# Advances in Bianisotropic GSTC-based Metasurfaces


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***Abstract*—We present an overview of some of our recent advances on bianisotropic metasurfaces synthesized using susceptibility-based General Sheet Transition Conditions (GSTCs). Specifically, we discuss a series of related applications, which highlight the versatility and power of this method.**


## I. Introduction

Metasurfaces have been shown to enable a variety wave transformations [1]. Since they are structures of deeply-subwavelength thickness, they can be modelled as zero-thickness discontinuities in space and characterized by the Generalized Sheet Transition Conditions [2] (GSTCs). Specifically, bianisotropic metasurfaces, which include magneto-electric coupling, have been shown to be of critical importance in several key applications [3], [4], [5].

Here, we recall the basics of GSTCs and illustrate their capabilities for synthesizing bianisotropic metasurfaces via a number of application examples developed in our group.

## II. Susceptibility-based GSTCs

The GSTCs read [1], [2]

$$\hat{z}\times\Delta\mathbf{H} = j\omega\mathbf{P}-\hat{z}\times\nabla M_z, \Delta\mathbf{E}\times\hat{z} = j\omega\mu\mathbf{M}+\frac{\hat{z}\times\nabla P_z}{\epsilon}, \quad (1)$$

where $\mathbf{P}$ and $\mathbf{M}$ are the electric and magnetic surface polarization densities induced on the metasurface, which can be written in terms of surface susceptibility tensors as

$$\mathbf{P} = \epsilon\overline{\overline{\chi}}_{\text{ee}}\mathbf{E}_{\text{av}} + \epsilon\eta\overline{\overline{\chi}}_{\text{em}}\mathbf{H}_{\text{av}},\ \mathbf{M} = \overline{\overline{\chi}}_{\text{mm}}\mathbf{H}_{\text{av}} + 1/\eta\overline{\overline{\chi}}_{\text{me}}\mathbf{E}_{\text{av}}, \quad (2)$$

where $\Delta$ and $_{\text{av}}$ denote the difference and average of the fields on either side of the metasurface. The surface susceptibility tensors $\overline{\overline{\chi}}_{\text{ee}}$, $\overline{\overline{\chi}}_{\text{mm}}$, $\overline{\overline{\chi}}_{\text{em}}$ and $\overline{\overline{\chi}}_{\text{me}}$ contain each $3\times3$ susceptibility components, which leads to a system with 36 free parameters, some of which are typically set to zero according assumptions such as reciprocity, nongyrotropy or purely tangential susceptibilities.

In many applications [1], the normal polarization components ($P_z$ and $M_z$) can be eliminated and the susceptibility tensors can be reduced to their transverse part, which simplifies Eq. (1), with the substitution of Eq. (2) to

$$\hat{z}\times\Delta\mathbf{H} = i\omega\epsilon\overline{\overline{\chi}}_{\text{ee}}\mathbf{E}_{\text{av}} + i\omega\overline{\overline{\chi}}_{\text{em}}\sqrt{\mu\epsilon}\mathbf{H}_{\text{av}}, \quad (3a)$$

$$\Delta\mathbf{E}\times\hat{z} = i\omega\mu\overline{\overline{\chi}}_{\text{me}}\sqrt{\frac{\epsilon}{\mu}}\mathbf{E}_{\text{av}} + i\omega\mu\overline{\overline{\chi}}_{\text{mm}}\mathbf{H}_{\text{av}}, \quad (3b)$$

where the susceptiblity tensors $\overline{\overline{\chi}}_{\text{ee}}$, $\overline{\overline{\chi}}_{\text{mm}}$, $\overline{\overline{\chi}}_{\text{em}}$ and $\overline{\overline{\chi}}_{\text{me}}$ now each contain $2\times2$ susceptiblity components, yielding 16 free parameters.

The synthesis procedure consists in specifying a desired wave transformation in terms of the fields on both sides of the metasurface, calculating the field differences and averages, and inserting these quantities into (3) to determine the susceptibilities [1]. It is possible to specify multiple independent wave transformations by repeating this procedure with the appropriate number of non-zero susceptibility components to form a full-rank system.

## III. Diffraction-less Refraction

Metasurfaces realizing generalized refraction have been extensively demonstrated using the phase-gradient concept. It has however been shown that such an approach leads to an imperfect structure with undesired diffraction due to impedance mismatch on the metasurface [6].

We showed in [7] that Using bianisotropic metasurfaces allows to realize a passive, lossless and reciprocal metasurface that refracts a wave without any undesired diffraction. For a p-polarized wave incident in the $xz-$plane, this metasurface can be realized using the purely tangential susceptibility components $\chi_{\text{ee}}^{xx}$, $\chi_{\text{mm}}^{yy}$, $\chi_{\text{em}}^{xy}$, and $\chi_{\text{me}}^{yx}$.

## IV. Generalized Brewster Effect

The Brewster effect is a well-known electromagnetic phenomenon where a p-polarized wave incident at a specific angle (given by the inverse tangent of the ratio of the refractive indices) on a dielectric is transmitted without reflection.

Inserting a properly designed purely tangential bianisotropic metasurface at the dielectric interface can control the Brewster effect. Such a metasurface allows to modify the Brewster angle for any arbitrary angle and polarization [8]. Thanks to anisotropy, it is possible to achieve the Brewster effect for both polarizations simultaneously for a given plane of incidence. For an incidence in the $xz-$plane, the Brewster angle for the p-polarization can be tuned using the tangential susceptibility components $\chi_{\text{ee}}^{xx}$, $\chi_{\text{mm}}^{yy}$, $\chi_{\text{em}}^{xy}$, and $\chi_{\text{me}}^{yx}$, and independently for the s-polarization using $\chi_{\text{ee}}^{yy}$, $\chi_{\text{mm}}^{xx}$, $\chi_{\text{em}}^{yx}$, and $\chi_{\text{me}}^{xy}$. This type of bianisotropic metasurfaces can be utilized as subwavelength matching layers between different media.

## V. Angle-Independent Wave Transformations

An interesting and potentially useful special case of the tangential bianisotropic GSTCs occurs in structures having $\overline{\overline{\chi}}_{\text{ee}} = \overline{\overline{\chi}}_{\text{mm}} = 0$. Under such a restriction, Eqs. (3) simplify to what we refer to as the heteroanisotropic GSTCs

$$\hat{z} \times \Delta\mathbf{H} = jk\overline{\overline{\chi}}_{\text{em}}\mathbf{H}_{\text{av}}, \quad \Delta\mathbf{E} \times \hat{z} = jk\overline{\overline{\chi}}_{\text{me}}\mathbf{E}_{\text{av}}. \qquad (4)$$

Such a metasurface, when surrounded by the same medium (e.g., air) at both sides, has the particular property of performing angle-independent wave transformations [9]. One exciting application is the realization of an angle-independent amplifier, which provides the same amount of gain for all incident waves.

## VI. Concentric Metawaveguide Cloaking

We present here a novel type of cloaking, based on a circular concentric metawaguide structure, where the metawaveguides are bent waveguides with porous metasurface walls [10]. Compared to similar topologies used to approximate the perfect voluminal transformation optics cloak, this cloak is precisely optimized to this topology and therefore offers dramatically enhanced performance.

## VII. Nonreciprocal Wave Transformations

Bianisotropic metasurfaces exhibit a nonreciprocal response if $\overline{\overline{\chi}}_{\text{ee}} \neq \overline{\overline{\chi}}_{\text{ee}}^{T}$ or $\overline{\overline{\chi}}_{\text{mm}} \neq \overline{\overline{\chi}}_{\text{mm}}^{T}$ or $\overline{\overline{\chi}}_{\text{em}} \neq -\overline{\overline{\chi}}_{\text{em}}^{\text{T}}$ [11]. The type $\overline{\overline{\chi}}_{\text{em}} \neq -\overline{\overline{\chi}}_{\text{em}}^{\text{T}}$ can be nongyrotropic, meaning that the polarization of the incident wave can be conserved, and opens the possibility to realize birefringent nonreciprocal metasurfaces that have independent nonreciprocal responses for orthogonal polarizations. In contrast, the types $\overline{\overline{\chi}}_{\text{ee}} \neq \overline{\overline{\chi}}_{\text{ee}}^{T}$ and $\overline{\overline{\chi}}_{\text{mm}} \neq \overline{\overline{\chi}}_{\text{mm}}^{T}$ always perform gyrotropic wave transformations.

### *A. Nonreciprocal Phase-gradient*

Leveraging the nongyrotropic bianisotropic susceptibility components, it is possible to realize nonreciprocal phase-shifting unit cells that provide different phases of transmission for forward and backward excitations [12]. Those nonreciprocal phase-shifting unit cells can then be assembled into supercells able to provide arbitrary different phase-gradients in the forward and backward directions. Such kinds of metasurfaces can realize different types of spatial circulators and radomes having independent receive and transmit radiation patterns.

### *B. Nonreciprocal Waveplates*

The bianisotropic phase-shifting unit cells described in the previous sections can be synthesize to provide independent responses for vertically and horizontally polarized waves. This opens the possibility to realize nonreciprocal waveplates that have different phase shifts depending on the polarization and direction of propagation of the incident wave. For instance, a metasurface unit cell can be designed to act as a halfwave plate (180° phase shift between the two orthogonal polarizations) in the forward direction and provide the same phase for both polarization in the backward direction. Those unit cells could then leverage the geometric phase concept to easily realize a circularly-polarized phase-gradient for forward propagating waves, hence deflecting the wave, while leaving backward propagating waves unaltered.

### *C. Specular Isolator*

A specular isolator is a nongyrotropic metasurface that specularly reflects an wave incident incident at an angle $\theta$ while absorbing without reflection the corresponding time-reversed wave, incident at the opposite angle $-\theta$ [13]. In contrast to the other metasurface examples presented in this paper, this operation requires the use of normal susceptibility components in order to break the spatial symmetry.

### *D. Gyrotropic Reflective Isolator*

This is a birefringent metasurface that reflects vertically polarized incident waves into a horizontally polarized waves, and absorbs horizontally polarized incident waves, hence providing isolation between the two orthogonal polarization [14].

## VIII. Conclusion

We have recalled the susceptibility-base GSTCs and presented a few examples of bianisotropic metasurfaces synthesized using them as synthesis method. The diversity of these applications highlights the versatility of this approach.

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