Sports Engineering manuscript No.
(will be inserted by the editor)

Online Software Allows Ethical Safety-Conscious Design of Terrain Park Jumps

Jason K. Moore · Bryn Cloud · Mont Hubbard · Christopher A. Brown

Received: date / Accepted: date

Abstract Most American snowsport resorts now have terrain parks and decades-long epidemiological evidence correlates terrain park use with injuries. Engineering design of jumps could reduce injuries by limiting equivalent fall heights (EFHs), which are proportional to dissipated landing impact energy. No evidence refutes making terrain park jumps safer in this way. We discuss case studies illustrating that large EFHs are significant factors in traumatic injuries on terrain park jumps. We argue that it is the ethical responsibility of engineers to ensure the safety, health, and welfare of the public when performing and presenting research on snowsport safety. Developing standards and adopting design tools for builders can make jumps safer. We introduce an online tool that can evaluate existing jumps as well as design jump profiles with safer equivalent EFHs to reduce injuries as an example proactive practice.

1 Introduction

Impacts with fixed surfaces can cause injury. Greater velocities, perpendicular to the surfaces, provide greater injury potential due to increased kinetic energy dissipation. Equivalent fall height (EFH) is a conceptually simple and

J. K. Moore
Delft University of Technology
Mekelweg 2, 2628 CD Delft, The Netherlands
E-mail: j.k.moore@tudelft.nl

B. Cloud & M. Hubbard
University of California, Davis
One Shields Ave., Davis, CA 95616 USA
E-mail: becloud@ucdavis.edu,mhubbard@ucdavis.edu

C. A. Brown
Worcester Polytechnic Institute
100 Institute Rd., Worcester, MA 01609 USA
E-mail: brown@wpi.edu

familiar measure of impact danger used in safety standards worldwide, from construction [1] to children's playground equipment [2]. EFHs of terrain park jumps can be calculated using techniques in [3] from Cartesian coordinates of jump profiles. These must include starting points, takeoff ramps, and landing hills, all along jumpers' paths. Limiting energy dissipation in human bodies, hence EFH on jumps, is ethical engineering because it reduces the likelihood and severity of injuries. EFH should be a primary attribute of jump design. It must be considered because it is clearly connected to injury risk and can be used to design and construct safer jumps.

Societal costs of jump injuries are discussed here with case studies that illustrate dangers if EFH is not limited appropriately. We also discuss papers presented as ski safety research, which attempt to sow doubt about EFH relevance and snow sport danger, written by authors that regularly provide expert testimony defending the ski industry in personal injury lawsuits. Proposals for improved safety are absent in the papers. We present a user-friendly web application that can facilitate jumping injury reduction by calculating EFH on current and future jumps.

1.1 History

Terrain park jumps are not new. Gradual introduction in the 1980's was accompanied by increased interest in aerial maneuvers and extreme sports participation. Jumps have proliferated since and are today nearly ubiquitous. Roughly 95% of US ski resorts include terrain parks. Unfortunately, this growth correlates with injuries. Two early longitudinal studies in the 1980's and early 1990's [4,5] already found significant increases in head injuries and concussions. Between 1993 and 1997 head injuries accompanied most skiing and snowboarding deaths [6]. Koehle et al. [7] stated "[S]eventy-seven percent of spinal injuries [8] and 30% of head injuries [9] in snowboarding were a result of jumps." Jackson et al. [10] determined that by 2004 snow skiing replaced football as the second leading cause of serious head and spinal cord injuries in America.

These early increasing injury assessments persisted. According to [11], "between 5% and 27% of skiing and snowboarding injuries occur[red] in terrain parks [12,13,14,15,16,17]". Incredibly, at the first Winter Youth Olympic Games more than a third of all snowboard half-pipe and slope-style competitors were injured [18]. Epidemiological research [19,20,21] continues to show that injuries on terrain park jumps are more likely and more severe than on normal slopes. Audet et. al [20] provides evidence that skiing or snowboarding in a terrain park is a risk factor for head, neck, back, and other severe injuries. Hosaka et. al [21] concludes that jumping is a main cause for serious spinal injuries, regardless of skill level, and suggests that, because spinal injuries incidence have not decreased over time, the ski industry should focus on designing fail-safe jump features to minimize risks of serious spinal injuries.

Similar suggestions have appeared in peer-reviewed literature for more than a decade [22,23,24,25,26,27,3,28,29].

2 Methods

2.1 Equivalent Fall Height

EFH, a common proxy measure for impact danger in industrial safety standards, is the weight-specific kinetic energy that must be dissipated on falling impact from height h [30,22,31]. Initial potential energy mgh is transformed to kinetic energy available to injure in non-rotating falls. Injury potential can be reduced by controlling impact circumstances, e.g. impact cushioning, and body orientation, configuration, and motion; however this energy must still be dissipated. Larger EFHs require more elaborate measures to reduce injury; reducing EFH does not.

EFH can be interpreted by the general public. People have an intuitive sense of danger when faced with potential falls from large heights and a strong experiential common sense for relating fall height to likelihood of injury. People sense increasing danger associated with falling from larger heights because injury severity increases with increasing fall height [32]. Ground, second, and third floor falls are about 2.6, 5.1, 8.8 m, respectively [33]. The German Society for Trauma Surgery's threshold for trauma team activation is a fall height of 3 m [34]. The US Occupational Safety and Health Administration has for decades required protection for heights greater than 1.2 m for general workplace safety [1]. Chalmers et al. [2] argues for 1.5 m maximum fall heights for playground equipment. The Swiss Council for Accident Prevention makes specific recommendations for EFHs below 1.5 m for jumps requiring basic skills [35]. Even with no standards in Olympic Nordic ski jumps, typical "equivalent landing height" [31] is only about 0.5 m.

EFH h of objects is formally defined as

$$h = \frac{v^2}{2g} \tag{1}$$

where v is impact velocity and g gravitational acceleration. Kinetic energy of objects moving at velocity v is transformed from potential energy at height h; indisputable, fundamental physics.

Beginning from equation 1 equivalent fall heights h can be determined for any surface, i.e., sloped landing profile or shape, after jumping [28]. The result, neglecting air drag, is

$$h = \left[\frac{x^2}{4(x\tan\theta_T - y)\cos^2\theta_T} - y\right]\sin^2\left[\tan^{-1}\left(\frac{2y}{x} - \tan\theta_T\right) - \tan^{-1}\frac{dy}{dx}\right] \tag{2}$$

a function only of takeoff angle θ_T, impact coordinates (x, y) relative to takeoff, and landing surface slope $\frac{dy}{dx}$, but not a function of takeoff speed [28]. To analyze jumps, one measures Cartesian coordinates of landing surfaces along jumpers' flight paths and takeoff angles. Slopes $\frac{dy}{dx}$ are computed from measured coordinates (x, y). Curvature for the last several meters before takeoffs must be near zero to avoid unintentional inversion, although this does not influence EFHs.

2.2 Software and Online Access

We presented the first version of software for designing ski jumps with a specified EFH in [29]. It comprises a general-purpose, extensible, object-oriented software library with tools for 2D skiing simulation. Using this code, a web application was developed for interactive jump design. The web application is designed for a non-technical end-user and operable on any desktop, tablet, or mobile device supporting a web browser.

We have extended the capabilities of the software in version 1.4.0 to assist the work described in this paper. New library features automate calculation of EFH for jump profiles described by a set of Cartesian coordinates. Additionally, a new "analysis" page allows users to upload measured jump profile coordinates in either a .CSV or Microsoft Excel spreadsheet file. The jump is then analyzed and the equivalent fall height is displayed graphically for interactive user manipulation and viewing. Figure 1 shows the web application with one of the case study jumps loaded for analysis and explains its primary features.

The software is written in Python and directly depends on popular packages including Cython [36], matplotlib [37], NumPy [38], pandas [39], Plotly & Dash [40], pycvodes [41], SciPy [42], SymPy [43], and xlrd. The software is open source and licensed under the MIT redistribution license. The source code is distributed on PyPi (`https://pypi.org/project/skijumpdesign`). Users can submit bug reports, feature requests, code improvements, and additions at the Gitlab repository (`https://gitlab.com/moorepants/skijumpdesign`). The software library is documented at `https://skijumpdesign.readthedocs.io`. Basic examples of using the library are provided in the documentation and this paper's supplementary materials. The web app is hosted for free use at `http://www.skijumpdesign.info`.

We do not view the software as the definitive ski jump design and analysis tool, but rather as a foundation. The tool has been released as open-source so that refinements and modifications are easy and encouraged. The software was carefully designed with extensibility and modularity in mind. New surface shapes such as different takeoff ramps are easily added by building upon the basic surface object using object-oriented programming principles. Similarly, new skier models can be added that incorporate more complex biomechanical features and actions. We make use of this flexible software design for the web application and for the calculations and visualizations presented in the following section.

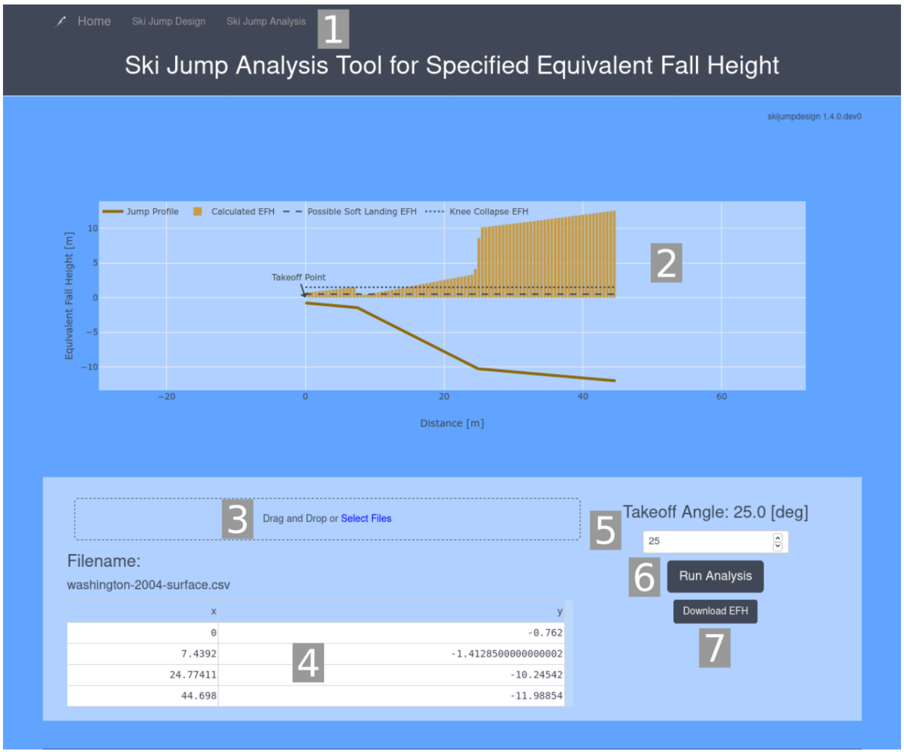

Fig. 1 Screenshot of the ski jump design and analysis web app To use the analysis portion of the app, a user selects "Ski Jump Analysis" from the primary menu [1], uploads a .CSV or .XLS file by dragging it onto the screen [3], inspects the input data for accuracy in the table [4], sets the takeoff angle [5], runs the analysis by pressing the "Run Analysis" button [6], views the results in the interactive plot [2], and downloads the results by pressing the "Download EFH" button [7].

3 Results

In these case studies of American lawsuits, juries ruled for injured plaintiffs. Negligent jump design and construction contributed significantly to injuries [44,45]. Simulations below use methods in [3], assuming skier mass, frontal area, and drag coefficient of 75 kg, 0.34 m^2, and 0.821, respectively.

3.1 Vine v. Bear Valley Ski Company

In April 2000, Ms. Vine's lower spine was injured when she landed badly skiing a jump at Bear Valley in California. The jump shape (Fig. 2) was a common form called a "table-top". Builders intend that jumpers completely clear the table, landing on down-slopes near a "sweet spot". The upper panel of Fig. 2 shows the measured jump surface from accident investigation. Vine landed short of the knuckle (end of the table-top). This table-top, typically flat and

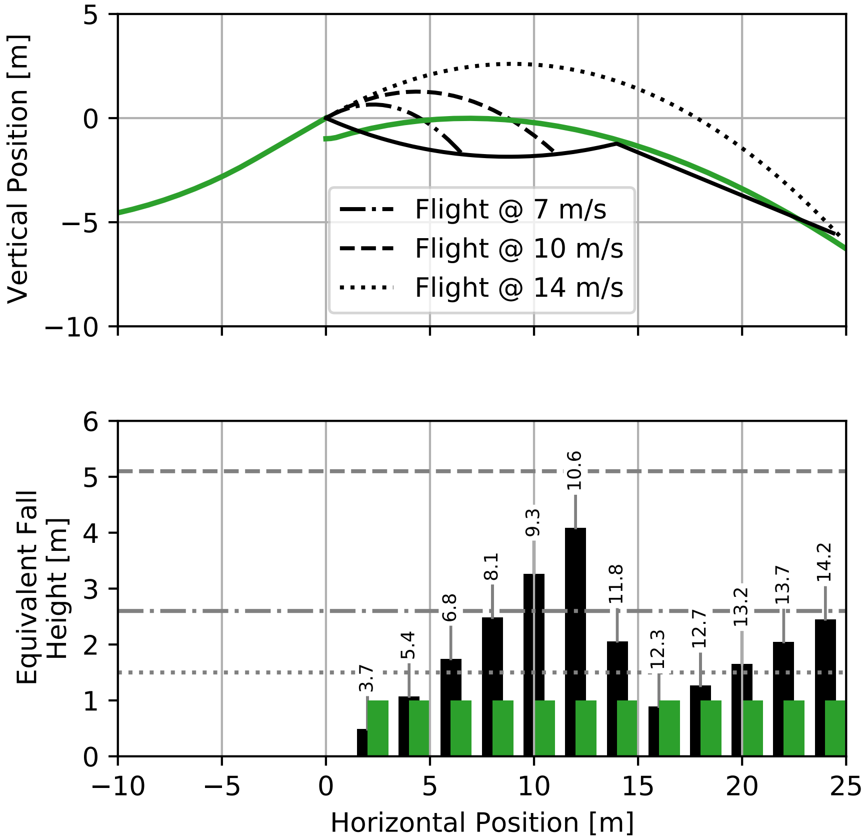

Fig. 2 Bear Valley jump compared to possible safer design Top: Measured landing surface (solid black) and jumper flight paths (intermittent black) from measured 30° takeoff angle. A 14 $\mathrm{m\,s^{-1}}$ takeoff speed is used as the design speed [3] for a comparison jump (solid green) shaped to have constant EFH of 1 m. Bottom: EFH for both jumps in corresponding colors at 2 m intervals. Numbers above bars indicate takeoff speeds required to land at that location. Intermittent horizontal gray lines indicate increasing relatable fall heights: knee collapse, average 1$^{\mathrm{st}}$ story fall, and average 2$^{\mathrm{nd}}$ story fall.

horizontal, was instead concave, compounding dangers of short landings. At the 11 m landing horizontal distance measured from takeoff, the surface sloped upwards approximately 5°. The concave shape emphasizes detrimental effects of not aligning surface tangents closer to jumper flight paths at impact.

The lower panel displays EFH at different landing locations, which is greatest just short of the knuckle. At the sweet spot just past the knuckle the EFH drops precipitously to about 1 m but landing in this narrow region requires jumpers to control takeoff speeds within 1 $\mathrm{m\,s^{-1}}$. Landing at 11 meters, Vine's EFH was almost 4 meters, equivalent to falling from between one and two stories [33]. She had also rotated backward in flight, landed on her lower spine and was paralyzed. A lower EFH could have decreased likelihood of injury, due to lower impact forces.

In contrast, landing surfaces designed to have smaller EFHs can be created at similar cost. The green jump profile in the upper panel of Fig. 2 shows a possible jump design, see [3], of similar size with similar flight times that ensures a constant (smaller) EFH of about 1 m. The convex shape of this jump is interestingly close to the original concave table-top inverted, showing that convex landing shapes are critically important for limiting EFHs. This alternative jump design would have lowered impact forces for landings at all locations. In 2002, the jury ruled in favor of Ms. Vine, agreeing that Bear Valley was responsible for not designing safer jumps.

3.2 Salvini v. Ski Lifts Inc.

In 2004, Mr. Salvini attempted a table-top jump on skis in the terrain park of The Summit at Snoqualmie Ski Resort, in Washington state. Salvini overshot the intended landing location while traveling at typical skiing speeds, rotated backward during flight and landed on his back, ultimately suffering quadriplegia. At his landing location of 30 m the EFH exceeded 10 meters, approximately a 3-story fall. Figure 3 shows the measured jump surface from the accident investigation. For takeoff speeds greater than 13 $\mathrm{m\,s^{-1}}$, the lower panel shows that the EFH is greater than 10 m and growing linearly with larger takeoff speeds. Severe injury is almost certain in falls this high, especially if landing body orientation loads the spine, as in this case.

The upper panel also shows a jump profile (green) designed to have a 1 m equivalent fall height for all speeds below 16 $\mathrm{m\,s^{-1}}$. This profile requires significantly more snow than the measured jump but alleviates dangerous impacts.

This jump highlights how extreme EFHs can become if jumps are not properly designed. Few recreational skiers will jump out three story windows, snow or not. Injuries are clearly likely. Our internal altimeter tells us so, but it's not easy to discern when visually assessing a jump's safety.

These two case studies clearly demonstrate that deficient jump landing shapes have devastating consequences and that engineering analysis and design, based on laws of mechanics, can be used to shape jump landings that limit EFHs. Designing jumps this way is based on well-established, centuries-old mechanics of Isaac Newton and Émilie du Châtelet [46], fundamental to physics and engineering education. Designing jumps to limit EFHs appropriately reduces injury risks by reducing impact energies, absolutely.

4 Discussion

4.1 Moral Imperative

"Hold paramount the safety, health and welfare of the public" [47], is the first canon of engineering ethics. Ethics is not a matter of opinion and should not be

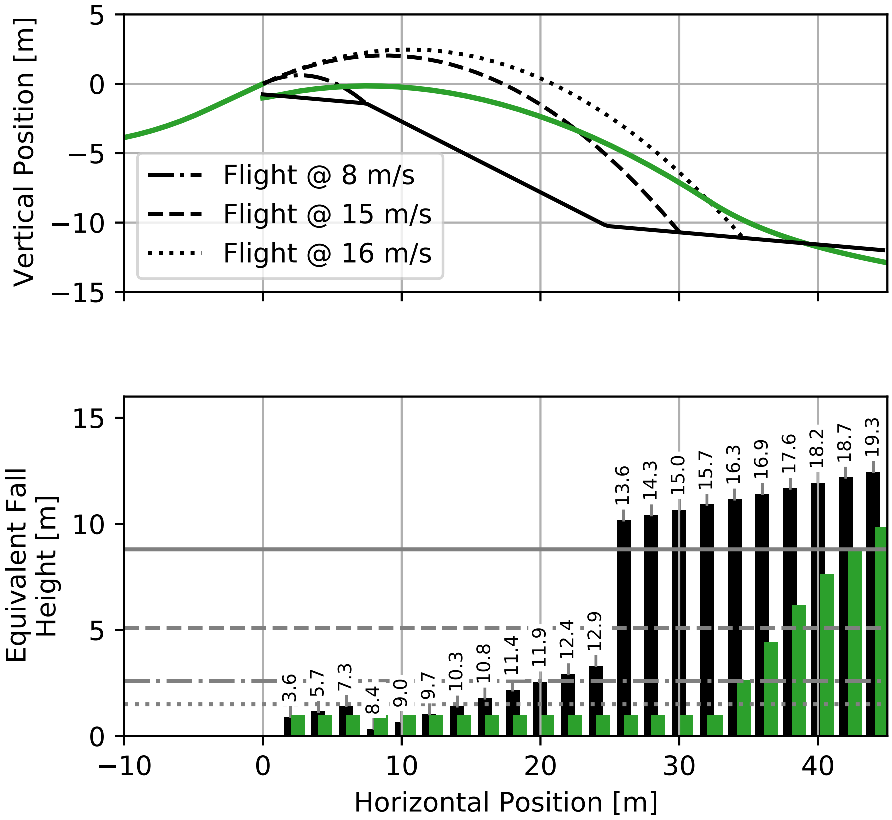

Fig. 3 Snoqualmie jump compared to possible safer design Top: Measured landing surface (solid black) and jumper flight paths (intermittent black) for measured 25° takeoff angle. The 16 $\mathrm{m\,s^{-1}}$ takeoff speed is used as the design speed for a comparison jump (solid green) with constant equivalent fall height of 1 m. Bottom: Equivalent fall height for both jumps in corresponding colors at 2 m intervals. Numbers above bars indicate takeoff speed required to land at that location. Intermittent horizontal gray lines indicate increasing relatable fall heights: knee collapse, average 1$^{\text{st}}$ story fall, average 2$^{\text{nd}}$ story fall, and average 3$^{\text{rd}}$ story fall.

optional. It is the foundation for engineering. The first canon compels engineers to use their technical expertise to protect snowsport participants from injuries. No one can rationally argue that reducing EFH increases likelihood of injury. Building well designed, safer jumps is no more laborious than building poorly designed, unsafe jumps. We see no reason not to control EFHs with good design methods. Nonetheless skiing industries and their insurance companies are reluctant to adopt and endorse such design methods, choosing instead to spend money and energy on litigation defense rather than on technology for constructing safer jumps. They hire expert witness engineers to sow doubt on simple, basic physics, while ignoring the first canon. What is their excuse for inaction?

In their book “Merchants of Doubt” [48], Oreskes and Conway have studied this problem more generally. They show that in numerous industries over the last 60 years, scientific evidence accumulated that commonly accepted industrial activities were harmful, either to individuals or society. But the industries had vested interests in continuing the status quo since operational changes would have led to significant, short-term costs. Examples carefully described and analyzed [48] are the use of DDT, smoking tobacco, acid rain due to coal-fired power plants, ozone hole caused by CFCs, second-hand tobacco smoke’s effects, and CO2-caused climate change, among others. Rather than using the proven science as a basis for changes in practice, strategic responses of industries have been to “emphasize the controversy among scientists and the need for continued research” [48].

This same strategy is used by the snowsport industry and its defense experts. To sow doubt and counter solid, fundamental, scientific concepts of landing hill design limiting EFH, defense experts introduce confounding factors to cloud and confuse basic issues. Consider as evidence three papers [49, 50, 51] co-authored by well-known skiing industry defense experts who have testified for the snowsport resorts and their insurance companies. We do not fundamentally question their empirical findings but we do reject their interpretation of the findings, namely their conclusion that greater fall height is not a cause of greater injury.

Shealy et. al [49] conducted an experimental study attempting to test the hypothesis that takeoff speed is a predictor of the distance from a jump take-off to landing. They reached the mechanically impossible conclusions both that there is “no statistically significant relationship between takeoff speed and the distance traveled” and that “takeoff speed is not a dominant or controlling factor (in how far a jumper travels)” [49]. These conclusions were used to question the soundness of analytical mechanical modeling of jumper flight used in [22, 25].

Some of these same authors later vouched for terrain park jump safety. Using data held by the National Ski Areas Association (NSAA), Shealy et. al [50] concluded that their “hypothesis that jumping features resulted in an increase risk of injury [was] not ... substantiated.” [50] This is the only study we are aware of with this conclusion. It is difficult to reconcile it with the voluminous contradictory research documenting the unique dangers posed by terrain park jumps in tens of other studies cited both herein and in [22, 23, 25, 26, 27, 3, 28, 29]. Although NSAA releases yearly the total of resort-related fatalities and catastrophic injuries, the raw data on which [50] was based is not even publicly available!

In a third experimental study (N=13) specifically designed “to evaluate injury mitigation potential of surfaces limiting EFH” [51], Scher et al. clearly show that body orientation, i.e. falling directly on one’s head (in all trials), can cause dangerous cervical spine compression loads [51], even at low fall heights. They report on effects of EFH but only test heights from 0.23 m to 1.52 m, committing a similar fault as in [49], restricting ranges of their independent variables, and ignoring fall heights known to have caused severe injuries regardless of body orientation. Yet they insinuate that EFH has no appreciable effect on injuries. The title, “Terrain Park Jump Design: Would Limiting Equivalent Fall Height Reduce Spinal Injuries?” implies that they appear to believe that falling from greater heights might *not* cause greater injuries. Why propose such mechanically flawed hypotheses?

Extending the scope of the findings in these ways are common statistical mistakes, but ones that should not be made by professional engineers. Fundamental laws cannot be disproved by these kinds of jumping experiments. If statistical or experimental results seem in conflict with predictions from classical mechanics, the problems are most certainly with the statistical or experimental design or their interpretations, but not fundamental laws of mechanics. Defending practices that lead to injuries helps prolong these dangerous practices, which leads to further injuries, clearly contradictory to ethical engineering. Ski industry defense engineering experts are complicit in the continued societal damage. As Upton Sinclair wrote “it is difficult to get a man to understand something when his salary depends on his not understanding it” [52].

It is not evident that these papers “hold paramount the safety, health and welfare of the public”. They are silent on how their findings can be used to reduce injuries. They obscure a scientifically fundamental, mechanically irrefutable fact that impacting surfaces at lower normal velocities is safer. They “create the appearance that the claims being promoted were scientific” [48, page 244]. Fundamental laws have made mechanics a science. Findings that contradict such fundamental laws should be carefully scrutinized and review processes accepting such articles should be questioned.

Organizations also merchandise doubt. A decade ago, NSAA argued [53] that, because of rider and snow variability, terrain park jump “standards are essentially impossible.” While it is true that the “virtually . . . infinite number of ways that a given feature may be used by an individual . . . varying speed, pop, body movement, takeoff stance, angles of approach, the attempting of different kinds of maneuvers, landing stance, and the type of equipment used (skis or snowboard) . . . create a wide variety of experiences for the users” [53], none of these in fact preclude analysis or design. This was shown clearly in reference [24] which examined quantitatively the effects of variations in factors actually involved in the mechanics: takeoff speed, snow friction, air drag, tail wind, snow melt and jumper pop. These so-called “uncontrollable factors” fell into three groups: (1) those for which there is zero sensitivity, i.e., an uncontrollable factor that makes no difference in the ability of the designed jump to deliver the designed EFH; (2) those for which fairly large parameter variations cause only insignificant maximum deviations in EFH, and (3) those for which the factor can be taken into account in the design process itself and its larger effect on EFH completely eliminated in the unsafe direction. The allegation that design of limited EFH surfaces is prevented by the complexity of the problem and by the large number and types of parameter variations away from nominal is false; in fact the allegation is just more merchandised doubt.

In injury cases, testifying for injured plaintiffs and testifying defending corporations are not ethically equivalent. The former attempts to address problems that cause injuries, holding paramount the public's safety, health and welfare. The latter attempts to defend practices that might have contributed to the injury, to limit financial losses of corporations. The idiom "two sides to every question", is not appropriate in science and engineering [48, page 268].

4.2 What Can Be Done?

Absolutely, the most important change will be to incorporate rigorous, rational processes and scientific principles that consider mechanical impact safety into designing freestyle jumps. At present a large fraction of, if not most, jumps in the USA are created in a formulaic way using two straight lines, a horizontal deck (tabletop) and nearly constant-slope landing region, linked by a curved knuckle. This design philosophy is recommended in the instructions provided by the NSAA [54] and is presumably followed by their members. Although such jumps are simple and thus easy to design, previous research has shown that jumps with bi-linear geometry have generally poor EFH behavior [23] and that they can have low EFH only in a small region just past the knuckle (called the "sweet spot"). In the more recent version of their freestyle terrain park notebook [54], the jump landing area is even termed the "landing plan" because it is envisioned to be planar! There is no reference whatsoever to any concept such as EFH or similar measure of impact or its effect on safety because the NSAA's strategy is to put the responsibility for safety fully on the jumper. There is no quantitative consideration of jump impact safety (e.g. from the point of view of EFH) beyond seat-of-the-pants experience of the designer. The skiing industry continues to resist more scientifically-based rational approaches to design, in spite of the fact that computer aided design (and even automated computer-controlled fabrication) of snow park jumps (see Figure 4) has been available from snow groomer manufacturers for more than 5 years [55]. The 2015 NSAA reference in [54] still contained the statement that "Standards are essentially impossible. . . " We expect the 2021 version may also.

Once the jump surface has been designed, the next most important change is to build accurately what was designed. Presently a dominant fraction of jumps are simply fabricated by groomer operators, based on perhaps a few measurements of distances and slopes (deck length, takeoff angle, landing region angle and length) during the process. But the design concepts are overly simple and do not incorporate or address quantitative indicators of safety such as EFH. The introduction of computer controlled grooming, similar to computer controlled (CNC) machining, will facilitate precise construction of more complex designed shapes. These would include the non-trivial constant EFH surfaces provided by our online ski jump design software that limit landing impulses to acceptable levels.

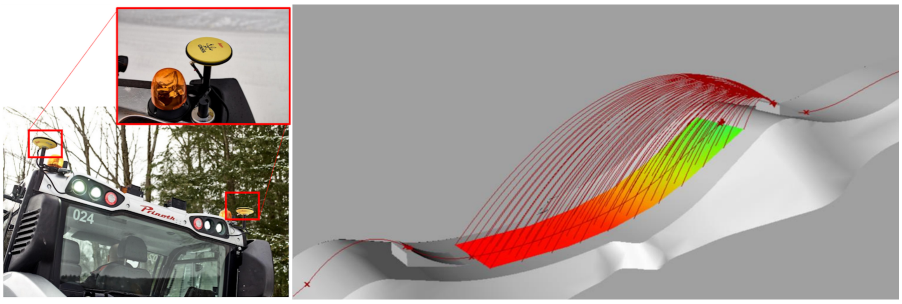

Fig. 4 Commercial availability of computer-aided design and computer-controlled fabrication of snow park surfaces began as early as 2016. The right panel shows a computer generated 3-D jump landing surface with a family of simulated jumper paths, even ones outside the central vertical bisecting plane, with the landing surface colored corresponding to the EFH incurred by the jumper at that landing point. The left panel shows a computer-controlled snow groomer fitted with two GNSS receivers that allow real time measurement of their position to an accuracy of about 2 cm, calculation of the yaw and roll of the groomer blade, and precise closed loop control of the snow addition and removal process. Images courtesy of Prinoth, official supplier of snow groomers for the winter Olympics in China 2022.

Every jumper (and parent of young jumpers) should be able to confirm that a jump is safe before trying it. Appropriate inspection, evaluation, and correction of existing jumps, and the design and construction of safer new jumps should be promoted. Postings should be required and include EFHs, the certified inspectors name, and when last inspected. Inspections should be frequent enough to ensure that jumps meet the standards, particularly regarding takeoffs and starting points to prevent inadvertent inversions. Standards need to be developed that limit EFHs in collaboration between industry and research engineers to design, build, inspect, maintain, and post safer jumps. An example of first steps in this area is a terrain park safety guide by the Swiss Council for Accident Prevention [35].

To complement standards, certification programs are needed for jump building, inspection, and maintenance. As an example of a successful certification program, around 1980 ASTM Committee F27 began to develop ski binding standards. Proponents were led by orthopedic surgeons and academic researchers [56]. Industry argued that standards were impossible because release value measurement was impossible by ski shops (industry now makes similar arguments about jumps [54]). Nevertheless certifications and inspection standards for bindings were developed, which led to far fewer lower-extremity equipment-related injuries. But now no medical professionals and almost no academics remain in F27. Efforts to create similar standards for terrain park ski jumps began in F27 more than a decade ago [57], yet no standards have yet been developed. The US skiing industry, aided by the NSAA, has been successful in slow-walking the process in order to prevent standards.

In parallel with standards development, assessing and possibly reshaping existing jumps to eliminate dangerous EFHs should be an straightforward route for ski resorts to proactively increase terrain park safety. Accurate enough measurements of existing surfaces can occur even with simple tools, e.g. tape measure and digital level, and consume relatively little time and effort per jump (see supplementary materials for details). Calculation and visualization of EFHs from these measurements can take some time without a computational program for calculating EFHs from hill profiles. The user-friendly, freely-accessible open-source online web application tool that we have made available for jump designers and builders has almost instantaneous calculation and visualization steps, solving this problem.

With this software jump builders can easily add safety assessment to their toolbox, even accessing it from a smartphone or tablet on hills. We see no reason that this basic assessment should not be part of jump construction processes. The only ethical decision is to adopt these methods; saving even one person from a life of paralysis, or even death, must be worth the relatively minor inconvenience of shaping jumps using the methods in reference [3].

5 Conclusion

There are, of course, more factors than jump takeoff and landing surface shapes that contribute to injuries on terrain park jumps. Yet normal impact velocity can be easily controlled with a properly designed and fabricated landing surface shape. There is no evidence that decreasing designed EFH increases injuries in falls; injuries only decrease. Thus we see no reason not to adopt constant low values of EFH for public-use jump designs. In fact, safety research [58] tells us that designing forgiving environments (i.e., limiting EFH at all possible landing locations) is more effective than forcing behavioral change (e.g., requiring the jumper to regulate their speed to ensure a landing only in a small safe region). Fabricators of jumps that are not designed with these principles in mind are negligent. The safety, health, and welfare of the public involved in this sport must be held paramount.

A real limitation to reducing injuries, especially paralysis, on terrain park jumps is that it cannot be accomplished just by publishing papers on scientific design methods and providing software. Industry must adopt ethical, user-focused procedures for designing, building, inspecting, and maintaining safer terrain park jumps. As shown in the case studies above, jumps can have EFHs comparable to falling from aerial ski lifts. Just like rules enforced by states and insurance companies to prevent falls from lifts, similar standards are needed for terrain park jumps. Public safety must be held paramount to short-term return-on-investment.

Engineers whose scholarly work ignores engineering's first canon of ethics in favor of merchandising doubt can diminish the scientific integrity of engineering journals and engineering conferences. Journal editors should recognize papers primarily intending to cast doubt on good science and engineering for what they are, tools of insurance companies for defending civil suits, and reject them. Papers helping to perpetuate dangerous practices do not belong in engineering journals or conference proceedings.

The methods and software we have illustrated in Section 2.2 provide a starting point for realizing EFH-conscious designs in terrain parks. We hope to see the design and analysis adopted by commercial grooming equipment manufacturers so that safety is made integral to jump design. Our software can grow and evolve through contributions from other researchers to incorporate many other nuances of injury prevention. We also see the methods providing a structure for standards development. And minimally, we see the software as an immediately usable tool for jump fabricators in the field.

Acknowledgements We thank Rado Dukalski for feedback on the web application and both Yumiko Henneberry and Lyn Taylor for feedback on the manuscript.

Declarations

Funding Not applicable

Conflict of interest MH served as a plaintiff's expert witness in the two case studies discussed above and in numerous other similar cases since. CB testifies occasionally on behalf of plaintiffs in ski and snowboard injury cases. He has collaborated with Shealy and C. D. Mote, Jr., Sher's doctoral advisor, on ski safety research, has participated in ASTM F27 since the 1980s on standards for bindings, boots, and skis, and holds patents on ski and snowboard binding designs intended to reduce injuries.

Availability of data and material All data is available at `https://gitlab.com/moorepants/skijumpdesign` and `https://gitlab.com/mechmotum/ski-jump-analysis-paper`.

Code availability The skijumpdesign version 1.4.0 source code is archived at `https://doi.org/10.5281/zenodo.4637076`. Additionally, it and the paper's source code is available at `https://gitlab.com/moorepants/skijumpdesign` and `https://gitlab.com/mechmotum/ski-jump-analysis-paper`.

Author's contributions JM and MH contributed to the study conception and design. Material preparation, data collection and analysis were performed by JM and MH. The first draft of the manuscript was written by JM, BC, MH, and CB. MH and CB were primarily responsible for drafting the parts on merchandising doubt and ethics, respectively. All authors read and approved the final manuscript. BC and JM wrote the accompanying software.

Ethics approval Not applicable

Consent for publication JM, BC, MH, and CB consent for publication.

References

1. Occupational Safety Health Adminstration, "Safety and health regulations for construction," tech. rep., US Dept. of Labor, Alexandria, Virginia, USA, 2021.

2. D. J. Chalmers, S. W. Marshall, J. D. Langley, M. J. Evans, C. R. Brunton, A. M. Kelly, and A. F. Pickering, "Height and surfacing as risk factors for injury in falls from playground equipment: A case-control study.," *Injury Prevention*, vol. 2, pp. 98–104, June 1996.
3. D. Levy, M. Hubbard, J. A. McNeil, and A. Swedberg, "A design rationale for safer terrain park jumps that limit equivalent fall height," *Sports Engineering*, vol. 18, pp. 227–239, Dec. 2015.
4. M. C. Deibert, D. D. Aronsson, R. J. Johnson, C. F. Ettlinger, and J. E. Shealy, "Skiing Injuries in Children, Adolescents, and Adults*," *The Journal of Bone & Joint Surgery*, vol. 80, no. 1, pp. 25–32, 1998.
5. M. Furrer, S. Erhart, A. Frutiger, H. Bereiter, A. Leutenegger, and T. Ruedi, "Severe Skiing Injuries: A Retrospective Analysis of 361 Patients Including Mechanism of Trauma, Severity of Injury, and Mortality," *Journal of Trauma and Acute Care Surgery*, vol. 39, pp. 737–741, Oct. 1995.
6. U.S. Consumer Product Safety Commission, "Skiing Helmets: An Evaluation of the Potential to Reduce Head Injury," tech. rep., Washington, D.C., Jan. 1999.
7. M. S. Koehle, R. Lloyd-Smith, and J. E. Taunton, "Alpine Ski Injuries and Their Prevention," *Sports Medicine*, vol. 32, pp. 785–793, Oct. 2002.
8. F. Tarazi, M. F. S. Dvorak, and P. C. Wing, "Spinal Injuries in Skiers and Snowboarders," *The American Journal of Sports Medicine*, vol. 27, pp. 177–180, Mar. 1999.
9. O. Fukuda, M. Takaba, T. Saito, and S. Endo, "Head Injuries in Snowboarders Compared with Head Injuries in Skiers: A Prospective Analysis of 1076 patients from 1994 to 1999 in Niigata, Japan*," *The American Journal of Sports Medicine*, vol. 29, pp. 437–440, July 2001.
10. A. B. Jackson, M. Dijkers, M. J. DeVivo, and R. B. Poczatek, "A demographic profile of new traumatic spinal cord injuries: Change and stability over 30 years," *Archives of Physical Medicine and Rehabilitation*, vol. 85, pp. 1740–1748, Nov. 2004.
11. K. Russell, W. H. Meeuwisse, A. Nettel-Aguirre, C. A. Emery, J. Wishart, N. T. R. Romanow, B. H. Rowe, C. Goulet, and B. E. Hagel, "Feature-specific terrain park-injury rates and risk factors in snowboarders: A case–control study," *British Journal of Sports Medicine*, vol. 48, pp. 23–28, Jan. 2014.
12. E. J. Bridges, F. Rouah, and K. M. Johnston, "Snowblading injuries in Eastern Canada," *British Journal of Sports Medicine*, vol. 37, pp. 511–515, Dec. 2003.
13. C. Goulet, D. Hamel, B. Hagel, and G. Légaré, "Risk Factors Associated with Serious Ski Patrol-reported Injuries Sustained by Skiers and Snowboarders in Snow-parks and on Other Slopes," *Canadian Journal of Public Health*, vol. 98, pp. 402–406, Sept. 2007.
14. C. Moffat, S. McIntosh, J. Bringhurst, K. Danenhauer, N. Gilmore, and C. L. Hopkins, "Terrain Park Injuries," *Western Journal of Emergency Medicine*, vol. 10, pp. 257–262, Nov. 2009.
15. M. W. Greve, D. J. Young, A. L. Goss, and L. C. Degutis, "Skiing and Snowboarding Head Injuries in 2 Areas of the United States," *Wilderness & Environmental Medicine*, vol. 20, pp. 234–238, Sept. 2009.
16. M. A. Brooks, M. D. Evans, and F. P. Rivara, "Evaluation of skiing and snowboarding injuries sustained in terrain parks versus traditional slopes," *Injury Prevention*, vol. 16, pp. 119–122, Apr. 2010.
17. G. Ruedl, M. Kopp, R. Sommersacher, T. Woldrich, and M. Burtscher, "Factors associated with injuries occurred on slope intersections and in snow parks compared to on-slope injuries," *Accident Analysis & Prevention*, vol. 50, pp. 1221–1225, Jan. 2013.
18. G. Ruedl, W. Schobersberger, E. Pococco, C. Blank, L. Engebretsen, T. Soligard, K. Steffen, M. Kopp, and M. Burtscher, "Sport injuries and illnesses during the first Winter Youth Olympic Games 2012 in Innsbruck, Austria," *British Journal of Sports Medicine*, vol. 46, pp. 1030–1037, Dec. 2012.
19. L. Carús and M. Escorihuela, "Feature-specific ski injuries in snow parks," *Accident Analysis & Prevention*, vol. 95, pp. 86–90, Oct. 2016.
20. O. Audet, A. K. Macpherson, P. Valois, B. E. Hagel, B. Tremblay, and C. Goulet, "Terrain park feature compliance with Québec ski area safety recommendations," *Injury Prevention*, Apr. 2020.

21. N. Hosaka, K. Arai, H. Otsuka, and H. Kishimoto, "Incidence of recreational snowboarding-related spinal injuries over an 11-year period at a ski resort in Niigata, Japan," *BMJ Open Sport & Exercise Medicine*, vol. 6, May 2020.
22. M. Hubbard, "Safer Ski Jump Landing Surface Design Limits Normal Impact Velocity," *Journal of ASTM International*, vol. 6, no. 1, 2009.
23. A. D. Swedberg and M. Hubbard, "Modeling Terrain Park Jumps: Linear Tabletop Geometry May Not Limit Equivalent Fall Height," in *Skiing Trauma and Safety: 19th Volume* (R. J. Johnson, J. E. Shealy, R. M. Greenwald, and I. S. Scher, eds.), pp. 120–135, 100 Barr Harbor Drive, PO Box C700, West Conshohocken, PA 19428-2959: ASTM International, Nov. 2012.
24. M. Hubbard and A. D. Swedberg, "Design of Terrain Park Jump Landing Surfaces for Constant Equivalent Fall Height Is Robust to "Uncontrollable" Factors," in *Skiing Trauma and Safety: 19th Volume* (R. J. Johnson, J. E. Shealy, R. M. Greenwald, and I. S. Scher, eds.), pp. 75–94, 100 Barr Harbor Drive, PO Box C700, West Conshohocken, PA 19428-2959: ASTM International, Nov. 2012.
25. J. A. McNeil, M. Hubbard, and A. D. Swedberg, "Designing tomorrow's snow park jump," *Sports Engineering*, vol. 15, pp. 1–20, Mar. 2012.
26. J. A. McNeil, "The Inverting Effect of Curvature in Winter Terrain Park Jump Take-offs," in *Skiing Trauma and Safety: 19th Volume* (R. J. Johnson, J. E. Shealy, R. M. Greenwald, and I. S. Scher, eds.), pp. 136–150, 100 Barr Harbor Drive, PO Box C700, West Conshohocken, PA 19428-2959: ASTM International, Nov. 2012.
27. M. Hubbard, J. A. McNeil, N. Petrone, and M. Cognolato, "Impact Performance of Standard Tabletop and Constant Equivalent Fall Height Snow Park Jumps," in *Skiing Trauma and Safety: 20th Volume* (R. J. Johnson, ed.), (West Conshohocken, PA), pp. 51–71, ASTM International, Feb. 2015.
28. N. Petrone, M. Cognolato, J. A. McNeil, and M. Hubbard, "Designing, building, measuring, and testing a constant equivalent fall height terrain park jump," *Sports Engineering*, vol. 20, pp. 283–292, Dec. 2017.
29. J. K. Moore and M. Hubbard, "Skijumpdesign: A Ski Jump Design Tool for Specified Equivalent Fall Height," *The Journal of Open Source Software*, vol. 3, p. 818, Aug. 2018.
30. W. Müller, D. Platzer, and B. Schmölzer, "Scientific approach to ski safety," *Nature*, vol. 375, no. 455, 1995.
31. H.-H. Gasser, "Jumping Hills, Construction Norm 2018, Implementing Provisions for Art. 411 of the ICR Ski Jumping," tech. rep., International Ski Federation, Nov. 2018.
32. C. Nau, M. Leiblein, R. D. Verboket, J. A. Hörauf, R. Sturm, I. Marzi, and P. Störmann, "Falls from Great Heights: Risk to Sustain Severe Thoracic and Pelvic Injuries Increases with Height of the Fall," *Journal of Clinical Medicine*, vol. 10, p. 2307, Jan. 2021.
33. N. L. Vish, "Pediatric window falls: Not just a problem for children in high rises," *Injury Prevention*, vol. 11, pp. 300–303, Oct. 2005.
34. Polytrauma Guideline Update Group, "Level 3 guideline on the treatment of patients with severe/multiple injuries : AWMF Register-Nr. 012/019," *European Journal of Trauma and Emergency Surgery: Official Publication of the European Trauma Society*, vol. 44, pp. 3–271, Apr. 2018.
35. B. Heer, F. Bürgi, and M. Weiler, "Terrain Parks," Tech. Rep. 2.081, Swiss Council for Accident Prevention, Bern, Switzerland, 2019.
36. S. Behnel, R. Bradshaw, C. Citro, L. Dalcin, D. S. Seljebotn, and K. Smith, "Cython: The best of both worlds," *Computing in Science & Engineering*, vol. 13, no. 2, pp. 31–39, 2011.
37. J. D. Hunter, "Matplotlib: A 2D graphics environment," *Computing in Science & Engineering*, vol. 9, no. 3, pp. 90–95, 2007.
38. T. E. Oliphant, *A Guide to NumPy*, vol. 1. Trelgol Publishing USA, 2006.
39. W. McKinney, "Pandas." Zenodo, Feb. 2020.
40. Plotly Technologies Inc., "Plotly: Collaborative data science," 2015.
41. B. Dahlgren, "Pyodesys: Straightforward numerical integration of ODE systems from Python," *Journal of Open Source Software*, vol. 3, p. 490, Jan. 2018.
42. P. Virtanen, R. Gommers, T. E. Oliphant, M. Haberland, T. Reddy, D. Cournapeau, E. Burovski, P. Peterson, W. Weckesser, J. Bright, *et al.*, "SciPy 1.0: Fundamental

algorithms for scientific computing in Python," *Nature methods*, vol. 17, no. 3, pp. 261–272, 2020.

43. A. Meurer, C. P. Smith, M. Paprocki, O. Čertík, S. B. Kirpichev, M. Rocklin, A. Kumar, S. Ivanov, J. K. Moore, S. Singh, T. Rathnayake, S. Vig, B. E. Granger, R. P. Muller, F. Bonazzi, H. Gupta, S. Vats, F. Johansson, F. Pedregosa, M. J. Curry, A. R. Terrel, Š. Roučka, A. Saboo, I. Fernando, S. Kulal, R. Cimrman, and A. Scopatz, "SymPy: Symbolic computing in Python," *PeerJ Computer Science*, vol. 3, Jan. 2017.
44. Superior Court San Francisco County, "Charlene Vine v. Bear Valley Ski Company," Dec. 2002.
45. King County Superior Court, "Kenneth Salvini v. Ski Lifts, Inc.," Oct. 2008.
46. J. P. Zinsser, *Emilie Du Chatelet: Daring Genius of the Enlightenment.* Penguin, Nov. 2007.
47. NSPE, "Code of Ethics for Engineers," tech. rep., National Society of Professional Engineers, Alexandria, Virginia, USA, July 2019.
48. N. Oreskes and E. M. Conway, *Merchants of Doubt: How a Handful of Scientists Obscured the Truth on Issues from Tobacco Smoke to Global Warming.* Bloomsbury Press, 2010.
49. J. Shealy, I. Scher, L. Stepan, and E. Harley, "Jumper Kinematics on Terrain Park Jumps: Relationship between Takeoff Speed and Distance Traveled," *Journal of ASTM International*, vol. 7, p. 10, Nov. 2010.
50. J. E. Shealy, I. Scher, R. J. Johnson, and J. A. Rice, "Jumping Features at Ski Resorts: Good Risk Management?," in *Skiing Trauma and Safety: 20th Volume* (R. J. Johnson, J. E. Shealy, and R. M. Greenwald, eds.), pp. 39–50, 100 Barr Harbor Drive, PO Box C700, West Conshohocken, PA 19428-2959: ASTM International, Jan. 2015.
51. I. Scher, J. Shealy, L. Stepan, R. Thomas, and R. Hoover, "Terrain Park Jump Design: Would Limiting Equivalent Fall Height Reduce Spine Injuries?," in *Skiing Trauma and Safety: 20th Volume* (R. J. Johnson, J. E. Shealy, and R. M. Greenwald, eds.), pp. 72–90, 100 Barr Harbor Drive, PO Box C700, West Conshohocken, PA 19428-2959: ASTM International, Feb. 2015.
52. U. Sinclair, *I, Candidate for Governor: And How I Got Licked.* University of California Press, Berkeley, 1994.
53. NSAA, "Freestyle Terrain Park Notebook," tech. rep., National Ski Areas Association, Lakewood, Colorado, USA, 2008.
54. NSAA, "Freestyle Terrain Park Notebook," tech. rep., National Ski Areas Association, Lakewood, Colorado, USA, 2015.
55. A. Muigg, "6th FIS Clinic 2019 hosted by Prinoth: Competitions Courses, Terrain Park and Fun Slopes," Sept. 2019.
56. E. Bahniuk, "Twenty Years' Development of ASTM Skiing Standards," in *Skiing Trauma and Safety: Tenth Volume*, (West Conshohocken, PA), pp. 15–22, ASTM International, 1996.
57. SAM, "ASTM F27 Committee Could Broaden Scope to Include Park Jumps Soon," *SAM: Ski Area Management, The Voice of the Mountain Resort Industry*, Oct. 2011.
58. L. S. Smith, N. J. Wilkins, and R. J. McClure, "A systemic approach to achieving population-level impact in injury and violence prevention," *Systems Research and Behavioral Science*, vol. 38, no. 1, pp. 21–30, 2020.

Supplementary Materials for Analysis and Ethical Design of Terrain Park Jumps for Snow Sports

Jason K. Moore Bryn Cloud Mont Hubbard
Christopher A. Brown

August 13, 2021

1 Example Software Library Use

The closed form equation

$$h = \left[\frac{x^2}{4(x\tan\theta_T - y)\cos^2\theta_T} - y\right]\sin^2\left[\tan^{-1}\left(\frac{2y}{x} - \tan\theta_T\right) - \tan^{-1}\frac{dy}{dx}\right] \tag{1}$$

is useful for understanding the fundamental relationship of equivalent fall height to the landing surface shape. It will predict EFH for small jumps but other factors may be useful to include in the model. For example, jumpers are subject to aerodynamic drag and this is not negligible for larger jumps. If drag is included there is no closed form solution for the equivalent fall height, but the equivalent fall height can be computed through iterative simulation [1]. The jumper's flight path is found by integrating the flight equations of motion at various takeoff velocities and computing the misalignment of jumper landing and slope angles to then compute the equivalent fall height. This more general simulation method is implemented in the software described herein and the results reflect the inclusion of both gravitational and drag forces. Even with drag incorporated, the calculating EFH still only require measurements of the landing surface cross-sectional profile coordinates (x, y) relative to the takeoff point and a measurement of the takeoff angle. Listing 1 demonstrates the new software library features creating a surface from some measured data points and then calculating the equivalent fall height at 0.2m increments.

2 Jump Shape Measurement

Calculating equivalent fall height requires the Cartesian coordinates and slope of the landing surface along the path of the jumper. There are a number of possible measurement techniques for collecting data adequate for the equivalent

```
>>> import numpy as np
>>> from skijumpdesign import Surface, Skier, plot_efh
>>> takeoff_ang = 10  # degrees
>>> takeoff_point = (0, 0)  # (x, y) in meters
>>> x_ft = np.array([-232.3,-203.7,-175.0,-146.3,-117.0,-107.4,
...     -97.7,-88.0,-78.2,-68.5,-58.8,-49.1,-39.4,-34.5,-29.7,
...     ...
...     38.8,43.3,47.8,52.3,56.8,61.5,66.2,70.9,75.7,80.6,85.5,
...     88.4,88.4])
...
>>> y_ft = np.array([55.5,46.4,37.7,29.1,22.2,19.7,17.2,14.8,
...     12.5,10.2,7.7,5.2,2.9,1.8,0.7,-0.2,-1.0,-1.2,-1.4,-1.6,
...     ...
...     -16.2,-18.1,-19.8,-21.4,-22.9,-24.0,-25.0,-25.6,-25.6])
...
>>> x_mt = x_ft*0.3048 # convert to meters
>>> y_mt = y_ft*0.3048 # convert to meters
>>> # create a surface from the data
>>> measured_surf = Surface(x_mt, y_mt)
>>> # create a skier
>>> skier = Skier(mass=75.0, area=0.34, drag_coeff=0.821)
>>> # calculate the equivalent fall height
>>> x, efh, v = measured_surf.calculate_efh(
...      np.deg2rad(takeoff_ang), takeoff_point, skier, increment=0.2)
...
>>> x  # display the x coordinates
array([ 0. ,  0.2,  0.4,  0.6,  0.8,  1. ,  1.2,  1.4,  1.6,  1.8,  2. ,
        2.2,  2.4,  2.6,  2.8,  3. ,  3.2,  3.4,  3.6,  3.8,  4. ,  4.2,
       ...
       24.2, 24.4, 24.6, 24.8, 25. , 25.2, 25.4, 25.6, 25.8, 26. , 26.2,
       26.4, 26.6, 26.8])
>>> efh  # display the equivalent fall height for each x coordinate
array([0.        , 0.02541035, 0.03479384, 0.03264587, 0.05956476,
       0.09096091, 0.12358184, 0.13702364, 0.15202999, 0.17018343,
       ...
       3.93910556, 3.97387212, 4.00891899, 4.04424779, 4.07984952,
       4.11573359, 4.68049185, 5.53413479, 6.45253722, 7.42628019])

>>> v  # display takeoff speeds to reach x positions
array([0.07373847, 0.13081777, 0.1878382 , 0.2447865 , 0.30166299,
       0.35851949, 0.41537661, 0.47221055, 0.52897197, 0.58564902,
       ...
       6.71699974, 6.76760188, 6.81816819, 6.86869777, 6.9191902 ,
       6.96962124, 7.02001551, 7.07037288, 7.1206941 ])
>>> # calculate and plot the efh curve
>>> plot_efh(measured_surf, takeoff_ang, takeoff_point, increment=0.2)
```

Listing 1: Python interpreter session illustrating how one could compute the equivalent fall height of a measured jump.

fall height calculation but the simplest method requires only a digital level [1], a flexible tape measure, and less than an hour's time from one person per jump. A tenth of a degree accuracy from the level and down to 25 cm accuracy from the tape measure should be more than sufficient for typical snowsport jumps.

To measure the jump, the takeoff point should be identified and the tape measure should then be draped over the contour of the landing surface along the projection of the expected flight path onto the landing surface. The origin of the tape measure should be aligned with the takeoff point. Starting with the takeoff point, the digital level should be used to record the absolute angle at regular increments along the tape. The increment can be varied between 25 cm and 100 cm, with the former used for steep slope changes and the later for less steep; 50 cm increments are appropriate for average jump shapes. Positive angles should be recorded for positive slope and negative angles for negative slope. The tabulated data should include the distance along the surface from the takeoff point, d_i, and the associated surface angle, θ_i, at each distance measurement for n measurements. Assuming θ_i is in radians, the Cartesian coordinates can be computed using the average angle to find the adjacent coordinates. The following equations show the calculation of the Cartesian coordinates from these two measures used in the software.

$$\frac{dy_i}{dx_i} = \tan^{-1}\theta_i \quad \text{for } i = 1 \ldots n \tag{2}$$

$$x_{i+1} = \begin{cases} 0 & \text{for } i = 0 \\ x_i + (d_{i+1} - d_i)\cos\frac{\theta_{i+1}+\theta_i}{2} & \text{for } i = 1 \ldots n-1 \end{cases} \tag{3}$$

$$y_{i+1} = \begin{cases} 0 & \text{for } i = 0 \\ y_i + (d_{i+1} - d_i)\sin\frac{\theta_{i+1}+\theta_i}{2} & \text{for } i = 1 \ldots n-1 \end{cases} \tag{4}$$

Listing 2 demonstrates calculating the landing surface's Cartesian coordinates from measured distance and angle data collected with the method described above.

References

[1] D. Levy, M. Hubbard, J. A. McNeil, and A. Swedberg, "A design rationale for safer terrain park jumps that limit equivalent fall height," *Sports Engineering*, vol. 18, pp. 227–239, Dec. 2015.

[1]Smartphone digital level measurement applications are likely sufficient and readily available.

```
>>> import numpy as np
>>> from skijumpdesign import cartesian_from_measurements
>>> dis = np.array([14.5, 15.0, 15.5, 16.0, 16.5, 17.0])  # meters
>>> ang = np.deg2rad([4.6, -7.4, -16.5, -9.7, -11, -6.9])  # radians
>>> x, y, to_point, to_angle = cartesian_from_measurements(dis, ang)
>>> print(x)  # meters
[0.         0.49985074 0.98901508 1.47600306 1.96786738 2.46177962]
>>> print(y)  # meters
[ 0.         -0.01221609 -0.1157451  -0.22907075 -0.31890113 -0.39668737]
>>> print(to_point)  # takeoff point in meters
(0.0, 0.0)
>>> print(to_angle)  # takeoff angle in radians
0.08028514559173916
```

Listing 2: Python interpreter session showing how one could compute the Cartesian coordinates from equivalent fall height of a measured jump.