Online Software and Ethical Issues for Safety-Conscious Design of Terrain Park Jumps

Jason K. Moore Bryn Cloud Mont Hubbard
Christopher A. Brown

J. K. Moore Delft University of Technology
Mekelweg 2, 2628 CD Delft, The Netherlands
j.k.moore@tudelft.nl

B. Cloud & M. Hubbard University of California, Davis
One Shields Ave., Davis, CA 95616 USA
becloud@ucdavis.edu,mhubbard@ucdavis.edu

C. A. Brown Worcester Polytechnic Institute
100 Institute Rd., Worcester, MA 01609 USA
brown@wpi.edu

October 5, 2021

Abstract

Many snowsport resorts now have terrain parks and decades-long epidemiological evidence correlates terrain park use with injuries. Engineering design of jumps could reduce injuries by limiting equivalent fall heights, which are proportional to dissipated landing impact energy. No evidence refutes making terrain park jumps safer in this way. We discuss case studies illustrating that large equivalent fall heights are significant factors in traumatic injuries on terrain park jumps. We argue that it is the ethical responsibility of engineers to ensure the safety, health, and welfare of the public when performing and presenting research on snowsport safety. Developing standards and adopting design tools for builders can make jumps safer. To reduce injuries, we introduce an online tool that can evaluate existing jumps as well as design jump profiles with safer equivalent fall heights.

1 Introduction

Impacts with fixed surfaces can cause injury. Greater velocities, perpendicular to the surfaces, provide greater injury potential due to increased kinetic energy dissipation. Equivalent fall height (EFH) is a conceptually simple and familiar measure of impact danger used in safety standards worldwide, from construction [1] to children's playground equipment [2]. EFHs of terrain park jumps can be calculated using techniques in [3] from Cartesian coordinates of jump profiles. These coordinates include starting points, takeoff ramps, and landing hills, all along jumpers' paths. Limiting energy dissipation in human bodies, hence EFH on jumps, reduces likelihoods of injuries and their severities. EFH should be considered because it is clearly connected to injury risk and can be used to design and construct safer jumps. In fact, safety research [4] tells us that designing forgiving environments (i.e., limiting EFH at all possible landing locations) is more effective than forcing behavioral change (e.g., requiring the jumper to regulate their speed to ensure a landing only in a small safe region).

Societal costs of jump injuries are discussed here with case studies that illustrate dangers if EFH is not limited appropriately. We also critique papers that question EFH relevance, written by authors that regularly provide expert testimony defending the ski industry in personal injury lawsuits. We present a web application that can facilitate jumping injury reduction by calculating EFH on current and future jumps.

1.1 History

Gradual introduction of terrain parks in the 1980's was accompanied by increased interest in aerial maneuvers and extreme sports participation. Jumps have proliferated since and are today nearly ubiquitous. Roughly 95% of US ski resorts include terrain parks. Unfortunately, this growth correlates with injuries. Two early longitudinal studies in the 1980's and early 1990's [5, 6] already found significant increases in head injuries and concussions. Between 1993 and 1997 head injuries accompanied most skiing and snowboarding deaths [7]. Koehle et al. [8] stated "[S]eventy-seven percent of spinal injuries [9] and 30% of head injuries [10] in snowboarding were a result of jumps." Jackson et al. [11] determined that by 2004 snow skiing replaced football as the second leading cause of serious head and spinal cord injuries in America.

These early increasing injury assessments persisted. According to [12], "between 5 and 27% of skiing and snowboarding injuries occur[red] in terrain parks [13, 14, 15, 16, 17, 18]". At the first Winter Youth Olympic Games over a third of all snowboard half-pipe and slope-style competitors were injured [19]. Epidemiological research [20, 21, 22] continues to show that injuries on terrain park jumps are more likely and more severe than on normal slopes. Audet et. al [21] provides evidence that skiing or snowboarding in a terrain park is a risk factor for head, neck, back, and other severe injuries. Hosaka et. al [22] concludes that jumping is a main cause for serious spinal injuries, regardless of skill level, and suggests that, because spinal injuries' incidence have not decreased over time, the ski industry should focus on designing fail-safe jump features to minimize risks of serious spinal injuries. Similar jump design suggestions have appeared in peer-reviewed literature for more than a decade [23, 24, 25, 26, 27, 28, 3, 29, 30].

2 Methods

2.1 Equivalent Fall Height

EFH, a common proxy measure for impact danger in industrial safety standards, is the weight-specific kinetic energy that must be dissipated on falling impact from height h [31, 23, 32]. Initial potential energy mgh is transformed to kinetic energy available to injure in non-rotating falls. Injury potential can be reduced by controlling impact circumstances, e.g. impact cushioning, and body orientation, configuration, and motion; however this energy must still be dissipated. Larger EFHs require more elaborate measures to reduce injury; reducing EFH does not.

EFH can be interpreted by the general public. People have an intuitive sense of danger when faced with potential falls from large heights and a strong experiential common sense for relating fall height to likelihood of injury. People sense increasing danger associated with falling from larger heights because injury severity increases with fall height [33]. Ground, second, and third floor falls are about 2.6, 5.1, 8.8 m, respectively [34]. The German Society for Trauma Surgery's threshold for trauma team activation is a fall height of 3 m [35]. The US Occupational Safety and Health Administration requires protection for heights over 1.2 m for general workplace safety [1]. Chalmers et al. [2] argues for 1.5 m maximum fall heights for playground equipment. The Swiss Council for Accident Prevention makes specific recommendations for EFHs below 1.5 m for terrain park jumps requiring basic skills [36]. Even with no standards in Olympic Nordic ski jumps, typical "equivalent landing height" [32] is only about 0.5 m.

EFH h of objects is formally defined as

$$h = \frac{v^2}{2g} \tag{1}$$

where v is impact velocity and g gravitational acceleration. Kinetic energy of objects moving at velocity v is transformed from potential energy at height h.

Beginning from equation 1 equivalent fall heights h can be determined for any surface, i.e., sloped landing profile or shape, after jumping [29]. The result, neglecting air drag, is

$$h = \left[\frac{x^2}{4(x\tan\theta_T - y)\cos^2\theta_T} - y\right]\sin^2\left[\tan^{-1}\left(\frac{2y}{x} - \tan\theta_T\right) - \tan^{-1}\frac{dy}{dx}\right] \tag{2}$$

a function only of takeoff angle θ_T, impact coordinates (x, y) relative to takeoff, and landing surface slope $\frac{dy}{dx}$, but not takeoff speed [29]. To analyze jumps, one measures Cartesian coordinates of landing surfaces along jumpers' flight paths and takeoff angles. Slopes $\frac{dy}{dx}$ are computed from measured coordinates (x, y). Positive curvatures (concavity) in takeoff ramps tend to cause skiers to rotate rearwards, inverting them in flight, so they might land in more dangerous body orientations [37], although ramp curvature does not influence EFHs.

2.2 Software and Online Access

We presented the first version of software for designing ski jumps with a specified EFH in [30]. It comprises a general-purpose, extensible, object-oriented software library with tools for 2D skiing simulation. Using this code, a web application was developed for interactive jump design. The web application is designed for a non-technical end-user and operable on any desktop, tablet, or mobile device supporting a web browser.

We have extended capabilities of this software in version 1.4.0 (March 25, 2021) to assist work described here. New library features automate calculation of EFH for jump profiles described by a set of Cartesian coordinates. Additionally, a new "analysis" page allows users to upload measured jump profile coordinates in either a comma separated value or Microsoft Excel spreadsheet file. Jumps are then analyzed and EFHs are displayed graphically for interactive user manipulation and viewing. Figure 1 shows the web application with one of the case study jumps (Salvini v. Ski Lifts Inc.) loaded for analysis and explains its primary features.

This software is written in Python and directly depends on popular packages including Cython [38], matplotlib [39], NumPy [40], pandas [41], Plotly & Dash [42], pycvodes [43], SciPy [44], SymPy [45], and xlrd. This software is open source and licensed under the MIT redistribution license. The source code is distributed on PyPi [1]. Users can submit bug reports, feature requests, code improvements, and additions at the Gitlab repository [2]. The software library's documentation is hosted via Read the Docs [3]. Basic examples of using the library are provided in the documentation and this paper's supplementary materials. We have also made the web application available for free use online. [4]

We do not view the software as the definitive ski jump design and analysis tool, but rather as a foundation. The tool has been released as open-source so that refinements and modifications are easy and encouraged. The software was designed to be extensible and modular. New surface shapes such as different takeoff ramps are easily added by building upon the basic surface object using object-oriented programming principles. Similarly, new skier models can be added that incorporate more complex biomechanical features and actions. We make use of this flexibility for the web application and for calculations and visualizations presented in the following section.

3 Results

In these case studies of American lawsuits, juries ruled for injured plaintiffs. Negligent jump design and construction contributed significantly to injuries [46, 47]. Simulations below use methods in [3], assuming the same skier mass, frontal area, and drag coefficient of 75 kg, 0.34 m^2, and 0.821, respectively.

[1] `https://pypi.org/project/skijumpdesign`
[2] `https://gitlab.com/moorepants/skijumpdesign`
[3] `https://skijumpdesign.readthedocs.io`
[4] `http://www.skijumpdesign.info`

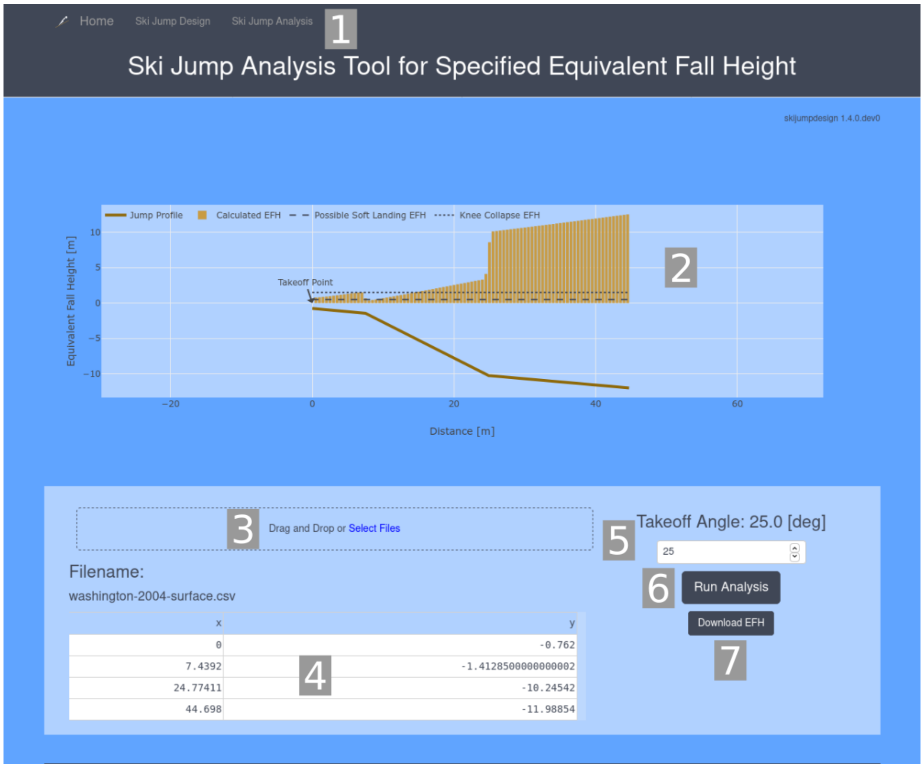

Figure 1: **Screenshot of the ski jump design and analysis web app** To use the analysis portion of the app, a user selects "Ski Jump Analysis" from the primary menu [1], uploads a .CSV or .XLS file by dragging it onto the screen [3], inspects the input data for accuracy in the table [4], sets the takeoff angle [5], runs the analysis by pressing the "Run Analysis" button [6], views results in an interactive plot [2], and downloads results by pressing the "Download EFH" button [7].

3.1 Vine v. Bear Valley Ski Company

In April 2000, Ms. Vine's lower spine was injured when she landed badly while jumping on skis at Bear Valley in California. The jump shape (Fig. 2) was a common form called a "table-top". Builders intend that jumpers completely clear the table, landing on down-slopes near a "sweet spot". The upper panel of Fig. 2 shows the measured jump surface from accident investigation. Vine landed short of the knuckle, defined as the end of the table-top. This table-top was not flat and horizontal as is typical. Instead it was concave, compounding dangers of short landings. At the 11 m landing horizontal distance measured from takeoff, the surface sloped upwards approximately 5°. Concave table tops exacerbate detrimental effects of failing to align landing zone tangents close to jumper flight paths at impact.

The lower panel displays EFHs at different landing locations. These are greatest just short of the knuckle. At the sweet spot, just past the knuckle, EFH drops precipitously to about 1 m, although landing in this narrow region requires jumpers to control takeoff speeds very accurately to within about 1 $\mathrm{m\,s^{-1}}$. Landing at 11 meters, Vine's EFH was instead almost 4 meters, equivalent to falling from between one and two stories [34]. She had also rotated backward in flight, landed on her lower spine and was paralyzed. Lower EFH would have decreased risk of injury, due to lower impact forces.

Jumps with smaller EFHs can be created at similar costs. The green jump profile in the upper panel of Fig. 2 shows a possible jump design, see [3], of similar size with similar flight times that ensures constant (small) EFHs of about 1 m. Interestingly, the convex shape of this jump is close to the original concave table-top inverted, showing that convex landing shapes are critically important for limiting EFHs. This alternative jump design would have lowered impact forces for landings at all locations. In 2002, the jury ruled in favor of Ms. Vine, agreeing that Bear Valley was responsible for providing unsafe jumps.

3.2 Salvini v. Ski Lifts Inc.

In 2004, Mr. Salvini attempted a table-top jump on skis in the terrain park of The Summit at Snoqualmie Ski Resort, in Washington state. Salvini overshot the intended landing location while traveling at typical skiing speeds [48], rotated backward during flight and landed on his back, ultimately suffering quadriplegia. The jury sided with Mr. Salvini and he was awarded a judgment of \$14M.

At his landing location of 30 m the EFH exceeded 10 meters, approximately a 3-story fall. Figure 3 shows the measured jump profile from the accident investigation. For takeoff speeds over 13 $\mathrm{m\,s^{-1}}$, the lower panel shows that the EFH is over 10 m and growing linearly with larger takeoff speeds. Severe injury is almost certain in falls this high, especially if landing body orientation loads the spine, as in this case.

The upper panel also shows a jump profile (green) designed to have a 1 m EFH for all speeds below 16 $\mathrm{m\,s^{-1}}$. This profile requires significantly more

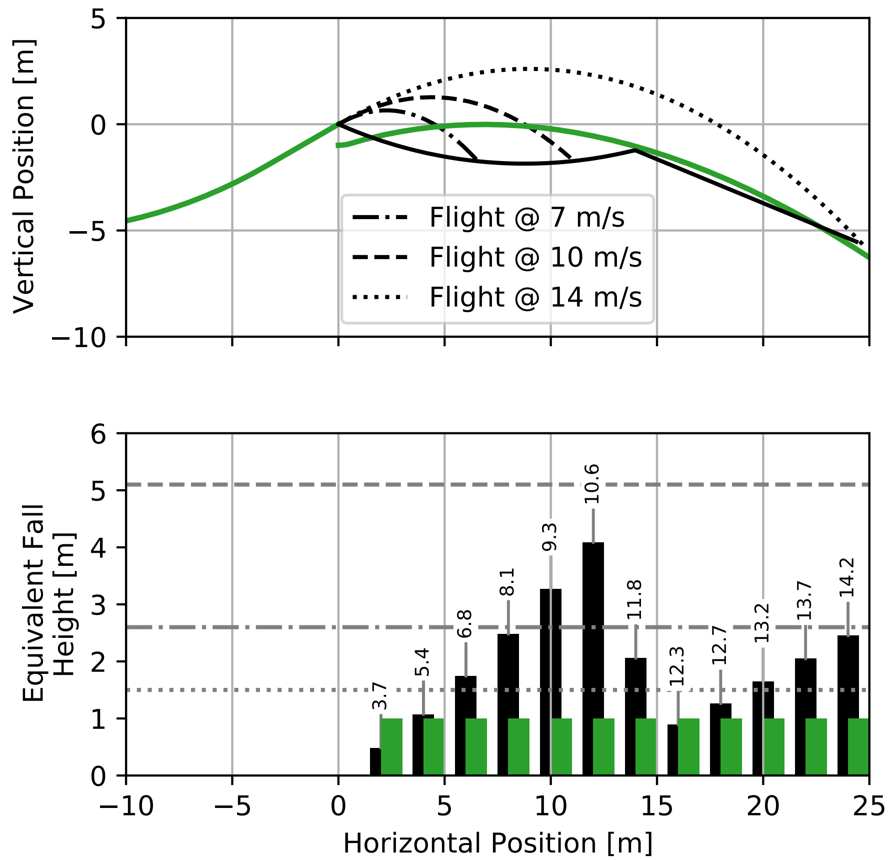

Figure 2: **Bear Valley jump compared to possible safer design** Top: Measured landing surface (solid black) and jumper flight paths (intermittent black) from measured 30° takeoff angle. A 14 $\mathrm{m\,s^{-1}}$ takeoff speed is used as the design speed [3] for a comparison jump (solid green) shaped to have constant EFH of 1 m. Bottom: EFH for both jumps in corresponding colors at 2 m intervals. Numbers above bars indicate takeoff speeds required to land at that location. Intermittent horizontal gray lines indicate increasing relatable fall heights: knee collapse, average 1^{st} story fall, and average 2^{nd} story fall.

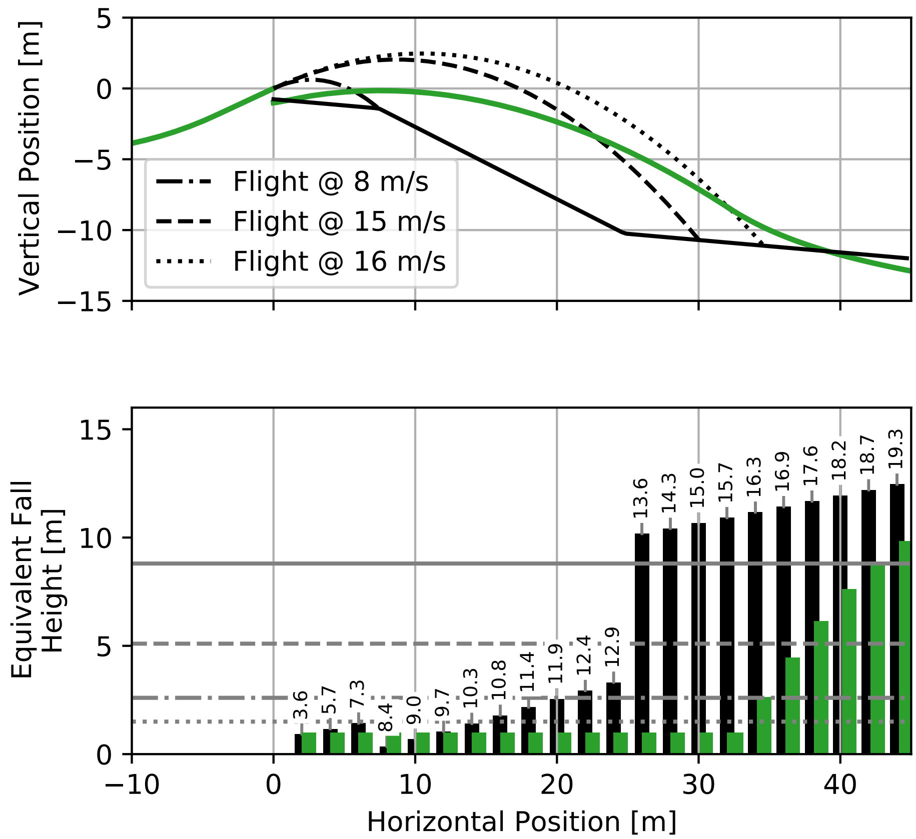

Figure 3: **Snoqualmie jump compared to possible safer design** Top: Measured landing surface (solid black) and jumper flight paths (intermittent black) for measured 25° takeoff angle. The 16 $\mathrm{m\,s^{-1}}$ takeoff speed is used as the design speed for a comparison jump (solid green) with constant EFH of 1 m. Bottom: Equivalent fall height for both jumps in corresponding colors at 2 m intervals. Numbers above bars indicate takeoff speed required to land at that location. Intermittent horizontal gray lines indicate increasing relatable fall heights: knee collapse, average 1$^{\text{st}}$ story fall, average 2$^{\text{nd}}$ story fall, and average 3$^{\text{rd}}$ story fall.

snow than the measured jump but limits EFH to 1 m. This jump highlights how extreme EFHs can become if jumps are not properly designed. Nobody would voluntarily jump out of three story windows, snow or not, as injuries are clearly likely. Our internal altimeter tells us so, but it is impossible for recreational skiers to evaluate EFHs simply by looking at jumps.

These two case studies demonstrate that deficient jump landing shapes have devastating consequences and that engineering analysis and design based on well-established laws of mechanics could be used to design jumps that limit EFHs safely. Designing jumps this way is based on mechanics elucidated centuries ago by Isaac Newton and Émilie du Châtelet [49], and fundamental to physics and engineering education. Designing jumps to limit EFHs unquestionably reduces injury risk by reducing impact energies and associated forces.

4 Discussion

4.1 Moral Imperative

"Hold paramount the safety, health and welfare of the public" [50], is the first canon of engineering ethics. Ethics is not a matter of opinion and should not be optional, but rather is the foundation for engineering. The first canon compels engineers to use their technical expertise to protect snowsport participants from injuries. Reducing EFHs cannot increase likelihoods of injuries. Building well designed, safer jumps is no more laborious than building poorly designed, unsafe jumps. There is no reason not to control EFHs with ethical design algorithms and software. Nonetheless ski industries and their insurance companies are reluctant to adopt and endorse such design methods, choosing instead to invest in litigation defense rather than technologies for constructing safer jumps. They hire engineers to profess doubt on the fundamental physics of EFHs during litigation. Publications cited by the defense in litigation to support these doubts provide little or nothing for the safety, health, and welfare of the public, that engineers should hold paramount.

In their book "Merchants of Doubt" [51], Oreskes and Conway have studied this problem more generally. They show that in numerous industries over the last 60 years, scientific evidence accumulated that commonly accepted industrial activities were harmful, to individuals and society. However, industries had vested interests in continuing practices that were dangerous to the public, perhaps because operational changes would have led to significant, short-term costs and inconvenience. Examples carefully described and analyzed [51] include using DDT, smoking tobacco, producing acid rain from coal-fired power plants, causing ozone holes from CFCs, damaging health with second-hand tobacco smoke, and changing our climate with CO2 emissions. Rather than using the scientific evidence as a basis for changes in practice, strategic responses of industries have been to "emphasize the controversy among scientists and the need for continued research" [51].

This same strategy is used by some snowsport industries and their defense experts, who disparage EFHs. To sow doubt and counter solid, fundamental, ethical, scientific concepts of jump designs limiting EFH, defense experts introduce confounding factors to cloud and confuse basic issues. Consider as evidence three papers [52, 53, 37] co-authored by well-known ski industry defense experts who have testified for snowsport resorts and their insurance companies. We do not fundamentally question their empirical findings but we do question their interpretation of the findings, namely their conclusion that greater fall height is not a basic indicator of greater risk of injury.

Shealy et al. [52] conducted an experimental study attempting to test the hypothesis that takeoff speed is a predictor of the distance from a jump take-off to landing. They reached the mechanically impossible conclusions both that there is "no statistically significant relationship between takeoff speed and the distance traveled" and that "takeoff speed is not a dominant or controlling factor (in how far a jumper travels)" [52]. These conclusions were used to question the soundness of analytical mechanical modeling of jumper flight used in [23, 26].

Some of these same authors later vouched for terrain park jump safety. Using data held by the National Ski Areas Association (NSAA), Shealy et al. [53] concluded that their "hypothesis that jumping features resulted in an increase risk of injury [was] not . . . substantiated." [53] This is the only study we are aware of with this conclusion. It is difficult to reconcile it with the voluminous contradictory research documenting the unique dangers posed by terrain park jumps in tens of other studies cited both herein and in [23, 24, 26, 27, 28, 3, 29, 30]. Although NSAA releases yearly totals of resort-related fatalities and catastrophic injuries, the raw data on which [53] was based is not publicly available, thus making these results unverifiable. The data was collected from press releases produced by the NSAA [53], which has an inherent conflict of interest, thus potentially introducing confounding bias.

In a third experimental study (N=13) specifically designed "to evaluate injury mitigation potential of surfaces limiting EFH" [37], Scher et al. clearly show that body orientation, i.e. falling directly on one's head (in all trials), can cause dangerous cervical spine compression loads [37], even at low fall heights. They report on effects of EFH but only test heights from 0.23 m to 1.52 m, similar to limitations in [52] by restricting ranges of their independent variables, and ignoring fall heights known to have caused severe injuries regardless of body orientation. Yet, they insinuate that EFH has no appreciable effect on injuries. The title, "Terrain Park Jump Design: Would Limiting Equivalent Fall Height Reduce Spinal Injuries?" implies that they appear to believe that falling from greater heights might *not* cause greater injuries. Why propose such mechanically flawed hypotheses? Sowing doubt on EFH as a basic indicator of risk appears to be paramount.

Extending the scope of findings is a common mistake, but one that should not be made by ethical, professional engineers when safety, health, and welfare of the public is at issue. Fundamental laws cannot be disproved by these kinds of jumping experiments. If statistical or experimental results seem in conflict with predictions from classical mechanics, the problems are probably with the statistical or experimental design or their interpretations, but not fundamental laws of mechanics. Defending dangerous practices that lead to injuries helps prolong these practices, which leads to further injuries, clearly contradictory to ethical engineering. Engineering experts defending ski industries and their practices could be complicit in continued societal damage, and in doing harm to the safety, health, and welfare of the public. As Upton Sinclair wrote "it is difficult to get a man to understand something when his salary depends on his not understanding it" [54].

It is not evident that these papers [52, 53, 37] "hold paramount the safety, health and welfare of the public". They are silent on how their findings can be used to reduce injuries. They obscure a scientifically fundamental, mechanically irrefutable fact that impacting surfaces at lower normal velocities is safer. They "create the appearance that the claims being promoted were scientific" [51, page 244]. Fundamental laws have made mechanics a science. Findings that contradict such fundamental laws should be carefully scrutinized and review processes accepting such articles should be questioned.

Organizations also merchandise doubt. A decade ago, NSAA argued [55] that, because of rider and snow variability, terrain park jump "standards are essentially impossible." While it is true that the "virtually ... infinite number of ways that a given feature may be used by an individual ... varying speed, pop, body movement, takeoff stance, angles of approach, the attempting of different kinds of maneuvers, landing stance, and the type of equipment used (skis or snowboard) ... create a wide variety of experiences for the users" [55], none of these in fact preclude EFH analysis and rational engineering design. This was shown clearly in reference [25] which examined quantitatively the effects of variations in factors actually involved in the mechanics: takeoff speed, snow friction, air drag, tail wind, snow melt and jumper pop. These so-called "uncontrollable factors" fell into three groups: (1) those for which there is zero sensitivity, i.e., an uncontrollable factor that makes no difference in the ability of the designed jump to deliver the designed EFH; (2) those for which fairly large parameter variations cause only insignificant maximum deviations in EFH, and (3) those for which the factor can be taken into account in the design process itself and its larger effect on EFH completely eliminated in the unsafe direction. The allegation that design of limited EFH surfaces is prevented by the complexity of the problem and by the large number and types of parameter variations away from nominal is false; in fact the allegation is just more merchandised doubt.

In snowsport injury cases, testifying for injured plaintiffs and testifying defending corporations are not ethically equivalent. The former attempts to address problems that cause injuries, holding paramount the public's safety, health, and welfare. The latter attempts to defend practices that might have contributed to the injury, to limit financial losses of corporations. The idiom "two sides to every question", is not appropriate in science and engineering [51, page 268].

Engineers whose scholarly work ignores engineering's first canon of ethics in favor of merchandising doubt can diminish the scientific integrity of engineering journals and engineering conferences. Journal editors should recognize submissions primarily intending to cast doubt on good science and engineering for what they are, tools of insurance companies for defending civil suits, and reject these submissions. Papers whose findings help to perpetuate dangerous practices for the short-term financial benefit of industry and which apparently do nothing for the safety, health, and welfare of the public, are unethical and do not belong in engineering journals or conference proceedings.

4.2 What Can Be Done?

Absolutely, the most important change will be to incorporate rigorous, rational processes and scientific principles that consider mechanical impact safety into designing freestyle jumps. At present a large fraction of, if not most, jumps in the USA are created in a formulaic way using two straight lines, a horizontal deck (tabletop) and nearly constant-slope landing region, linked by a curved knuckle. This design philosophy is recommended in the instructions provided by the NSAA [56] and is presumably followed by their members. Although such jumps are simple and thus easy to design, research has shown that jumps with bi-linear geometry have generally poor EFH behavior [24], i.e. that they can have low EFH only in a small region just past the knuckle (called the "sweet spot"). In a recent version of their freestyle terrain park notebook [56], the jump landing area is even termed the "landing plane" because it is envisioned to be planar. There is no reference to any concept such as EFH or similar measure of impact or its effect on safety because the NSAA's strategy is to put the responsibility for safety fully on the jumper. There is no quantitative consideration of jump impact safety (e.g. from the point of view of EFH) beyond the experience of the designer. The skiing industry continues to resist more scientifically-based rational approaches to design, in spite of the fact that computer aided design (and even computer-assisted fabrication and maintenance) of snow park jumps (see Figure 4) has been available from snow groomer manufacturers for over 5 years [57]. The 2015 NSAA reference in [56] still contained the statement that "Standards are essentially impossible . . . ".

Once the jump surface has been designed, the next most important change is to build accurately what was designed. Presently a dominant fraction of jumps are simply fabricated by groomer operators, based on perhaps a few measurements of distances and slopes (deck length, takeoff angle, landing region angle and length) during the process. But the design concepts are overly simple and do not incorporate or address quantitative indicators of safety such as EFH. The introduction of computer controlled grooming (see Figure 4), similar to computer aided manufacturing (CAM) and machining (CNC), will facilitate construction of more complex designed shapes precisely and accurately to within ten centimeters. These would include the non-trivial constant EFH surfaces provided by our online ski jump design software that limit landing impulses to acceptable levels.

Every jumper (and parent of young jumpers) should be able to confirm that a jump is not unsafe before trying it. Appropriate inspection, evaluation, correction, and maintenance of existing jumps, and the design and construction of safer new jumps should be promoted. Postings should be required and include

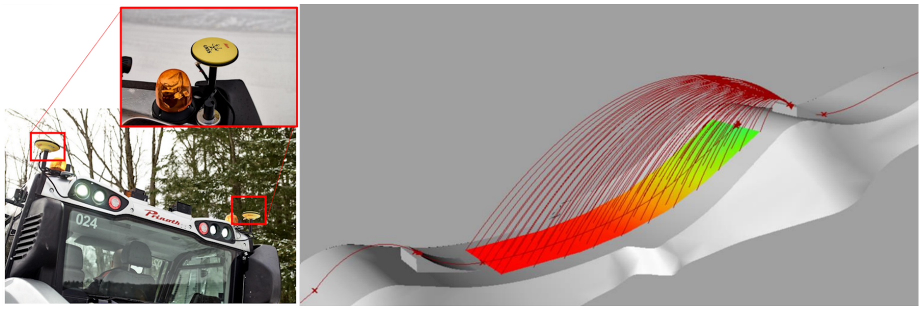

Figure 4: **Commercial availability of computer-aided design and computer-controlled fabrication of snow park surfaces began as early as 2016.** The right panel shows Prinoth's computer generated 3-D jump landing surface with their family of simulated jumper paths, even ones outside the central vertical bisecting plane, with the landing surface colored corresponding to the EFH incurred by the jumper at that landing point. The left panel shows a computer-controlled snow groomer fitted with two GNSS receivers that allow real time measurement of their position to an accuracy of about 2 cm, calculation of the yaw and roll of the groomer blade, and precise closed loop control of the snow addition and removal process. Images courtesy of Prinoth, supplier of snow groomers for the winter Olympics in China 2022.

EFHs, the certified inspectors name, and when last inspected and maintained. Inspections should be frequent enough to ensure that jumps meet safe design standards, particularly regarding takeoffs and starting points to prevent inadvertent inversions due to take off ramp curvature. Standards need to be developed that limit EFHs in collaboration between industry and research engineers to design, build, inspect, maintain, and post safer jumps. An example of first steps in this area is a terrain park safety guide by the Swiss Council for Accident Prevention [36].

To complement standards, certification programs are needed for jump building, inspection, and maintenance. ASTM, an American organization for a wide variety of consensus standards, provides a historical example of a successful certification program. ASTM Committee F27 was created in 1982 for skiing safety and began to develop ski binding standards. Proponents were led by orthopedic surgeons and academic researchers [58]. Industry argued that standards were unattainable because release value measurement was impossible by ski shops, just as industry now makes similar arguments about jumps [56]. Nevertheless certifications and inspection standards for bindings were developed, which led to fewer lower-extremity equipment-related injuries [58].

Now however no medical professionals and almost no academics remain in F27. Efforts to create similar standards for terrain park ski jumps began in F27 more than a decade ago [59], yet no standards have yet resulted with any appearance of increasing safety for the public. The US skiing industry, aided by the NSAA, has been successful in delaying the implementation of standards.

In parallel with standards development, assessing and possibly reshaping existing jumps to eliminate dangerous EFHs should be a straightforward route for ski resorts to proactively increase terrain park safety. Accurate enough measurements of existing surfaces can occur even with simple tools, e.g. tape measure and digital level, and consume relatively little time and effort per jump (see supplementary materials for details). Calculation and visualization of EFHs from these measurements can take some time without a computational program for calculating EFHs from hill profiles. The user-friendly, freely-accessible open-source online web application tool that we have made available for jump designers and builders has almost instantaneous calculation and visualization steps, solving this problem.

With this software, jump builders can add safety assessment to their toolbox, even accessing it from a smartphone or tablet on hills. We see no reason that this basic assessment should not be part of every jump construction process. The only ethical decision is to adopt these methods; saving even one person from a life of paralysis, or even death, must be worth the relatively minor inconvenience of shaping jumps using the methods in reference [3].

5 Conclusion

There are, of course, more factors than jump takeoff and landing profiles that contribute to injuries on terrain park jumps. Yet normal impact velocity can be easily controlled with a properly designed and fabricated jump. There is no evidence that decreasing designed EFH increases injuries in falls; injuries can only decrease. Thus we see no reason not to adopt constant low values of EFH for public-use jump designs. Builders of jumps that are not designed as forgiving environments are negligent. Public safety must be held paramount to short-term return-on-investment.

The methods implemented in the software illustrated in Section 2.2 provide a starting point for realizing EFH-conscious designs in terrain parks. We hope to see the design and analysis adopted by commercial grooming equipment manufacturers so that safety is made integral to jump design. Our software can grow and evolve through contributions from other researchers to incorporate many other nuances of injury prevention. We also see the methods providing a structure for standards development. And minimally, we see the software as an immediately usable tool for jump fabricators in the field.

Acknowledgements

We thank Rado Dukalski for feedback on the web application and Yumiko Henneberry, Lyn Taylor, Andy Ruina, and Ton van den Bogert for feedback on the manuscript.

Declarations

Funding Not applicable

Conflict of interest MH served as a plaintiff's expert witness in the two case studies discussed above and in numerous other similar cases since. CB testifies occasionally on behalf of plaintiffs in ski and snowboard injury cases. He has collaborated with Shealy and C. D. Mote, Jr., Sher's doctoral advisor, on ski safety research, has participated in ASTM F27 since the 1980s on standards for bindings, boots, and skis, and holds patents on ski and snowboard binding designs intended to reduce injuries.

Availability of data and material All data is available at `https://gitlab.com/moorepants/skijumpdesign` and `https://gitlab.com/mechmotum/ski-jump-analysis-paper`.

Code availability The skijumpdesign version 1.4.0 source code is archived at `https://doi.org/10.5281/zenodo.4637076`. Additionally, it and the paper's source code is available at `https://gitlab.com/moorepants/skijumpdesign` and `https://gitlab.com/mechmotum/ski-jump-analysis-paper`.

Author's contributions JM and MH contributed to the study conception and design. Material preparation, data collection and analysis were performed by JM and MH. The first draft of the manuscript was written by JM, BC, MH, and CB. MH and CB were primarily responsible for drafting the parts on merchandising doubt and ethics, respectively. All authors read and approved the final manuscript. BC and JM wrote the accompanying software.

Ethics approval Not applicable

Consent for publication JM, BC, MH, and CB consent for publication.

References

[1] Occupational Safety Health Adminstration, "Safety and health regulations for construction," tech. rep., US Dept. of Labor, Alexandria, Virginia, USA, 2021.

[2] D. J. Chalmers, S. W. Marshall, J. D. Langley, M. J. Evans, C. R. Brunton, A. M. Kelly, and A. F. Pickering, "Height and surfacing as risk factors for injury in falls from playground equipment: A case-control study.," *Injury Prevention*, vol. 2, pp. 98–104, June 1996.

[3] D. Levy, M. Hubbard, J. A. McNeil, and A. Swedberg, "A design rationale for safer terrain park jumps that limit equivalent fall height," *Sports Engineering*, vol. 18, pp. 227–239, Dec. 2015.

[4] L. S. Smith, N. J. Wilkins, and R. J. McClure, "A systemic approach to achieving population-level impact in injury and violence prevention," *Systems Research and Behavioral Science*, vol. 38, no. 1, pp. 21–30, 2020.

[5] M. C. Deibert, D. D. Aronsson, R. J. Johnson, C. F. Ettlinger, and J. E. Shealy, "Skiing Injuries in Children, Adolescents, and Adults*," *The Journal of Bone & Joint Surgery*, vol. 80, no. 1, pp. 25–32, 1998.

[6] M. Furrer, S. Erhart, A. Frutiger, H. Bereiter, A. Leutenegger, and T. Ruedi, "Severe Skiing Injuries: A Retrospective Analysis of 361 Patients Including Mechanism of Trauma, Severity of Injury, and Mortality," *Journal of Trauma and Acute Care Surgery*, vol. 39, pp. 737–741, Oct. 1995.

[7] U.S. Consumer Product Safety Commission, "Skiing Helmets: An Evaluation of the Potential to Reduce Head Injury," tech. rep., Washington, D.C., Jan. 1999.

[8] M. S. Koehle, R. Lloyd-Smith, and J. E. Taunton, "Alpine Ski Injuries and Their Prevention," *Sports Medicine*, vol. 32, pp. 785–793, Oct. 2002.

[9] F. Tarazi, M. F. S. Dvorak, and P. C. Wing, "Spinal Injuries in Skiers and Snowboarders," *The American Journal of Sports Medicine*, vol. 27, pp. 177–180, Mar. 1999.

[10] O. Fukuda, M. Takaba, T. Saito, and S. Endo, "Head Injuries in Snowboarders Compared with Head Injuries in Skiers: A Prospective Analysis of 1076 patients from 1994 to 1999 in Niigata, Japan*," *The American Journal of Sports Medicine*, vol. 29, pp. 437–440, July 2001.

[11] A. B. Jackson, M. Dijkers, M. J. DeVivo, and R. B. Poczatek, "A demographic profile of new traumatic spinal cord injuries: Change and stability over 30 years," *Archives of Physical Medicine and Rehabilitation*, vol. 85, pp. 1740–1748, Nov. 2004.

[12] K. Russell, W. H. Meeuwisse, A. Nettel-Aguirre, C. A. Emery, J. Wishart, N. T. R. Romanow, B. H. Rowe, C. Goulet, and B. E. Hagel, "Feature-specific terrain park-injury rates and risk factors in snowboarders: A case–control study," *British Journal of Sports Medicine*, vol. 48, pp. 23–28, Jan. 2014.

[13] E. J. Bridges, F. Rouah, and K. M. Johnston, "Snowblading injuries in Eastern Canada," *British Journal of Sports Medicine*, vol. 37, pp. 511–515, Dec. 2003.

[14] C. Goulet, D. Hamel, B. Hagel, and G. Légaré, "Risk Factors Associated with Serious Ski Patrol-reported Injuries Sustained by Skiers and Snowboarders in Snow-parks and on Other Slopes," *Canadian Journal of Public Health*, vol. 98, pp. 402–406, Sept. 2007.

[15] C. Moffat, S. McIntosh, J. Bringhurst, K. Danenhauer, N. Gilmore, and C. L. Hopkins, "Terrain Park Injuries," *Western Journal of Emergency Medicine*, vol. 10, pp. 257–262, Nov. 2009.

[16] M. W. Greve, D. J. Young, A. L. Goss, and L. C. Degutis, "Skiing and Snowboarding Head Injuries in 2 Areas of the United States," *Wilderness & Environmental Medicine*, vol. 20, pp. 234–238, Sept. 2009.

[17] M. A. Brooks, M. D. Evans, and F. P. Rivara, "Evaluation of skiing and snowboarding injuries sustained in terrain parks versus traditional slopes," *Injury Prevention*, vol. 16, pp. 119–122, Apr. 2010.

[18] G. Ruedl, M. Kopp, R. Sommersacher, T. Woldrich, and M. Burtscher, "Factors associated with injuries occurred on slope intersections and in snow parks compared to on-slope injuries," *Accident Analysis & Prevention*, vol. 50, pp. 1221–1225, Jan. 2013.

[19] G. Ruedl, W. Schobersberger, E. Pocecco, C. Blank, L. Engebretsen, T. Soligard, K. Steffen, M. Kopp, and M. Burtscher, "Sport injuries and illnesses during the first Winter Youth Olympic Games 2012 in Innsbruck, Austria," *British Journal of Sports Medicine*, vol. 46, pp. 1030–1037, Dec. 2012.

[20] L. Carús and M. Escorihuela, "Feature-specific ski injuries in snow parks," *Accident Analysis & Prevention*, vol. 95, pp. 86–90, Oct. 2016.

[21] O. Audet, A. K. Macpherson, P. Valois, B. E. Hagel, B. Tremblay, and C. Goulet, "Terrain park feature compliance with Québec ski area safety recommendations," *Injury Prevention*, Apr. 2020.

[22] N. Hosaka, K. Arai, H. Otsuka, and H. Kishimoto, "Incidence of recreational snowboarding-related spinal injuries over an 11-year period at a ski resort in Niigata, Japan," *BMJ Open Sport & Exercise Medicine*, vol. 6, May 2020.

[23] M. Hubbard, "Safer Ski Jump Landing Surface Design Limits Normal Impact Velocity," *Journal of ASTM International*, vol. 6, no. 1, 2009.

[24] A. D. Swedberg and M. Hubbard, "Modeling Terrain Park Jumps: Linear Tabletop Geometry May Not Limit Equivalent Fall Height," in *Skiing Trauma and Safety: 19th Volume* (R. J. Johnson, J. E. Shealy, R. M. Greenwald, and I. S. Scher, eds.), pp. 120–135, 100 Barr Harbor Drive, PO Box C700, West Conshohocken, PA 19428-2959: ASTM International, Nov. 2012.

[25] M. Hubbard and A. D. Swedberg, "Design of Terrain Park Jump Landing Surfaces for Constant Equivalent Fall Height Is Robust to "Uncontrollable" Factors," in *Skiing Trauma and Safety: 19th Volume* (R. J. Johnson, J. E.

Shealy, R. M. Greenwald, and I. S. Scher, eds.), pp. 75–94, 100 Barr Harbor Drive, PO Box C700, West Conshohocken, PA 19428-2959: ASTM International, Nov. 2012.

[26] J. A. McNeil, M. Hubbard, and A. D. Swedberg, "Designing tomorrow's snow park jump," *Sports Engineering*, vol. 15, pp. 1–20, Mar. 2012.

[27] J. A. McNeil, "The Inverting Effect of Curvature in Winter Terrain Park Jump Takeoffs," in *Skiing Trauma and Safety: 19th Volume* (R. J. Johnson, J. E. Shealy, R. M. Greenwald, and I. S. Scher, eds.), pp. 136–150, 100 Barr Harbor Drive, PO Box C700, West Conshohocken, PA 19428-2959: ASTM International, Nov. 2012.

[28] M. Hubbard, J. A. McNeil, N. Petrone, and M. Cognolato, "Impact Performance of Standard Tabletop and Constant Equivalent Fall Height Snow Park Jumps," in *Skiing Trauma and Safety: 20th Volume* (R. J. Johnson, ed.), (West Conshohocken, PA), pp. 51–71, ASTM International, Feb. 2015.

[29] N. Petrone, M. Cognolato, J. A. McNeil, and M. Hubbard, "Designing, building, measuring, and testing a constant equivalent fall height terrain park jump," *Sports Engineering*, vol. 20, pp. 283–292, Dec. 2017.

[30] J. K. Moore and M. Hubbard, "Skijumpdesign: A Ski Jump Design Tool for Specified Equivalent Fall Height," *The Journal of Open Source Software*, vol. 3, p. 818, Aug. 2018.

[31] W. Müller, D. Platzer, and B. Schmölzer, "Scientific approach to ski safety," *Nature*, vol. 375, no. 455, 1995.

[32] H.-H. Gasser, "Jumping Hills, Construction Norm 2018, Implementing Provisions for Art. 411 of the ICR Ski Jumping," tech. rep., International Ski Federation, Nov. 2018.

[33] C. Nau, M. Leiblein, R. D. Verboket, J. A. Hörauf, R. Sturm, I. Marzi, and P. Störmann, "Falls from Great Heights: Risk to Sustain Severe Thoracic and Pelvic Injuries Increases with Height of the Fall," *Journal of Clinical Medicine*, vol. 10, p. 2307, Jan. 2021.

[34] N. L. Vish, "Pediatric window falls: Not just a problem for children in high rises," *Injury Prevention*, vol. 11, pp. 300–303, Oct. 2005.

[35] Polytrauma Guideline Update Group, "Level 3 guideline on the treatment of patients with severe/multiple injuries : AWMF Register-Nr. 012/019," *European Journal of Trauma and Emergency Surgery: Official Publication of the European Trauma Society*, vol. 44, pp. 3–271, Apr. 2018.

[36] B. Heer, F. Bürgi, and M. Weiler, "Terrain Parks," Tech. Rep. 2.081, Swiss Council for Accident Prevention, Bern, Switzerland, 2019. 10.13100/BFU.2.081.08.2019.

[37] I. Scher, J. Shealy, L. Stepan, R. Thomas, and R. Hoover, "Terrain Park Jump Design: Would Limiting Equivalent Fall Height Reduce Spine Injuries?," in *Skiing Trauma and Safety: 20th Volume* (R. J. Johnson, J. E. Shealy, and R. M. Greenwald, eds.), pp. 72–90, 100 Barr Harbor Drive, PO Box C700, West Conshohocken, PA 19428-2959: ASTM International, Feb. 2015.

[38] S. Behnel, R. Bradshaw, C. Citro, L. Dalcin, D. S. Seljebotn, and K. Smith, "Cython: The best of both worlds," *Computing in Science & Engineering*, vol. 13, no. 2, pp. 31–39, 2011.

[39] J. D. Hunter, "Matplotlib: A 2D graphics environment," *Computing in Science & Engineering*, vol. 9, no. 3, pp. 90–95, 2007.

[40] T. E. Oliphant, *A Guide to NumPy*, vol. 1. Trelgol Publishing USA, 2006.

[41] W. McKinney, "Pandas." Zenodo, Feb. 2020.

[42] Plotly Technologies Inc., "Plotly: Collaborative data science," 2015.

[43] B. Dahlgren, "Pyodesys: Straightforward numerical integration of ODE systems from Python," *Journal of Open Source Software*, vol. 3, p. 490, Jan. 2018.

[44] P. Virtanen, R. Gommers, T. E. Oliphant, M. Haberland, T. Reddy, D. Cournapeau, E. Burovski, P. Peterson, W. Weckesser, J. Bright, *et al.*, "SciPy 1.0: Fundamental algorithms for scientific computing in Python," *Nature methods*, vol. 17, no. 3, pp. 261–272, 2020.

[45] A. Meurer, C. P. Smith, M. Paprocki, O. Čertík, S. B. Kirpichev, M. Rocklin, A. Kumar, S. Ivanov, J. K. Moore, S. Singh, T. Rathnayake, S. Vig, B. E. Granger, R. P. Muller, F. Bonazzi, H. Gupta, S. Vats, F. Johansson, F. Pedregosa, M. J. Curry, A. R. Terrel, Š. Roučka, A. Saboo, I. Fernando, S. Kulal, R. Cimrman, and A. Scopatz, "SymPy: Symbolic computing in Python," *PeerJ Computer Science*, vol. 3, Jan. 2017.

[46] Superior Court San Francisco County, "Charlene Vine v. Bear Valley Ski Company," Dec. 2002. San Francisco, No. 317766.

[47] King County Superior Court, "Kenneth Salvini v. Ski Lifts, Inc.," Oct. 2008. Seattle, No. 60211-0-I.

[48] J. Shealy, C. Ettlinger, and R. Johnson, "How Fast Do Winter Sports Participants Travel on Alpine Slopes?," *Journal of ASTM International*, vol. 2, no. 7, p. 12092, 2005.

[49] J. P. Zinsser, *Emilie Du Chatelet: Daring Genius of the Enlightenment.* Penguin, Nov. 2007.

[50] NSPE, "Code of Ethics for Engineers," tech. rep., National Society of Professional Engineers, Alexandria, Virginia, USA, July 2019.

[51] N. Oreskes and E. M. Conway, *Merchants of Doubt: How a Handful of Scientists Obscured the Truth on Issues from Tobacco Smoke to Global Warming.* Bloomsbury Press, 2010.

[52] J. Shealy, I. Scher, L. Stepan, and E. Harley, "Jumper Kinematics on Terrain Park Jumps: Relationship between Takeoff Speed and Distance Traveled," *Journal of ASTM International*, vol. 7, p. 10, Nov. 2010.

[53] J. E. Shealy, I. Scher, R. J. Johnson, and J. A. Rice, "Jumping Features at Ski Resorts: Good Risk Management?," in *Skiing Trauma and Safety: 20th Volume* (R. J. Johnson, J. E. Shealy, and R. M. Greenwald, eds.), pp. 39–50, 100 Barr Harbor Drive, PO Box C700, West Conshohocken, PA 19428-2959: ASTM International, Jan. 2015.

[54] U. Sinclair, *I, Candidate for Governor: And How I Got Licked.* University of California Press, Berkeley, 1994.

[55] NSAA, "Freestyle Terrain Park Notebook," tech. rep., National Ski Areas Association, Lakewood, Colorado, USA, 2008.

[56] NSAA, "Freestyle Terrain Park Notebook," tech. rep., National Ski Areas Association, Lakewood, Colorado, USA, 2015.

[57] A. Muigg, "6th FIS Clinic 2019 hosted by Prinoth: Competitions Courses, Terrain Park and Fun Slopes," Sept. 2019.

[58] E. Bahniuk, "Twenty Years' Development of ASTM Skiing Standards," in *Skiing Trauma and Safety: Tenth Volume*, (West Conshohocken, PA), pp. 15–22, ASTM International, 1996.

[59] SAM, "ASTM F27 Committee Could Broaden Scope to Include Park Jumps Soon," *SAM: Ski Area Management, The Voice of the Mountain Resort Industry*, Oct. 2011.

Supplementary Materials for Online Software and Ethical Issues for Safety-Conscious Design of Terrain Park Jumps

Jason K. Moore Bryn Cloud Mont Hubbard
Christopher A. Brown

October 5, 2021

1 Example Software Library Use

The closed form equation

$$h = \left[\frac{x^2}{4(x\tan\theta_T - y)\cos^2\theta_T} - y\right]\sin^2\left[\tan^{-1}\left(\frac{2y}{x} - \tan\theta_T\right) - \tan^{-1}\frac{dy}{dx}\right] \tag{1}$$

is useful for understanding the fundamental relationship of equivalent fall height (EFH) to the landing surface shape. It will predict EFH for small jumps but other factors may be useful to include in the model. For example, jumpers are subject to aerodynamic drag and this is not negligible for larger jumps. If drag is included there is no closed form solution for the EFH, but the EFH can be computed through iterative simulation [1]. The jumper's flight path is found by integrating the flight equations of motion at various takeoff velocities and computing the misalignment of jumper landing and slope angles to then compute the EFH. This more general simulation method is implemented in the software described herein and the results reflect the inclusion of both gravitational and drag forces. Even with drag incorporated, the calculating EFH still only require measurements of the landing surface cross-sectional profile coordinates (x, y) relative to the takeoff point and a measurement of the takeoff angle. Listing 1 demonstrates the new software library features creating a surface from some measured data points and then calculating the EFH at 0.2m increments.

2 Jump Shape Measurement

Calculating EFH requires the Cartesian coordinates and slope of the landing surface along the path of the jumper. There are a number of possible measurement techniques for collecting data adequate for the EFH calculation but the

```
>>> import numpy as np
>>> from skijumpdesign import Surface, Skier, plot_efh
>>> takeoff_ang = 10  # degrees
>>> takeoff_point = (0, 0)  # (x, y) in meters
>>> x_ft = np.array([-232.3,-203.7,-175.0,-146.3,-117.0,-107.4,
...     -97.7,-88.0,-78.2,-68.5,-58.8,-49.1,-39.4,-34.5,-29.7,
...     ...
...     38.8,43.3,47.8,52.3,56.8,61.5,66.2,70.9,75.7,80.6,85.5,
...     88.4,88.4])
...
>>> y_ft = np.array([55.5,46.4,37.7,29.1,22.2,19.7,17.2,14.8,
...     12.5,10.2,7.7,5.2,2.9,1.8,0.7,-0.2,-1.0,-1.2,-1.4,-1.6,
...     ...
...     -16.2,-18.1,-19.8,-21.4,-22.9,-24.0,-25.0,-25.6,-25.6])
...
>>> x_mt = x_ft*0.3048 # convert to meters
>>> y_mt = y_ft*0.3048 # convert to meters
>>> # create a surface from the data
>>> measured_surf = Surface(x_mt, y_mt)
>>> # create a skier
>>> skier = Skier(mass=75.0, area=0.34, drag_coeff=0.821)
>>> # calculate the EFH
>>> x, efh, v = measured_surf.calculate_efh(
...      np.deg2rad(takeoff_ang), takeoff_point, skier, increment=0.2)
...
>>> x  # display the x coordinates
array([ 0. ,  0.2,  0.4,  0.6,  0.8,  1. ,  1.2,  1.4,  1.6,  1.8,  2. ,
        2.2,  2.4,  2.6,  2.8,  3. ,  3.2,  3.4,  3.6,  3.8,  4. ,  4.2,
       ...
       24.2, 24.4, 24.6, 24.8, 25. , 25.2, 25.4, 25.6, 25.8, 26. , 26.2,
       26.4, 26.6, 26.8])
>>> efh  # display the EFH for each x coordinate
array([0.        , 0.02541035, 0.03479384, 0.03264587, 0.05956476,
       0.09096091, 0.12358184, 0.13702364, 0.15202999, 0.17018343,
       ...
       3.93910556, 3.97387212, 4.00891899, 4.04424779, 4.07984952,
       4.11573359, 4.68049185, 5.53413479, 6.45253722, 7.42628019])

>>> v  # display takeoff speeds to reach x positions
array([0.07373847, 0.13081777, 0.1878382 , 0.2447865 , 0.30166299,
       0.35851949, 0.41537661, 0.47221055, 0.52897197, 0.58564902,
       ...
       6.71699974, 6.76760188, 6.81816819, 6.86869777, 6.9191902 ,
       6.96962124, 7.02001551, 7.07037288, 7.1206941 ])
>>> # calculate and plot the efh curve
>>> plot_efh(measured_surf, takeoff_ang, takeoff_point, increment=0.2)
```

Listing 1: Python interpreter session illustrating how one could compute the EFH of a measured jump.

simplest method requires only a digital level [1], a flexible tape measure, and less than an hour's time from one person per jump. A tenth of a degree accuracy from the level and down to 25 cm accuracy from the tape measure should be more than sufficient for typical snowsport jumps.

To measure the jump, the takeoff point should be identified and the tape measure should then be draped over the contour of the landing surface along the projection of the expected flight path onto the landing surface. The origin of the tape measure should be aligned with the takeoff point. Starting with the takeoff point, the digital level should be used to record the absolute angle at regular increments along the tape. The increment can be varied between 25 cm and 100 cm, with the former used for steep slope changes and the later for less steep; 50 cm increments are appropriate for average jump shapes. Positive angles should be recorded for positive slope and negative angles for negative slope. The tabulated data should include the distance along the surface from the takeoff point, d_i, and the associated surface angle, θ_i, at each distance measurement for n measurements. Assuming θ_i is in radians, the Cartesian coordinates can be computed using the average angle to find the adjacent coordinates. The following equations show the calculation of the Cartesian coordinates from these two measures used in the software.

$$\frac{dy_i}{dx_i} = \tan^{-1}\theta_i \quad \text{for } i = 1 \ldots n \tag{2}$$

$$x_{i+1} = \begin{cases} 0 & \text{for } i = 0 \\ x_i + (d_{i+1} - d_i)\cos\frac{\theta_{i+1}+\theta_i}{2} & \text{for } i = 1 \ldots n-1 \end{cases} \tag{3}$$

$$y_{i+1} = \begin{cases} 0 & \text{for } i = 0 \\ y_i + (d_{i+1} - d_i)\sin\frac{\theta_{i+1}+\theta_i}{2} & \text{for } i = 1 \ldots n-1 \end{cases} \tag{4}$$

Listing 2 demonstrates calculating the landing surface's Cartesian coordinates from measured distance and angle data collected with the method described above.

References

[1] D. Levy, M. Hubbard, J. A. McNeil, and A. Swedberg, "A design rationale for safer terrain park jumps that limit equivalent fall height," *Sports Engineering*, vol. 18, pp. 227–239, Dec. 2015.

[1] Smartphone digital level measurement applications are likely sufficient and readily available.

```
>>> import numpy as np
>>> from skijumpdesign import cartesian_from_measurements
>>> dis = np.array([14.5, 15.0, 15.5, 16.0, 16.5, 17.0])  # meters
>>> ang = np.deg2rad([4.6, -7.4, -16.5, -9.7, -11, -6.9])  # radians
>>> x, y, to_point, to_angle = cartesian_from_measurements(dis, ang)
>>> print(x)  # meters
[0.         0.49985074 0.98901508 1.47600306 1.96786738 2.46177962]
>>> print(y)  # meters
[ 0.         -0.01221609 -0.1157451  -0.22907075 -0.31890113 -0.39668737]
>>> print(to_point)  # takeoff point in meters
(0.0, 0.0)
>>> print(to_angle)  # takeoff angle in radians
0.08028514559173916
```

Listing 2: Python interpreter session showing how one could compute the Cartesian coordinates from EFH of a measured jump.