



# Proposing a Fuzzy Supply Chain Network Model for an Integrated Distribution System

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## Abstract

Fuzzy Supply Chain Network (FSCN) is one of the modern attitudes that lead to reduce system costs, losses in shipping products, and to reduce inventory levels. The queuing model used in this study obeys from M/G/1 queue that the arrival rate has a Poisson probability distribution, and the service rate has the general probability distribution. In this study, the fuzzy supply chain is analysed in three levels. The suppliers are placed on the first level, the distribution centres are at the second level and the local warehouse (consumer) is on the third level. The objective function of the presented model is based on profit maximization which the resulted benefit from products sale obtained from the difference between total revenues and total costs. The presented model in the fuzzy set is converted to the deterministic mathematical model. The appropriate results can be obtained from the analysis of the presented model. The model is solved by using Maple 12 software. Finally, this approach leads to the appropriate management of the market, increasing customer satisfaction, and ultimately, increasing the efficiency and effectiveness of the activities in the supply chain network.

*Keywords:* fuzzy supply chain network, queuing theory, mathematical modeling, optimization

## 1. Introduction

Today, according to complexity of decision making at the mass level, traditional management cannot have necessary efficiency and effectiveness. Therefore, supply chain management [1] can be considered as a new management approach. One of the popular methods in supply chain management belong to location allocation models. Initially, the location allocation models were presented by [2] since extensive research have been done in this area. Industries progress in the specialized field has led to confront with complex dimensions issues, uncertain in the real situation which the specific policies must be obtained in this regard. Therefore, supply chain has been modelled by fuzzy set theory which is presented for the first time by La Zadeh [3].

Supply Chain Management is comprised of three main components that one of these components is logistics management or inventory and material flow management in the supply chain. Logistics has different levels that each of the components plays an important role in the whole system. [4] [5] presented the mathematical modelling of sustainable closed-loop supply chain in uncertainty situation. They have solved the model by genetic algorithm. [6] studied the designing of service delivery systems in the communication network which minimize the cost of installation, operation and expectation and used from innovative algorithms and Lagrangian methods in solution. [7] used the fuzzy set theory to present the mathematical models to design a three-level supply chain. [8] studied the numbers of optimized service providers in any facilitation upon the queue that it has the arrival rates of the exponential distribution function and the distribution service function that has the Erlang distribution and the limited population can entry the queue. [9] considered the multiple targets in a mathematical model for locating distribution centres and allocation of customer demand in problems of supply chain network design. [10] studied a combinate algorithm to develop the problems of distribution network design. [11] studied the mathematical models in distributed systems models by using the location allocation models that it is the only part of the performed research in this area. See these references for more publications [4] [12] [13] [14] [5].

In this study, the various aspects of fuzzy mathematical models in production and distribution systems at the three-level supply chain have been studied. The structure of the objective function is based on maximizing the resulted profits of total income and the cost of transportation, construction of distribution centres, maintenance costs, shortages costs, and manpower costs. The desired results can be obtained from the model are very necessary and useful which include:

- allocate the suppliers to the distribution centers and in the case of allocation, the optimal amount of demand that must be estimated in distribution centers and similarly,
- allocate the distribution centers to the local warehouses (consumer) and in the case of allocation, the optimal amount of demand that need estimated for local storage (consumer).

In the second part, the service delivery model and available relations in queuing theory are presented. In the third part, the optimization model of supply chain is presented. In the fourth section the presented mathematical model of the fuzzy mathematical model is converted to a deterministic mathematical model. Finally, in the fifth part, the proposed model has been solved.

## 2. Formulating service delivery model

Queuing theory somehow are studied the queuing behavior in economic systems. The most obvious service systems can be indicated to the queue systems which in this system the customers get out of the system after entering the system and receiving the services. The criteria that stated in this area are queue length, the waiting time for service, productivity rates in the queue system, the average number of people in the queue and system, and the average waiting time in the queue and the system [10] [15].

### 2.1. Assumptions and notation

$\lambda$ : Shipment arrival rate in a group  
 $\mu$ : the service rates at each center (distribution centers)  
 $\rho$ : coefficient of system efficiency  
 $L_q$ : The average number of shipments in the queue  
 $L$ : The average number of shipments in the system  
 $W_q$ : The average waiting time of shipment in queue  
 $W$ : the average waiting time of shipment in system  
 $E(N)$ : the average number of shipments  
 $E(S)$ : average time for service  
 $N$ : shipment volume

$$\rho = \lambda \cdot E(N) / E(S) = \lambda \cdot E(N) / \mu \quad (1)$$

$$W_q = L_q / \lambda \quad (2)$$

$$W = W_q + 1 / \mu \quad (3)$$

$$L_q = (\rho^2 + [\lambda \cdot E(N)]^2 \text{Var}(S) + \rho[(E(N^2)) / (E(N)) - 1]) / (2(1 - \rho)) \quad (4)$$

$$L = L_q + \rho \quad (5)$$

$$E(S) = 1 / \mu \quad (6)$$

## 3. Optimizations Model

In this section we proposed the mathematical model of the paper. Before that, we need to propose the assumptions, indices, parameters, and variables of the optimization model:

### 3.1. Assumption

- The cost of employing labor (service provider) in various period in distribution centers and local warehouses are the same.
- The demand in distribution centers which has limit is absolute (upper limit).
- Demand at local storage is constant and deterministic.
- The capacity of distribution centers and local warehouses is a phasic amount.

### 3.2. Index

$l$ : Manufacturers Index ( $l=1, 2, \dots, L$ )  
 $j$ : Index of distribution centers ( $j=1, 2, \dots, J$ )  
 $n$ : The local stock indices(consumer) and ( $n=1, 2, \dots, N$ )  
 $p$ : Shipment index ( $p=1, 2, \dots, P$ )  
 $t$ : Time Periods ( $t=1, 2, \dots, T$ )

### 3.3. Parameters

- $K_{pl}$ : purchase price of products type  $p$  in supplier  $l$   
 $H_{pjt}$ : holding cost of products type  $p$  in distribution center  $j$  in period  $t$   
 $H_{pnt}$ : holding cost of products type  $p$  in warehouse  $n$  in period  $t$   
 $\pi_{pnt}$ : Shortage cost of products type  $p$  in warehouse  $n$  in period  $t$   
 $g_j$ : Cost of using the manpower in distribution center  $j$  for moving shipment and service delivery (according to M/G/1queue model)  
 $g_n$ : Cost of employing manpower in local warehouse  $n$  for moving shipment and service delivery (according to M/G/1queue model)  
 $\Delta_j$ : Capacity of servicing in Distribution Center  $j$  (fuzzy numbers)  
 $\Delta_n$ : capacity of servicing in warehouse  $n$  (fuzzy numbers)  
 $LT_{plj}$ : Arrival time of products type  $p$  from supplier  $l$  to distribution center  $j$   
 $LT_{pjn}$ : Arrival time of products type  $p$  from distribution center  $j$  to warehouse  $n$   
 $D_{pj}$ : The demand of products type  $p$  at distribution center  $j$   
 $D_{pn}$ : The demand of products type  $p$  at warehouse  $n$   
 $T_{plj}$ : Transportation cost of per unit of product type  $p$  from manufacturers  $l$  to distribution centers  $j$  at period  $t$ .  
 $T_{pjnt}$ : Transportation cost of per unit of product type  $p$  from distribution centers  $j$  to local warehouse of consumer  $n$  at period  $t$   
 $C_j$ : Fixed cost of distribution center  $j$   
 $M_j$ : Number of employed service providers at distribution center  $j$   
 $O_n$ : Number of employed service providers at warehouse  $n$   
 $\alpha$ : waste percentage in first stage (From suppliers to distribution centers)  
 $\gamma$ : waste percentage of second stage (from distribution centers to warehouses of factories)  
 $R_{pnt}$ : sale price of product type  $p$  at warehouse  $n$  at period  $t$   
 $D$ : Fixed number of distribution centers

### 3.4. Variables

- $S_{lj}$ : if supplier  $l$  send shipment to the distribution center  $j$ . (0-1)  
 $F_{jn}$ : if distribution center  $j$  send shipment to warehouse  $n$ . (0-1)  
 $W_j$ : if distribution center  $j$  to be constructed. (0-1)  
 $Z_{plj}$ : Number of product type  $p$  shipments from supplier  $l$  to distribution center  $j$   
 $X_{pjn}$ : Number of sent product type  $p$  shipments from distribution center  $j$  to warehouse  $n$   
 $Q_{pj}$ : Number of receiving product type  $p$  shipments in distribution center  $j$   
 $B_{pn}$ : Number of receiving product type  $p$  shipments in warehouse  $n$

### 3.5. Fuzzy mathematical modeling

$$\begin{aligned}
 \text{Maxprofit} = & \sum_p \sum_n \sum_t R_{pnt} \cdot B_{pn} - \sum_p \sum_t \sum_j H_{pjt} Q_{pj} - \sum_p \sum_t \sum_n (H_{pnt} + \pi_{pnt}) B_{pn} \\
 & - \sum_j C_j W_j - \sum_j M_j g_j W_j - \sum_j \sum_l M_j g_j S_{lj} - \sum_j \sum_n g_n O_n F_{jn} - \sum_p \sum_l \sum_j \sum_t (K_{pl} + LT_{plj} \cdot T_{pljt}) \\
 & - \sum_p \sum_j \sum_n \sum_t (LT_{pjn} \cdot T_{pjnt}) X_{pjn}
 \end{aligned} \tag{7}$$

Subject to:

$$W_j \geq S_{lj} \tag{8}$$

$$W_j \geq F_{jn} \tag{9}$$

$$(1 - \alpha) \sum_p \sum_l Z_{plj} = \sum_p Q_{pj}, \forall j \tag{10}$$

$$(1 - \gamma) \sum_p \sum_j X_{pjn} = \sum_p B_{pn}, \forall n \tag{11}$$

$$\sum_p Q_{pj} \leq \tilde{\Delta}_j, \forall j \tag{12}$$

$$\sum_p B_{pn} \leq \tilde{\Delta}_n, \forall n \tag{13}$$

$$\sum_j W_j = D \tag{14}$$

$$\sum_p \sum_l Z_{plj} \geq \sum_p D_{pj}, \forall j \tag{15}$$

$$\sum_p \sum_j X_{pjn} \geq \sum_p D_{pn}, \forall n \tag{16}$$

$$S_{lj}, F_{jn}, W_j \in \{0, 1\}, \forall l, j, n \tag{17}$$

$$X_{pjn}, Z_{plj}, Q_{pj}, B_{pn} \geq 0, \forall p, j, n, l \quad (18)$$

The first part of the objective function describes the sales results of shipments. The second part of the objective function presents the maintenance cost of per shipment in distribution center. The third part provides the cost of maintenance and shortage of each shipment in distribution centers and local warehouses. The fourth and sixth and seventh provide the cost of service providers in local Warehouses, distribution centers. Part five provides the construction cost of distribution centers. Part eight demonstrates the purchase and transportation costs from suppliers to distribution centers. Ninth Part presents transportation costs from the distribution centers to local warehouse. Sixth and seventh limitations check the construction and non-construction of distribution centers. The Eight limitation emphasizes that the total sent unharmed shipments from suppliers to distribution centers must be equal to the number of receiving shipments. Ninth limitation highlights total sent unharmed shipments from distribution centers to warehouses of factories must be equal to the number of receiving shipments. Tenth and Eleventh restrictions are relating to capacity of each node in distribution centers and warehouses of factory. Twelve restriction says that only a limited number of distribution centers should be established. Limitations of the thirteenth and fourteenth are related to customer demand that their requirements should be answered in two and three level. Limitations of the fifteenth and sixteenth discuss about variables of the model.

#### 4. Converting to deterministic mathematical modeling

The presented model in Section 3.5, are fuzzy models that need to be converted to the deterministic model. So, we have:

$$\begin{aligned} \text{Max} &= \sum_j C_j \cdot X_j \\ \sum_i A_{ij} \cdot X_j &\leq \tilde{B}_i \\ X_j &\geq 0 \end{aligned}$$

According to the fuzzy model number, the final model is as follows [13] [14].

$$\begin{aligned} \text{Max} Z &= \beta \\ \beta(Z_U - Z_L) - Z &\leq -Z_L, \\ \beta \cdot P_i + \sum_j A_{ij} \cdot X_j &\leq B_i + P_i \\ \beta &\geq 0, X_j \geq 0 \end{aligned}$$

Therefore, fuzzy mathematical model of the main problem converted to a deterministic mathematical model in this form (upper limit):

$$\begin{aligned} \text{Max} Z_u &= V \\ \text{S. t. } W_j &\geq S_{lj} \\ W_j &\geq F_{jn} \\ (1 - \alpha) \sum_p \sum_l Z_{plj} &= \sum_p Q_{pj} \\ (1 - \gamma) \sum_p \sum_j X_{pjn} &= \sum_p B_{pn}, \\ \sum_j Q_{pj} &\leq \Delta_j, \\ \sum_p \sum_l Z_{plj} &\geq \sum_j D_{pj}, \\ X_{pjn}, Z_{plj}, Q_{pj}, B_{pn} &\geq 0, \\ \sum_p \sum_l S_{lj} \cdot Z_{plj} &\leq \Delta_j + R_j \\ \sum_p \sum_l F_{jn} \cdot X_{pjn} &\leq \Delta_n + R_n \\ \sum_p Q_{pj} &\leq \Delta_j + R_j \\ \sum_p B_{pn} &\leq \Delta_n + R_n \end{aligned}$$

Then  $Z_L$  (Lower limit of the objective function) can be calculated [13] [14].

$$\begin{aligned}
 & \text{Min} Z_l = V \\
 & \text{S. t. } W_j \geq S_{lj}, \\
 & W_j \geq F_{jn}, \\
 & (1 - \alpha) \sum_p \sum_l Z_{plj} = \sum_p Q_{pj} \\
 & (1 - \gamma) \sum_p \sum_j X_{pjn} = \sum_p B_{pn}, \\
 & \sum_j Q_{pj} \leq \tilde{\Delta}_j, \\
 & \sum_p \sum_l Z_{plj} \geq \sum_j D_{pj}, \\
 & \sum_p \sum_l S_{lj} \cdot Z_{plj} \leq \Delta_j, \\
 & \sum_p \sum_l F_{jn} \cdot X_{pjn} \leq \Delta_n \\
 & \sum_p Q_{pj} \leq \Delta_j, \\
 & \sum_p B_{pn} \leq \Delta_n, \\
 & X_{pjn}, Z_{plj}, Q_{pj}, B_{pn} \geq 0,
 \end{aligned}$$

Finally, it is replaced in the model number 18 and the phasic mathematical model converts to a deterministic mathematical programming model.

### 5. Solution

After the defuzzification of the optimized model, we need to solve it in a deterministic space. The presented model is analyzed by Maple 12 software. The output of suppliers designated model to the distribution centers and similarly, the allocation of distribution centers to the retailers which are located on the third level. The model problem examined as a case study which in this study considered the diversity shipments in three types and for each one of suppliers, distributors and retailer warehouse considered a suggested location that its results are described in below.

$$W_1=W_2=1, W_3=0$$

$B_{pn}$	n=1	n=2	n=3
p=1	0	3000	0
p=2	300	4999	0

$Q_{pj}$	j=1	j=2	j=3
p=1	0	0	0
p=2	599	1199	0

For P=1, we have:

$Z_{plj}$	j=1	j=2	j=3
L=1	3000	0	0
L=2	0	0	0

For P=2, we have:

$Z_{plj}$	j=1	j=2	j=3
L=1	0	0	0
L=2	0	0	0
L=3	0	0	0

For P=1, we have:

$X_{pjn}$	n=1	n=2	n=3
j=1	3000	0	0
j=2	0	0	0
j=3	0	4000	0

For P=2, we have:

$X_{pjn}$	n=1	n=2	n=3
j=1	0	0	0
j=2	0	0	0
j=3	0	0	0

$S_{lj}$	j=1	j=2	j=3
L=1	1	0	0
L=2	1	0	0
L=3	0	1	0

$F_{jn}$	n=1	n=2	n=3
j=1	1	0	0
j=2	1	0	0
j=3	0	1	0

## 6. Conclusions

This study explained the overall structure of the supply chain network in fuzzy set that considered different costs in real world. The structure of objective function has been considered to maximize the difference between total revenue and available different expenditure. The output of model is the optimal volume of sending and receiving shipment from suppliers to distribution centers (liaison between suppliers and retailers) as well as the optimal size of the sending and receiving of Shipment from distribution centers to the retailers that used to queuing model M/G/1. To apply and analyze this issue can have a rightful effects on being economical and commercial of distribution systems leads to proper management of market, to reduce the system costs, to increase the customers' satisfaction, to prevent damage, corruption and performance and to increase the efficiency effectiveness of the distribution systems in general level. In the future research, the multicriteria decision-making method can be used to prioritize the distribution centers according to various criteria.

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