Pulse Width Modulation (PWM) Signals Based on Spiking Neuronal Networks

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***Abstract*—This paper proposes a digital construction of Pulse Width Modulation (PWM) signals based on the Izhikevich neuron model using a Field Programmable Gate Array (FPGA) platform. The signals are intended for use in diverse electronics applications such as robotics and power converters. A spiking pattern was used to generate the input data and produce the PWM signals. A comparator was used to compare between the spiking pattern data and DC level parameters. The results validate that the proposed hardware can reproduce PWM signals with duty cycles from 0% to 100%.**

***Index Terms*—Spiking Neural Network, Biological Neuron Model, Pulse Width Modulation.**

I. INTRODUCTION

Pulse Width Modulation (PWM) has become a widely known control method in analog and digital circuits. The methods for generating a PWM signal in an analog circuit are complex and sensitive, hence, the digital techniques of producing PWM signals are more common and simple [1][2]. One of the simple techniques utilizes a sawtooth and data signal. A comparator evaluates two values and produces a PWM signal [3]. In this paper, spiking wave-forms of the biological neuron models were used as an alternative of sawtooth waveforms.

Various models have been proposed to validate the dynamic behaviors of neuronal networks. These models are based on the biological detection of the neuron compositions and generally are represented by differential equations.

One of the well-known models is the Hodgkin Huxley (HH) model where the membranes surface ionic structure is taken into consideration [4][9]. FitzHugh-Nagumo (FHN) is another simplified model of the HH model [8]. The Morris-Lecar (ML) is a conductance based model that describes the oscillations in a barnacle muscle fibers [9].

The Hindmarsh-Rose (HR) model [10] can reproduce several neuronal behaviors with precise correlation between the generated frequency and the stimulus current. Although it lacks a reliable description, the Izhikevich neuron model [11] contains neuronal dynamics compared with the HH neuron model.

In all models, there is a trade-off between accuracy and computational complexity: conductance-based models have high biological precision while spiking based models describe temporal behavior of the cortical spikes. In this paper, hardware implementation of the PWM signal generator based on Izhikevich neuron model on FPGA platform is proposed. The rest of this paper is organized as follows: section II presents the Izhikevich model and the PWM signal. The proposed models and error analysis are discusses in section III. Section IV presents the simulation and hardware design of the PWM structure. The proposed block diagram of the PWM generator is presented in section V. Implementation results are presented in section VI. Conclusion is summarized in section VII.

II. BACKGROUND

A. Izhikevich Neuron Model

In [11], [14], a model is proposed, which consists of two coupled differential equations:

$$\frac{dv}{dt} = 0.04V^2 + 5V + 140 - u + I \qquad (1)$$

$$\frac{du}{dt} = a(bV - u) \qquad (2)$$

with the auxiliary reset equations

$$\text{if } V \geq V_{\text{th}} \text{ then } \begin{cases} V \leftarrow c \\ u \leftarrow u + d \end{cases} \qquad (3)$$

where V, is the membrane potential in mV, and u is the membrane recovery variable. The auxiliary reset equations of the membrane voltage and the recovery variables are reset according to equation 3 [11], [14].

B. Pulse Width Modulation (PWM)

Pulse width modulation (PWM) is a powerful technique for controlling analog circuits with digital outputs. It is employed in a wide variety of applications, ranging from measurement and communications to power control and conversion. The applications of PWM are used in control systems like DC-DC converters have been explained in references [15], [16], [17] where the PWM signals were used to convert the unregulated DC voltage to regulated or variable DC voltage. The main parameter is the duty cycle (D); which can be defined as the ratio of ON time over the period as shown in Fig. 1. The duty cycles is given by equation 3.

$$\text{Duty cycle} = \frac{t_{on}}{t_{all}} \times 100 \qquad (4)$$

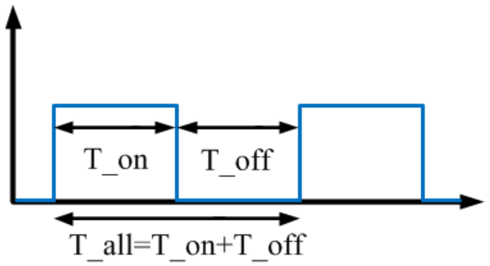

Fig. 1. The PWM signal with duty cycle.

TABLE I
THE COEFFICIENTS VALUES OF THE IMLPWL6 MODEL.

coefficient	values
(m_0, k_0)	(25,-1)
(m_1, k_1)	(1.75,76)
(m_2, k_2)	(0.625,20.5)
(m_3, k_3)	(-0.25,-32.3)
(m_4, k_4)	(-1.75,-142.5)
(m_5, k_5)	(-0.625,-57.5)
(a, b)	(0.015,0.25)

III. MODIFIED MODEL

In order to achieve computational efficiency and attain a low expenditure implementation, the main mathematical model has been modified. The key modification was the computational overhead for the modified design. The expression 0.04V 2 + 5V + 140 is chosen so that V is in mV and time is in ms. However, there are still various obstacles in understanding the model on a digital platform. Hardware implementation of this model is difficult due to the polynomial term of the model that is demonstrated by the parabolic line in Fig. 2. In this paper, The Izhikevich- Multiplier Less Piecewise Linear 6 (IMLPWL6) model was used to realize the neuron. The differential equations are given in equations 5 and 6:

$$\frac{dv}{dt} = F(v) - u + I \tag{5}$$

$$\frac{du}{dt} = a(bV - u) \tag{6}$$

where F is a quadratic function which can be approximated by six PWL segments (shown with red dotted lines), representing linear and exponential terms in the equation. Accordingly, the IMLPWL6 model is given by:

$$F(v) = \begin{cases} m_0 v + k_0 \, ; \; v1 < v < v2 \\ m_1 v + k_1 \, ; \; v2 < v < v3 \\ m_2 v + k_2 \, ; \; v3 < v < v4 \\ m_3 v + k_3 \, ; \; v4 < v < v5 \\ m_4 v + k_4 \, ; \; v5 < v < v6 \\ m_5 v + k_5 \, ; \; \text{else}. \end{cases} \tag{7}$$

where m_i are the slopes of the lines in the IMLPWL6 approximation of the F function. Consequently, the exponential term isis approximated by a line, which has a significantly lower cost compared to the original term. The approximated membrane potential with values are given in Table I.

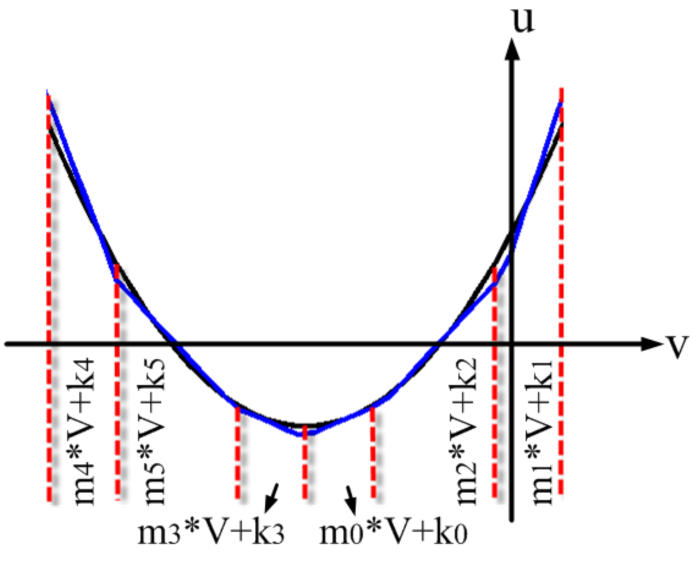

Fig. 2. The accuracy of the proposed model.

TABLE II
MAE COMPUTATIONS FOR DIFFERENT STIMULUS CURRENTS WITH TIME STEP=0.25 MS.

Stimulus current	MAE
0 (uA)	0
5 (uA)	0.0033
7 (uA)	0.0034
10(uA)	0.0040
12.2(uA)	0.0047
25(uA)	0.0075
Mean Error	**0.0038**

IV. SIMULATION

In order to achieve a better comparison between the main and the proposed model, various spiking patterns were simulated using the identical stimulus currents. Fig. 3 confirms that the proposed model mimics the main spiking patterns. In Table II, the Mean Absolute Error (MAE) [4] is calculated at the time step of 0.25 ms.

V. THE PROPOSED BLOCK DIAGRAM OF THE PWM GENERATOR

The block diagram of the proposed architecture is shown in Fig. 4. The system includes: neuron unit, DC level register, and a comparator based on the desired PWM duty cycle. The comparator compares the membrane potential, V, and the output value of the DC level. When the membrane potential output is less than the DC level value, the comparators output will be reset to zero, and, the input stimulus current, I_{app} is used to change the frequency of the membrane potential. Therefore, the duty cycle and the generated PWM frequency are controlled by the proposed PWM generation path. Fig. 5 shows the proposed PWM data path with different external excitation. Fig. 6, and Fig. 7 show the digital implementations of the PWM generation process based on the original Izhikevich and IMLPWL6 models.

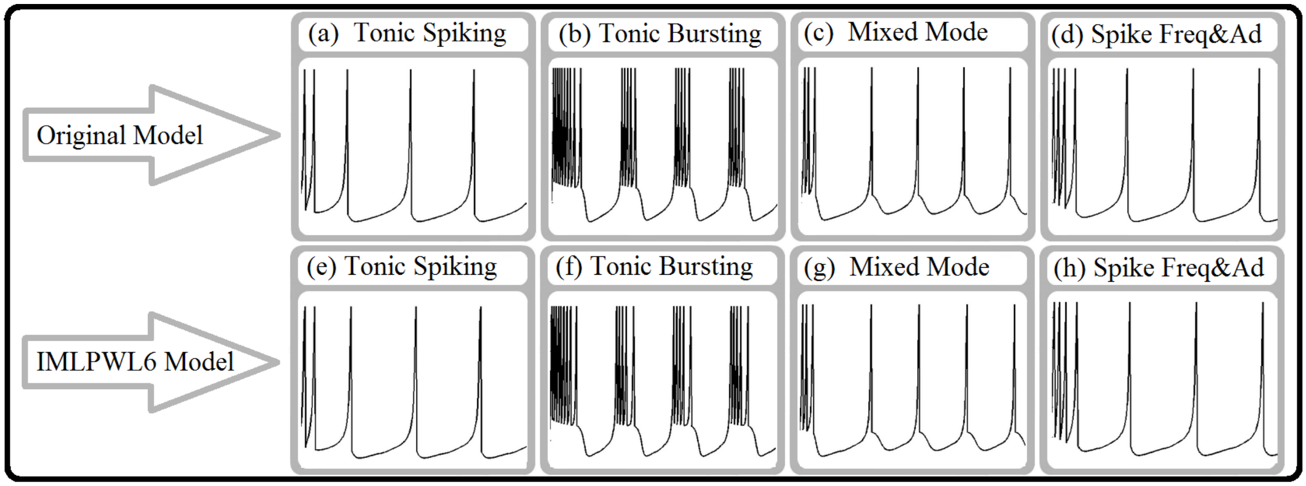

Fig. 3. Detailed comparison between main and proposed spiking ouput patterns. (a) Tonic spiking. (b) Tonic bursting. (c) Mixed mode. (d) Spike frequency adapting. (e) Tonic spiking. (f) Tonic bursting. (g) Mixed mode. (h) Spike frequency adapting.

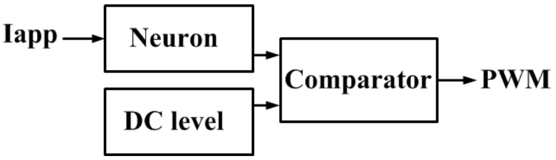

Fig. 4. The proposed PWM data path with diffrent external excitations.

A. PWM and Its application

DC motors are frequently used in robotics platforms. In this section, the proposed PWM generator is used to control the speed of the L298N motor driver module. This module has an H-bridge driver chip to control the internal drive mechanisms of the robots. The prototype circuit of the motor speed control is shown in Fig. 8. The proposed PWM generator is also used to control the luminous intensity of an LED. The measurement results are summarized in Table III.

VI. IMPLEMENTATION RESULT

The proposed and original models are implemented on a XILINX Spartan-6 (XC6SLX9) board, which are shown in Fig. 9. The device utilization of the models are summarized in Table IV.

VII. CONCLUSION

In this paper, Pulse Width Modulation (PWM) signals were generated based on biological models. The implementation results validate that the proposed model can reproduce the PWM signals with a variable duty cycle 0% to 100%, successfully.

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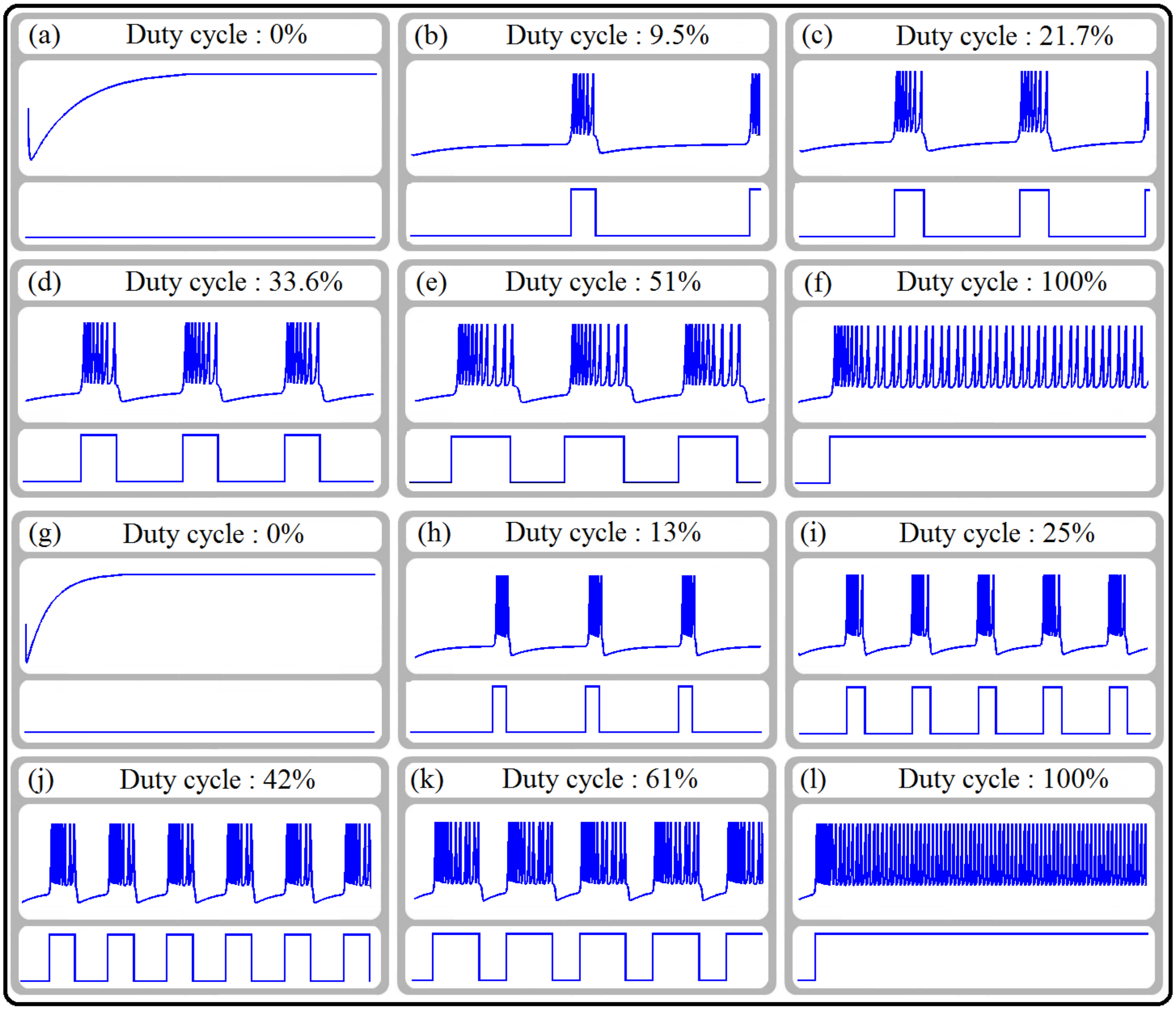
(a) Duty cycle : 0%
(b) Duty cycle : 9.5%
(c) Duty cycle : 21.7%
(d) Duty cycle : 33.6%
(e) Duty cycle : 51%
(f) Duty cycle : 100%
(g) Duty cycle : 0%
(h) Duty cycle : 13%
(i) Duty cycle : 25%
(j) Duty cycle : 42%
(k) Duty cycle : 61%
(l) Duty cycle : 100%

Fig. 5. Output of the biological Izhikevich neuron model and PWM signal simulation. (a) Stimulus current=0μA. (b) Stimulus current=5μA and dt=0.25. (c) Stimulus current=7μA and dt=0.25. (d) Stimulus current=10μA and dt=0.25. (e) Stimulus current=12.2μA and dt=0.25. (f) Stimulus current=25μA and dt=0.25. (g) Stimulus current=0μA and dt=0.5. (h) Stimulus current=5μA and dt=0.5. (i) Stimulus current=7μA and dt=0.5. (j) Stimulus current=10μA and dt=0.5. (k) Stimulus current=12.2μA and dt=0.5. (l) Stimulus current=25μA and dt=0.5.

TABLE III
DUTY CYCLE, V_{RMS}, RPM AND LUX MEASURMENT FOR DIFFERENT STIMULUS CURRENTS BASED ON $dt = 0.25$ MS.

Iapp (μA)	Duty cycle %	V_{RMS} (V)	Motor Speed (RPM)	Illuminance (Lux)
0	0	0	0	0
5	10.28	0.4	99	83
7	16.05	0.7	158	118
10	21.42	1	208	150
12.2	31.7	1.7	309	226
25	100	2.5	987	782

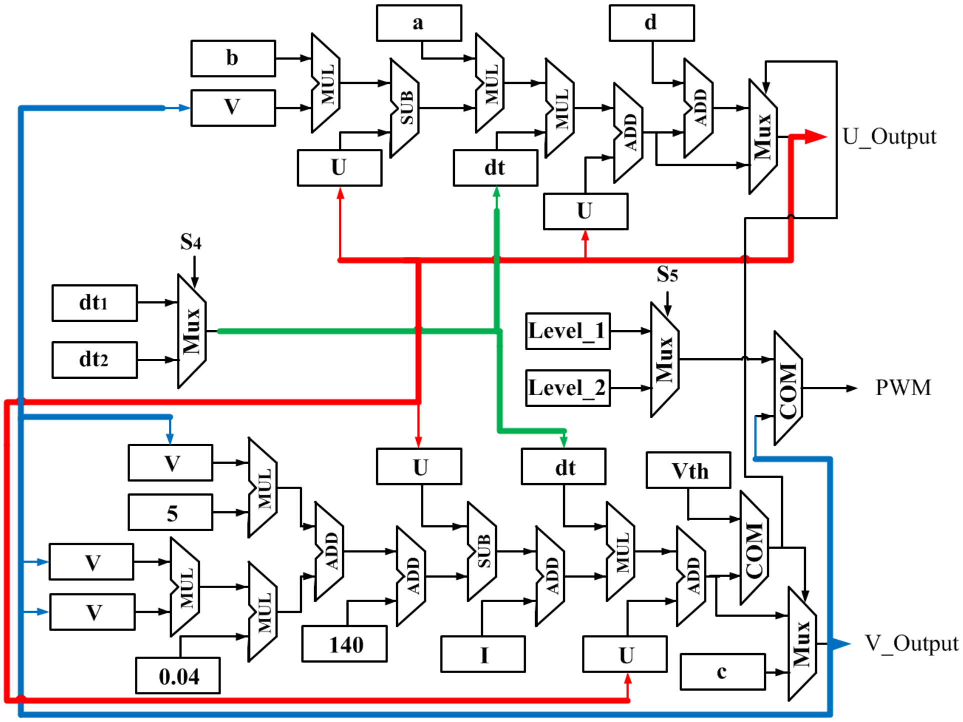

Fig. 6. The architectural synthesis of the PWM based Izhikevich neuron model. Abbreviations: Adder (ADD), Subtractor (SUB), Comparator (COM), Multiplier (MUL), DC Level (level$_1$ and Level$_2$), and Multiplexer (MUX), V , in mV, is the membrane potential, Vth is the apex of the membrane potential, dt represents the time step, u is a membrane recovery variable, a, b, c, d, and 0.04 are controlling parameters and coefcient of the Izhikevich neuron model, respectively.

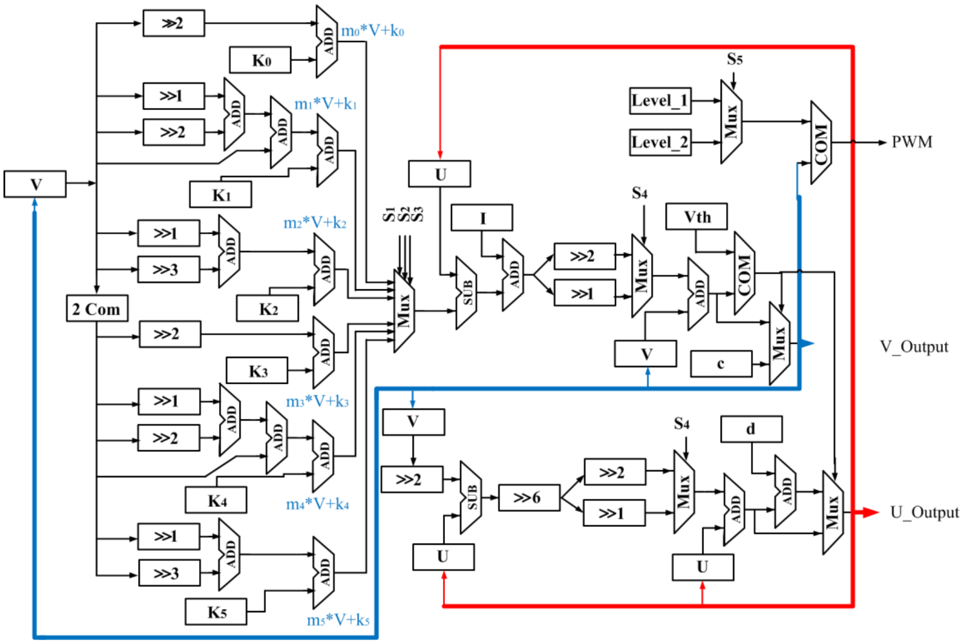

Fig. 7. The architectural synthesis of the PWM based IMLPWL6 neuron model. Abbreviations: Adder (ADD), Subtractor (SUB), Comparator (COM), Two's complement (2 Com), DC Level (level$_1$ and Level$_2$), and Multipelxer (MUX), V, in mV, is the membrane potential, Vth is the apex of the membrane potential, u is a membrane recovery variable, c, and d are controlling parameters and coefcient of the Izhikevich neuron model, respectively. $m0$, $k0$, $m1$, $k1$, $m2$, $k2$, $m3$, $k3$, $m4$, $k4$, $m5$, and $k5$ are cofficient of IMLPWL6 model.

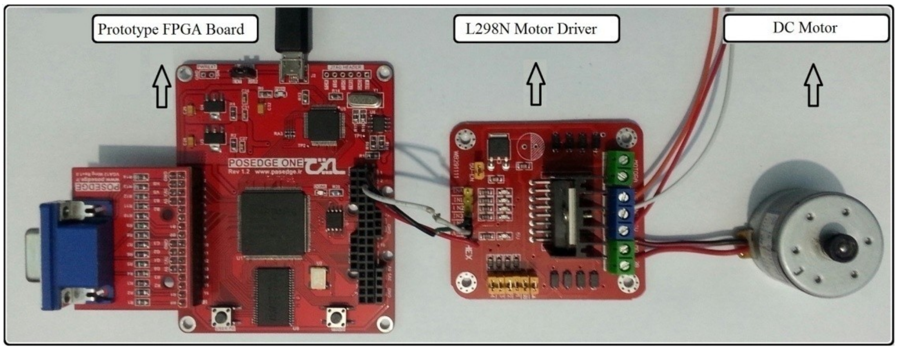

Fig. 8. The prototype circuit of the motor speed control using the proposed PWM generation process.

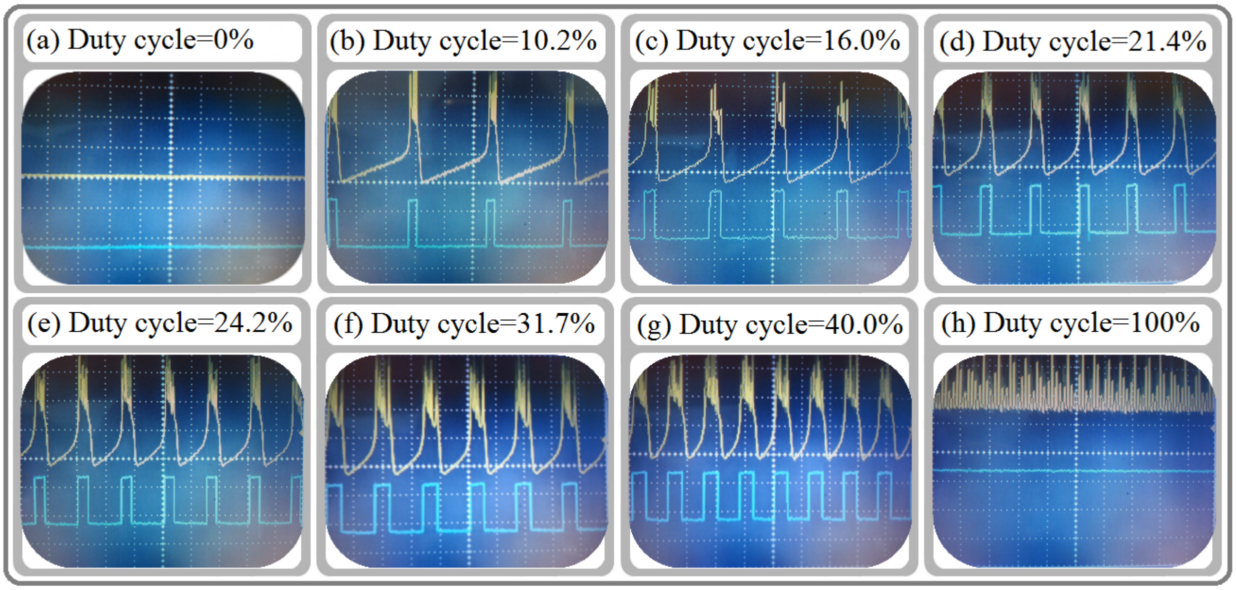

Fig. 9. The output of the PWM signal implemented based on proposed. (a) Stimulus current=0μA. (b) Stimulus current=5μA. (c) Stimulus current=7μA. (d) Stimulus current=10μA. (e) Stimulus current=11.5μA. (f) Stimulus current=12.2μA. (h) Stimulus current=25μA. (time scale = 100 μs, volt. scale = 200 mv.

TABLE IV
DEVICE UTILIZATION OF THE XILINX SPARTAN-6 (XC6SLX9). ABBREVIATIONS: RESOURCE (RES.), UTILIZATION (UTIL.), AVAILABLE (AV.), FF SLICE (FF S.), 4 INPUT LUTS (LUTS), BONDED IOBS (IOBS), DIGITAL SIGNAL PROCESSIG (DSPS), AND FREQUENCY (FREQ.).

*	PWM based Izhikevich			PWM based IMLPWL6		
Res.	Used	Av.	Util.	Used	Av.	Util.
Slices	499	11440	4%	330	11440	2%
FF'S	304	5720	5%	654	5720	11%
LUTs	165	707	25%	209	707	26%
IOBs	10	102	9%	10	102	9%
DSPs(48A1)	12	16	75%	0	16	0%
GCLKs	1	16	6%	1	16	6%
Freq.	57.527 MHZ			107.519 MHZ		