

Cloaking using Spacetime Curvature Induced by Perturbation

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Abstract

We present here a novel technique of cloaking. The related cloak is based on spacetime curvature induced by an accelerated perturbation, without any transfer of matter. First, we point out an accelerated moving perturbation modulation is an alternative to mass and moving matter for spacetime curving. Next, we outline the theory of this cloak which, following the general theory of relativity, consists in mapping the wave cloaking trajectories to a spacetime-modulation metric so as to determine the required spacetime refractive index profile. Then, we describe a practical implementation of the cloak, consisting in an SiO₂ slab modulated by a circular wave pump. Compared to previously reported cloaks, the proposed cloak is immune of the issues of narrow bandwidth, single polarization, restricted cloaking size, and complex or impossible implementation. Given its unique features, this cloak should soon find its way to real-world applications. This work represents, to the best of our knowledge, the first report of spacetime curving based on perturbation modulation, without any motion of matter.

1 Introduction

Electromagnetic cloaking is a technique that makes objects invisible by judiciously deviating the trajectory of light around them. While such light deviation is a simple effect of refractive index gradient in natural phenomena, such as mirages, and optical components, such as Luneburg or Eaton lenses, its realization for an operation as demanding as cloaking is not trivial. Three main approaches have been reported so far to achieve such a feat: transformation optics [Schurig et al. \(2006\)](#), scattering cancellation [Alù and Engheta \(2007\)](#), and matter acceleration [Leonhardt and Philbin \(2010\)](#).

Despite their spectacular success in the metamaterial community, these approaches are hindered by serious drawbacks that have been stalling their application to real-world problems. Transformation optics suffers from high complexity, narrow bandwidth and single polarization. Scattering cancellation is in principle simpler to realize, but it is restricted to electrically small objects, while also suffering from narrow bandwidth. Matter acceleration overcomes the issue of narrow bandwidth but would require a level of fluid acceleration that is practically unrealistic. Research of other cloaking techniques is therefore highly desirable.

We introduce here a cloaking technique that is seemingly immune to all the aforementioned drawbacks. This technique is inspired by the matter acceleration approach, but it eliminates the unpractical requirement of ultra-fast matter transfer by utilizing a *perturbation modulation* (as opposed to ‘matter modulation’ with moving media) acceleration.

2 Modulation Versus Gravitation Induced Curvature

According to Einstein’s theory of general relativity, the trajectories of electromagnetic waves are bent in gravitational fields, which are typically produced by massive stellar objects, such as stars and black holes [Einstein \(1915\)](#); [Misner et al. \(1974\)](#). The related *curvature of spacetime* is in this case due to mass.

However, mass is not the only way to curve spacetime. Indeed, Einstein’s equivalence principle states that there is no fundamental difference between gravity and acceleration [Einstein \(1915\)](#). This principle offers then an extra option (than mass) to deviate light, and this is what underpins the Leonhardt matter acceleration cloak: in this device, light follows geodesics of the spacetime curvature induced by the fluid *acceleration* around the object to cloak [Leonhardt and Piwnicki \(1999\)](#), theoretically without any bandwidth or polarization restriction. In fact, the same effect, as far as light (as opposed to matter) is con-

cerned, occurs when the accelerating fluid is replaced by an accelerated perturbation, since the curvature of spacetime is fundamentally a consequence of acceleration, irrespectively to its origin. So far, only *uniform*, i.e., constant-velocity, spacetime-perturbed media have been extensively studied in the context of metamaterials [Deck-Léger et al. \(2019\)](#); [Caloz and Deck-Léger \(2019b\)](#). However, as we shall show in the next section, medium perturbations may also be nonuniform, or *accelerated*, and specifically engineered to produce a cloaking curvature of spacetime. This curvature is the same as in Leonhardt matter acceleration [Leonhardt and Piwnicki \(1999\)](#), but it does not involve any transfer of matter, and it is therefore presumably amenable to real-world applications.

3 Theory

Figure 1 depicts the proposed spacetime perturbation-induced cloak, which is here considered, as is most often the case, to be two-dimensional. The related spacetime interval [Carroll \(2019\)](#) is

$$ds^2 = -\frac{c^2 dt^2}{n^2(\rho, \phi; t)} + d\rho^2 + \rho^2 d\phi^2, \quad (1)$$

where $n(\rho, \phi; t)$ is the spacetime-dependent refractive index of the cloak.

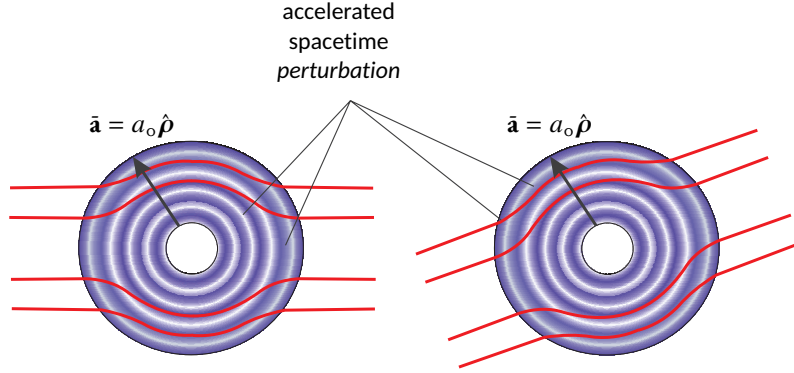


Figure 1: Proposed spacetime perturbation-induced cloak. The acceleration here is considered to be radial, for consistency with the implementation in Sec. 4

From this point, the cloak may be designed according to the following procedure:

1. Compute the metric, and the resulting geodesic equation [Carroll \(2019\)](#), corresponding to the spacetime interval (1);
2. Write in mathematical format the cloaking wave trajectories in Fig. 1;
3. Equate the geodesic equation from 2 to the trajectory equation, and solve for the refractive index function, $n(\rho, \phi; t)$, which must be, by construction, an accelerated profile, i.e., $\ddot{n}(\rho, \phi; t) \neq 0$;
4. Devise a medium modulation technique that provides the so-found accelerated spacetime refractive index $n(\rho, \phi; t)$.

4 Implementation

Figure 2 shows an example of an experimental implementation of proposed cloak with the accelerated spacetime function, $n(\rho, \phi; t)$, obtained in 4 of the procedure delineated in the previous section. Specifically, we assume the *radial* acceleration indicated in Fig. 1. In this implementation, the cloak consists in an SiO_2 slab that is modulated by a circular harmonic pump source placed below it. The pump is positioned at a distance to the cloak that is calculated so that its phase fronts hit the SiO_2 slab to produce the synthesized $n(\rho, \phi; t)$ profile, according to the ‘guillotine’ effect [Caloz and Deck-Léger \(2019a\)](#), where the velocity of the perturbation is inversely proportional to the angle, ϕ , between the slab and the incoming phase fronts. Note that the velocity in this device decreases from the center to the periphery of the cloak, corresponding to a radial deceleration ($a_o < 0$), with velocity reaching of course the velocity of the surrounding medium at the limit with it.

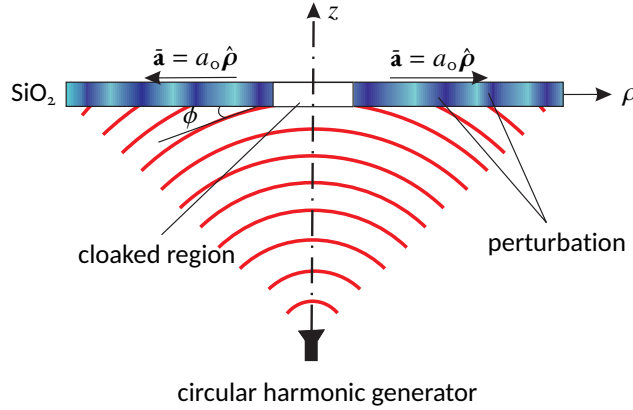


Figure 2: Proposed experimental implementation (side view) to realize the cloak in Fig. 1.

5 Conclusion

We have presented a novel type of cloaking, based on spacetime curvature induced by an accelerated perturbation. This cloak offers the benefits of the matter acceleration cloak, in particular a theoretically unlimited operation bandwidth, while being perfectly realizable in practice. We have proposed a specific implementation that may be tested in the lab and that is expected to result in real-world applications. Simulation results will be presented at the conference.

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