

The effect of geometric imperfections on the mechanical response of isotropic closed-cell plate-lattices

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Abstract

Cubic+octet plate-lattices, whose unit cell comprises plates aligned along simple-cubic and face-centered-cubic planes of crystal structures, have attracted scientists and engineers because of their isotropic, near-optimal mass-specific performance that reaches theoretical upper bounds on stiffness and strength at low density. While their structural efficiency has been recently examined analytically, numerically and experimentally, their sensitivity to geometric imperfections has remained elusive. Here, using finite element simulations, we present sensitivity of the macroscopic mechanical properties of the plate-lattices to two types of geometric imperfections: namely, periodically distributed defects, characterizing plate waviness and displaced intersections, and randomly dispersed defects, representing missing plates, observed in their additively manufactured samples. Our results show that the randomly dispersed imperfections lead to a greater reduction in the Young's modulus and yield strength than its counterpart across all relative densities under consideration, while their scaling relations with the relative density remains linear, confirming their structural efficiency attributed to stretching-dominated behavior even with the presence of the imperfections. This study sheds light on previously elusive sensitivity of the plate-lattices to the geometric imperfections and provides understanding of their defect-dependent mechanical performance.

Keywords: plate-lattice; imperfection sensitivity; mechanical response; finite element modeling

1. Introduction

In the past two decades, architected cellular materials, comprising a periodic arrangement of a material phase and a void phase, have attracted scientists and engineers for designing lightweight applications because their macroscopic (effective) Young's modulus, E , and yield strength, σ_y , scale with the relative density $\bar{\rho}$ through a power law relation ($E \propto \bar{\rho}^m$ and $\sigma_y \propto \bar{\rho}^n$, where m and n are the scaling exponents). The most mechanical efficient topologies are stretching-dominated (materials deform majorly through tension and compression of cell walls) with $m = n = 1$, while bending-dominated topologies (materials deform majorly through the bending of cell walls) are less efficient with $m = 2$ (3 in some stochastic aerogels) and $n = 1.5$ [1–3]. However, these topologies are often challenging to fabricate especially at low relative densities ($\bar{\rho} \ll 0.2$ [4]).

Thanks to the recent advancement in additive manufacturing, the fabrication of these mechanically efficient yet complex topologies can now be realized across multi-length scales. Most notably, large area micro-stereolithography and direct laser writing have successfully printed numerous samples of distinct topologies, e.g., periodic strut-based octets and octahedrons [5,6], periodic plate-based cubic+octet (CO) lattices [7,8] and stochastic minimal surface-based spinodal shells [9,10] with smallest feature (i.e., strut diameter, plate thickness, or shell thickness) sizes down to micro- and nano-scale. Among these topologies, closed-cell isotropic CO plates lattice have been studied extensively in the past few years for their ability to achieve both Hashin-Shtrikman [11] and Suquet [12,13] upper bounds [7,8,14]; these upper bounds represent the theoretical limits for the effective Young's modulus, E_{HSU} , and yield strength, $\sigma_{y,\text{SU}}$, of an isotropic cellular material respectively and are expressed below:

$$\frac{E_{\text{HSU}}}{E_s} = \frac{2\bar{\rho} (5\nu_s - 7)}{13\bar{\rho} + 2\nu_s (6 - \bar{\rho}) + 15\nu_s^2 (1 - \bar{\rho}) - 27} \quad (1)$$

$$\frac{\sigma_{y,\text{SU}}}{\sigma_{ys}} = \frac{6\bar{\rho}}{\sqrt{36 + 33(1 - \bar{\rho})}} \quad (2)$$

where E_s , σ_{ys} and ν_s are the Young's modulus, yield strength and Poisson's ratio of the solid constituent material, respectively. Despite reaching the upper bounds, it is suspected that geometric imperfections due to process variability (e.g., plate waviness, mass agglomeration, misaligning plates at intersections and missing plates) can result in significant knockdown on E and σ_y of these micro-architected CO plate-lattices due to the instability and irregularity introduced by such flaws in ideal periodic lattices [15,16]. For example, the macroscopic mechanical properties of open-cell cellular materials suffer noticeable degradation attributed to misplaced nodes, wavy and missing struts [16–19].

In this work, we seek to quantify and understand the effect of geometric imperfections (induced by process variability) on the Young's modulus and yield strength of the isotropic CO plate-lattices using finite element simulations. We will achieve this by numerically examining two kinds of imperfections: (i) periodically distributed defects, representing plate waviness and displaced intersections through eigenmode perturbations and (ii) randomly dispersed defects (e.g., missing plates) through random plate removals to closely simulate the defect locations during real

fabrication. The knockdown on E and σ_y and their scaling relations with the relative density, caused by these imperfections, are then presented. While imperfections formulated in this study may only capture fractions of true flaws or defects in additively manufactured samples, they comprehensively reveal the defect-dependent mechanical performance of the plate-lattices.

2. Methods

2.1. Synopsis

To study the effect of geometric imperfections on the mechanical properties of isotropic cubic+octet (CO) plate-lattices, we first constructed the plate-lattices with the relative densities, $\bar{\rho}$, of 0.2, 0.3 and 0.4 according to the topological designs from previous works [7,8,14] using a commercial finite element software (Abaqus 2018). The unit cell of these CO plate-lattices, composed of a cubic- and octet-architecture, were represented via shell models, where the thickness of each shell model is defined through the section properties per the isotropic condition [14]. These shell models are meshed with quadrilateral elements (S4R) with an average element size as 5% of the unit cell size (Fig. 1). All results determined from this mesh size were found to be within 5% of each other in our mesh convergence study. We then imposed two kinds of geometric flaws (i.e., imperfections) separately in the shell models: (i) periodically distributed defects illustrated by eigenmode imperfections, hence representing small manufacturing deviations (e.g., plate waviness and displaced intersections) from the perfect mesh and (ii) randomly dispersed defects through random plate removals, representing relatively large defects like gaps. In the following subsections, we describe the imposition of these imperfections in detail, allowing us to extract the macroscopic mechanical properties of the plate-lattices under consideration.

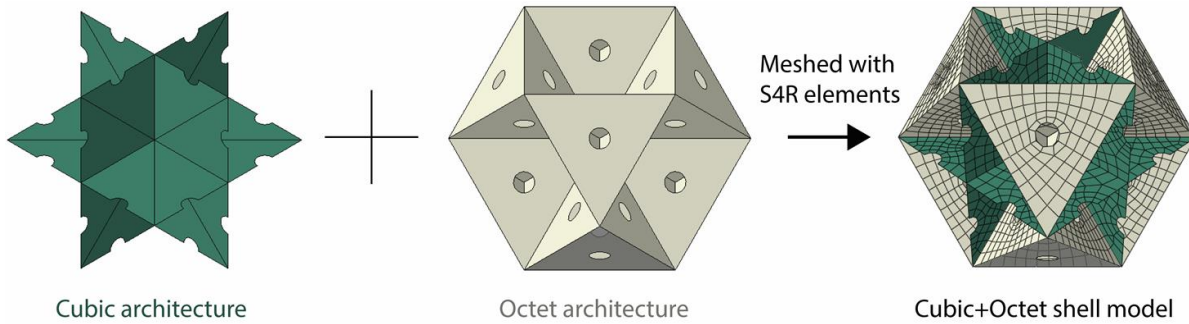


Figure 1. Finite element shell model representing a cubic+octet unit cell composed of a cubic- and octet-architecture.

2.2. Periodically distributed imperfections

To realize the periodically distributed imperfections, we extracted the first 20 eigenmodes by performing a linear perturbation buckling analysis on a perfect $3 \times 3 \times 3$ shell model, where the constituent material is assumed to be linearly elastic. We then superimposed these 20 eigenmodes with common imperfection scale factors on to the mesh of perfect shell models to dislocate nodes of the perfect model, leading to creation of the imperfect shell models. The scale factors were chosen such that the resulting imperfection size, ψ , defined as the average nodal deviation in the imperfect model from the perfect model divided by the averaged plate thickness (that is, $\psi = d_{avg}/t_{avg}$, where $t_{avg} = (t_{cubic} + t_{octet})/2$), was equal to 0.1, 0.2, and 0.3. For example, an imperfect model with ψ of 0.3 represents the perfect model distorted by 30% of the averaged plate thickness, where the shape of the distortion complies with the first 20 superposed eigenmodes. Hence, this strategy adequately represents plate waviness and dislocated intersections seen in fabricated samples. The schematic of this process is shown in Fig. 2a.

2.3. Randomly dispersed imperfections

To impose the randomly dispersed imperfections, we randomly removed plates within the plate-lattices as follow. We first numbered each plate in each the cubic architecture (1-24 plates) and octet architecture (1-32 plates) of the CO plate unit cell separately. We then randomly generated one, two, or three numbers between 1-24 for the cubic architecture and between 1-32 for the octet architectures. These one, two, or three numbers corresponded to the removal locations of one, two or three plates from each architecture in the unit cell. This removal process was repeated 27 times for 27 unit cells. These 27 unit cells were then randomly combined to form a $3 \times 3 \times 3$ plate-lattice shell model. To further capture the effect of randomness, the random combining process was repeated three times (labeled as “Test 1”, “Test 2, and “Test 3”) for each of the one, two, or three plate removals. The schematic for the plate removals is shown in Fig. 2b.

2.4. Finite element simulations to extract Young’s modulus and yield strength

We imported the imperfect shell models obtained from sec. 2.2 and 2.3 into Abaqus and performed finite element simulations to obtain their macroscopic mechanical properties under uniaxial compressive loading. All these simulations were performed using quasi-static analyses with geometric nonlinearity via Abaqus explicit solver. The constituent material is assumed to be linearly elastic, perfectly plastic with Young’s modulus $E_s = 123$ GPa, Poisson’s ratio $\nu_s = 0.3$ and yield strength $\sigma_{ys} = 932$ MPa; this leads to the yield strain of a typical metal ~ 0.01 . To model the compressive response of infinite imperfect lattices, we applied quasi-periodic boundary conditions (QPBCs), that force all boundary faces of $3 \times 3 \times 3$ plate-lattices to remain planar after deformation, hence simulating similar strain fields imposed by the true periodic boundary conditions but can be implemented in a much simpler manner. The detailed implementation of QPBCs can be found in [9,20]

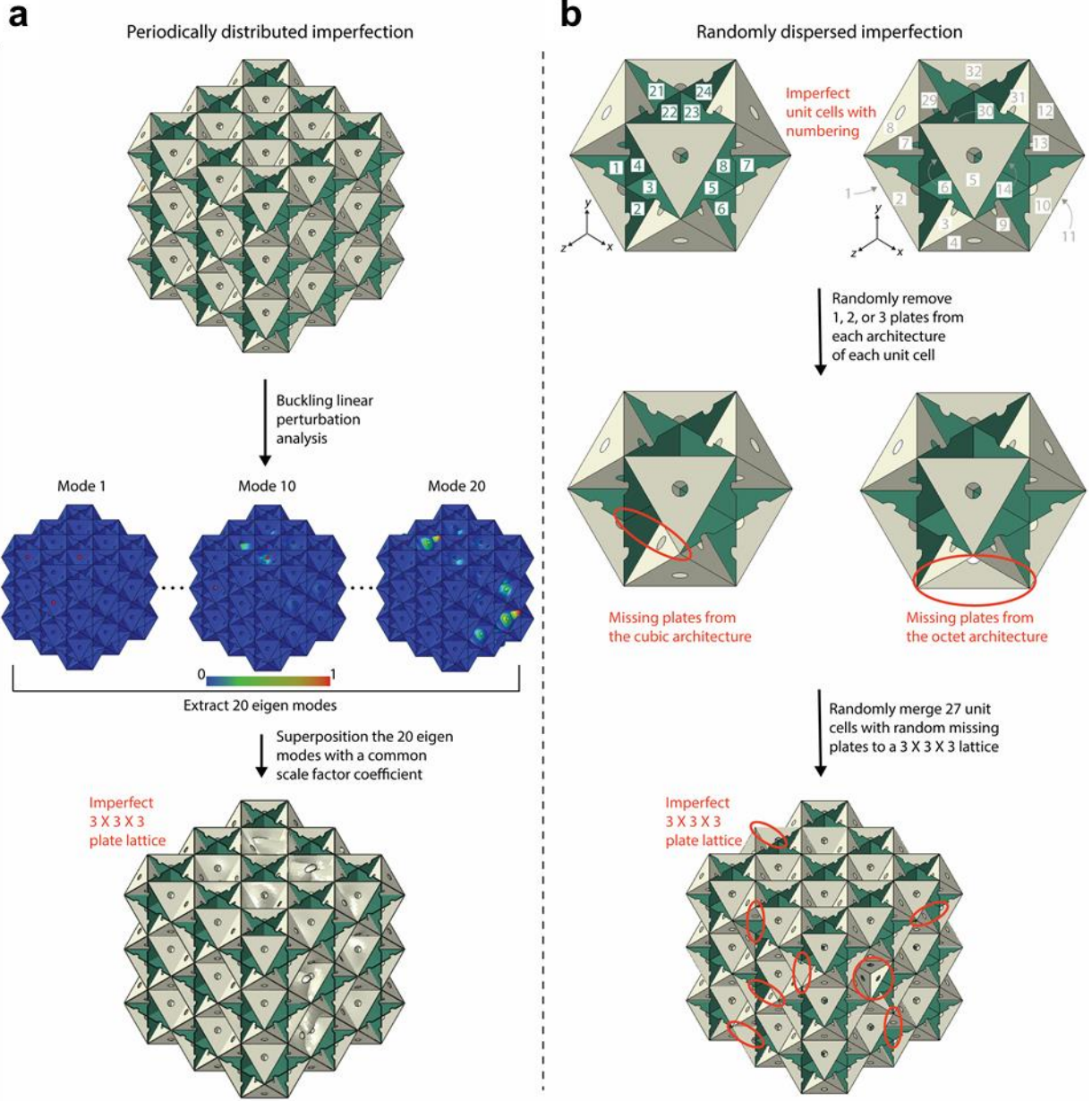


Figure 2. Schematic of the implementation of (a) periodically distributed imperfections induced by eigen-modes and (b) randomly dispersed imperfections by random removal of plates. Imperfection are not drawn to scale.

3. Results and discussions

Figure 3 depicts effects of two types of geometric imperfections on the normalized Young's modulus, E/E_s , of the plate-lattices and their scaling relations across the relative density from 0.2 to 0.4. The periodically distributed defects, representing plate waviness and displaced intersections, led to a gradual, marginal decrease in the Young's modulus of the plate-lattices with an increase of imperfection size, ψ , while the percentage of such knockdown remained the same regardless of the relative density (Fig. 3a). For example, when ψ was varied from 0.1 to 0.3 with an increment

of 0.1, the percentages of the knockdown in the Young's modulus were ~ 4 , ~ 7 and $\sim 10\%$ across all relative densities under consideration with respect to that of an ideal structure. However, this knockdown revealed a trivial effect on the scaling exponent, m , of the modulus-relative density relation (i.e., $E/E_s \propto \bar{\rho}^m$) across all imperfection sizes ($m = 0.99 - 1$) obtained from least square fitting (Fig. 3c). This indicates that a dominant deformation of the plate-lattices is preserved to be a combination of tensile and compressive load-bearing plates even with the presence of localized bending deformation induced by the periodically distributed geometric imperfections degrading their stiffness. By contrast, removals of plates, rendering randomly dispersed defects, resulted in a significant reduction in the Young's modulus of the plate-lattices, in which a greater reduction was observed with a higher percentage of removed plates, while the locations of the random removals within the lattice displayed a negligible effect on the magnitude of modulus reduction (Fig. 3b). For example, removing ~ 4 , ~ 7 and $\sim 10\%$ of the plates led to ~ 7 , ~ 15 and 22% reductions in the modulus, respectively, and such a degradation was found to be invariant with a change in the relative density and locations of removed plates. This decrease did not affect their efficient near-linear scaling of the modulus ($m = 1 - 1.01$), confirming its stretching-dominated deformation regardless of different percentages of missing plates (Fig. 3d). Note that the extracted moduli from the simulations are presented by the averaged value at each relative density whereas the error bars, indicating their standard deviation from Test 1 to Test 3, are not intentionally shown because of their small magnitudes.

The normalized yield strength of the plate-lattices, σ/σ_y , subjected to the periodically distributed defects and randomly dispersed defects are shown in Fig. 4. Similar to inferences drawn for the Young's modulus, a reduction in the yield strength in consequence of the two imperfections was proportional to the degree of imperfections (i.e., ψ and the percentage of plate removal) for all relative densities, however an overall knockdown in the yield strength was noticeably larger than that of the Young's modulus. In particular, varying ψ from 0.1, to 0.2 and to 0.3 led to ~ 6 , ~ 10 and $\sim 13\%$ reduction in the yield strength as compared to that of the perfect model, respectively, whereas ~ 8 , ~ 17 and $\sim 25\%$ degradations were seen for ~ 4 , ~ 7 and $\sim 10\%$ removal of the plates; for both cases, these percentages did not vary across the relative densities. In addition, the scaling exponent, n , of the yield strength-relative density relation (i.e., $\sigma_y/\sigma_{ys} \propto \bar{\rho}^n$) for both cases was varied with a slightly larger degree than those for the modulus but still exhibited near-linear scaling ($n = 1 - 1.05$) (Figs. 4c and 4d), indicating that a major collapse of the plate-lattice is predominantly due to the stretching-dominated behavior.

It is worth to note that our simulation results lie slightly below theoretical bounds on the stiffness and strength (Harshin-Shtrikman and Suquet bounds, respectively) as compared to reported literature [7,8,14]. We believe this can be attributed to two factors: (i) the holes at each face of the plates and (ii) shell models used in the simulations. First, we verified that Young's modulus of shell models without the holes are very close to analytical solutions [14] across all relative densities under consideration. This confirms our models are properly created and proves an additional knockdown on mechanical properties due to the presence of the holes, which has affected somewhat greater than previously reported works. In addition, shell models used in this study slightly underestimates mechanical properties of the structure because they are inherently incapable of capturing the plate intersection geometry where volume accumulation may result in

additional stiffness and strength. Although these two factors contributed to the reduced mechanical properties compared with the theoretical bounds and the reported studies, our study highlights previously unexplored mechanical efficiency linked to geometric imperfections as well as understanding of defect-dependent mechanical performance of the plate-based architected materials.

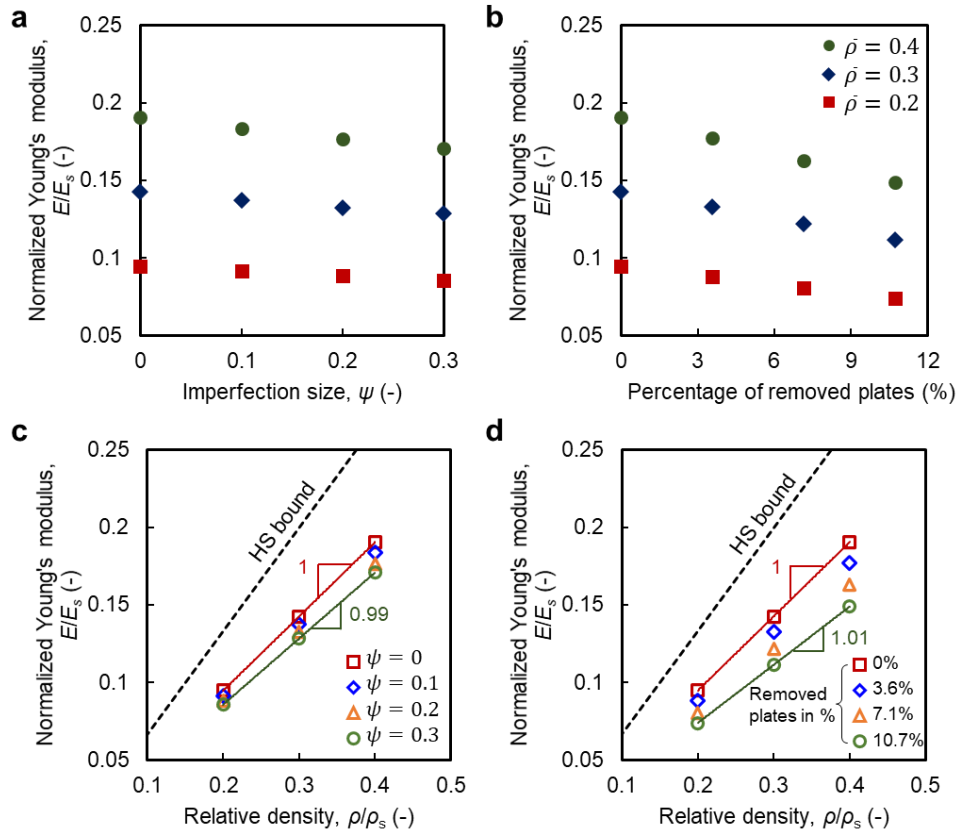


Figure 3. Normalized Young's modulus of CO plate-lattices and their scaling against relative density with two types of imperfections: distributed defects (e.g., plate waviness and displaced intersections) imposed by superposition of different eigenmodes (a and c); (ii) randomly dispersed defects (e.g., missing plates) (b and d).

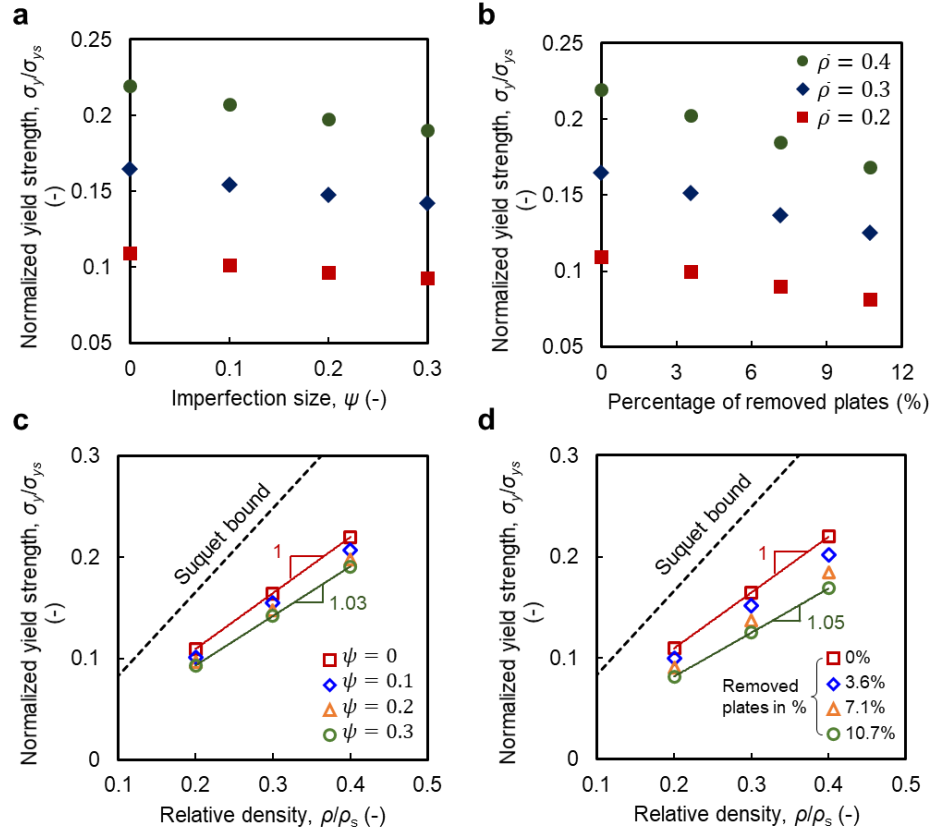


Figure 4. Normalized yield strength of CO plate-lattices and their scaling against relative density with two types of imperfections: distributed defects (e.g., plate waviness and displaced intersections) imposed by superposition of different eigenmodes (a and c); (ii) randomly dispersed defects (e.g., missing plates) (b and d).

4. Conclusions

In this work, we study the imperfection sensitivity of the Young's modulus and yield strength of the cubic+octet plate-lattices to two types of geometric imperfections using finite element simulations. These imperfections render two typical defects observed in additively manufactured specimens, including periodically distributed defects (e.g., plate waviness and displaced intersections) and randomly dispersed defects (e.g., missing plates). Our results showed that the macroscopic mechanical properties of the plate-lattices degrade with an increase in the degree of the two imperfections while being more sensitive to the randomly dispersed defects than the periodically distributed defects. The obtained near-linear scaling on the modulus and yield strength of the plate-lattices with relative densities under consideration indicates that their efficient stretching-dominated behavior is preserved even with the presence of the two disparate imperfections, hence highlighting their structural efficiency.

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Declaration of interests

☒ The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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