.4	84.8	-	-
Transverse tensile, Y_t (MPa)		25.9	13.2		
In-plane shear, S (MPa)		37.7	5.7		
Longitudinal compression, X_c (MPa)	Uniform	-	-	110	136.9
Transverse compression, Y_c (MPa)				76	100

Another uncertainty source is the misalignment of ply orientation (θ) that incurred during the layup process. In the uncertainty analysis of the composite, the orientation of each ply is considered as an independent normal random variable as

$$\theta_i \sim \mathcal{N}(\mu_{\theta_i}, \sigma_{\theta_i}^2), \quad i \in \{1, 2, \dots, N\} \quad (19)$$

where μ_{θ} is the nominal ply orientation, σ_{θ} is the standard deviation of ply orientation, and N is the number of plies. In contrast to the assumption of deterministic values for the standard deviation of material properties, the ply orientation variability (σ_{θ}) is non-deterministic and subject to uncertainties due to uncertainties in the processing of fibers as well as uncertainties during the lay-up. Hence, a parametric sensitivity analysis concerning the dispersion of the ply orientation is required. For this purpose, the uncertainty analysis is conducted using four

different values of $\pi/360$, $\pi/180$, $\pi/90$, and $\pi/60$ for the standard deviation of the ply orientation.

In the subsequent sections, the resultant uncertainties in the failure load and natural frequencies of a UD flax/epoxy composite plate are evaluated, and the validity of the PCE-Legendre method in assessing the uncertainties in the material response of natural fiber composites with high-dimensional uncertain input parameters are demonstrated. The analysis is performed on a desktop computer with an Intel Core™ i7-4790 CPU and 16 GB RAM.

3.1. Prediction of failure load of a UD flax /epoxy composite laminate

As mentioned in the previous section, there are considerable uncertainties in the primitive variables of flax fiber composites. It is necessary to consider the uncertainty of these variables when simulating the response of a flax fiber composite. In this section, the probability distribution of the first-ply failure load of a flax/epoxy composite laminate is predicted using the PCE-Legendre based RSM. The lay-up of the laminate is $[0]_{12}$ and it is subjected to the tensile load N_x per unit length, as shown in Figure 1.

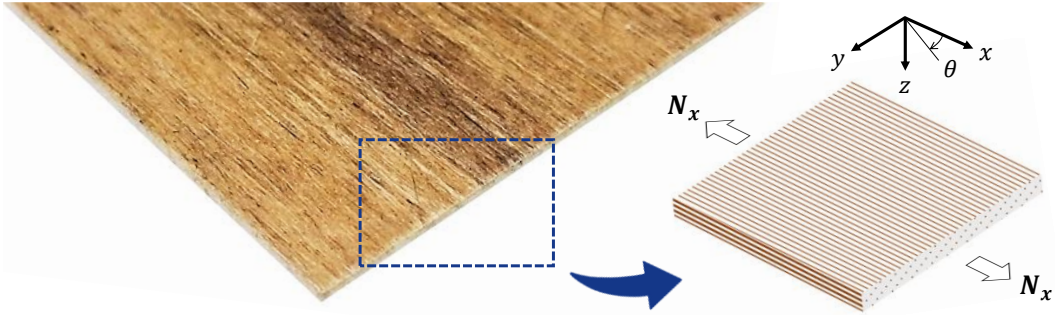


Figure 1. UD flax/epoxy laminate and the loading direction

Considering the classical laminate theory and the plane stress assumption, the deterministic stresses in the k^{th} ply with a misalignment of θ_k can be calculated as follows

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{Bmatrix}^{(k)} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix}^{(k)} \begin{Bmatrix} \varepsilon_x^0 + z\kappa_x \\ \varepsilon_y^0 + z\kappa_y \\ \gamma_{xy}^0 + z\kappa_{xy} \end{Bmatrix} \quad (20)$$

where $\bar{Q}_{ij}$ are the components of the transformed lamina stiffness matrix of the k^{th} ply, and $(\varepsilon_x^0 \ \varepsilon_y^0 \ \gamma_{xy}^0)^T$ and $(\kappa_x \ \kappa_y \ \kappa_{xy})^T$ are the in-plane strains and curvatures associated with the middle surface of the laminate, respectively. The strain and curvature components of the middle surface can be obtained based on the tensile load N_x as

$$\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix}^{-1} \begin{Bmatrix} N_x \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \quad (21)$$

where the laminate coefficients A_{ij} , B_{ij} , and D_{ij} are defined in terms of the transformed lamina stiffnesses $\bar{Q}_{ij}$ as

$$(A_{ij}, B_{ij}, D_{ij}) = \sum_{k=1}^N \int_{z_k}^{z_{k+1}} \bar{Q}_{ij}^{(k)}(1, z, z^2) dz \quad (22)$$

where N is the number of plies. The components of the transformed lamina stiffness matrix are obtained as a function of the lamina stiffnesses matrix and the ply orientation as

$$\begin{aligned} \bar{Q}_{11}^{(k)} &= Q_{11} \cos^4 \theta_k + 2(Q_{12} + 2Q_{66}) \sin^2 \theta_k \cos^2 \theta_k + Q_{22} \sin^4 \theta_k \\ \bar{Q}_{12}^{(k)} &= (Q_{11} + Q_{22} - 4Q_{66}) \sin^2 \theta_k \cos^2 \theta_k + Q_{12}(\sin^4 \theta_k + \cos^4 \theta_k) \\ \bar{Q}_{22}^{(k)} &= Q_{11} \sin^4 \theta_k + 2(Q_{12} + 2Q_{66}) \sin^2 \theta_k \cos^2 \theta_k + Q_{22} \cos^4 \theta_k \\ \bar{Q}_{16}^{(k)} &= (Q_{11} - Q_{12} - 2Q_{66}) \sin \theta_k \cos^3 \theta_k \\ &\quad + (Q_{12} - Q_{22} + 2Q_{66}) \sin^3 \theta_k \cos \theta_k \\ \bar{Q}_{26}^{(k)} &= (Q_{11} - Q_{12} - 2Q_{66}) \sin^3 \theta_k \cos \theta_k \\ &\quad + (Q_{12} - Q_{22} + 2Q_{66}) \sin \theta_k \cos^3 \theta_k \\ \bar{Q}_{66}^{(k)} &= (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66}) \sin^2 \theta_k \cos^2 \theta_k + Q_{66}(\sin^4 \theta_k \\ &\quad + \cos^4 \theta_k) \end{aligned} \quad (23)$$

The components of the lamina stiffness matrix, Q_{ij} , are related to the uncertain effective elastic properties of Eq.(17) by

$$\begin{aligned} Q_{11} &= \frac{E_1}{1 - \nu_{12}\nu_{21}} \\ Q_{12} &= \frac{\nu_{12}E_2}{1 - \nu_{12}\nu_{21}} = \frac{\nu_{21}E_1}{1 - \nu_{12}\nu_{21}} \\ Q_{22} &= \frac{E_2}{1 - \nu_{12}\nu_{21}} \\ Q_{66} &= G_{12} \\ \frac{\nu_{12}}{E_1} &= \frac{\nu_{21}}{E_2} \end{aligned} \quad (24)$$

One can plug Eq.(21)-(24) into Eq.(20) and use a failure criterion to predict the value of N_x corresponding to the first-ply failure of the laminate. Here, the Tsai-Wu criterion is applied to

determine the failure of a lamina. According to this criterion, ply failure occurs when the following inequality is satisfied

$$F_1\sigma_1 + F_2\sigma_2 + F_{11}\sigma_1^2 + F_{22}\sigma_2^2 + F_{66}\sigma_{12}^2 + 2F_{12}\sigma_1\sigma_2 \geq 1 \quad (25)$$

where the strength coefficients are defined in terms of the lamina strengths as

$$\begin{aligned} F_1 &= \frac{1}{X_t} - \frac{1}{X_c}, F_2 = \frac{1}{Y_t} - \frac{1}{Y_c}, F_{11} = \frac{1}{X_t X_c}, F_{22} = \frac{1}{Y_t Y_c}, F_{12} \\ &= -\frac{1}{2} \sqrt{F_{11} F_{22}}, F_{66} = \frac{1}{S^2} \end{aligned} \quad (26)$$

in which the lamina strengths are random variables, as given in Table 2. For a given set of input parameters, the nonlinear Eq.(25) can be solved for N_x to obtain the failure load of each lamina, where the least value corresponds to the first-ply failure load.

The PCE-Legendre method described in section 2 is applied to construct the response surface for N_x in MATLAB, considering all 23 uncertain uniform and normal variables, including fiber elastic properties (Table 1), lamina strengths (Table 2), and the misalignment of ply orientations, i.e. $\theta_i, i \in \{1, 2, \dots, 12\}$. Based on the level of accuracy, the number of sample points is determined and applied to solve the Eq.(25) to obtain the corresponding outputs. In this problem, the required accuracy was achieved when the order of Hermite polynomial chaos (p) and the collocation level were equal to 4 and 2, respectively.

The distributions of the first-ply failure load when the standard deviation of the ply orientation (σ_θ) is $\pi/180$, obtained from the direct MCS and the PCE-Legendre based RSM are compared in Figure 2. Based on the central limit theorem, it can be concluded that the variation of N_x due to the multi-type uncertainties in elastic properties, strengths, and the ply orientation is normal. This enables us to use the mean value and the standard deviation to describe the N_x .

As can be seen, the distributions of N_x obtained from the direct MCS and the PCE-Legendre method are in an excellent correlation. It should be noted, however, the required numbers of solving the nonlinear inequality of Eq.(25) to obtain N_x are considerably different in each method. The number of sample points required to construct the response surface model using the PCE-Legendre is 3065, while the direct MCS needs at least 40000 sample points to reach the desired convergence. Therefore, the computational efficiency of the PCE-Legendre RSM is significantly higher than the direct MCS.

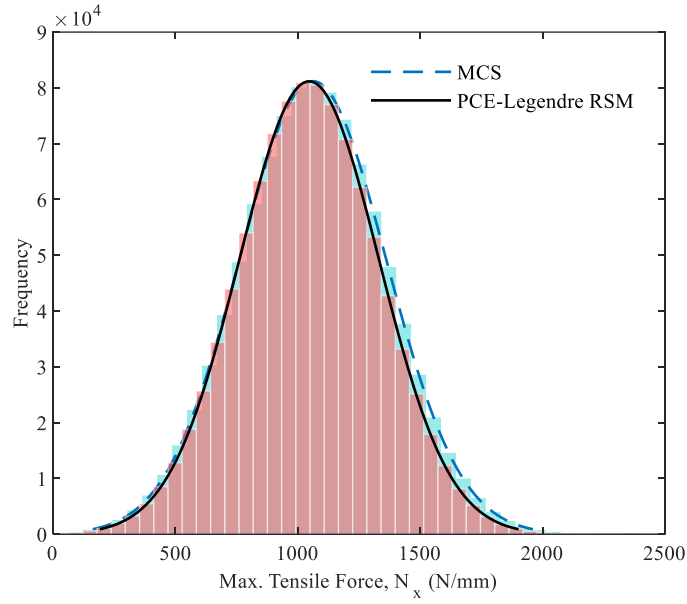


Figure 2. Probability density function of the first-ply failure load (N_x).

The number of sample points and the corresponding means and standard deviations (SD) of N_x for four different values of σ_θ are compared in *Table 3*. Although the difference between the results of the RSM and MCS methods is increased by increasing the standard deviation of the ply orientation, the magnitude of the relative errors, particularly for the mean values, is not significant. Consequently, considering the number of uncertain input parameters, it can be said that the PCE-Legendre response surface model is able to represent the uncertainties of the failure load of the flax fiber composite to a good accuracy, with much less computational cost. Due to the lack of knowledge about the degree of the ply misalignment, one can repeat the uncertainty analysis of N_x for various values of σ_θ to evaluate its effects on the strength of the laminate. The sensitivity of the mean value and standard deviation of N_x to the ply misalignment dispersion using the PCE-Legendre method is shown in *Figure 3a*. The results of the direct MCS are also given for validation. Both the mean value and the standard deviation of N_x show a dependence on σ_θ , registering a reduction of up to 4.5% and 8.2%, respectively, when σ_θ is increased from 0.5 to 3 degrees.

The reduction predicted by the direct MCS is slightly less than that of the PCE-Legendre method – 4.1% and 6.4% in the mean and the standard deviation of N_x , respectively. An interesting fact seen in *Figure 3a* is that the standard deviation of N_x decreases by increasing the dispersion of the ply misalignment. This means that the uncertainty of N_x is inversely related to the uncertainty of the ply orientations. It must be noted that this correlation is only observed when all the uncertainties (i.e. fiber properties, lamina strengths, and misalignment of ply

orientations), are considered in the analysis. In other words, an opposite behavior (increasing the standard deviation of N_x by increasing σ_θ) is obtained when the uncertain inputs are limited to ply orientations. This can be clearly seen from *Figure 3b* in which all input parameters are deterministic except the ply orientations. The mean values are used for the deterministic inputs.

Table 3. Uncertainty analysis results of the first-ply failure load (N_x) for different standard deviation of ply orientation using RSM and MCS.

σ_θ	PCE-Legendre based RSM			MCS			Error (%)	
	No. of simulations	Mean (N/mm)	SD (N/mm)	No. of simulations	Mean (N/mm)	SD (N/mm)	Mean	SD
$\frac{\pi}{360}$	3065	1105.87	314.64	30000	1106.1	320.23	0.02	1.75
$\frac{\pi}{180}$		1092.53	309.21	40000	1097.04	316.03	0.41	2.16
$\frac{\pi}{90}$		1076.92	300.1	60000	1080.48	309.66	0.33	3.09
$\frac{\pi}{60}$		1055.89	288.75	70000	1060.94	299.75	0.48	3.67

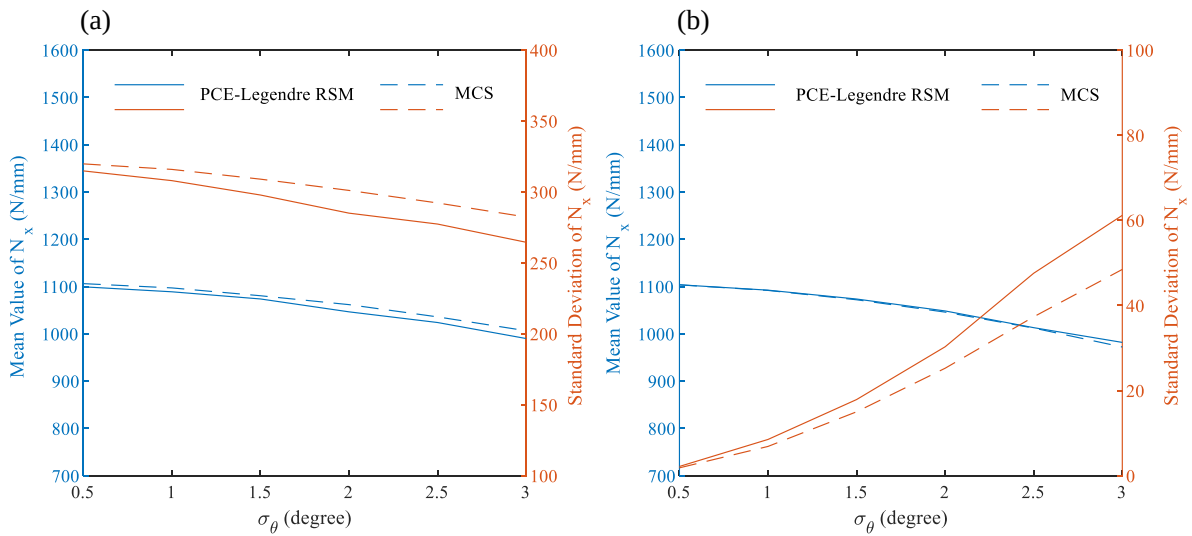


Figure 3. Mean and standard deviation of N_x against the standard deviation of ply orientation considering: (a) all uncertainties and (b) only uncertainties in ply orientations.

3.2. Uncertainty analysis of natural frequencies of a UD flax /epoxy composite laminate

In the second case study, the effect of uncertainties on the natural frequencies of a $[0]_{12}$ flax/epoxy laminate is investigated. The fiber material properties (*Table 1*), the fiber volume fraction, and the misalignment of ply orientations are considered as the stochastic input

variables. In this problem, the only uniform variable is the Poisson's ratio of the fiber, ν_f , and all other inputs are normally distributed. The effective elastic properties of the individual lamina are obtained using Eq. (17). Thus, the natural frequency problem can be stated as

$$\omega_n = \omega_n(E_f, G_f, V_f, \nu_f, \rho_f, \theta_1, \theta_2, \dots, \theta_{12}) \quad (27)$$

where ω_n is the n^{th} natural frequency. To obtain the frequency response of the composite, an $11\text{mm} \times 11\text{mm}$ laminate with 12 layers is modeled using the conventional shell element in Abaqus/Explicit. After a mesh convergence study, the finite element analysis (FEA) results using the mean values as inputs were compared with experimental data from the literature [65] to validate the model. Table 4 compares the first four natural frequencies obtained from the FEA and experimental tests of a $[0]_{12}$ flax/epoxy laminate. The comparison of FEA and experimental results indicates that the model is able to capture the frequency response of the composite with acceptable accuracy.

Table 4. FEA and experimental results of the natural frequency analysis of the flax/epoxy laminate.

Mode No.	Experimental (Hz)	FEM (Hz)	Error (%)
1	58.82	59.87	1.79
2	125.7	127.69	1.58
3	377.96	371.67	1.66
4	481.35	488.30	1.44

A Python script in conjunction with a MATLAB code is developed to construct the RSM following the approach described for the PCE-Legendre method and based on the specified accuracy level, i.e. the order of Hermite polynomial chaos and the collocation level. In this fully automatic procedure, the sample points of the uniform and normal parameters are generated by the MATLAB code employing the LHS and SCC sampling strategies. Then, the Python script runs the FEA in Abaqus for each set of the sample points and save the results for post-processing in the MATLAB. Finally, the RSM is constructed by calculating the coefficients of Eq.(10) and the sensitivity analysis is implemented. This model is then used for the uncertainty quantification.

In order to conduct the direct MCS, a Python script is written capable of generating stochastic input parameters, running the FEA, and performing statistical analysis on the outputs. In this uncertainty analysis, the order of Hermite polynomial chaos and the collocation level are considered equal to 4 and 2, respectively, to achieve the desired precision.

The results of the uncertainty analysis of the first four natural frequencies are given in Table 5. The results of the direct MCS are also presented for validation purposes. As can be seen, the results of the PCE-Legendre method are in good agreement with the direct MCS results. The relative errors of the mean values and standard deviations calculated by the PCE-Legendre response surface model are less than 0.15% and 1.5%, respectively. Figure 4 compares the cumulative distribution functions (CDFs) of the natural frequencies obtained from both the methods, which confirms that the results of the RSM are highly correlated to the direct MCS results.

Table 5. Uncertainty analysis results of the first four natural frequencies using the RSM and MCS.

Mode No.	PCE-Legendre based RSM (Hz)		MCS (Hz)		Error (%)	
	Mean	SD	Mean	SD	Mean	SD
1	52.38	9.88	52.46	9.72	0.14	1.51
2	122.4	9.34	122.43	9.36	0.02	0.15
3	326.12	60.54	325.74	60.45	0.12	0.14
4	454.68	50.86	455.39	50.31	0.15	1.09

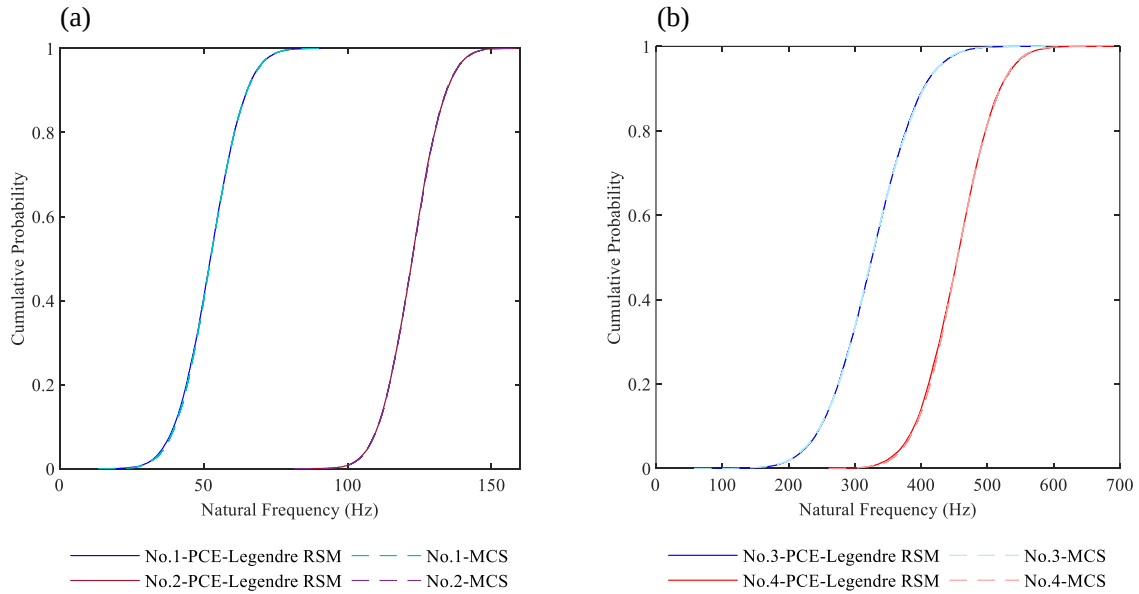


Figure 4. Probability of natural frequencies, (a) mode 1 and 2, (b) mode 3 and 4.

To demonstrate the efficiency of the RSM, the computation time of the uncertainty analysis using the direct MCS and the PCE-Legendre response surface are calculated and compared in Figure 5. The initial computation time of the RSM indicates the total time of running the FEA

of the sample points to construct the surrogate model. Based on the chosen order of Hermite polynomial chaos, the collocation level, and the LHS in this problem, the number of sample points is 1635 and the corresponding initial computation time is 369.7 minutes. The difference between the computation time of the MCS and PCE-Legendre method is more noticeable as the number of simulations increases. Therefore, the PCE-Legendre method is significantly more efficient than the direct MCS in terms of the computational cost, particularly for high dimension uncertainties where the convergence rate of the MCS is very slow.

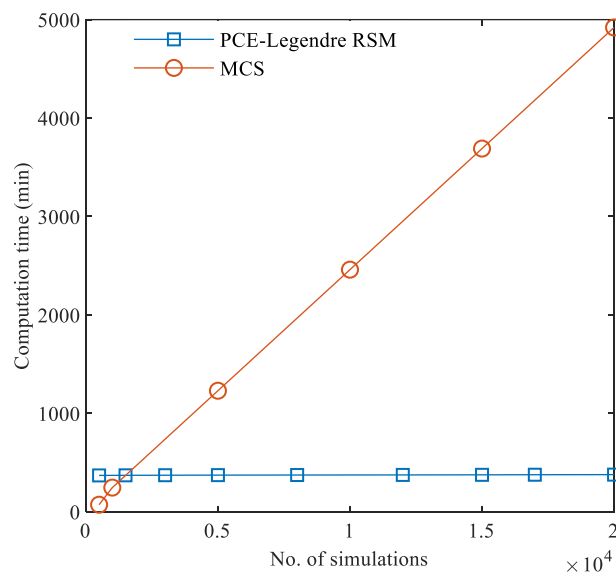


Figure 5. Computation time of the uncertainty analysis of frequency response using the MCS and the RSM.

As mentioned in the previous section, the sensitivity of the uncertainty analysis to the dispersion of the ply misalignment needs to be evaluated. Thus, the influence of the standard deviation of the ply orientation on the frequency response is investigated by means of the PCE-Legendre RSM. *Figure 6* illustrates the variation of mean values and standard deviations of the first four natural frequencies with respect to various standard deviations of the ply orientation (σ_f), ranging from 0.5 to 3 degrees. The other uncertainties (i.e. material properties) are the same as discussed earlier.

The uncertainties analysis results show that the mean and standard deviation of all the first four natural frequencies remain almost unchanged with increasing the standard deviation of the ply orientation up to 3 degrees. In other words, considering all the sources of the uncertainty (i.e. fiber material properties, fiber volume fraction, and ply orientation), the effect of small misalignment of ply orientations on the frequency response is negligible. This can be attributed to the high uncertainty in the fiber properties compared to the uncertainty considered for the

ply orientations. Therefore, it can be concluded that the frequency response of the UD flax/epoxy laminate is not sensitive to small misalignments of the ply orientation.

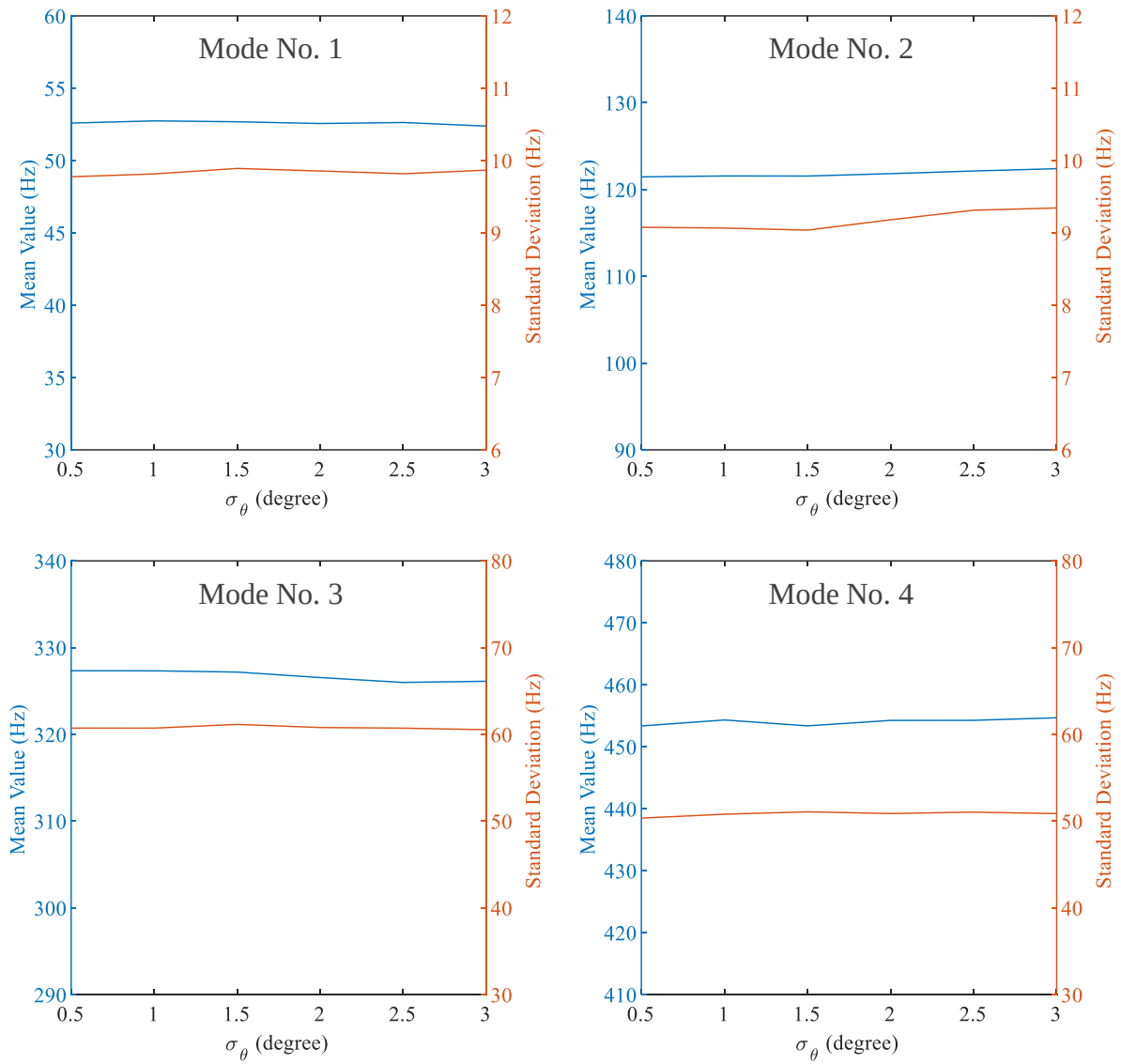


Figure 6. Mean and standard deviation of the first four natural frequencies against the standard deviation of ply orientation.

4. Conclusion

The natural origin of fibers and unavoidable defects during the extraction stage, combined with random manufacturing flaws, have always given rise to considerable uncertainties in the properties of natural fiber reinforced composites. Therefore, the prediction of the uncertainty of the mechanical and structural behavior of natural fiber composites is crucial for the design process.

This work presents an uncertainty analysis of tensile strength and frequency response of a UD flax/epoxy composite, as one of the widely studied natural fiber composites. The variability of the fiber elastic properties, fiber density, lamina strengths, fiber volume fraction, and the misalignment of ply orientations are considered as the uncertainty sources. The uncertain input variables are divided into normal and uniform variables by means of the Anderson-Darling test, based on the various experimental data acquired from the literature. Due to the high number of uncertain input variables, a computationally efficient RSM based on the PCE is adopted and modified accordingly to take the multi-type stochasticity of the inputs into account. For each case study, a direct MCS is performed to validate the accuracy of the stochastic model.

In both cases, with a different number of non-deterministic input variables, the results of PCE-Legendre RSM demonstrate a strong agreement with the direct MCS results. Therefore, it is concluded that the modified RSM is capable of capturing the stochastic mechanical behavior of natural fiber composites to good accuracy, with significantly less computational effort in comparison with the direct MCS. Besides, a parametric sensitivity analysis with respect to the dispersion of the ply misalignment is conducted to identify the effect of uncertainties introduced during the lay-up. It is found that the mean and standard deviation of the maximum tensile force decrease with increasing the standard deviation of the ply orientation. However, the frequency response shows almost no dependence on the standard deviation of the ply orientation up to 3 degrees.

The uncertainty analysis of natural fiber composites can help to characterize the tails of the distribution of mechanical properties, allowing evaluation of their reliability and enabling their use in engineering designs.

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