Dynamic shear instabilities in metallic sheets subjected to shear-compression loading

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Abstract

This paper presents an investigation on the formation of adiabatic shear bands in metallic sheets subjected to dynamic shear-compression loading under plane-stress conditions. For that purpose, we have developed a theoretical model based on perturbation analysis, and have performed finite element calculations. The material behavior has been modeled with von Mises plasticity, and the evolution of the yield stress has been considered dependent on plastic strain, plastic strain rate, and temperature. The theoretical model extends the linear stability analysis for simple-shear formulated by Molinari (1997) to shear-compression loading. The numerical simulations have been performed in ABAQUS/Explicit (2016) using a unit-cell model based on the work of Rodríguez-Martínez et al. (2015) in which the shear band formation is favored introducing geometric and material imperfections. Moreover, we have developed a calibration procedure for the stability analysis that enables to make qualitative and quantitative comparisons with the finite element calculations. Both stability analysis predictions and finite element results display the same overall trends, and show that any small negative triaxiality has a profound stabilizing effect on the material behavior delaying, or even preventing, the formation of a shear band. This key outcome has been substantiated for a wide range of strain rates and for materials with different strain hardening coefficients, strain rate sensitivities, and thermal softening behaviors.

Keywords:

Shear-compression loading, Adiabatic shear bands, **Inertia**, Linear stability analysis, Finite element calculations

1. Introduction

The formation of shear localization bands generally precedes ductile failure of metals and alloys subjected to shear-dominated dynamic loading scenarios, e.g. during machining (Molinari et al., 2002; Cai and Dai, 2014), high-speed penetration (Borvik et al., 2001) and high-velocity metalworking operations (Chen et al.,

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2003). The interested reader can find a wealth of experimental evidence and additional examples in which shear bands lead to catastrophic failure of structures in Chapter 2 of the book edited by Bai and Dodd's (Bai and Dodd, 2012). Molinari and Clifton (1987) divided shear localization bands into two types: *those in which thermal softening plays a negligible role in their formation and those in which thermal softening plays a primary role*. In the first case, the shear bands, which are often referred to as *isothermal shear bands*, are formed as a result of microstructural softening mechanisms such as dynamic recrystallization (Rittel et al., 2008; Osovski et al., 2012, 2013) or deformation texture evolution (Kuroda and Tvergaard, 2007; Kuroda and Needleman, 2019). The second type of shear localization bands correspond to the so-called *adiabatic shear bands*, which are formed when the thermal softening caused by the local increase of temperature under adiabatic conditions of deformation overcomes the strain hardening of the material (Zener and Hollomon, 1944). While both types of shear bands, *isothermal* and *adiabatic*, have been extensively studied over the past 80 years, our interest in this paper is limited to those of the latter type.

The experiments of Hartley et al. (1987) and Marchand and Duffy (1988) marked a turning point in the research on adiabatic shear bands. These authors twisted thin-walled tubes made of AISI 1018 cold rolled steel, AISI 1020 hot rolled steel and HY-100 structural steel at shear strain rates ranging from 1000 s^{-1} to 5000 s^{-1} using a torsional Kolsky bar and they measured, for the first time, both local strains and local temperatures during the specimen deformation. The experimental evidence enabled to identify three stages in the initiation and growth of an adiabatic shear band: in the first step, the strain in the specimen was homogeneous; in the second step, after the maximum stress was reached, the strain started to localize with a progressive decrease in the width of the region over which localization was occurring; and in the third stage, the strain localized further into an ever narrower shear band, leading to an important increase in the material temperature and a sharp drop in the flow stress, with the subsequent fracture of the specimen. Duffy and co-workers (Cho et al., 1990; Duffy, 1991; Duffy and Chi, 1992; Liao and Duffy, 1998) published additional experimental results for two tempers of AISI 4340 VAR steel of varying hardness and Ti6Al4V titanium alloy. All these papers concluded that the mechanisms involved in the development of the adiabatic shear band are cumulative, i.e. the high temperature band becomes progressively narrower, and ever higher strain rates and temperatures are attained in this narrowing band, which accelerates the localization process. This *autocatalytic behavior*, as it was called by Molinari and Clifton (1987), has also been observed by other authors over the years (e.g. Wang et al. (1988); Zhou et al. (1996b,a); Bonnet-Lebouvier and Klepaczko (2002)), and has motivated that the adiabatic shear bands were accepted as a thermomechanical (material) instability.

Bai (1982) published what is probably the first paper in the open literature in which the formation of

adiabatic shear bands is modeled using linear stability theory. The technique consists of evaluating the rate of growth of an infinitesimal spatio-temporal perturbation which is added to the fundamental solution of the problem at different loading times, i.e. for different background strains, to determine whether shear localization can occur: if there is any perturbation mode that grows with time, the fundamental solution is unstable, and a shear band can be formed. The assumption is that the perturbation growth represents the first stages of the shear localization process. Bai (1982) demonstrated that for isotropic materials with yield stress dependent on strain, strain rate and temperature, the condition for the shear instability to occur, i.e. for the initiation of the shear band, is that the thermal softening caused by plastic work overcomes the work hardening of the material, in agreement with the classical explanation of Zener and Hollomon (1944). Batra and Chen (2001) demonstrated that for adiabatic simple-shearing deformations, the critical strain obtained from the instability criterion of Bai (1982) equals that given by the maximum stress condition derived from the Considère (1885) criterion. Moreover, Molinari (1988) used the linear stability analysis to calculate the critical strain leading to the onset of shear localization in AISI 1018 cold rolled steel, and found very good agreement with the experimental results reported by Hartley et al. (1987) for a shear strain rate of 1000 s^{-1} . Molinari (1988) also performed a qualitative evaluation of the dynamical and heat conduction effects on the perturbation growth, and showed that the stabilizing effects of inertia and thermal conductivity on small and large perturbation modes, respectively, promote the speed of growth of intermediate modes. The perturbation mode that grows the fastest is referred to as the *critical perturbation mode*, and the corresponding perturbation growth as the *critical perturbation grow rate*. Molinari (1997) showed for AISI 1018 steel and titanium materials, modeled as isotropic solids with yield stress described by a thermo-viscoplastic power-law constitutive equation, that the critical perturbation grow rate evolves with the strain of the fundamental solution such that it first increases, reaches a maximum, and then decreases. It was assumed that the background strain associated to the largest critical perturbation grow rate corresponds to the strain for which shear bands form in the experiments of Nesterenko et al. (1995b,a), in which thick-walled cylinders were collapsed radially at strain rates of the order of 10000 s^{-1} . Good agreement between theoretical predictions and experimental results was found for both steel and titanium materials. In these experiments, in which the material is subjected to pure-shear deformations, the shear bands emanate from the inner boundary of the cylinder, with spiral trajectories and regular spacing. Molinari (1997) also showed that the average bands spacing in these tests finds good agreement with the predictions of the stability analysis for the wavelength associated to the perturbation mode corresponding to the largest critical perturbation grow rate. Following the work of Molinari (1997), Daridon et al. (2004) and Batra and Wei (2006) used the stability analysis to explore the constitutive sensitivity of the average shear bands spacing in the dynamic radial collapse

of thick-walled cylinders, and showed that the spacing increases with the thermal conductivity and the strain rate sensitivity, and decreases with the strain hardening of the material. The effect of material behavior on the shear bands spacing was further confirmed in the experiments performed by Xue et al. (2002, 2004) and Lovinger et al. (2011, 2015) with various metallic materials including Ti6Al4V, AISI 304L steel and Mg-AM50. Other recent works that have used the linear stability theory to investigate the formation of shear bands in metallic materials are the papers of Rodríguez-Martínez et al. (2015) and Longère (2018), who analyzed the respective roles of thermal softening and dynamic recrystallization in the initiation of shear bands in Ti6Al4V, and the article of Gurrutxaga-Lerma (2018), who studied the conditions that dislocation generation and motion must fulfill to promote the development of adiabatic shear bands in crystalline metals. In addition, Arriaga et al. (2015, 2016) recast the stability analysis as a generalized eigenvalue problem that can be implemented in a finite element code to determine the onset of shear localization, damage and fracture, at the element level, in general multidimensional applications. Numerical results on a 1D problem representative of a dynamic torsion experiment showed that the eigenvalue analysis recovers the instability point predicted by the classical analytical criterion of Bai (1982). The eigenvalue analysis was later used in the finite element simulation of a series of 2D problems, including the Kalthoff experiment, bringing out the capacity of this approach to detect the formation and propagation of cracks, even in complex situations like crack branching (Arriaga and Waisman, 2017, 2018a,b).

Today, almost 40 years after Bai's paper (Bai, 1982), it is widely accepted that the stability analysis is a very useful tool for identifying the essential material features that control the formation of adiabatic shear bands, and that it provides results which are in qualitative and quantitative agreement with the experimental evidence. However, all the analytical stability analyses cited in this introduction are concerned only with one-dimensional simple-shear deformations and, to the authors' knowledge, the application of the linear perturbation theory to investigate analytically the formation of shear bands under two-dimensional shear-dominated loadings is still missing. This is precisely the goal of this paper, in which the linear stability analysis for simple-shear deformations developed by Molinari (1997) is extended to consider dynamic shear-compression loadings under plane-stress conditions. The reason to investigate such stress state is given by the dynamic shear experiments in sheet materials performed by Peirs et al. (2012) and Xu et al. (2017, 2018, 2019), among others, who showed that, despite the efforts to improve the specimens design to obtain stress homogeneity and zero triaxiality in the gauge, undesired normal stresses frequently arise in the shear zone of the samples during the tests. In this regard, the linear stability analysis developed in this paper shows that any small negative triaxiality has a profound stabilizing effect on the material behavior delaying, or even preventing, the formation of a shear band. The

theoretical predictions have been compared with finite element calculations performed in ABAQUS/Explicit (2016) for von Mises materials with different strain hardening coefficients, strain rate sensitivities, and thermal softening behaviors. A key outcome of this investigation is that both stability analysis predictions and finite element results display the same overall trends, and the agreement is also quantitative for some of the strain rates and material behaviors investigated.

2. Constitutive framework

The constitutive model follows von Mises plasticity. The strain rate tensor $\mathbf{d}$ is decomposed as the sum of an elastic $\mathbf{d}^e$ and a plastic part $\mathbf{d}^p$:

$$\mathbf{d} = \mathbf{d}^e + \mathbf{d}^p \quad (1)$$

The relation between the elastic strain rate and the stress rate is given by the following hypo-elastic law:

$$\dot{\boldsymbol{\sigma}} = \mathbf{C} : \mathbf{d}^e \quad (2)$$

where $\dot{\boldsymbol{\sigma}}$ is an objective derivative of the Cauchy stress tensor and $\mathbf{C}$ is the tensor of isotropic elastic moduli given by:

$$\mathbf{C} = 2G\tilde{\mathbf{I}} + K\mathbf{1} \otimes \mathbf{1} \quad (3)$$

where G is the elastic shear modulus, K is the bulk modulus, $\mathbf{1}$ is the second-order unit tensor and $\tilde{\mathbf{I}}$ is the fourth-order deviatoric unit tensor.

Moreover, the plastic part of the strain rate tensor is given by the following associated flow rule:

$$\mathbf{d}^p = \dot{\kappa} \frac{\partial f}{\partial \boldsymbol{\sigma}} \quad (4)$$

where $\dot{\kappa}$ is the rate of the plastic multiplier and $f = \bar{\sigma} - \sigma_Y$ is the yield **condition**, with $\bar{\sigma} = \sqrt{\frac{3}{2} \mathbf{s} : \mathbf{s}}$ being the equivalent stress, where $\mathbf{s} = \boldsymbol{\sigma} - \sigma_h : \mathbf{1}$ and $\sigma_h = \frac{1}{3} \boldsymbol{\sigma} : \mathbf{1}$. Moreover, σ_Y is the yield stress of the material defined as:

$$\sigma_Y(\bar{\varepsilon}^p, \dot{\bar{\varepsilon}}^p, \theta) = \sigma_Y^0 + \sigma_K (\bar{\varepsilon}^p)^n \left(\frac{\dot{\bar{\varepsilon}}^p}{\dot{\varepsilon}_{ref}} \right)^m \left(\frac{\theta}{\theta_{ref}} \right)^\nu \quad (5)$$

where $\bar{\varepsilon}^p = \int_0^t \dot{\bar{\varepsilon}}^p(\tau) d\tau$ is the equivalent plastic strain, $\dot{\bar{\varepsilon}}^p = \sqrt{\frac{2}{3} \mathbf{d}^p : \mathbf{d}^p}$ is the equivalent plastic strain rate and θ is the material temperature. The parameters σ_Y^0 and σ_K are the initial yield stress and the plastic modulus, respectively. Moreover n , m and ν are constants which represent the strain, strain rate and temperature sensitivities of the material. The parameters $\dot{\varepsilon}_{ref}$ and θ_{ref} are the reference strain rate and temperature, respectively.

The values of the material constants used in the constitutive model are taken from Vaz-Romero et al. (2015), and correspond to the AISI 430 steel, see Table 1. Unless otherwise noted, this set of parameters is used in the stability analysis calculations and finite element simulations presented in this paper.

Symbol	Property and units	Value
ρ_o	Initial density (kg/m ³), Eq. (21)	7740
C_p	Specific heat (J/kg K), Eq. (21)	460
k	Thermal conductivity (W/m K), Eq. (21)	26.1
α	Thermal expansion coefficient (K ⁻¹)	0.00001
K	Bulk modulus (GPa), Eq. (3)	196
G	Shear modulus (GPa), Eq. (3)	75
σ_Y^0	Initial yield stress (MPa), Eq. (5)	175.67
σ_K	Strain hardening modulus (MPa), Eq. (5)	530.13
n	Strain hardening exponent, Eq. (5)	0.167
$\dot{\varepsilon}_{ref}$	Reference strain rate (s ⁻¹), Eq. (5)	0.01
m	Strain rate sensitivity exponent, Eq. (5)	0.0118
θ_{ref}	Reference temperature (K), Eq. (5)	300
ν	Temperature sensitivity exponent, Eq. (5)	-0.51
χ	Taylor-Quinney coefficient, Eq. (21)	0.9

Table 1: *Conventional material constants and parameters used in the constitutive model for AISI 430 steel. Data after Vaz-Romero et al. (2015).*

3. Theoretical model

We develop a linear stability analysis for the theoretical modelling of shear instabilities in von Mises materials subjected to shear-compression loading. A rectangular sheet of initial thickness b^0 and edges of initial length L_x^0 and L_y^0 , with $L_x^0 \gg L_y^0 \gg b^0$, is considered for the analysis. Note that the superscript $()^0$ is used to denote quantities at time $t = 0$. As illustrated in Fig. 1, the sheet is subjected to **prescribed** velocities on opposed sides, V_y and V_x . It is assumed that this loading condition is always satisfied. Moreover, the elastic deformations are neglected and from now on $\bar{\varepsilon} = \bar{\varepsilon}^p$ will be named equivalent strain and $\dot{\bar{\varepsilon}} = \dot{\bar{\varepsilon}}^p$ equivalent strain rate. Note that (X, Y, Z) and (X', Y', Z') are the Lagrangian Cartesian frames associated to the applied velocity field and the shear plane, respectively, and (x, y, z) and (x', y', z') are their Eulerian counterparts. Note also that $Z = Z'$ and $z = z'$, being zero the stresses in this direction (plane-stress condition). The shear plane, which contains the x' axis, is determined by the condition $V_{x'}|_{y'=0} = 0$. The orientation of the shear plane with respect to the x axis is defined by the angle β . Note that this angle varies with the loading time t . Note also that, while the imposed velocity field is expressed as a function of the Lagrangian coordinates, the problem will be formulated in the (x', y', z') frame. The kinematics of the problem in the (x, y, z) and (x', y', z') frames is shown in Appendix A.

Figure 1: Scheme of a rectangular sheet of initial thickness b^0 and edges of initial length L_x^0 and L_y^0 subjected to an imposed velocity field given by $V_x = \dot{\varepsilon}_x^0 X + 2\dot{\varepsilon}_{xy}^0 Y$ and $V_y = -\dot{\varepsilon}_y^0 Y$. Note that (X, Y, Z) and (X', Y', Z') are the Lagrangian Cartesian frames associated to the applied velocity field and the shear plane, respectively, and (x, y, z) and (x', y', z') are their Eulerian counterparts. Note also that $Z = Z'$ and $z = z'$, being zero the stresses in this direction (plane-stress condition). The shear plane is determined by the condition $V_{x'}|_{y'=0} = 0$. The orientation of the shear plane with respect to the x axis is defined by the angle β . This angle varies with the loading time.

3.1. Governing equations

The unknowns of the problem are: velocities ($V_{x'}$, $V_{y'}$), strain rates ($d_{x'x'}$, $d_{y'y'}$, $d_{z'z'}$, $d_{x'y'}$), thickness (b), stresses ($\sigma_{x'x'}$, $\sigma_{y'y'}$, $\sigma_{x'y'}$), equivalent stress ($\bar{\sigma}$), yield stress (σ_Y), equivalent strain ($\bar{\varepsilon}$), equivalent strain rate ($\dot{\bar{\varepsilon}}$), rate of plastic multiplier ($\dot{\kappa}$) and temperature (θ). The fundamental equations governing the loading process are presented below:

- Rate of deformation tensor

$$d_{x'x'} = \frac{\partial V_{x'}}{\partial x'} = 0 \quad (6)$$

$$d_{y'y'} = \frac{\partial V_{y'}}{\partial y'} \quad (7)$$

$$d_{x'y'} = \frac{1}{2} \left(\frac{\partial V_{x'}}{\partial y'} + \frac{\partial V_{y'}}{\partial x'} \right) \quad (8)$$

where the explicit expressions of $V_{x'}$ and $V_{y'}$ as a function of x' and y' are given in Appendix A.2, see equation A.14.

- Incompressibility

$$d_{x'x'} + d_{y'y'} + d_{z'z'} = 0 \quad (9)$$

- Thickness

$$d_{z'z'} = \frac{\dot{b}}{b} \quad (10)$$

Previous expression is obtained from the kinematics of the problem described in Appendix A.

- Equivalent strain rate and equivalent strain

$$\dot{\bar{\varepsilon}} = \sqrt{\frac{2}{3} \left(d_{x'x'}^2 + d_{y'y'}^2 + d_{z'z'}^2 + 2d_{x'y'}^2 \right)} \quad (11)$$

$$\bar{\varepsilon} = \int_0^t \dot{\bar{\varepsilon}}(\tau) d\tau \quad (12)$$

- Yield condition

$$f = \bar{\sigma} - \sigma_Y = 0 \quad (13)$$

where, as reported in Section 2, $\bar{\sigma}$ and σ_Y are the equivalent stress and the yield stress, respectively.

- Equivalent stress

$$\bar{\sigma} = \sqrt{\sigma_{x'x'}^2 + \sigma_{y'y'}^2 - \sigma_{x'x'}\sigma_{y'y'} + 3\sigma_{x'y'}^2} \quad (14)$$

- Yield stress

$$\sigma_Y(\bar{\varepsilon}, \dot{\bar{\varepsilon}}, \theta) = \sigma_Y^0 + \sigma_K(\bar{\varepsilon})^n \left(\frac{\dot{\bar{\varepsilon}}}{\dot{\bar{\varepsilon}}_{ref}} \right)^m \left(\frac{\theta}{\theta_{ref}} \right)^\nu \quad (15)$$

Previous expression is obtained replacing $\bar{\varepsilon}^p$ and $\dot{\bar{\varepsilon}}^p$, by $\bar{\varepsilon}$ and $\dot{\bar{\varepsilon}}$, in equation (5).

- Flow rule

$$d_{x'x'} = \dot{\kappa} \frac{2\sigma_{x'x'} - \sigma_{y'y'}}{2\bar{\sigma}} = 0 \quad (16)$$

$$d_{y'y'} = \dot{\kappa} \frac{2\sigma_{y'y'} - \sigma_{x'x'}}{2\bar{\sigma}} \quad (17)$$

$$d_{x'y'} = \dot{\kappa} \frac{3\sigma_{x'y'}}{2\bar{\sigma}} \quad (18)$$

- Balance of linear momentum

$$\frac{\partial}{\partial x'} (b\sigma_{x'x'}) + \frac{\partial}{\partial y'} (b\sigma_{x'y'}) = \rho b \frac{DV_{x'}}{Dt} \quad (19)$$

$$\frac{\partial}{\partial x'} (b\sigma_{x'y'}) + \frac{\partial}{\partial y'} (b\sigma_{y'y'}) = \rho b \frac{DV_{y'}}{Dt} \quad (20)$$

The momentum balance equation in the out-of-plane direction has been disregarded.

- Energy

$$c\dot{\theta} = k \left(\frac{\partial^2 \theta}{\partial x'^2} + \frac{\partial^2 \theta}{\partial y'^2} \right) + \frac{k}{b} \left(\frac{\partial \theta}{\partial x'} \frac{\partial b}{\partial x'} + \frac{\partial \theta}{\partial y'} \frac{\partial b}{\partial y'} \right) + \chi \bar{\sigma} \dot{\bar{\epsilon}} \quad (21)$$

where $c = \rho C_p$, k and χ are the heat capacity per unit volume, the thermal conductivity and the Taylor-Quinney coefficient, respectively. Recall that the material is considered incompressible, and thus $\rho = \rho_0$. Note that in previous expression we have assumed no heat flow at the specimen's boundaries and neglected the contribution of thermoelastic effects.

The set of equations (6)-(21) is to be solved under the following initial and boundary conditions expressed in the (X, Y, Z) frame:

$$\begin{aligned}
V_x(X, Y, 0) &= \dot{\varepsilon}_x^0 X + 2\dot{\varepsilon}_{xy}^0 Y, & V_y(X, Y, 0) &= -\dot{\varepsilon}_y^0 Y, & \sigma_{ij}(X, Y, 0) &= \sigma_Y^0 \\
\bar{\varepsilon}(X, Y, 0) &= 0, & \theta(X, Y, 0) &= \theta^0, & b(X, Y, 0) &= b^0
\end{aligned} \tag{22}$$

$$\begin{aligned}
V_x(X, \pm \frac{L_y^0}{2}, t) &= \dot{\varepsilon}_x^0 X \pm \dot{\varepsilon}_{xy}^0 L_y^0, & V_x(\pm \frac{L_x^0}{2}, Y, t) &= \pm \dot{\varepsilon}_x^0 \frac{L_x^0}{2} + 2\dot{\varepsilon}_{xy}^0 Y \\
V_y(X, \pm \frac{L_y^0}{2}, t) &= \mp \dot{\varepsilon}_y^0 \frac{L_y^0}{2}, & V_y(\pm \frac{L_x^0}{2}, Y, t) &= -\dot{\varepsilon}_y^0 Y \\
\frac{\partial \theta}{\partial X} \Big|_{X=\pm L_x^0/2} &= \frac{\partial \theta}{\partial Y} \Big|_{Y=\pm L_y^0/2} = 0
\end{aligned} \tag{23}$$

where we take $\theta^0 = \theta_{ref}$ and $b^0 = 2$ mm. The constants $\dot{\varepsilon}_x^0 \geq 0$, $\dot{\varepsilon}_y^0 \geq 0$ and $\dot{\varepsilon}_{xy}^0 > 0$ define the values of the initial strain rates in the sheet. In this paper we limit our attention to shear-dominated loading conditions such that $\dot{\varepsilon}_{xy}^0 \gg \dot{\varepsilon}_x^0$ and $\dot{\varepsilon}_{xy}^0 \gg \dot{\varepsilon}_y^0$, and assume that the sheet does not buckle due to the load applied in the Y direction. Note that if $\dot{\varepsilon}_x^0 = \dot{\varepsilon}_y^0 = 0$, we retrieve the simple-shear problem studied, for instance, by Wright (1995), Molinari (1997), Daridon et al. (2004) and Gurrutxaga-Lerma (2018). The applied loading is selected through the input value of the initial equivalent strain rate $\dot{\bar{\varepsilon}}^0$, and the initial strain rates $\dot{\varepsilon}_x^0$ and $\dot{\varepsilon}_y^0$. Moreover, the value of $\dot{\varepsilon}_{xy}^0$ which enters into the initial and boundary conditions is obtained from the following expression:

$$\dot{\varepsilon}_{xy}^0 = \sqrt{\frac{3}{4} (\dot{\bar{\varepsilon}}^0)^2 - (\dot{\varepsilon}_x^0)^2 - (\dot{\varepsilon}_y^0)^2 - \dot{\varepsilon}_x^0 \dot{\varepsilon}_y^0} \tag{24}$$

which is derived from the definition of the equivalent strain rate at $t = 0$ using the incompressibility condition.

3.2. Linear stability analysis

Let us denote by $\mathbb{S}(x', y', t)$ the fundamental solution of the problem. The solution at $t = t^1$ is:

$$\mathbb{S}^1 = (V_{x'}^1, V_{y'}^1, d_{x'x'}^1, d_{y'y'}^1, d_{z'z'}^1, d_{x'y'}^1, \bar{\varepsilon}^1, \dot{\bar{\varepsilon}}^1, \dot{\kappa}^1, b^1, \theta^1, \sigma_{x'x'}^1, \sigma_{y'y'}^1, \sigma_{x'y'}^1, \bar{\sigma}^1, \sigma_Y^1)^T \tag{25}$$

All the unknowns of the problem at time t^1 can be readily obtained by integration of the governing equations stated in Section 3.1, satisfying the initial and boundary conditions given in (22) and (23). Note that the

superscript $()^1$ is used to denote quantities evaluated at t^1 . Note also that this fundamental solution considers large strains and deformations, as can be seen in the kinematics of the problem detailed in Appendix A.

Next, a small perturbation $\delta\mathbb{S}$ given by:

$$\delta\mathbb{S}(y', t)_{t^1} = \delta\mathbb{S}^1 e^{i\xi y'} e^{\eta(t-t^1)} \quad (26)$$

is imposed over the fundamental solution, where:

$$\delta\mathbb{S}^1 = (\delta V_{x'}, \delta V_{y'}, \delta d_{x'x'}, \delta d_{y'y'}, \delta d_{z'z'}, \delta d_{x'y'}, \delta \bar{\varepsilon}, \delta \dot{\bar{\varepsilon}}, \delta \dot{\kappa}, \delta b, \delta \theta, \delta \sigma_{x'x'}, \delta \sigma_{y'y'}, \delta \sigma_{x'y'}, \delta \bar{\sigma}, \delta \sigma_Y)^T \quad (27)$$

is the perturbation amplitude, ξ is the wavenumber and η is the growth rate of the perturbation at time t^1 . Note that t^1 is the time at which the perturbation is inserted. Note also that η will be referred to as instantaneous instability index everywhere in this paper. Moreover, while in this paper we have assumed the classical frozen coefficients theory and the exponential form of the time dependency of the perturbation for the sake of simplicity, Xavier et al. (2020) have recently developed a promising method to relax these assumptions. Furthermore, we seek only for shear instability modes such that the perturbation is imposed in the Eulerian coordinate y' . For the current shear-compression problem, we assume that no other instabilities like necking can develop (necking requires thickness reduction). On the other hand, note that there are several papers in the literature in which specific stability analyses have been formulated to study the formation of dynamic necks in tensile dominated loadings, e.g. Fressengeas and Molinari (1985, 1994), Guduru and Freund (2002), Mercier and Molinari (2003, 2004) and N'souglo et al. (2020). In future works, in order to investigate the formation of both shear and necking instabilities, the stability analysis should be extended to consider shear-tensile loadings.

The solution of the problem is:

$$\mathbb{S} = \mathbb{S}^1 + \delta\mathbb{S} \quad (28)$$

with $|\delta\mathbb{S}| \ll |\mathbb{S}^1|$. By substituting equation (28) into equations (6)-(21), and retaining only first-order terms in

the increments $\delta\mathbb{S}^1$, linearized equations are obtained.

In order to scale the problem and bring out the key variables which control the loading process, we introduce the following nondimensional groups:

$$\begin{aligned}\hat{V}_{x'} &= \frac{2}{L_y^0} \frac{V_{x'}}{\dot{\bar{\varepsilon}}^1}, & \hat{V}_{y'} &= \frac{2}{L_y^0} \frac{V_{y'}}{\dot{\bar{\varepsilon}}^1}, & \hat{d}_{x'x'} &= \frac{d_{x'x'}}{\dot{\bar{\varepsilon}}^1}, & \hat{d}_{y'y'} &= \frac{d_{y'y'}}{\dot{\bar{\varepsilon}}^1}, & \hat{d}_{z'z'} &= \frac{d_{z'z'}}{\dot{\bar{\varepsilon}}^1}, & \hat{d}_{x'y'} &= \frac{d_{x'y'}}{\dot{\bar{\varepsilon}}^1} \\ \hat{\bar{\varepsilon}} &= \bar{\varepsilon}, & \hat{\dot{\bar{\varepsilon}}} &= \frac{\dot{\bar{\varepsilon}}}{\dot{\bar{\varepsilon}}^1}, & \hat{\kappa} &= \frac{\dot{\kappa}}{\dot{\bar{\varepsilon}}^1}, & \hat{b} &= \frac{b}{b^0}, & \hat{\theta} &= \frac{\theta}{\theta^0} \\ \hat{\sigma}_{x'x'} &= \frac{\sigma_{x'x'}}{\sigma_Y^0}, & \hat{\sigma}_{y'y'} &= \frac{\sigma_{y'y'}}{\sigma_Y^0}, & \hat{\sigma}_{x'y'} &= \frac{\sigma_{x'y'}}{\sigma_Y^0}, & \hat{\bar{\sigma}} &= \frac{\bar{\sigma}}{\sigma_Y^0}, & \hat{\sigma}_Y &= \frac{\sigma_Y}{\sigma_Y^0}\end{aligned}$$

Likewise, the nondimensional variables defining the perturbation are:

$$\hat{\eta} = \frac{\eta}{\dot{\bar{\varepsilon}}^1}, \quad \hat{\xi} = \xi \frac{L_y^0}{2}$$

that yield to the following linearized equations.

- Rate of deformation tensor

$$\delta\hat{d}_{x'x'} = 0 \tag{29}$$

$$\delta\hat{d}_{y'y'} - i\hat{\xi}\delta\hat{V}_{y'} = 0 \tag{30}$$

$$\delta\hat{d}_{x'y'} - \frac{i\hat{\xi}}{2}\delta\hat{V}_{x'} = 0 \tag{31}$$

- Incompressibility

$$\delta \hat{d}_{x'x'} + \delta \hat{d}_{y'y'} + \delta \hat{d}_{z'z'} = 0 \quad (32)$$

- Thickness

$$\left(\hat{d}_{z'z'}^1 - \hat{\eta} \right) \delta \hat{b} + \hat{b}^1 \delta \hat{d}_{z'z'} = 0 \quad (33)$$

- Equivalent strain rate and equivalent strain

$$\delta \hat{\varepsilon} - \frac{2}{3} \left(\hat{d}_{x'x'}^1 \delta \hat{d}_{x'x'} + \hat{d}_{y'y'}^1 \delta \hat{d}_{y'y'} + \hat{d}_{z'z'}^1 \delta \hat{d}_{z'z'} + 2 \hat{d}_{x'y'}^1 \delta \hat{d}_{x'y'} \right) = 0 \quad (34)$$

$$\delta \hat{\varepsilon} - \hat{\eta} \delta \hat{\varepsilon} = 0 \quad (35)$$

- Yield condition

$$\delta \hat{\sigma} - \delta \hat{\sigma}_Y = 0 \quad (36)$$

- Equivalent stress

$$\hat{\sigma}^1 \delta \hat{\sigma} + \frac{\hat{\sigma}_{y'y'}^1 - 2 \hat{\sigma}_{x'x'}^1}{2} \delta \hat{\sigma}_{x'x'} + \frac{\hat{\sigma}_{x'x'}^1 - 2 \hat{\sigma}_{y'y'}^1}{2} \delta \hat{\sigma}_{y'y'} - 3 \hat{\sigma}_{x'y'}^1 \delta \hat{\sigma}_{x'y'} = 0 \quad (37)$$

- Yield stress

$$\delta\hat{\sigma}_Y - \hat{H}\delta\hat{\varepsilon} - \hat{S}\delta\hat{\dot{\varepsilon}} - \hat{P}\delta\hat{\theta} = 0 \quad (38)$$

where $\hat{H} = \frac{1}{\sigma_Y^0} \left. \frac{\partial\sigma_Y}{\partial\bar{\varepsilon}} \right|_{\mathfrak{t}^1}$, $\hat{S} = \frac{\dot{\bar{\varepsilon}}^1}{\sigma_Y^0} \left. \frac{\partial\sigma_Y}{\partial\dot{\bar{\varepsilon}}} \right|_{\mathfrak{t}^1}$ and $\hat{P} = \frac{\theta^0}{\sigma_Y^0} \left. \frac{\partial\sigma_Y}{\partial\theta} \right|_{\mathfrak{t}^1}$.

- Flow rule

$$\hat{\sigma}^1 \delta \hat{d}_{x'x'} + \hat{d}_{x'x'}^1 \delta \hat{\sigma} - \hat{\kappa}^1 \delta \hat{\sigma}_{x'x'} + \frac{\hat{\kappa}^1 \delta \hat{\sigma}_{y'y'}}{2} - \frac{2\hat{\sigma}_{x'x'}^1 - \hat{\sigma}_{y'y'}^1}{2} \delta \hat{\kappa} = 0 \quad (39)$$

$$\hat{\sigma}^1 \delta \hat{d}_{y'y'} + \hat{d}_{y'y'}^1 \delta \hat{\sigma} - \hat{\kappa}^1 \delta \hat{\sigma}_{y'y'} + \frac{\hat{\kappa}^1 \delta \hat{\sigma}_{x'x'}}{2} - \frac{2\hat{\sigma}_{y'y'}^1 - \hat{\sigma}_{x'x'}^1}{2} \delta \hat{\kappa} = 0 \quad (40)$$

$$\hat{\sigma}^1 \delta \hat{d}_{x'y'} + \hat{d}_{x'y'}^1 \delta \hat{\sigma} - \frac{3\hat{\kappa}^1}{2} \delta \hat{\sigma}_{x'y'} - \frac{3\hat{\sigma}_{x'y'}^1}{2} \delta \hat{\kappa} = 0 \quad (41)$$

- Balance of linear momentum

$$i\hat{\xi} \left(\hat{b}^1 \delta \hat{\sigma}_{x'y'} + \hat{\sigma}_{x'y'}^1 \delta \hat{b} \right) - \hat{\eta} \frac{\hat{b}^1}{\hat{L}^2} \delta \hat{V}_{x'} = 0 \quad (42)$$

$$i\hat{\xi} \left(\hat{b}^1 \delta \hat{\sigma}_{y'y'} + \hat{\sigma}_{y'y'}^1 \delta \hat{b} \right) - \hat{\eta} \frac{\hat{b}^1}{\hat{L}^2} \delta \hat{V}_{y'} = 0 \quad (43)$$

where $\hat{L}^{-1} = \frac{\dot{\bar{\varepsilon}}^1 L_y^0}{2} \sqrt{\frac{\rho}{\sigma_Y^0}}$ is the inertial resistance to motion.

- Energy

$$\left(\hat{\eta}\hat{c} + \hat{\xi}^2\hat{k}\right)\delta\hat{\theta} - \chi\hat{\sigma}^1\delta\hat{\varepsilon} - \chi\delta\hat{\sigma} = 0 \quad (44)$$

$$\text{where } \hat{c} = c \frac{\theta^0}{\sigma_Y^0} \text{ and } \hat{k} = k \left(\frac{2}{L_y^0}\right)^2 \frac{\theta^0}{\sigma_Y^0 \hat{\varepsilon}^1}.$$

A non-trivial solution for $\delta\mathbb{S}^1$ can only be obtained if the determinant of the matrix of coefficients of the system of linear algebraic equations (29)-(44) is equal to zero. Application of this condition leads to a fourth-degree polynomial in $\hat{\eta}$:

$$\textcolor{red}{B}_4(\mathbb{S}^1)\hat{\eta}^4 + B_3(\mathbb{S}^1, \hat{\xi})\hat{\eta}^3 + B_2(\mathbb{S}^1, \hat{\xi})\hat{\eta}^2 + B_1(\mathbb{S}^1, \hat{\xi})\hat{\eta} + B_0(\mathbb{S}^1, \hat{\xi}) = 0 \quad (45)$$

where the coefficients $\textcolor{red}{B}_i$, with $i = 0, \dots, 4$, are not shown explicitly for the sake of brevity. Equation (45) has four roots in $\hat{\eta}$. The requisite for unstable growth of the perturbation is given by the real part of the instantaneous instability index $\hat{\eta}$ to be positive. The perturbation growth represents the first stage of the shear localization process, i.e. if the perturbation grows a shear band can be formed. The root of equation (45) with the greatest positive real part is the physical solution of the problem and it is considered for the analysis (Dudzinski and Molinari, 1991). Equation (45) shows that the dimensionless instantaneous instability index $\hat{\eta}$ depends on the fundamental solution $\mathbb{S}^1$ (i.e. on the loading time t and thus on the equivalent strain in the material, see Fig. 3 in Section 5) and the dimensionless wavenumber $\hat{\xi}$. For a given value of the loading time t , the stabilizing effect of inertia on short wavenumbers, and of strain rate sensitivity and thermal conductivity on large wavenumbers, promotes the growth of intermediate modes (see Section 5). The mode with the greatest value of $\hat{\eta}$ is referred to as the critical instantaneous wavenumber $\hat{\xi}_{\hat{\eta}}^c$, and the corresponding grow rate of the perturbation is referred to as the critical instantaneous grow rate $\hat{\eta}^c$. The value of $\hat{\eta}^c$, which evolves with the loading time (i.e. with the equivalent strain in the material, see Fig. 5 in Section 5), is generally used as an indicator of the material stability in shear band problems: the greater the critical instantaneous grow rate, the more prone the material is to develop shear localization (see also the works of Molinari (1997) and Rodríguez-Martínez et al. (2015)). An alternative indicator of the stability of the solution was proposed by Fressengeas and Molinari (1994) who defined a cumulative instability index, $\hat{I} = \int_0^t \hat{\eta} \dot{\varepsilon}^1 dt$, which tracks the history of the instantaneous grow rate of all the growing modes. The cumulative instability index has been

used by several authors to predict the formation of necks at high strain rates (see Petit et al. (2005), El Maï et al. (2014) and N'souglo et al. (2020)). To calculate $\hat{I}$, we introduce the perturbation at different times and sum the **dimensionless** instantaneous instability index $\hat{\eta}$ obtained for each loading time. The mode with the greatest value of the cumulative instability index $\hat{I}$ is called the critical cumulative wavenumber $\hat{\xi}_f^c$, and the corresponding cumulative index is referred to as the critical cumulative grow rate $\hat{I}^c$. Like the critical instantaneous grow rate, $\hat{I}^c$ also evolves with the loading time, and thus with the equivalent strain in the material (see Fig. 5 in Section 5).

Results of the stability analysis for both **dimensionless** instantaneous and cumulative instability indexes are shown in Section 5. Moreover, the calibration of the linear stability analysis and the comparison between stability analysis predictions and numerical simulations for the shear bands formation are presented in Section 7.

4. Finite element model

This section describes the 2D finite element model developed in ABAQUS/Explicit (2016) to simulate shear localization under shear-compression loading. Accordingly to the theoretical model, the problem consists of a long thin sheet extended in the X direction and subjected to plane-stress conditions, see Fig. 2. Material points are referred to using a Cartesian coordinate system with positions in the reference configuration denoted as (X, Y, Z) . The origin of coordinates is located in the center of mass of the specimen. The initial and boundary conditions are the same used in the theoretical model (see Fig. 1 and equations (22) and (23) in Section 3.1). Following the work of Molinari and Clifton (1987), a sinusoidal spatial imperfection has been introduced in the finite element model to promote early shear localization:

$$L_x = L_x^0 - \delta \left[1 + \cos \left(\frac{2\pi Y}{L_y^0} \right) \right] \quad (46)$$

where L_x is the perturbed length of the specimen and δ is the amplitude of the imperfection. Consistent with the notation used in the theoretical model presented in Section 3, L_x^0 and L_y^0 are the unperturbed initial length and initial width of the sample, respectively. We have chosen $\delta = \frac{L_x^0}{100}$ and $L_x^0 = 23.5 L_y^0$, with $L_y^0 = 0.508$ mm as in Rodríguez-Martínez et al. (2015). The dimensions of the finite element model are based on the (typical) slot of the Shear-Compression Specimen (Vural et al., 2011). Notice that the value of L_y^0 is similar to the average spacing between shear bands in the experimental and analytical results of Nesterenko et al. (1995b)

and Molinari (1997).

Moreover, in order to break the fundamental solution and favor shear localization, the initial yield stress of the material σ_Y^0 (see Table 1) has been perturbed in a narrow strip of width γ , extended in the X direction and located in the center of the specimen (see Fig. 2). Four elements on each side of the X axis have been perturbed (i.e. $\gamma = 0.1$ mm). The number of perturbed elements has been chosen arbitrarily. The results reported in Appendix B.1 show the effect of the number of elements perturbed in the shear band formation (i.e. the effect of γ). The initial yield stress of each perturbed element is a random number generated using the Marsaglia polar method according to a normal distribution function. The mean of this distribution is $\mu = \sigma_Y^0 C_u$ and the standard deviation is $s = \sigma_Y^0 C_v$, where C_u determines the average perturbed yield stress and C_v the maximum dispersion of the generated random values of perturbed yield stress. The finite element calculations have been performed for $C_u = 0.975$ and $C_v = 0.01$. The specific influence of C_u in the shear band formation is shown in Appendix B.2.

Figure 2: Finite element model. Mesh and boundary conditions. (For interpretation of the references to colour in the text, the reader is referred to the web version of this article.)

The finite element model is meshed with four-node bilinear plane-stress **coupled displacement-temperature elements** with reduced integration and hourglass control (CPS4RT in ABAQUS notation), with initial dimensions $\approx 25 \times 12.5 \mu\text{m}^2$. A mesh convergence study has been performed, in which the time evolution of the stress and strain fields in the specimen were compared for different mesh sizes. There is some mesh sensitivity in the numerical calculations, however it does not affect significantly the finite element results presented in this paper, neither quantitatively nor qualitatively (note that we do not investigate the width of the *fully-developed shear band* which, indeed, it is mesh dependent). This is attributed to the regularizing effect of inertia, strain rate sensitivity and thermal conductivity (see also Jacques et al. (2012), McAuliffe and Waisman (2013), Rodríguez-Martínez et al. (2015), Czarnota et al. (2017) and Svolos et al. (2020)). The constitutive model presented in Section 2 has been implemented in ABAQUS/Explicit (2016) through a VUMAT user subroutine using the integration algorithm developed by Zaera and Fernández-Sáez (2006).

Selected finite element simulations which show the stabilizing effect of compression on shear band formation are reported in Section 6. In addition, the comparison of the finite element results with the stability analysis predictions for the critical strain which leads to the shear bands formation is presented Section 7.

5. Salient features of the stability analysis

This section shows the main outcomes of the linear stability analysis. The initial equivalent strain rate in all the calculations presented in this section is $\dot{\bar{\epsilon}}^0 = 10000 \text{ s}^{-1}$. **Note that we have checked that the fundamental solution perturbed in the stability analysis, equation (25), matches with the solution obtained from the finite element model until localization occurs in the numerical simulations.**

Fig. 3 shows the instantaneous instability index $\hat{\eta}$ versus the wavenumber $\hat{\xi}$ for different values of the equivalent strain $\bar{\epsilon}$, namely: 1.25, 1.5, 2, 2.5 and 3.

Results corresponding to $\dot{\bar{\epsilon}}_x^0 = \dot{\bar{\epsilon}}_y^0 = 0 \text{ s}^{-1}$ (simple-shear) are shown in Fig. 3(a). The instantaneous instability index $\hat{\eta}$ first increases with ξ , reaches a maximum, and then decreases. Molinari (1997) demonstrated that inertia slows down the growth of small wavenumbers, and strain rate sensitivity and thermal conductivity slow down the growth of large wavenumbers, see Section 3.2. Recall also from Section 3.2 that the maximum value of the instantaneous instability index is referred to as the critical instantaneous grow rate $\hat{\eta}^c$, and the wavenumber that grows the fastest is the critical instantaneous wavenumber $\hat{\xi}_{\hat{\eta}}^c$. Moreover, note that while the increase of the equivalent strain from 1.25 to 2 shifts the $\hat{\eta} - \hat{\xi}$ curves upwards, a further increase of $\bar{\epsilon}$ leads to a continuous drop in the instantaneous instability index. Note that the perturbation analyses performed

by Molinari (1997) and Daridon et al. (2004) for different materials modeled with von Mises plasticity and subjected to simple-shear also predicted a non-monotonic influence of the equivalent strain on the instantaneous instability index.

Results corresponding to $\dot{\varepsilon}_x^0 = 100 \text{ s}^{-1}$ and $\dot{\varepsilon}_y^0 = 200 \text{ s}^{-1}$ are shown in Fig. 3(b). Notice that, as $\dot{\varepsilon}_x^0$ and $\dot{\varepsilon}_y^0$ increase, the initial shear strain rate $\dot{\varepsilon}_{xy}^0$ decreases (because $\dot{\varepsilon}^0$ is kept constant). Notice also that all the shear-compression loading cases analyzed in this paper fulfill the relation $\dot{\varepsilon}_y^0/\dot{\varepsilon}_x^0 = 2$ (in absence of the shear loading this ratio is close to uniaxial compression). A detailed analysis on the influence of the ratio $\dot{\varepsilon}_y^0/\dot{\varepsilon}_x^0$ on the formation of shear bands under shear-compression loading is left for a future work. On the one hand, similarly to the results reported in Fig. 3(a), the $\hat{\eta}-\hat{\xi}$ curves display a concave-downward shape, with a maximum $\hat{\eta}^c$ for an intermediate wavenumber. As in the simple-shear case, the value of $\hat{\eta}^c$ is a non-monotonic function of the equivalent strain, and it shows a maximum for an intermediate value of $\bar{\varepsilon}$. On the other hand, the comparison with the results shown in Fig. 3(a) reveals that the compression reduces the range of growing perturbation modes and decreases the value of the instantaneous instability index. We have checked that the decrease in the grow rate of the perturbation is not induced by the decrease of the shear strain rate (for a fixed shear strain rate, the perturbation grow also decreases with the increase of the compression rate). These results suggest that the compression loading stabilizes material behavior, retarding or preventing shear band formation, as will be shown in Sections 6 and 7 using finite element simulations.

Figure 3: Linear stability analysis. Instantaneous instability index $\hat{\eta}$ versus wavenumber $\hat{\xi}$ for an initial equivalent strain rate $\dot{\varepsilon}^0 = 10000 \text{ s}^{-1}$. Different values of the equivalent strain are considered, $\bar{\varepsilon} = 1.25, 1.5, 2, 2.5$ and 3 . (a) $\dot{\varepsilon}_x^0 = \dot{\varepsilon}_y^0 = 0 \text{ s}^{-1}$ (simple-shear) and (b) $\dot{\varepsilon}_x^0 = 100 \text{ s}^{-1}$ and $\dot{\varepsilon}_y^0 = 200 \text{ s}^{-1}$.

Fig. 4 shows the cumulative instability index $\hat{I}$ versus the wavenumber $\hat{\xi}$ for the same values of equivalent

strain $\bar{\varepsilon} = 1.25, 1.5, 2, 2.5$ and 3 . As the instantaneous instability index (see Fig. 3), $\hat{I}$ first increases with $\hat{\xi}$, reaches a maximum (referred to as the critical cumulative grow rate $\hat{I}^c$, see Section 3.2), and then decreases. The comparison between the results presented in Figs. 4(a) and 4(b), for $\dot{\varepsilon}_x^0 = \dot{\varepsilon}_y^0 = 0 \text{ s}^{-1}$ (simple-shear) and $\dot{\varepsilon}_x^0 = 100 \text{ s}^{-1}$ and $\dot{\varepsilon}_y^0 = 200 \text{ s}^{-1}$, respectively, shows that the compression decreases the value of the cumulative instability index (in the same way that it decreases the value of the instantaneous instability index, see Fig. 3). Moreover, unlike the $\hat{\eta} - \bar{\varepsilon}$ curves, the cumulative instability index increases monotonically with $\bar{\varepsilon}$ because $\hat{I}$ is the temporal integration of $\hat{\eta}$, and the instantaneous instability index is positive.

Figure 4: Linear stability analysis. Cumulative instability index $\hat{I}$ versus wavenumber $\hat{\xi}$ for an initial equivalent strain rate $\dot{\varepsilon}^0 = 10000 \text{ s}^{-1}$. Different values of the equivalent strain are considered, $\bar{\varepsilon} = 1.25, 1.5, 2, 2.5$ and 3 . (a) $\dot{\varepsilon}_x^0 = \dot{\varepsilon}_y^0 = 0 \text{ s}^{-1}$ (simple-shear) and (b) $\dot{\varepsilon}_x^0 = 100 \text{ s}^{-1}$ and $\dot{\varepsilon}_y^0 = 200 \text{ s}^{-1}$.

Fig. 5 shows the evolution of the critical instantaneous grow rate $\hat{\eta}^c$ and the critical cumulative grow rate $\hat{I}^c$ with the equivalent strain $\bar{\varepsilon}$ for different values of the pair $\{\dot{\varepsilon}_x^0, \dot{\varepsilon}_y^0\}$. Notice that the corresponding values of the critical instantaneous and cumulative wavenumbers, $\hat{\xi}_{\hat{\eta}}^c$ and $\hat{\xi}_{\hat{I}}^c$, also evolve with the strain (nevertheless, the $\hat{\xi}_{\hat{\eta}}^c - \bar{\varepsilon}$ and $\hat{\xi}_{\hat{I}}^c - \bar{\varepsilon}$ curves are not shown here for the sake of brevity).

The results corresponding to the critical instantaneous grow rate are shown in Fig. 5(a). The $\hat{\eta}^c - \bar{\varepsilon}$ curves display a concave-downwards shape, with a maximum for $\bar{\varepsilon} \approx 2$ for all the loading cases investigated. As the compression loading increases, the $\hat{\eta}^c - \bar{\varepsilon}$ curve is shifted downwards, the minimum equivalent strain for the perturbation to grow increases, and the range of equivalent strains for which the perturbation grows decreases, which reinforces the idea that the compression contributes to material stabilization, and therefore delays or prevents shear band formation. The results corresponding to the critical cumulative grow rate are shown in Fig. 5(b). The $\hat{I}^c - \bar{\varepsilon}$ curves show a monotonic increase of the critical cumulative perturbation grow with the

equivalent strain. Similarly to the results obtained for the critical instantaneous growth rate, the value of $\hat{I}^c$ decreases with the increase of the compression loading, indicating the stabilization of the material.

The specific criteria to determine when a shear band is formed attending to the critical instantaneous and cumulative growth rates, $\hat{\eta}^c$ and $\hat{I}^c$, are given in Section 7.

Figure 5: Linear stability analysis. (a) Critical instantaneous growth rate $\hat{\eta}^c$ and (b) critical cumulative growth rate $\hat{I}^c$ versus equivalent strain $\bar{\epsilon}$ for an initial equivalent strain rate $\dot{\epsilon}^0 = 10000 \text{ s}^{-1}$. Results for different values of the pair $\{\dot{\epsilon}_x^0, \dot{\epsilon}_y^0\}$ are presented: $\{0 \text{ s}^{-1}, 0 \text{ s}^{-1}\}$, $\{50 \text{ s}^{-1}, 100 \text{ s}^{-1}\}$, $\{100 \text{ s}^{-1}, 200 \text{ s}^{-1}\}$, $\{150 \text{ s}^{-1}, 300 \text{ s}^{-1}\}$ and $\{200 \text{ s}^{-1}, 400 \text{ s}^{-1}\}$.

Fig. 6 shows the evolution of the stress triaxiality σ_h and the orientation of the shear plane β (see Fig. 1) with the equivalent strain $\bar{\epsilon}$ for the same values of the pair $\{\dot{\epsilon}_x^0, \dot{\epsilon}_y^0\}$ considered in Fig. 5. Recall that the stress triaxiality is defined as the ratio between the hydrostatic and the equivalent stress.

The results for the stress triaxiality are presented in Fig. 6(a). For simple-shear, the value of σ_h is (obviously) zero. On the other hand, for the shear-compression loading cases the triaxiality is negative, and it decreases with the evolution of the equivalent strain. As an example, for $\dot{\epsilon}_x^0 = 200 \text{ s}^{-1}$ and $\dot{\epsilon}_y^0 = 400 \text{ s}^{-1}$, the triaxiality evolves from -0.0147 for $\bar{\epsilon} = 1.3$ (onset of unstable behavior) to -0.0173 for $\bar{\epsilon} = 4$. Similar values of triaxiality have been reported in shear-dominated experiments used to investigate the shear failure of metallic sheets (e.g. see Peirs et al. (2012)).

The results for the orientation of the shear plane β are shown in Fig. 6(b). For $\dot{\epsilon}_x^0 = 0 \text{ s}^{-1}$ and $\dot{\epsilon}_y^0 = 0 \text{ s}^{-1}$, the shear plane (obviously) contains the x axis (see Fig. 1). Moreover, for the shear-compression loading

cases, the value of β is slightly positive (see Fig. 1) and decreases with the equivalent strain. For instance, for $\dot{\epsilon}_x^0 = 200 \text{ s}^{-1}$ and $\dot{\epsilon}_y^0 = 400 \text{ s}^{-1}$ the orientation of the shear band plane evolves from 0.627° for $\bar{\epsilon} = 1.3$ (onset of unstable behavior) to 0.560° for $\bar{\epsilon} = 4$.

Figure 6: Linear stability analysis. Evolution of (a) stress triaxiality σ_h and (b) orientation of the shear plane β with equivalent strain $\bar{\epsilon}$ for an initial equivalent strain rate $\dot{\epsilon}^0 = 10000 \text{ s}^{-1}$. Results for different values of the pair $\{\dot{\epsilon}_x^0, \dot{\epsilon}_y^0\}$ are presented: $\{0 \text{ s}^{-1}, 0 \text{ s}^{-1}\}$, $\{50 \text{ s}^{-1}, 100 \text{ s}^{-1}\}$, $\{100 \text{ s}^{-1}, 200 \text{ s}^{-1}\}$, $\{150 \text{ s}^{-1}, 300 \text{ s}^{-1}\}$ and $\{200 \text{ s}^{-1}, 400 \text{ s}^{-1}\}$.

In summary, the stability analysis predictions presented in this section for both instantaneous and cumulative indexes (see Figs. 3, 4 and 5) suggest that any small negative triaxiality (see Fig. 6) has a profound stabilizing effect on the material behavior delaying, or even preventing, the formation of a shear band. This conclusion is further investigated in the next section using finite element calculations.

6. Key finite element results

This section shows the main outcomes of the finite element simulations. As in previous section, the initial equivalent strain rate is set to $\dot{\epsilon}^0 = 10000 \text{ s}^{-1}$.

Fig. 7 shows the average equivalent strain in path A , denoted by $\bar{\epsilon}_A^{avg}$, versus the average equivalent strain in path B , denoted by $\bar{\epsilon}_B^{avg}$, for the same values of the pair $\{\dot{\epsilon}_x^0, \dot{\epsilon}_y^0\}$ considered in Figs. 5 and 6. The paths A and B are indicated in Fig. 2 with dashed orange lines. As the loading process starts, the strain field in the

specimen is fairly uniform so that $\bar{\varepsilon}_A^{avg}/\bar{\varepsilon}_B^{avg} \approx 1$ (despite of the perturbation of the yield stress in the center of the sample, see Section 4). With the continuation of the loading process, the average equivalent strain in path A starts to grow faster than in path B due to the formation of strain gradients along the width of the specimen. For all the loading cases investigated, the greatest deformations are located in the zone of the finite element model which contains the perturbed elements (see Fig. 2). The contours of equivalent strain shown in Fig. 8 reveal that the gradients of strain along the width of the sample are less severe as the compression rate increases. As an example, for simple-shear (black solid line in Fig. 7), the ratio $\bar{\varepsilon}_A^{avg}/\bar{\varepsilon}_B^{avg}$ increases very rapidly and without bound, which reveals that path B no longer deforms, and that plastic flow has localized in the center of the specimen leading to the formation of a shear band (the specific criterion to determine when a shear band is formed in the numerical simulations is given in Section 7). In the equivalent strain contours of Fig. 8(a), the shear band corresponds to the red strip located in the center of the specimen and extended along the longitudinal direction. Note that the shear band is not originated at the boundaries of the specimen, as follows from the fact that the maximum strains are located far from the ends of the sample. This suggests that the shear band formation is not much affected by the boundary conditions and the size of the finite element model. In addition, we have performed simulations with longer specimens and have checked that the $\bar{\varepsilon}_A^{avg} - \bar{\varepsilon}_B^{avg}$ curves are similar for all cases, see Appendix C. This is consistent with the theoretical model which assumes that $L_x^0 \gg L_y^0$ (see Section 3). Moreover, for $\dot{\varepsilon}_y^0 = 400 \text{ s}^{-1}$ (dotted orange line in Fig. 7), the increase of the ratio $\bar{\varepsilon}_A^{avg}/\bar{\varepsilon}_B^{avg}$ is gradual and steady, which indicates that path B is still deforming, and that shear localization has not occurred. To be noted that the finite element simulations for $\dot{\varepsilon}_x^0 > 0$ and $\dot{\varepsilon}_y^0 > 0$ predict a local increase in thickness, within the localization band, which is not considered in the stability analysis.

The finite element results shown in this section are in qualitative agreement with the stability analysis predictions presented in Section 5, and reinforce the idea that the formation of shear bands is delayed, or even prevented, under shear-compression loading. In the next section of the paper, we carry out quantitative comparisons between the stability analysis predictions and the finite element results.

7. Comparison between linear stability results and finite element calculations

The quantitative comparison between stability analysis and finite element calculations requires to define criteria to determine when a shear band is formed. The criterion used in the linear stability analysis, which relies on the concept of *effective instability* introduced by Dudzinski and Molinari (1991), considers that a shear band is incepted when the critical instantaneous and cumulative grow rates reach specific values $\hat{\eta}^c = \hat{\eta}_{SB}^c$ and $\hat{I}^c = \hat{I}_{SB}^c$ (to be determined). We are aware that this is a rather crude assumption, but it enables to carry out

Figure 7: Finite element simulations. Average equivalent strain in path A, denoted by $\bar{\varepsilon}_A^{avg}$, versus average equivalent strain in path B, denoted by $\bar{\varepsilon}_B^{avg}$, for an initial equivalent strain rate $\dot{\varepsilon}^0 = 10000 \text{ s}^{-1}$. Results for different values of the pair $\{\dot{\varepsilon}_x^0, \dot{\varepsilon}_y^0\}$ are presented: $\{0 \text{ s}^{-1}, 0 \text{ s}^{-1}\}$, $\{50 \text{ s}^{-1}, 100 \text{ s}^{-1}\}$, $\{100 \text{ s}^{-1}, 200 \text{ s}^{-1}\}$, $\{150 \text{ s}^{-1}, 300 \text{ s}^{-1}\}$ and $\{200 \text{ s}^{-1}, 400 \text{ s}^{-1}\}$. (For interpretation of the references to colour in the text, the reader is referred to the web version of this article.)

Figure 8: Finite element simulations. Contours of equivalent strain $\bar{\epsilon}$ for an initial equivalent strain rate $\dot{\bar{\epsilon}}^0 = 10000 \text{ s}^{-1}$. Results for different values of the pair $\{\dot{\epsilon}_x^0, \dot{\epsilon}_y^0\}$ are presented: (a) $\{0 \text{ s}^{-1}, 0 \text{ s}^{-1}\}$, (b) $\{50 \text{ s}^{-1}, 100 \text{ s}^{-1}\}$, (c) $\{100 \text{ s}^{-1}, 200 \text{ s}^{-1}\}$, (d) $\{150 \text{ s}^{-1}, 300 \text{ s}^{-1}\}$ and (e) $\{200 \text{ s}^{-1}, 400 \text{ s}^{-1}\}$. The average strain in path B is $\bar{\epsilon}_B^{\text{avg}} = 1$. (For interpretation of the references to colour in the text, the reader is referred to the web version of this article.)

quantitative comparisons with the finite element results and to check the capacity of the stability analysis to capture the mechanisms which control shear band formation under shear-compression loading (nevertheless, consistent with other works published in the literature, e.g. Molinari (1997) and Daridon et al. (2004), the results presented in this section show that there are not universal **critical** instability indexes valid for all materials). The equivalent strains for which the conditions $\hat{\eta}^c = \hat{\eta}_{SB}^c$ and $\hat{I}^c = \hat{I}_{SB}^c$ are met (for the first time in the case of $\hat{\eta}^c$), see Fig. 5, are referred to as *theoretical critical strains* and they are denoted by $\bar{\varepsilon}_{\hat{\eta}}^c$ and $\bar{\varepsilon}_{\hat{I}}^c$, respectively. Similarly, the criterion used in the finite element simulations, which is based on the definition of shear localization reported by Molinari and Clifton (1987), establishes that a shear band is formed when the slope of the $\bar{\varepsilon}_A^{avg} - \bar{\varepsilon}_B^{avg}$ curve reaches a given value $d\bar{\varepsilon}_A^{avg}/d\bar{\varepsilon}_B^{avg} = FEM_{SB}^c$ (to be determined). The average equivalent strain in path B for which the condition $d\bar{\varepsilon}_A^{avg}/d\bar{\varepsilon}_B^{avg} = FEM_{SB}^c$ is met for the first time is named the *numerical critical strain* $\bar{\varepsilon}_{FEM}^c$. We are aware that considering the average strain in Path A in the criterion used in the finite element simulations amounts to assume that the shear band is aligned with the X axis (see Fig. 2). However, as shown in Fig. 6(b), this is not the case if $\dot{\varepsilon}_y^0 > 0$. Nevertheless, since the angle formed by the shear band plane and the X axis is less than 1° for all the compression strain rates and equivalent strains investigated (see Fig. 6(b)), we assume that using $\bar{\varepsilon}_A^{avg}$ is a good approximation which makes easier to obtain and interpret the finite element results.

Fig. 9 shows the evolution of the theoretical critical strains, $\bar{\varepsilon}_{\hat{\eta}}^c$ and $\bar{\varepsilon}_{\hat{I}}^c$, and the numerical critical strain, $\bar{\varepsilon}_{FEM}^c$, with the initial strain rate along Y direction, $\dot{\varepsilon}_y^0$. The stability analysis results obtained with the instantaneous and cumulative indexes correspond to $\hat{\eta}_{SB}^c = 2, 2.5$ and 3 , and $\hat{I}_{SB}^c = 0.1, 1$ and 2 , respectively. The finite element results correspond to $FEM_{SB}^c = 5, 10$ and 20 . The imposed initial equivalent strain rate is $\dot{\varepsilon}^0 = 10000 \text{ s}^{-1}$.

The comparison of the theoretical critical strain obtained with the instantaneous instability index $\bar{\varepsilon}_{\hat{\eta}}^c$ with the numerical critical strain $\bar{\varepsilon}_{FEM}^c$ is shown in Fig. 9(a). The $\bar{\varepsilon}_{\hat{\eta}}^c - \dot{\varepsilon}_y^0$ curves display a concave-upward shape with increasing slope as the compression rate increases. The slope of these curves, which is initially mild, eventually tends to infinity, such that there is a value of $\dot{\varepsilon}_y^0$ beyond which the criterion $\hat{\eta}^c = \hat{\eta}_{SB}^c$ is not met (this is because the $\hat{\eta}^c - \bar{\varepsilon}$ curves in Fig. 5(a) present a maximum). Note that, as $\hat{\eta}_{SB}^c$ increases, the $\bar{\varepsilon}_{\hat{\eta}}^c - \dot{\varepsilon}_y^0$ curves are shifted upwards, and the range of compression rates for which the condition $\hat{\eta}^c = \hat{\eta}_{SB}^c$ is met decreases. Moreover, the numerical critical strain $\bar{\varepsilon}_{FEM}^c$ also increases nonlinearly with the compression rate. The increase of FEM_{SB}^c shrinks the range of values of $\dot{\varepsilon}_y^0$ for which the criterion $d\bar{\varepsilon}_A^{avg}/d\bar{\varepsilon}_B^{avg} = FEM_{SB}^c$ is met (the finite element simulations are performed with increments of 10 s^{-1} for the initial strain rate

along Y direction until the criterion $d\bar{\varepsilon}_A^{avg}/d\bar{\varepsilon}_B^{avg} = FEM_{SB}^c$ is not met). Note that the fact that the condition $d\bar{\varepsilon}_A^{avg}/d\bar{\varepsilon}_B^{avg} = FEM_{SB}^c$ is not met implies that an adiabatic shear band is not formed (attending to the criterion used in this paper). In other words, the numerical simulations show that the compression strain rate stabilizes material behavior delaying shear localization and suppressing the formation of shear bands for large values of $\dot{\varepsilon}_y^0$. Moreover, the value of FEM_{SB}^c makes practically no difference for the numerical critical strain $\bar{\varepsilon}_{FEM}^c$ (when the criterion $d\bar{\varepsilon}_A^{avg}/d\bar{\varepsilon}_B^{avg} = FEM_{SB}^c$ is met). The best agreement between theoretical predictions and finite element calculations is obtained with $\hat{\eta}_{SB}^c = 2.5$ and $FEM_{SB}^c = 5$. The theoretical and numerical critical strains, and the slopes of the $\bar{\varepsilon}_{\hat{\eta}}^c - \dot{\varepsilon}_y^0$ and $\bar{\varepsilon}_{FEM}^c - \dot{\varepsilon}_y^0$ curves, are very similar. The maximum compression rate for which the conditions $\hat{\eta}_{SB}^c = 2.5$ and $FEM_{SB}^c = 5$ are met is similar too: $\approx 238 \text{ s}^{-1}$ in the linear stability analysis and $\approx 230 \text{ s}^{-1}$ in the finite element calculations.

The comparison of the theoretical critical strain obtained with the cumulative instability index $\bar{\varepsilon}_I^c$ with the numerical critical strain $\bar{\varepsilon}_{FEM}^c$ is presented in Fig. 9(b). The $\bar{\varepsilon}_I^c - \dot{\varepsilon}_y^0$ curves, which are shifted upwards as $\hat{I}_{SB}^c$ increases, display a concave-upward shape with increasing slope as the compression rate increases. However, unlike the $\bar{\varepsilon}_{\hat{\eta}}^c - \dot{\varepsilon}_y^0$ curves, the slope does not become infinite within the range of compression rates investigated (this is because the $\hat{I}^c - \bar{\varepsilon}$ curves in Fig. 5(b) do not display a maximum). Note that, in comparison with the results presented in Fig. 9(a), the theoretical critical strains calculated with the cumulative index $\bar{\varepsilon}_I^c$ show worse agreement with the numerical critical strains $\bar{\varepsilon}_{FEM}^c$. The predictions of the stability analysis for $\bar{\varepsilon}_I^c$ do not capture neither the slope of the $\bar{\varepsilon}_{FEM}^c - \dot{\varepsilon}_y^0$ curves nor the fact that the compression rate prevents the formation of a shear band for the larger values of $\dot{\varepsilon}_y^0$ considered (attending to the criterion used in this paper in the finite element calculations).

In the rest of this paper, we only present theoretical predictions based on the instantaneous instability index since they show better agreement with the critical strains calculated with the finite element simulations (this is not the case in most dynamic necking problems, see El Maï et al. (2014) and Vaz-Romero et al. (2017)). Specifically, in the following sections, the theoretical and numerical critical strains obtained with the criteria $\hat{\eta}_{SB}^c = 2.5$ and $FEM_{SB}^c = 5$ (best agreement in Fig. 9(a)) are compared for different loading rates, strain hardening exponents, strain rate sensitivity parameters, and values of the Taylor-Quinney coefficient. We consider that $\hat{\eta}_{SB}^c = 2.5$ is large enough for the frozen coefficient theory and the exponential form of the perturbation to be correct hypotheses. Nevertheless, we are aware that this is a strong assumption, and we refer the reader to the recent work of Xavier et al. (2020) to obtain further details about an alternative procedure to consider the temporal evolution of the perturbation coefficients. Moreover, we acknowledge that assuming the

Figure 9: Comparison between linear stability analysis (LSA) and finite element simulations (FEM). Evolution of the theoretical critical strains, $\bar{\epsilon}_{\eta}^c$ and $\bar{\epsilon}_f^c$, and the numerical critical strain, $\bar{\epsilon}_{FEM}^c$, with the initial strain rate along Y direction, $\dot{\epsilon}_y^0$. The initial equivalent strain rate is $\dot{\epsilon}^0 = 10000 \text{ s}^{-1}$. The finite element results correspond to $FEM_{SB}^c = 5, 10$ and 20 . The stability analysis predictions correspond to: (a) $\hat{\eta}_{SB}^c = 2, 2.5$ and 3 and (b) $\hat{\eta}_{SB}^c = 0.1, 1$ and 2 .

same values of $\hat{\eta}_{SB}^c$ and FEM_{SB}^c for different strain rates and material behaviors is also a strong hypothesis. However, it facilitates the comparison between finite element results and linear stability analysis predictions, providing guidelines regarding the effect of material behavior and loading rate on shear bands formation under shear-compression loading (see N'souglo et al. (2020) where similar methodology was adopted to study dynamic necking instabilities).

7.1. The effect of loading rate

Fig. 10 shows the theoretical critical strain, $\bar{\epsilon}_{\eta}^c$, and the numerical critical strain, $\bar{\epsilon}_{FEM}^c$, as a function of the initial strain rate along Y direction, $\dot{\epsilon}_y^0$, for three different initial equivalent strain rates: $\dot{\epsilon}^0 = 5000 \text{ s}^{-1}$, 10000 s^{-1} (reference case shown in Fig. 9) and 20000 s^{-1} . Similar strain rates were investigated in the shear-dominated experiments performed by Xu et al. (2017, 2019).

The stability analysis predictions are in qualitative and quantitative agreement with the finite element simulations. The critical strain for $\dot{\epsilon}_y^0 = 0$ (i.e. simple-shear) is roughly same for the three values of $\dot{\epsilon}^0$ considered. These results are consistent with the stability analysis predictions and finite element computations reported by Rodríguez-Martínez et al. (2015) who modeled shear band localization in Ti6Al4V under simple-shear conditions. In contrast, the slope of the $\bar{\epsilon}_{\eta}^c - \dot{\epsilon}_y^0$ and $\bar{\epsilon}_{FEM}^c - \dot{\epsilon}_y^0$ curves decreases with the equivalent strain rate. Thus, the maximum compression rate for which the shear band is formed increases from $\approx 110 \text{ s}^{-1}$ for $\dot{\epsilon}^0 = 5000 \text{ s}^{-1}$ to $\approx 480 \text{ s}^{-1}$ for $\dot{\epsilon}^0 = 20000 \text{ s}^{-1}$. These results show that the increase of the equivalent strain rate reduces the stabilizing effect of the compression rate on the shear band formation (because the shear strain rate increases), favoring earlier localization.

Figure 10: Comparison between linear stability analysis (LSA) and finite element simulations (FEM). Theoretical critical strain $\bar{\epsilon}_{\eta}^c$ and numerical critical strain $\bar{\epsilon}_{FEM}^c$ versus initial strain rate along Y direction $\dot{\epsilon}_y^0$. The stability analysis predictions correspond to $\hat{\eta}_{SB}^c = 2.5$ and the finite element results to $FEM_{SB}^c = 5$. Results are shown for three different initial equivalent strain rates: $\dot{\epsilon}^0 = 5000 \text{ s}^{-1}$, 10000 s^{-1} (reference case) and 20000 s^{-1} .

7.2. The effect of strain hardening

Fig. 11 shows the evolution of the theoretical and numerical critical strains, $\bar{\epsilon}_{\eta}^c$ and $\bar{\epsilon}_{FEM}^c$, with the initial strain rate along Y direction, $\dot{\epsilon}_y^0$, for three different values of the strain hardening coefficient: $n = 0.1$, 0.167 (reference case shown in Fig. 9) and 0.2 . These are typical values of the strain hardening coefficient for metals and alloys (e.g. Rusinek et al. (2009) and Knezevic et al. (2013)).

The stability analysis predictions are in qualitative agreement with the finite element simulations: the increase of the strain hardening exponent increases the critical strain and reduces the range of compression rates for which a shear band is formed. These results are in accordance with previous findings of Bai (1982), Molinari (1988) and Shawki and Clifton (1989), among others, who showed that the strain hardening stabilizes shear localization under simple-shear conditions. However, the theoretical predictions show quantitative differences with the finite element results: for $n = 0.1$ the stability analysis provides $\approx 35\%$ smaller value for the maximum compression rate leading to shear band formation, and for $n = 0.2$ it underestimates by $\approx 25\%$ the numerical critical strain.

7.3. The effect of strain rate sensitivity

Fig. 12 displays the theoretical and numerical critical strains, $\bar{\epsilon}_{\eta}^c$ and $\bar{\epsilon}_{FEM}^c$, versus the compression rate, $\dot{\epsilon}_y^0$, for three different values of the strain rate sensitivity exponent: $m = 0.001$, 0.0118 (reference case shown

Figure 11: Comparison between linear stability analysis (LSA) and finite element simulations (FEM). Theoretical critical strain $\bar{\epsilon}_{\eta}^c$ and numerical critical strain $\bar{\epsilon}_{FEM}^c$ versus initial strain rate along Y direction $\dot{\epsilon}_y^0$. The stability analysis predictions correspond to $\hat{\eta}_{SB}^c = 2.5$ and the finite element results to $FEM_{SB}^c = 5$. Results are shown for three different strain hardening coefficients: $n = 0.1, 0.167$ (reference case) and 0.2 . The initial equivalent strain rate is $\dot{\epsilon}^0 = 10000 \text{ s}^{-1}$.

in Fig. 9) and 0.02. Similar values for this parameter have been reported in the literature for different metals and alloys (e.g. Hutchinson and Neale (1977) and Nielsen et al. (2010)).

The stability analysis results show qualitative agreement with the finite element simulations. The material strain rate sensitivity increases the critical strain and decreases the maximum compression rate for which the shear band is formed. These results are consistent with the works of Molinari (1988) and Shawki and Clifton (1989), among others, who showed the stabilizing effect of strain rate sensitivity on shear localization under simple-shear conditions. On the other hand, there are meaningful quantitative differences between theoretical and numerical results. Namely, **for the criteria used in this paper**, the stability analysis predicts greater influence of the strain rate sensitivity parameter on the maximum compression rate leading to shear band formation, and greater values of the critical strain for most of the compression rates.

7.4. The effect of thermal softening

Fig. 13 shows the theoretical critical strain $\bar{\epsilon}_{\eta}^c$ and the numerical critical strain $\bar{\epsilon}_{FEM}^c$ versus the initial strain rate along Y direction for three different values of the Taylor-Quinney coefficient: $\chi = 0.6, 0.75$ and 0.9 (reference case shown in Fig. 9). Several works have shown that the Taylor-Quinney coefficient of most metals and alloys lies within this range of values (e.g. Kapoor and Nemat-Nasser (1998), Galán et al. (2013) and Rittel et al. (2017)).

The predictions of the stability analysis and the finite element calculations show that the decrease of the

Figure 12: Comparison between linear stability analysis (LSA) and finite element simulations (FEM). Theoretical critical strain $\bar{\epsilon}_{\eta}^c$ and numerical critical strain $\bar{\epsilon}_{FEM}^c$ versus initial strain rate along Y direction $\dot{\epsilon}_y^0$. The stability analysis predictions correspond to $\hat{\eta}_{SB}^c = 2.5$ and the finite element results to $FEM_{SB}^c = 5$. Results are shown for three different strain rate sensitivity exponents: $m = 0.001$, 0.0118 (reference case) and 0.02 . The initial equivalent strain rate is $\dot{\epsilon}^0 = 10000 \text{ s}^{-1}$.

Taylor-Quinney coefficient increases the critical strain and shrinks the range of values of $\dot{\epsilon}_y^0$ for which the shear band is formed. The decrease of χ decreases the proportion of plastic work converted into heat, reducing the material thermal softening and stabilizing material behavior. Note that the destabilizing effect of enhanced thermal softening on shear band formation under simple-shear conditions was shown by Shawki and Clifton (1989) and Rodríguez-Martínez et al. (2015), among others. Moreover, there are quantitative differences between the theoretical predictions and the finite element calculations. The stability analysis predicts critical strains which are greater than the numerical results, and values for the maximum compression rate leading to shear band formation which are smaller in the cases of $\chi = 0.6$ and $\chi = 0.75$. The differences increase as the Taylor-Quinney coefficient decreases.

In summary, this section shows that the stability analysis provides predictions for the effect that strain rate, strain hardening, strain rate hardening and thermal softening have on the critical strain leading to shear band formation which are in qualitative agreement with the finite element results. The quantitative differences between stability analysis results and finite element simulations are partially attributed to the simplicity of the analytical model which includes a series of hypotheses (linear nature, frozen perturbation coefficients, etc.) to make it mathematically tractable and to the simple criterion used to determine when a shear band is formed (which is based on a constant value of the instantaneous instability index). In agreement with Molinari (1997)

Figure 13: Comparison between linear stability analysis (LSA) and finite element simulations (FEM). Theoretical critical strain $\bar{\epsilon}_{\eta}^c$ and numerical critical strain $\bar{\epsilon}_{FEM}^c$ versus initial strain rate along Y direction $\dot{\epsilon}_y^0$. The stability analysis predictions correspond to $\hat{\eta}_{SB}^c = 2.5$ and the finite element results to $FEM_{SB}^c = 5$. Results are shown for three different Taylor-Quinney coefficients: $\chi = 0.6$, 0.75 and 0.9 (reference case). The initial equivalent strain rate is $\dot{\epsilon}^0 = 10000 \text{ s}^{-1}$.

and Daridon et al. (2004), the results obtained in this paper show that there is not a universal instability index valid for all materials. Moreover, the simple criterion used to determine the formation of the shear band in the finite element simulations (a constant value of the ratio between the strain increments inside and outside the localization band) is, most likely, another source for the quantitative differences between the theoretical and the numerical results. In this regard, note that the identification of more refined criteria to capture the formation of the shear bands (e.g. dependent on material properties), both in the stability analysis and the numerical computations, is left for a future work.

8. Summary and conclusions

In this paper, we have investigated the formation of adiabatic shear bands under shear-compression loading using a two-pronged approach which includes linear stability analysis and finite element calculations. The linear stability analysis, which is an original contribution of this paper, extends the one-dimensional formulation developed by Molinari (1997) for simple-shearing deformations to shear-compression loadings. Both instantaneous and cumulative instability indexes, $\hat{\eta}$ and $\hat{I}$, have been used to monitor the stability of the solution and assess the susceptibility of the material to shear localization. On the other hand, the finite element calculations have been performed with a unit-cell model based on the earlier work of Rodríguez-Martínez et al. (2015), which has been enhanced in this paper to consider the same initial and boundary conditions used in the stabil-

ity analysis. The finite element model includes geometric and material imperfections to favor the formation of the shear band. Moreover, the material has been modeled using von Mises plasticity, with associated flow rule and isotropic hardening. The yield stress evolution has been considered dependent on strain, strain rate and temperature. In addition, we have developed specific criteria to determine the critical strains leading to the shear band inception both in the finite element simulations and the stability analysis. Namely, in the numerical calculations we have assumed that the shear band is formed when the ratio between the strain increments inside and outside the localization band reaches a critical value FEM_{SB}^c . Similarly, the criterion used in the stability analysis considers that a shear band is formed when the critical instantaneous and cumulative grow rates reach specific values, $\hat{\eta}_{SB}^c$ and $\hat{I}_{SB}^c$, respectively. We have found that the critical strains obtained with the critical instantaneous index $\hat{\eta}_{SB}^c$ have better qualitative and quantitative agreement with the finite element results than the predictions obtained with the critical cumulative index $\hat{I}_{SB}^c$. Specifically, using $\hat{\eta}_{SB}^c = 2.5$ and $FEM_{SB}^c = 5$, we have calculated the theoretical and numerical critical strains leading to shear band formation for different compression loadings, a wide range of strain rates, and different values of the strain hardening coefficient, the strain rate sensitivity exponent and the Taylor-Quinney factor. **Attending to the shear band criteria used in this paper, and for the loading conditions and material behaviors investigated in this work**, both theoretical and finite element results have suggested that the compression loading increases the critical strain leading to shear localization, preventing shear band formation even at low values of negative triaxiality. The stability analysis and the finite element calculations have also shown that the increase of the strain and strain rate exponents, and the decrease of the Taylor-Quinney factor, increase the critical strain and reduces the range of compression rates for which shear localization occurs. The key outcome of this paper is the identification of **some of** the main factors that control adiabatic shear band formation in isotropic metallic sheets subjected to dynamic shear-compression loading. **On the other hand, a limitation of this work is that the proposed stability analysis is valid only for the specific shear-compression problem defined by the kinematics of Appendix A and the governing equations of Section 3. For a complex structural problem, involving parts of complex geometry and transient loading conditions, it is difficult to obtain analytical solutions, and numerical methods are needed. In this regard, note that in the papers of Arriaga et al. (2015, 2016) and Arriaga and Waisman (2017, 2018a,b) a stability analysis is implemented in a finite element code to determine the onset of shear localization, damage and fracture in different dynamic structural problems. We shall explore this technique in future works.**

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Appendix A. Kinematics of the fundamental solution

The kinematics of the problem in the (x, y, z) and (x', y', z') frames is presented in Appendix A.1 and Appendix A.2, respectively.

Appendix A.1. Kinematics of the fundamental solution in the (x, y, z) frame

As illustrated in Fig. 1, we assume the following in-plane velocity field referred to the (X, Y, Z) Lagrangian Cartesian frame:

$$\begin{aligned} V_x &= \dot{\varepsilon}_x^0 X + 2\dot{\varepsilon}_{xy}^0 Y \\ V_y &= -\dot{\varepsilon}_y^0 Y \end{aligned} \tag{A.1}$$

where $\dot{\varepsilon}_x^0$, $\dot{\varepsilon}_y^0$ and $\dot{\varepsilon}_{xy}^0$ are the initial strain rates in the specimen, as mentioned in Section 3.1. This velocity field leads to a null acceleration field, which guarantees a uniform stress field for the fundamental solution. The in-plane displacement field, obtained by temporal integration of previous expressions, is:

$$\begin{aligned} U_x &= \dot{\varepsilon}_x^0 t X + 2\dot{\varepsilon}_{xy}^0 t Y \\ U_y &= -\dot{\varepsilon}_y^0 t Y \end{aligned} \tag{A.2}$$

Moreover, the (x, y) Eulerian coordinates are expressed as a function of the (X, Y) Lagrangian coordinates as:

$$\begin{aligned} x &= X + U_x = (1 + \dot{\varepsilon}_x^0 t) X + 2\dot{\varepsilon}_{xy}^0 t Y \\ y &= Y + U_y = (1 - \dot{\varepsilon}_y^0 t) Y \end{aligned} \tag{A.3}$$

Using equation (A.3), the in-plane velocity field can be referred to the (x, y, z) Eulerian Cartesian frame:

$$\begin{aligned} V_x &= \frac{\dot{\varepsilon}_x^0}{(1 + \dot{\varepsilon}_x^0 t)} x + \frac{2\dot{\varepsilon}_{xy}^0}{(1 + \dot{\varepsilon}_x^0 t)(1 - \dot{\varepsilon}_y^0 t)} y \\ V_y &= -\frac{\dot{\varepsilon}_y^0}{(1 - \dot{\varepsilon}_y^0 t)} y \end{aligned} \quad (\text{A.4})$$

which provides the deformation rates in the plane of the specimen:

$$\begin{aligned} d_{xx} &= \frac{\partial V_x}{\partial x} = \frac{\dot{\varepsilon}_x^0}{1 + \dot{\varepsilon}_x^0 t} \\ d_{yy} &= \frac{\partial V_y}{\partial y} = -\frac{\dot{\varepsilon}_y^0}{1 - \dot{\varepsilon}_y^0 t} \\ d_{xy} &= \frac{1}{2} \left(\frac{\partial V_x}{\partial y} + \frac{\partial V_y}{\partial x} \right) = \frac{\dot{\varepsilon}_{xy}^0}{(1 + \dot{\varepsilon}_x^0 t)(1 - \dot{\varepsilon}_y^0 t)} \end{aligned} \quad (\text{A.5})$$

The incompressibility condition, $d_{xx} + d_{yy} + d_{zz} = 0$, is used to obtain the deformation rate in the perpendicular direction to the sheet:

$$d_{zz} = -\frac{\dot{\varepsilon}_x^0}{1 + \dot{\varepsilon}_x^0 t} + \frac{\dot{\varepsilon}_y^0}{1 - \dot{\varepsilon}_y^0 t} \quad (\text{A.6})$$

which is integrated to obtain the corresponding velocity:

$$V_z = \left(-\frac{\dot{\varepsilon}_x^0}{1 + \dot{\varepsilon}_x^0 t} + \frac{\dot{\varepsilon}_y^0}{1 - \dot{\varepsilon}_y^0 t} \right) z \quad (\text{A.7})$$

where the Eulerian coordinate z is related to the Lagrangian coordinate Z by the following expression:

$$z = \frac{Z}{(1 + \dot{\varepsilon}_x^0 t)(1 - \dot{\varepsilon}_y^0 t)} \quad (\text{A.8})$$

which allows to obtain the out-of-plane displacement:

$$U_z = \left(\frac{1}{(1 + \dot{\varepsilon}_x^0 t)(1 - \dot{\varepsilon}_y^0 t)} - 1 \right) Z \quad (\text{A.9})$$

and the out-of-plane velocity:

$$V_z = \left(-\frac{\dot{\varepsilon}_x^0}{(1 + \dot{\varepsilon}_x^0 t)^2 (1 - \dot{\varepsilon}_y^0 t)} + \frac{\dot{\varepsilon}_y^0}{(1 + \dot{\varepsilon}_x^0 t)(1 - \dot{\varepsilon}_y^0 t)^2} \right) Z \quad (\text{A.10})$$

as function of the Lagrangian coordinate.

Appendix A.2. Kinematics of the fundamental solution in the (x', y', z') frame

The relation between the (x, y) and (x', y') coordinates is:

$$\begin{aligned} x' &= x \cos \beta - y \sin \beta \\ y' &= x \sin \beta + y \cos \beta \end{aligned} \quad (\text{A.11})$$

and the relation between the corresponding velocities is:

$$\begin{aligned} V_{x'} &= V_x \cos \beta - V_y \sin \beta - x \dot{\beta} \sin \beta - y \dot{\beta} \cos \beta \\ V_{y'} &= V_x \sin \beta + V_y \cos \beta + x \dot{\beta} \cos \beta - y \dot{\beta} \sin \beta \end{aligned} \quad (\text{A.12})$$

where β determines the orientation of the shear plane, as mentioned in Section 3. Using equations (A.4) and (A.11), expressions (A.12) are rewritten as:

$$\begin{aligned}
V_{x'} &= \left(\frac{\dot{\varepsilon}_x^0}{1 + \dot{\varepsilon}_x^0 t} \cos^2 \beta - \frac{2\dot{\varepsilon}_{xy}^0 \cos \beta + \dot{\varepsilon}_y^0 (1 + \dot{\varepsilon}_x^0 t) \sin \beta}{(1 - \dot{\varepsilon}_y^0 t) (1 + \dot{\varepsilon}_x^0 t)} \sin \beta \right) x' \\
&\quad - \left(-\frac{\dot{\varepsilon}_x^0}{1 + \dot{\varepsilon}_x^0 t} \sin \beta \cos \beta - \frac{2\dot{\varepsilon}_{xy}^0 \cos \beta + \dot{\varepsilon}_y^0 (1 + \dot{\varepsilon}_x^0 t) \sin \beta}{(1 - \dot{\varepsilon}_y^0 t) (1 + \dot{\varepsilon}_x^0 t)} \cos \beta + \dot{\beta} \right) y' \\
V_{y'} &= \left(\frac{\dot{\varepsilon}_x^0}{1 + \dot{\varepsilon}_x^0 t} \sin \beta \cos \beta - \frac{2\dot{\varepsilon}_{xy}^0 \sin \beta - \dot{\varepsilon}_y^0 (1 + \dot{\varepsilon}_x^0 t) \cos \beta}{(1 - \dot{\varepsilon}_y^0 t) (1 + \dot{\varepsilon}_x^0 t)} \sin \beta + \dot{\beta} \right) x' \\
&\quad + \left(\frac{\dot{\varepsilon}_x^0}{1 + \dot{\varepsilon}_x^0 t} \sin^2 \beta + \frac{2\dot{\varepsilon}_{xy}^0 \sin \beta - \dot{\varepsilon}_y^0 (1 + \dot{\varepsilon}_x^0 t) \cos \beta}{(1 - \dot{\varepsilon}_y^0 t) (1 + \dot{\varepsilon}_x^0 t)} \cos \beta \right) y'
\end{aligned} \tag{A.13}$$

Inserting the shear plane condition, $V_{x'}|_{y'=0} = 0$, in the above equations provides the following expressions for the in-plane velocities as a function of the (x', y') coordinates:

$$\begin{aligned}
V_{x'} &= \left(\frac{\dot{\varepsilon}_x^0}{1 + \dot{\varepsilon}_x^0 t} \cot \beta - \dot{\beta} \right) y' \\
V_{y'} &= \left(\frac{\dot{\varepsilon}_y^0}{1 - \dot{\varepsilon}_y^0 t} \tan \beta + \dot{\beta} \right) x' + \left(\frac{\dot{\varepsilon}_x^0}{1 + \dot{\varepsilon}_x^0 t} - \frac{\dot{\varepsilon}_y^0}{1 - \dot{\varepsilon}_y^0 t} \right) y'
\end{aligned} \tag{A.14}$$

and a second-degree polynomial to calculate the shear plane orientation β :

$$\frac{\dot{\varepsilon}_y^0}{1 - \dot{\varepsilon}_y^0 t} \tan^2 \beta + \frac{2\dot{\varepsilon}_{xy}^0}{(1 - \dot{\varepsilon}_y^0 t) (1 + \dot{\varepsilon}_x^0 t)} \tan \beta - \frac{\dot{\varepsilon}_x^0}{1 + \dot{\varepsilon}_x^0 t} = 0 \tag{A.15}$$

We consider that the positive root of equation (A.15) is the one of interest since it provides the greatest value of the shear strain rate. Note that the positive root is smaller in absolute value than the negative one. Note also that previous expression shows that the value of β evolves with the loading time.

Using equation (A.14), the expressions for the in-plane strain rates **referred to the (x', y', z') frame** are obtained:

$$\begin{aligned}
d_{x'x'} &= \frac{\partial V_{x'}}{\partial x'} = 0 \\
d_{y'y'} &= \frac{\partial V_{y'}}{\partial y'} = \frac{\dot{\varepsilon}_x^0}{1 + \dot{\varepsilon}_x^0 t} - \frac{\dot{\varepsilon}_y^0}{1 - \dot{\varepsilon}_y^0 t} \\
d_{x'y'} &= \frac{1}{2} \left(\frac{\partial V_{x'}}{\partial y'} + \frac{\partial V_{y'}}{\partial x'} \right) = \frac{1}{2} \left(\frac{\dot{\varepsilon}_x^0}{1 + \dot{\varepsilon}_x^0 t} \cot \beta + \frac{\dot{\varepsilon}_y^0}{1 - \dot{\varepsilon}_y^0 t} \tan \beta \right)
\end{aligned} \tag{A.16}$$

The incompressibility condition, equation (9), leads to the following expression for the deformation rate in the direction perpendicular to the sheet:

$$d_{z'z'} = -\frac{\dot{\varepsilon}_x^0}{1 + \dot{\varepsilon}_x^0 t} + \frac{\dot{\varepsilon}_y^0}{1 - \dot{\varepsilon}_y^0 t} \tag{A.17}$$

which obviously coincides with equation (A.6). Similarly, the velocity in the direction perpendicular to the sheet is:

$$V_{z'} = \left(-\frac{\dot{\varepsilon}_x^0}{1 + \dot{\varepsilon}_x^0 t} + \frac{\dot{\varepsilon}_y^0}{1 - \dot{\varepsilon}_y^0 t} \right) z' \tag{A.18}$$

which also coincides with equation (A.7).

Appendix B. The effect of yield stress perturbation in the finite element results

The effect of the yield stress perturbation in the finite element results has been investigated performing calculations in which the number of elements perturbed and the amplitude of the yield stress perturbation is varied.

Appendix B.1. The effect of the number of elements perturbed

Fig. B.14 shows the numerical critical strain $\bar{\varepsilon}_{FEM}^c$ versus the initial strain rate along Y direction $\dot{\varepsilon}_y^0$ for an initial equivalent strain rate of $\dot{\varepsilon}^0 = 10000 \text{ s}^{-1}$. The finite element results correspond to $FEM_{SB}^c = 5$. Results are shown for simulations in which the number of perturbed elements along Y direction is: $N_{pert} = 4$, 8 (reference case shown in Fig. 9) and 16. The increase of N_{pert} leads to a drop in the critical strain and increases the maximum compression rate for which the shear band is formed. Nevertheless, we have checked

that, while the number of elements perturbed influences the finite element results, the trends and conclusions presented in Sections 6 and 7 remain essentially the same.

Figure B.14: Finite element simulations (FEM). Numerical critical strain $\bar{\epsilon}_{FEM}^c$ versus initial strain rate along Y direction $\dot{\epsilon}_y^0$. The finite element results correspond to $FEM_{SB}^c = 5$. Results are shown for simulations in which the number of perturbed elements is: $N_{pert} = 4$, 8 (reference case) and 16. The initial equivalent strain rate is $\dot{\epsilon}^0 = 10000 \text{ s}^{-1}$.

Appendix B.2. The effect of the amplitude of the yield stress perturbation

Fig. B.15 presents the evolution of the numerical critical strain $\bar{\epsilon}_{FEM}^c$ versus the compression rate $\dot{\epsilon}_y^0$ for an initial equivalent strain rate of $\dot{\epsilon}^0 = 10000 \text{ s}^{-1}$. The finite element results correspond to $FEM_{SB}^c = 5$. Results are shown for simulations with three different values of the average perturbed yield stress: $C_u = 0.9$, 0.95 and 0.975 (reference case shown in Fig. 9). The increase of the average perturbed yield stress leads to an increase in the critical strain and reduces the maximum compression rate for which the shear band is formed. On the other hand, while the results presented in Fig. B.15 reveal an important effect of C_u in the finite element results, we have checked that the qualitative analysis developed in Sections 6 and 7 remains valid for different values of C_u .

Appendix C. The effect of the specimen size in the finite element results

The effect of the specimen size in the finite element results has been investigated carrying out simulations for different values of the length-to-width ratio L_x^0/L_y^0 . The width of the specimen is kept constant $L_y^0 = 0.508 \text{ mm}$, and the imposed initial equivalent strain rate is $\dot{\epsilon}^0 = 10000 \text{ s}^{-1}$. Fig. C.16 shows the evolution of the critical

Figure B.15: Finite element simulations (FEM). Numerical critical strain $\bar{\epsilon}_{FEM}^c$ versus initial strain rate along Y direction $\dot{\epsilon}_y^0$. The finite element results correspond to $FEM_{SB}^c = 5$. Results are shown for three different values of the average perturbed yield stress: $C_u = 0.9$, 0.95 and 0.975 (reference case). The initial equivalent strain rate is $\dot{\epsilon}^0 = 10000 \text{ s}^{-1}$.

strain $\bar{\epsilon}_{FEM}^c$ with the compression rate $\dot{\epsilon}_y^0$ for $L_x^0/L_y^0 = 23.5$ (reference case shown in Fig. 9), 47 and 70.5. The finite element results correspond to $FEM_{SB}^c = 5$. The differences in the $\bar{\epsilon}_{FEM}^c - \dot{\epsilon}_y^0$ curves, which are negligible for $\dot{\epsilon}_y^0 = 0$ (i.e. simple-shearing), increase gradually as the compression rate increases. These results reveal the increasing influence of the specimen size and the boundary conditions on the shear band formation as the compression rate increases. Nevertheless, we have checked that all the trends and conclusions obtained in the analysis developed in Sections 6 and 7 are virtually independent of the size of the specimen.

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Figure C.16: Finite element simulations (FEM). Numerical critical strain $\bar{\epsilon}_{FEM}^c$ versus initial strain rate along Y direction $\dot{\epsilon}_y^0$. The finite element results correspond to $FEM_{SB}^c = 5$. Calculations are shown for three specimen sizes: $L_x^0/L_y^0 = 23.5$ (reference case), 47 and 70.5. The width of the specimen is $L_y^0 = 0.508$ mm. The initial equivalent strain rate is $\dot{\epsilon}^0 = 10000$ s $^{-1}$.

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