

Arbitrary Pulse Shaping using Nonuniform Spacetime Modulation

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Abstract

This paper introduces and demonstrates a pulse shaping technique based on *nonuniform* spacetime modulation. This technique allows *arbitrary* pulse shaping and is in this regard unique, to the knowledge of the authors. Although we restrict here our attention to a 1+1D (1 dimension of time + 1 dimension of space) system, this idea can be extended to 1+2D (and 1+3D) systems, which will feature richer scattering phenomenology given the simultaneous transformations of the temporal vectorial spatial spectra of pulses.

1 Introduction

Pulse shaping is the process of altering the waveforms of pulses to make them better suited for given applications [Saleh and Teich \(2019\)](#). Some of the most common applications are chirped power amplification [Strickland and Mourou \(1985\)](#), dispersion compensation [Lemoff and Barty \(1993\)](#), optical coherence tomography [Crespo and Rosa \(2008\)](#), analog signal processing [Caloz et al. \(2013\)](#), and attosecond physics [Bucksbaum \(2003\)](#).

The main techniques for pulse shaping in electromagnetics are group velocity dispersion (GVD) engineering [Froehly et al. \(1983\)](#); [Caloz \(2011\)](#), nonlinear processing [Assion et al. \(1998\)](#); [Agrawal \(2000\)](#) and spacetime modulation [Dugan et al. \(1997\)](#); [Weiner \(2000\)](#). Each of these techniques have intrinsic limitations insofar as *arbitrary* pulse shaping is concerned. GVD engineering is restricted to modulated pulses and can compress only prechirped pulses. Nonlinear processing is dependent on the intensity of the input pulses and may induce spurious hysteresis. Spacetime modulation is immune of these limitations. Moreover, it is more versatile than these two techniques, being able to perform a diversity of compansion (compression or expansion) and space-reversal transformations [Caloz and Deck-Léger \(2019a,b\)](#). However, in its conventional, *uniform regime*, i.e., regime where the spacetime modulation velocity is constant, it cannot perform arbitrary transformations¹. This paper removes the pulse shaping restrictions of the uniform spacetime modulation technique, by making the modulation *nonuniform*, and offers hence an unprecedented technology for *fully arbitrary* pulse shaping.

2 Nonuniform Pulse Shaping Concept

For the sake of simplicity, we shall consider here a modulation that is formed by a perfectly reflecting mirror set in nonuniform motion, as shown in Fig. 1. 'Nonuniform' refers here to motion that includes *non-constant velocity*, or *acceleration*, i.e., specifically, acceleration and deceleration, and hence possibly back-and-forth displacement. Such a system may be designed to reshape, as suggested in the figure, any pulse waveform, $f(t)$, into a specified arbitrary pulse waveform, $g(t)$, upon reflecting the incident pulse from the nonuniformly moving mirror.

The nonuniformly moving mirror can indeed arbitrarily transform the spectrum of the incident pulse. This may be intuitively understood by thinking of the Doppler effect, which induces, on each component of the spectrum of the pulse, upshifting, downshifting and chirping for approaching, receding and accelerating motions, respectively. In the case of an unmodulated pulse, as in Fig. 1, the mirror compresses the reflected

¹For instance, if the modulation is set to compress a pulse, it necessarily compresses it entirely, and could not, for instance, compress part of it and expand the rest of it so as to transform an originally symmetric pulse into an asymmetric one.

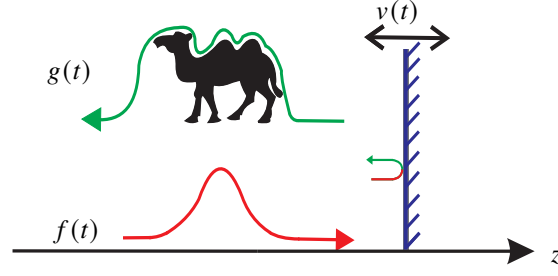


Figure 1: Arbitrary pulse shaping upon reflection from a moving mirror with nonuniform (including back-and-forth) motion.

pulse when approaching it and expands it when receding from it, and properly designing the trajectory of the mirror in spacetime will then allow arbitrary reshaping of the pulse.

3 Pulse Shaping Synthesis

Figure 2 presents a realization of the arbitrary pulse shaping system depicted in Fig. 1. Figure 1(a) shows the spacetime trajectory of the mirror, $\gamma(z, t)$, that compands the incident pulse $f(t)$ so as to reshape it into the specified reflected pulse $g(t)$. Figure 1(b) shows an electronic implementation of the system under the form of an artificial transmission line loaded by shorting diodes that are driven by a Field Programmable Gate Array (FPGA) switching the diodes on for short-circuit full reflection, hence providing the desired mirror function, with motion corresponding to the design of Fig. 1(a).

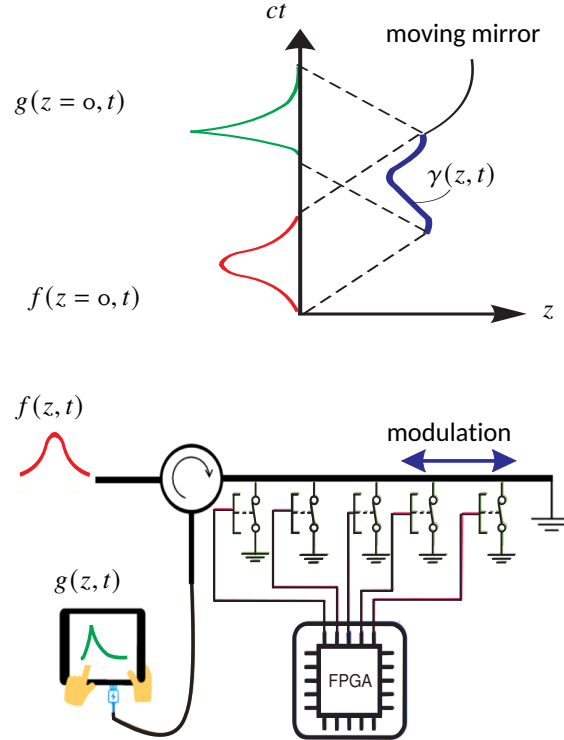


Figure 2: Realization of the arbitrary pulse shaping operation depicted in Fig. 1. (a) Spacetime design. (b) Experimental implementation, in the form of an artificial transmission line loaded by short-circuiting diodes driven by a Field Programmable Gate Array (FPGA).

The design illustrated in Fig. 1(a) consists in determining the trajectory of the mirror, $\gamma(z, t)$, that reshapes the input waveform $f(t)$ into the specified reflected waveform $g(t)$. The wave reflected by the mirror, assumed here to be a Perfect Electric Conductor (PEC) and moving with the nonuniform velocity $v(t)$, has the general waveform Deck-Léger et al. (2019)

$$E_r(z, t) = E_i \left(\frac{1 - \frac{v(z, t)}{c}}{1 + \frac{v(z, t)}{c}} \right) \exp \left[i \left(\omega_i t + k_i z \right) \left(\frac{1 - \frac{v(z, t)}{c}}{1 + \frac{v(z, t)}{c}} \right) \right], \quad (1)$$

where the subscript 'i' refers to the input waveform. The reflected waveform (1) generalizes for nonharmonic waves to

$$g(z, t) = \left(\frac{1 - \frac{v(z, t)}{c}}{1 + \frac{v(z, t)}{c}} \right) f \left(\frac{1 - \frac{v(z, t)}{c}}{1 + \frac{v(z, t)}{c}} t \right). \quad (2)$$

Solving this equation for $v(z, t)$ yields then the time-dependent mirror velocity

$$v(z, t) = \frac{1 - \frac{1}{t} F^{-1}(tg(z, t))}{1 + \frac{1}{t} F^{-1}(tg(z, t))}, \quad (3)$$

where $F(x) = xf(x)$, and hence $\gamma(z, ct) = c/v(z, ct)$.

4 FDTD Simulation

Figure 3b provides an FDTD proof of concept of the system described in Sec. 3, with the FDTD algorithm incorporating a PEC wall that moves in spacetime Kriegsmann (1989) according to the synthesized velocity in Eq. (3).

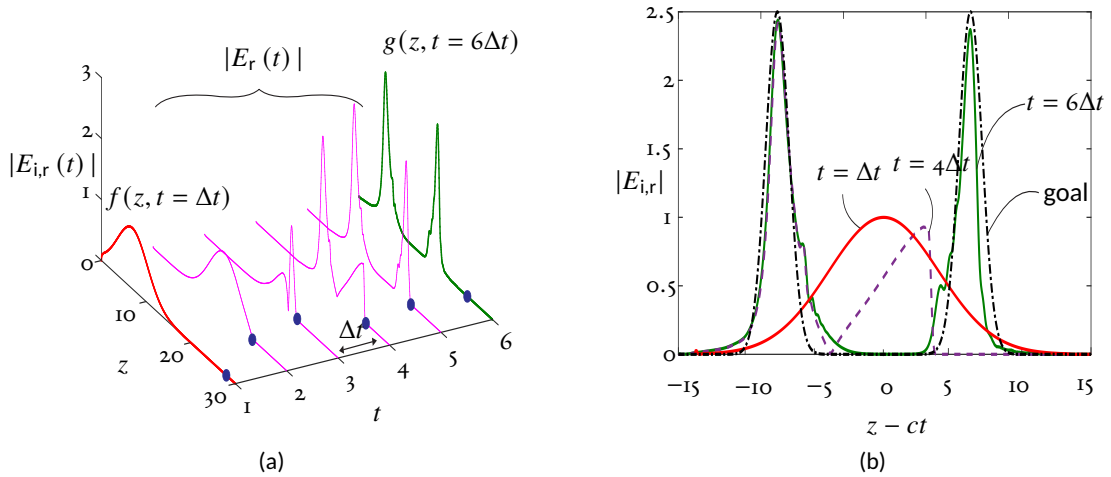


Figure 3: Full-wave FDTD demonstration of the system described in Fig. 2 for reshaping the Gaussian waveform $f(z = 0, t) = \exp[-(t/5)^2]$ into the waveform $g(z = 0, t) = 2.5 \exp[-(t - 7)^2] + 2.5 \exp[-(t + 7)^2]$. (a) Evolution of the pulse in spacetime during the reflection process. The blue dots represent samples of the spacetime trajectory of the mirror. (b) Pulse magnitude variation in the retarded frame.

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