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Analysis and Ethical Design of Terrain Park Jumps for Snow Sports

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Abstract Most American snowsport resorts now have terrain parks and decades-long epidemiological evidence correlates terrain park use with injuries. Engineering design of jumps could reduce injuries by limiting equivalent fall heights (EFHs), which indicate dissipated landing impact energy. No evidence refutes making terrain park jumps safer in this way. We discuss case studies illustrating that large EFHs are significant factors in traumatic injuries on terrain park jumps. Standards and design tools for builders can make jumps safer. We introduce a tool that can evaluate existing jumps as well as design jump profiles with safer equivalent EFHs to reduce injuries.

1 Introduction

Impacts with fixed surfaces can cause injury. Greater velocities, perpendicular to the surfaces, provide greater injury potential due to increased kinetic energy dissipation. Equivalent fall height (EFH) is a conceptually simple and familiar measure of impact danger used in safety standards worldwide, from construction [1] to children's playground equipment [2]. EFHs of terrain park jumps can be calculated using techniques in [3] from Cartesian coordinates of

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jump profiles. Profiles must include starting points, takeoff ramps, and landing hills, along jumpers' paths. Controlling energy dissipation in human bodies, hence EFH on jumps, is ethical engineering because it reduces the likelihood and severity of injuries. EFH should be a primary attribute of jump design. It must be considered because it is clearly connected to injury risk and can be used to design and construct safer jumps.

Societal costs of jump injuries are discussed here with case studies that illustrate dangers if EFH is not limited appropriately. We also discuss papers purporting to be ski safety research, which attempt to sow doubt about EFH relevance, written by authors that regularly provide expert testimony for defending the ski industry in personal injury lawsuits. Proposals for improved safety are absent in the papers. We present a user-friendly web application that can facilitate jumping injury reduction by calculating EFH on current and future jumps.

1.1 History

Terrain park jumps are not new. Gradual introduction in the 1980's was accompanied by increased interest in aerial maneuvers and extreme sports participation. Jumps have proliferated since and are today nearly ubiquitous. Roughly 95% of US ski resorts include terrain parks. Unfortunately, this growth correlates with injuries. Two early longitudinal studies in the 1980's and early 1990's [4,5] already found significant increases in head injuries and concussions. Between 1993 and 1997 head injuries accompanied most skiing and snowboarding deaths [6]. Koehle et al. [7] stated "[S]eventy-seven percent of spinal injuries [8] and 30% of head injuries [9] in snowboarding were a result of jumps." Jackson et al. [10] determined that by 2004 snow skiing replaced football as the second leading cause of serious head and spinal cord injuries in America.

These early increasing injury assessments persisted. According to [11], "between 5% and 27% of skiing and snowboarding injuries occur[red] in terrain parks [12,13,14,15,16,17]". Incredibly, at the first Winter Youth Olympic Games more than a third of all snowboard half-pipe and slope-style competitors were injured [18]. Epidemiological research [19,20,21] continues to show that injuries on terrain park jumps are more likely and more severe than on normal slopes. Audet et. al [20] provides evidence that skiing or snowboarding in a terrain park is a risk factor for head, neck, back, and severe injuries. Hosaka et. al [21] concludes that jumping is a main cause for serious spinal injuries, regardless of skill level, and suggests that, because spinal injuries incidence have not decreased over time, the ski industry should focus on designing fail-safe jump features to minimize risks of serious spinal injuries. Similar suggestions appeared in peer-reviewed literature for more than a decade [22,23, 24,25,26,3,27,28].

2 Equivalent Fall Height

EFH, a common proxy measure for impact danger in industrial safety standards, is the weight-specific kinetic energy that must be dissipated on falling impact from height h [29,22,30]. Initial potential energy mgh is transformed to kinetic energy available to injure in non-rotating falls. Injury potential can be reduced by controlling impact circumstances, e.g. impact cushioning, and body orientation, configuration, and motion; however this energy must still be dissipated. Larger EFHs require more elaborate measures to reduce injury; reducing EFH does not.

EFH can be interpreted by the general public. People have an intuitive danger sense when faced with potential falls from large heights and a strong experiential common sense for relating fall height to likelihood of injury. People sense increasing danger associated with falling from larger heights. Ground, second, and third floor falls are about 2.6, 5.1, 8.8 m, respectively [31]. The US Occupational Safety and Health Administration requires protection for heights greater than 1.2 m for general workplace safety [1]. Chalmers et al. [2] argues for 1.5 m maximum fall heights for playground equipment. The Swiss Council for Accident Prevention makes specific recommendations for EFHs to be below 1.5 m for jumps requiring basic skills [32]. Even with no standards in Olympic Nordic ski jumps, typical "equivalent landing height" [30] is only about 0.5 m.

EFH h of objects is formally defined as

$$h = \frac{v^2}{2g} \tag{1}$$

where v is impact velocity and g gravitational acceleration. Kinetic energy of objects moving at velocity v is transformed from potential energy at height h; indisputable, fundamental physics.

From equation 1 equivalent fall heights h can be determined for any surface, i.e., sloped landing profile or shape, after jumping [27]. The result, neglecting air drag, is

$$h = \left[\frac{x^2}{4(x\tan\theta_T - y)\cos^2\theta_T} - y\right]\sin^2\left[\tan^{-1}\left(\frac{2y}{x} - \tan\theta_T\right) - \tan^{-1}\frac{dy}{dx}\right] \tag{2}$$

a function only of takeoff angle θ_T, impact coordinates (x, y) relative to takeoff, and landing surface slope $\frac{dy}{dx}$, but not a function of takeoff speed [27]. To analyze jumps, one measures Cartesian coordinates of landing surfaces along jumpers' flight paths and takeoff angles. Slopes $\frac{dy}{dx}$ are computed from measured coordinates (x, y). Curvature for the last several meters before takeoffs must be near zero to avoid unintentional inversion, although this does not influence EFHs.

3 Case Studies

In these American lawsuits juries ruled for injured plaintiffs. Negligent jump design and construction contributed significantly to injuries [33,34]. Simulations below use methods in [3], assuming skier mass, frontal area, and drag coefficient of 75 kg, 0.34 m^2, and 0.821, respectively.

3.1 Vine v. Bear Valley Ski Company

In April 2000, Ms. Vine's lower spine was injured when she landed badly on a jump at Bear Valley in California. The jump shape (Fig. 1) was a common form called a "table-top". Builders intend that jumpers completely clear the table, landing on down-slopes near a "sweet spot". The upper panel of Fig. 1 shows the measured jump surface from accident investigation. Vine landed short of the knuckle (end of the table-top). This table-top, typically flat and horizontal, was instead concave, compounding dangers of short landings. At the 11 m measured landing horizontal distance from takeoff, the surface sloped upwards approximately 5°. The concave shape emphasizes detrimental effects of not aligning surface tangents closer to jumper flight paths at impact.

The lower panel displays EFH at different landing locations, which is greatest just short of the knuckle. At the sweet spot just past the knuckle the EFH drops precipitously to about 1 m but landing in this narrow region requires jumpers to control takeoff speeds within 1 $\mathrm{m\,s^{-1}}$. Landing at 11 meters, Vine's EFH was almost 4 meters, equivalent to falling from between one and two stories [31]. She had also rotated backward in flight, landed on her lower spine and was paralyzed. A lower EFH could have decreased likelihood of injury, due to lower impact forces.

In contrast, landing surfaces designed to have smaller EFHs can be created at similar cost. The green jump profile in the upper panel of Fig. 1 shows a possible jump design, see [3], of similar size with similar flight times that ensures a constant (smaller) EFH of about 1 m. The convex shape of this jump is interestingly close to the original concave table-top inverted, showing that convex landing shapes are critically important for limiting EFHs. This alternative jump design would have lowered impact forces for landings at all locations. In 2002, the jury ruled in favor of Ms. Vine, agreeing that Bear Valley was responsible for not designing safer jumps.

3.2 Salvini v. Ski Lifts Inc.

In 2004, Mr. Salvini attempted a table-top jump on skis in the terrain park of The Summit at Snoqualmie Ski Resort, in Washington state. Salvini overshot the intended landing location while traveling at typical skiing speeds, rotated backward during flight and landed on his back, ultimately suffering quadriplegia. At his landing location of 30 m the EFH was over 10 meters,

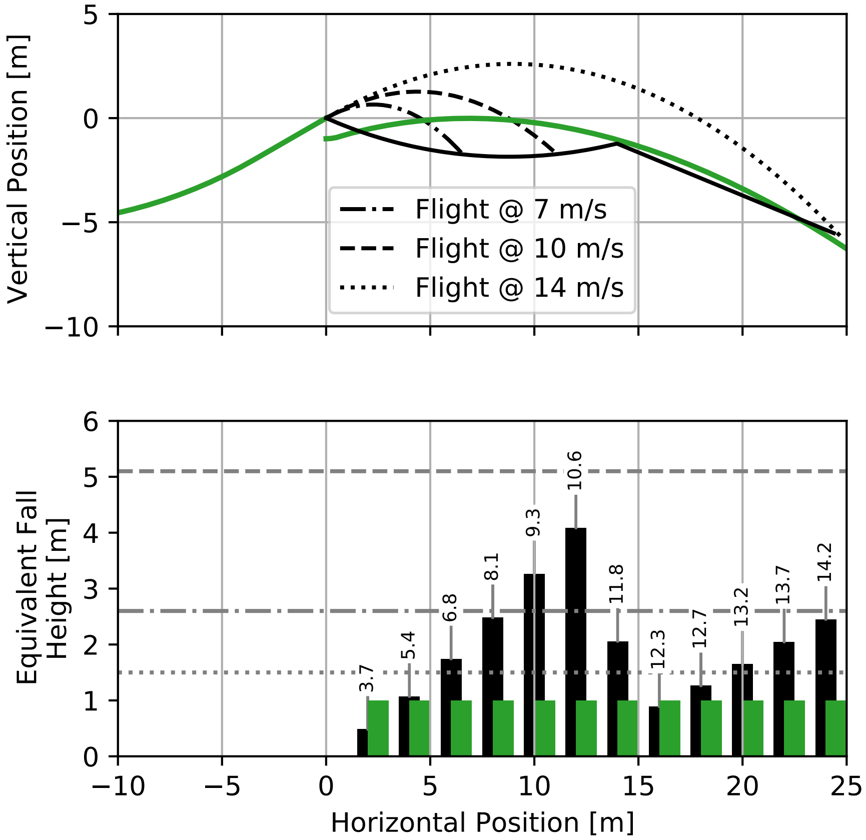

Fig. 1 Bear Valley jump compared to possible safer design Top: Measured landing surface (solid black) and jumper flight paths (intermittent black) from measured 30° takeoff angle. A 14 $\mathrm{m\,s^{-1}}$ takeoff speed is used as the design speed [3] for a comparison jump (solid green) shaped to have constant EFH of 1 m. Bottom: EFH for both jumps in corresponding colors at 2 m intervals. Numbers above bars indicate takeoff speeds required to land at that location. Intermittent horizontal gray lines indicate increasing relatable fall heights: knee collapse, average 1$^{\text{st}}$ story fall, and average 2$^{\text{nd}}$ story fall.

approximately a 3-story fall. Figure 2 shows the measured jump surface from the accident investigation. For takeoff speeds greater than 13 $\mathrm{m\,s^{-1}}$, the lower panel shows that the EFH is greater than 10 m and grows linearly with larger takeoff speeds. Severe injury is almost certain in falls this high, especially if landing body orientation loads the spine, as in this case.

The upper panel also shows a jump profile (green) designed to have a 1 m equivalent fall height for all speeds below 16 $\mathrm{m\,s^{-1}}$. This profile requires significantly more snow than the measured jump but alleviates dangerous impacts.

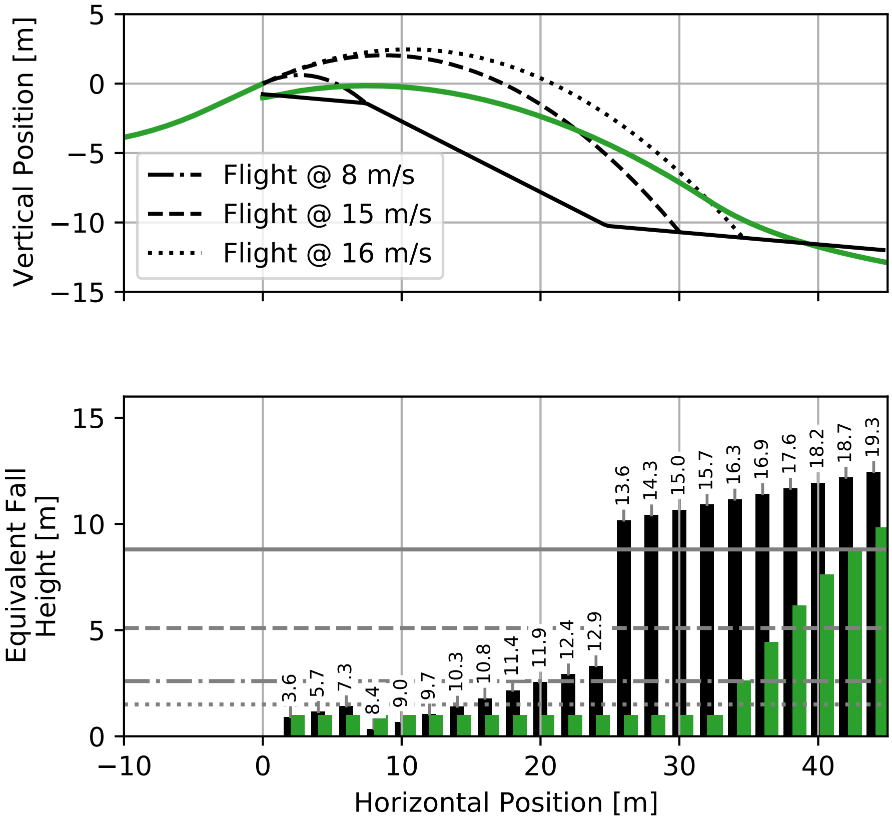

Fig. 2 Snoqualmie jump compared to possible safer design Top: Measured landing surface (solid black) and jumper flight paths (intermittent black) for measured 25° takeoff angle. The 16 $\mathrm{m\,s^{-1}}$ takeoff speed is used as the design speed for a comparison jump (solid green) with constant equivalent fall height of 1 m. Bottom: Equivalent fall height for both jumps in corresponding colors at 2 m intervals. Numbers above bars indicate takeoff speed required to land at that location. Intermittent horizontal gray lines indicate increasing relatable fall heights: knee collapse, average 1$^{\text{st}}$ story fall, average 2$^{\text{nd}}$ story fall, and average 3$^{\text{rd}}$ story fall.

This jump highlights how extreme EFHs become if jumps are not properly designed. Few recreational skiers will jump out three story windows, snow or not. Injuries are clearly likely. Our internal altimeter tells us so, but it's not easy to discern when visually assessing a jump's safety.

These two case studies clearly demonstrate that deficient jump shapes have devastating consequences and that engineering analysis and design, based on laws of mechanics, can be used to shape jump landings that limit EFHs. Designing jumps this way is based on well-established, centuries-old mechanics of

Newton and Émilie du Châtelet [35], fundamental to physics and engineering education. Designing jumps to limit EFHs appropriately reduces injury risks by reducing impact energies, absolutely.

4 Moral Imperative

"Hold paramount the safety, health and welfare of the public" [36], is the first canon of engineering ethics. The moral imperative of engineering ethics is motivating. The first canon compels engineers to use technical expertise to protect snowsport participants from injuries. No one can rationally argue that reducing EFH increases likelihood of injury. Building well designed, safe jumps is no more laborious than building poorly designed, unsafe jumps. There is no reason not to control EFHs with good design methods. Nonetheless skiing industries and their insurance companies are reluctant to adopt and endorse such design methods. They hire expert witnesses engineers to sow doubt on simple, basic physics, while ignoring the first canon. Why?

In their book "Merchants of Doubt" [37], Oreskes and Conway have studied this problem more generally. They show that in numerous industries over the last 60 years, scientific evidence accumulated that commonly accepted industrial activities were harmful, either to individuals or society. But the industries had vested interests in continuing the status quo since operational changes would have led to significant, short-term costs. Examples carefully described and analyzed [37] are the use of DDT, smoking tobacco, acid rain due to coal-fired power plants, ozone hole caused by CFCs, second-hand tobacco smoke's effects, and CO2-caused climate change, among others. Rather than using the proven science as a basis for changes in practice, strategic responses of industries have been to "emphasize the controversy among scientists and the need for continued research" [37].

This same strategy is used by the snowsport industry and its defense experts. To sow doubt and counter solid, fundamental, scientific concepts of landing hill design limiting EFH, defense experts introduce confounding factors to cloud and confuse basic issues. Consider three papers [38,39,40] coauthored by well-known skiing industry defense experts who have testified for the snowsport resorts and their insurance companies.

Shealy et. al [38] conducted an experimental study attempting to test the hypothesis that takeoff speed is a predictor of the distance from a jump take-off to landing. They reached the mechanically preposterous conclusions both that there is "no statistically significant relationship between takeoff speed and the distance traveled" and that "takeoff speed is not a dominant or controlling factor (in how far a jumper travels)" [38]. These conclusions were used to question the soundness of analytical mechanical modeling of jumper flight used in [22,24].

Some of these same authors later vouched for terrain park jump safety. Using data held by the National Ski Areas Association (NSAA), Shealy et. al [39] concluded that their "hypothesis that jumping features resulted in an increase risk of injury [was] not ... substantiated." [39] This is the only study we are aware of with this conclusion. It is difficult to reconcile it with the voluminous contradictory research documenting the unique dangers posed by terrain park jumps in tens of other studies cited both herein and in [22, 23, 24, 25, 26, 3, 27, 28]. Although NSAA releases yearly the total of resort-related fatalities and catastrophic injuries, the raw data on which [39] was based is not publicly available.

In a third experimental study (N=13) specifically designed "to evaluate injury mitigation potential of surfaces limiting EFH" [40], Scher et al. clearly show that body orientation, i.e. falling directly on one's head (in all trials), can cause dangerous cervical spine compression loads [40], even at low fall heights. They report on effects of EFH but only test heights from 0.23 m to 1.52 m, committing a similar fault as in [38], restricting ranges of their independent variables, and ignoring fall heights known to have caused severe injuries regardless of body orientation. Yet they insinuate that EFH has no appreciable effect on injuries. The title, "Terrain Park Jump Design: Would Limiting Equivalent Fall Height Reduce Spinal Injuries?" implies that they appear to believe that falling from greater heights might *not* cause greater injuries. Why propose such mechanically flawed hypotheses?

Poorly executed, limited experiments, no matter how expensive the instrumentation, cannot disprove the fundamental laws of mechanics. If statistics or experimental results seem to conflict with predictions from classical mechanics, the problems must be with the statistical or experimental design or its interpretation, but not with fundamental laws of mechanics. Defending practices that lead to injuries helps prolong these dangerous practices, which leads to further injuries, clearly contradictory to ethical engineering. It is difficult to get otherwise intellectually competent engineers to appreciate the damage they do in their defense work. As Upton Sinclair wrote "it is difficult to get a man to understand something when his salary depends on his not understanding it" [41].

None of these papers was written to reduce injuries. No suggestions are made how their findings can be used to promote public safety. Instead they attempt to obscure the mechanically irrefutable fact that impacting surfaces at lower velocities is safer. Introducing bogus science is as prevalent in American skiing litigation defense, as it was in defending tobacco [37].

American legal, healthcare and insurance systems corrupt technical literature for legal defense of unsafe practices. Peer-reviewed technical literature is used to support testimony in lawsuits. Both plaintiffs and defendants hire experts, authors of technical papers, to testify. When supporting the defense, this can result in denying compensation for injuries and prolong unsafe practices. American healthcare systems can leave families of paralysis victims destitute with little recourse but to sue snowsport resorts. American health insurance companies also sue resorts to recover their losses in injury cases. Conflicts of interest go undeclared in papers and presentations, for organizing and chairing meetings, and editing publications. Financial support for publications gets routed through consulting companies providing plausible deniability of actual conflicts.

In injury cases, testifying for injured plaintiffs and testifying defending insurance companies are not ethically equivalent. One attempts to address problems that cause injuries, holding paramount the public's safety, health and welfare. The other attempts to defend practices that might have contributed to the injury, to limit financial losses of insurance companies. The proverbial two sides to every question is not valid in science and engineering.

5 What Can Be Done?

Societal needs should be addressed. Preventable injuries on terrain park jumps should cease. Injured people require appropriate care that should not depend on prolonged adversarial litigation clouded by deceptive engineering. Papers helping to perpetuate dangerous practices do not belong in engineering journals or conferences.

Everything possible should be done to eliminate dangerous jumps. Ski area operators and their grooming staff must be educated and given tools. Standards are needed for jump designs with limited EFHs, specifying verification, inspection, and maintenance during use. Certification programs are needed for jump building, inspecting, and maintenance.

Around 1980 ASTM Committee F27 began to develop ski binding standards. Proponents were led by orthopedic surgeons and academic researchers [42]. Industry argued that standards were impossible because release value measurement was impossible by ski shops (industry now makes similar arguments about jumps). Nevertheless certifications and inspection standards for bindings were developed, and now there are far fewer lower-extremity equipment-related injuries. Now in F27, no medical professionals and almost no academics remain.

In parallel to standards development, assessing and reshaping existing jumps eliminating dangerous EFH should be an easy route for ski resorts to increase proactively terrain park safety. Accurate enough measurements of existing slopes can occur with simple tools, e.g. tape measure and digital level, consuming relatively little time per jump, see Appendix B. Calculation and visualization of EFHs from these measurements can take some time without a program for calculating EFHs from hill profiles. We have made a user-friendly, freely-accessible online web application available with the calculation and visualization steps almost instantaneous for jump builders. With the tool described in the next section, jump builders can easily add this safety assessment to their toolbox, even using it from a smartphone on hills. We see no reason that this basic assessment should not be part of jump construction processes. The only ethical decision is to adopt these methods; saving even one person from a life of paralysis, or even death, must be worth the relatively minor inconvenience of shaping jumps using the methods in reference [3].

5.1 Software and Online Access

We presented the first version of software for designing ski jumps with a specified EFH in [28]. It comprises a general-purpose, extensible, object-oriented software library with tools for 2D skiing simulation. Using this code, a web application was developed for interactive jump design. The web application is designed for a non-technical end-user and operable on any desktop, tablet, or mobile device supporting a web browser.

We have extended the capabilities of the software in version 1.4.0 to assist the work described in this paper. New library features automate calculation of EFH for jump profiles described by a set of Cartesian coordinates. Additionally, a new “analysis” page allows users to upload measured jump profile coordinates in either a .CSV or Microsoft Excel spreadsheet file. The jump is then analyzed and the equivalent fall height is displayed graphically for interactive user manipulation and viewing. Figure 3 shows the web application with one of the case study jumps loaded for analysis and explains its primary features.

The software is written in Python and directly depends on popular packages including Cython [43], matplotlib [44], NumPy [45], pandas [46], Plotly & Dash [47], pycvodes [48], SciPy [49], SymPy [50], and xlrd. The software is open source and licensed under the MIT redistribution license. The source code is distributed on PyPi (`https://pypi.org/project/skijumpdesign`. Users can submit bug reports, feature requests, code improvements, and additions at the Gitlab repository (`https://gitlab.com/moorepants/skijumpdesign`). The software library is documented at `https://skijumpdesign.readthedocs.io`. Basic examples of using the library are provided in the documentation and this paper’s appendix. The web app is hosted for free use at `http://www.skijumpdesign.info`.

6 Conclusion

There are, of course, more factors than jump landing surface shape that contribute to injuries on terrain park jumps. Yet impact velocity can be easily controlled with a designed landing surface shape. There is no evidence that decreasing EFH increases injuries in falls; injuries can only decrease. Thus there is no reason not to adopt constant low values of EFH for public-use jump designs. Common sense is really all that is needed to believe that falling from higher heights will increase injuries, other factors held constant. Any person that must fall would surely choose to do so from the lowest height. Constructors of jumps that are not designed with these facts in mind are negligent. The safety, health, and welfare of the public involved in this sport should be held paramount.

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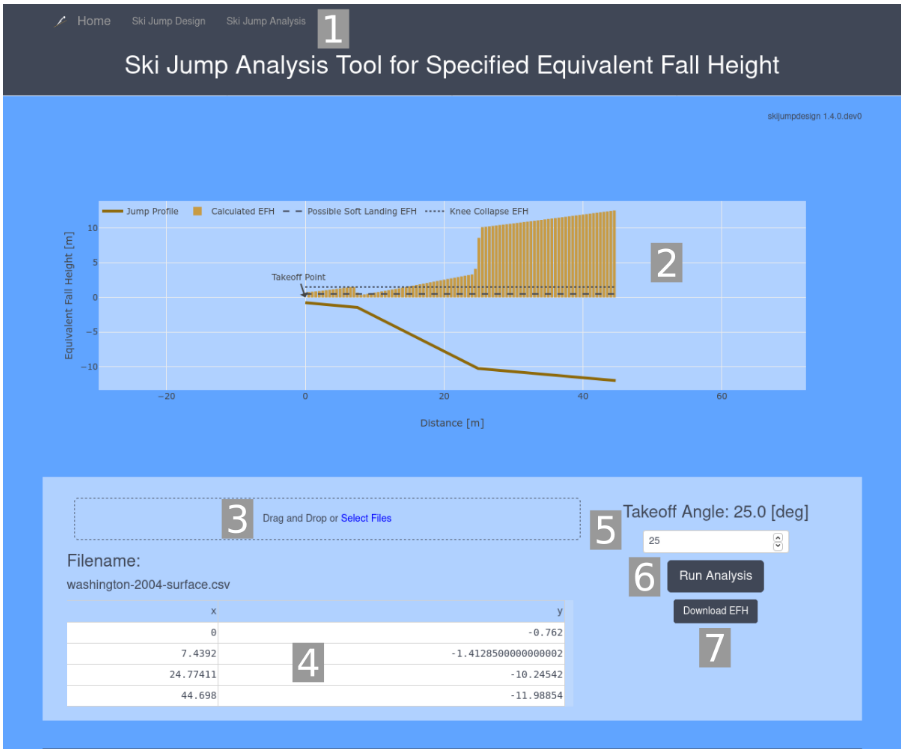

Fig. 3 Screenshot of the ski jump design and analysis web app To use the analysis portion of the app, a user selects "Ski Jump Analysis" from the primary menu [1], uploads a .CSV or .XLS file by dragging it onto the screen [3], inspects the input data for accuracy in the table [4], sets the takeoff angle [5], runs the analysis by pressing the "Run Analysis" button [6], views the results in the interactive plot [2], and downloads the results by pressing the "Download EFH" button [7].

Declarations

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Conflict of interest MH served as a plaintiff's expert witness in the two case studies discussed above. CB testifies occasionally on the side of plaintiffs in ski and snowboard injury cases, has collaborated with Shealy, and C. D. Mote, Jr., Sher's doctoral advisor, on ski safety research, has participated in ASTM F27 since the 1980s on standards for bindings, boots, and skis, and he holds patents on ski and snow board binding designs, intended to reduce injuries.

Availability of data and material All data is available at `https://gitlab.com/moorepants/skijumpdesign` and `https://gitlab.com/mechmotum/ski-jump-analysis-paper`.

Code availability The skijumpdesign version 1.4.0 source code is archived at `https://doi.org/10.5281/zenodo.4637076`. Additionally, it and the paper's source code is available at `https://gitlab.com/moorepants/skijumpdesign` and `https://gitlab.com/mechmotum/ski-jump-analysis-paper`.

Author's contributions JM and MH contributed to the study conception and design. Material preparation, data collection and analysis were performed by JM and MH. The first draft of the manuscript was written by JM, BC, MH, and CB. CB was primarily responsible for drafting the sections on ethics. All authors read and approved the final manuscript. BC and JM wrote the accompanying software.

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Consent for publication JM, BC, MH, and CB consent for publication.

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A Example Software Library Use

The closed form equation 2 is useful for understanding the fundamental relationship of equivalent fall height to the landing surface shape. It will predict EFH for small jumps but other factors may be useful to include in the model. For example, jumpers are subject to aerodynamic drag and this is not negligible for larger jumps. If drag is included there is no closed form solution for the equivalent fall height, but the equivalent fall height can be computed through iterative simulation [3]. The jumper's flight path is found by integrating the flight equations of motion at various takeoff velocities and computing the misalignment of jumper landing and slope angles to then compute the equivalent fall height. This more general simulation method is implemented in the software described herein and the results reflect the inclusion of both gravitational and drag forces. Even with drag incorporated, the calculating EFH still only require measurements of the landing surface cross-sectional profile coordinates (x, y) relative to the takeoff point and a measurement of the takeoff angle. Listing 1 demonstrates the new software library features creating a surface from some measured data points and then calculating the equivalent fall height at 0.2m increments.

```
>>> import numpy as np
>>> from skijumpdesign import Surface, Skier, plot_efh
>>> takeoff_ang = 10  # degrees
>>> takeoff_point = (0, 0)  # (x, y) in meters
>>> x_ft = np.array([-232.3,-203.7,-175.0,-146.3,-117.0,-107.4,
...     -97.7,-88.0,-78.2,-68.5,-58.8,-49.1,-39.4,-34.5,-29.7,
...     ...
...     38.8,43.3,47.8,52.3,56.8,61.5,66.2,70.9,75.7,80.6,85.5,
...     88.4,88.4])
...
>>> y_ft = np.array([55.5,46.4,37.7,29.1,22.2,19.7,17.2,14.8,
...     12.5,10.2,7.7,5.2,2.9,1.8,0.7,-0.2,-1.0,-1.2,-1.4,-1.6,
...     ...
...     -16.2,-18.1,-19.8,-21.4,-22.9,-24.0,-25.0,-25.6,-25.6])
...
>>> x_mt = x_ft*0.3048 # convert to meters
>>> y_mt = y_ft*0.3048 # convert to meters
>>> # create a surface from the data
>>> measured_surf = Surface(x_mt, y_mt)
>>> # create a skier
>>> skier = Skier(mass=75.0, area=0.34, drag_coeff=0.821)
>>> # calculate the equivalent fall height
>>> x, efh, v = measured_surf.calculate_efh(
...      np.deg2rad(takeoff_ang), takeoff_point, skier, increment=0.2)
...
>>> x  # display the x coordinates
array([ 0. ,  0.2,  0.4,  0.6,  0.8,  1. ,  1.2,  1.4,  1.6,  1.8,  2. ,
        2.2,  2.4,  2.6,  2.8,  3. ,  3.2,  3.4,  3.6,  3.8,  4. ,  4.2,
       ...
       24.2, 24.4, 24.6, 24.8, 25. , 25.2, 25.4, 25.6, 25.8, 26. , 26.2,
       26.4, 26.6, 26.8])
>>> efh  # display the equivalent fall height for each x coordinate
array([0.        , 0.02541035, 0.03479384, 0.03264587, 0.05956476,
       0.09096091, 0.12358184, 0.13702364, 0.15202999, 0.17018343,
       ...
       3.93910556, 3.97387212, 4.00891899, 4.04424779, 4.07984952,
       4.11573359, 4.68049185, 5.53413479, 6.45253722, 7.42628019])

>>> v  # display takeoff speeds to reach x positions
array([0.07373847, 0.13081777, 0.1878382 , 0.2447865 , 0.30166299,
       0.35851949, 0.41537661, 0.47221055, 0.52897197, 0.58564902,
       ...
       6.71699974, 6.76760188, 6.81816819, 6.86869777, 6.9191902 ,
       6.96962124, 7.02001551, 7.07037288, 7.1206941 ])
>>> # calculate and plot the efh curve
>>> plot_efh(measured_surf, takeoff_ang, takeoff_point, increment=0.2)
```

Listing 1: Python interpreter session illustrating how one could compute the equivalent fall height of a measured jump.

```
>>> import numpy as np
>>> from skijumpdesign import cartesian_from_measurements
>>> dis = np.array([14.5, 15.0, 15.5, 16.0, 16.5, 17.0])  # meters
>>> ang = np.deg2rad([4.6, -7.4, -16.5, -9.7, -11, -6.9])  # radians
>>> x, y, to_point, to_angle = cartesian_from_measurements(dis, ang)
>>> print(x)  # meters
[0.         0.49985074 0.98901508 1.47600306 1.96786738 2.46177962]
>>> print(y)  # meters
[ 0.         -0.01221609 -0.1157451  -0.22907075 -0.31890113 -0.39668737]
>>> print(to_point)  # takeoff point in meters
(0.0, 0.0)
>>> print(to_angle)  # takeoff angle in radians
0.08028514559173916
```

Listing 2: Python interpreter session showing how one could compute the Cartesian coordinates from equivalent fall height of a measured jump.

B Jump Shape Measurement

Calculating equivalent fall height requires the Cartesian coordinates and slope of the landing surface along the path of the jumper. There are a number of possible measurement techniques for collecting data adequate for the equivalent fall height calculation but the simplest method requires only a digital level [1], a flexible tape measure, and less than an hour's time from one person per jump. A tenth of a degree accuracy from the level and down to 25 cm accuracy from the tape measure should be more than sufficient for typical snowsport jumps.

To measure the jump, the takeoff point should be identified and the tape measure should then be draped over the contour of the landing surface along the projection of the expected flight path onto the landing surface. The origin of the tape measure should be aligned with the takeoff point. Starting with the takeoff point, the digital level should be used to record the absolute angle at regular increments along the tape. The increment can be varied between 25 cm and 100 cm, with the former used for steep slope changes and the later for less steep; 50 cm increments are appropriate for average jump shapes. Positive angles should be recorded for positive slope and negative angles for negative slope. The tabulated data should include the distance along the surface from the takeoff point, d_i, and the associated surface angle, θ_i, at each distance measurement for n measurements. Assuming θ_i is in radians, the Cartesian coordinates can be computed using the average angle to find the adjacent coordinates. The following equations show the calculation of the Cartesian coordinates from these two measures used in the software.

$$\frac{dy_i}{dx_i} = \tan^{-1}\theta_i \quad \text{for } i = 1 \dots n \tag{3}$$

$$x_{i+1} = \begin{cases} 0 & \text{for } i = 0 \\ x_i + (d_{i+1} - d_i)\cos\frac{\theta_{i+1}+\theta_i}{2} & \text{for } i = 1 \dots n-1 \end{cases} \tag{4}$$

$$y_{i+1} = \begin{cases} 0 & \text{for } i = 0 \\ y_i + (d_{i+1} - d_i)\sin\frac{\theta_{i+1}+\theta_i}{2} & \text{for } i = 1 \dots n-1 \end{cases} \tag{5}$$

Listing 2 demonstrates calculating the landing surface's Cartesian coordinates from measured distance and angle data collected with the method described above.

[1] Smartphone digital level measurement applications are likely sufficient and readily available.