Ocean Wave Powered Boats.

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August 3, 2021

Abstract

A brief outline of a feasibility study based on computer modeling of an ocean wave powered boat is presented. The boat consists of a main vessel and a trailing float with attached beams that are connected to the sides of the main vessel hull by revolute joints. The computer model is based on a Lagrangian formulation subject to the constraints of motion of the two bodies that move radially about the revolute joints. Of special interest is the performance of the vessel-float system in moderate ocean conditions.

1 Introduction.

Power obtained from ocean wave energy has been an area of active research for some decades with a number of power stations already in operation in various parts of the world. Ocean wave power stations are limited by the expense of construction of the many individual units needed to generate enough power to compete with other forms of power generation. While this limitation may be less attractive from the perspective of commercial use in supplying power grids, an individual unit based on some kind of flotation principle can generate enough power to operate a single less demanding device. The idea here is to consider a single flotation unit in isolation that is entirely dedicated to the supply of power to a single device. Here we will consider one such device in the form of an ocean vessel that has the special property that in itself acts as a power generating unit that in turn employs the power it generates to propel itself.

Consider a boat powered by an electric motor connected to a series of batteries. The electric motor rotates a propeller that provides the forward thrust of the vessel through the water. A float is placed at the rear of the main vessel and connected by beams to the sides of the main vessel hull by revolute joints as shown in Figure 1. In such a configuration the float is constrained to move radially about the revolute joints.

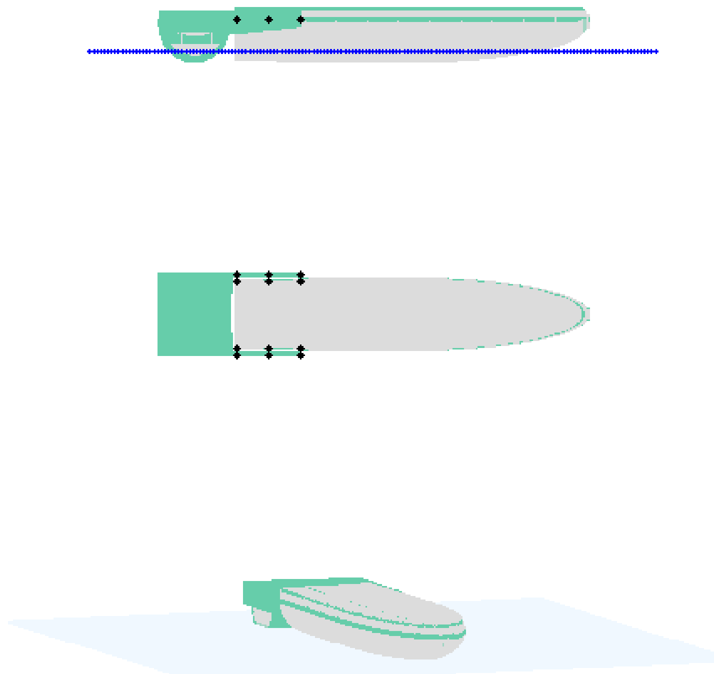


Figure 1: Side, plan and three dimensional view of the vessel-float system. The bullets at the rear of the main vessel hull and the beams of the float represent the positions of the revolute joints (center) and the two load points either side of the revolute joints. (The beams that connect the float to the sides of the main vessel hull are not shown in the three dimensional view.)

Figure 2: Three dimensional view of the float showing the extended beams that are attached to the sides of the main vessel hull by revolute joints. There is an opening at the lower center of the float for water flow induced by the propeller situated at the bottom rear of the main vessel hull.

The float and hull of the main vessel are subject to buoyancy forces that act in the positive vertical direction. Buoyancy forces are countered by gravitational forces that act in the negative vertical direction. As ocean waves propagate along the length of the vessel the float and main vessel hull will be set into an oscillatory motion. The oscillatory motions of the float and the main vessel will not be synchronous. In this way there will be a variation in the angular displacement between deck of the main vessel and the beams that connect the float to the sides of the main vessel.

The variation in angular displacement between the beams of the float and the deck of the main vessel lends itself to the generation of mechanical power that in turn can be converted into electrical energy. There are a number of ways that this can be achieved. The basic principle here may involve the translation of the angular displacement into a rotary motion that drives an alternator. The current induced by the alternator can be used to charge the batteries that supply power to the electric motor.

When the vessel is anchored there is no power being drained from the propulsion unit and the entire current induced by the alternator can be used to charge up the batteries. When the boat is being propelled through the water the electric motor will drain charge from the batteries. The extent to which the alternator can compensate for the charge being drained from the battery by the propulsion unit will depend on a number factors and these need to be examined.

The objective of this study is to investigate the effectiveness of this system under various ocean conditions for both anchored and propulsion states. To asses the potential for power production we will simply apply a load that works against the angular displacement of the float relative to the main vessel. The load is transferred to each of the float and vessel hulls at load points situated on either side of the revolute joints.

It can be expected that power production will increase as the ocean conditions become rougher. Of particular interest is the performance of this system for relatively calm ocean conditions. For the purposes of demonstration it will be sufficient to consider the idealized vessel-float design shown in Figure 1. Here we are interested in main vessel dimensions typically of length $10m$ but there is a potential to scale up to larger vessels. We start by describing the basic theory upon which the computer model is based.

2 Mathematical Model.

The vessel-float system described in the previous section consists of 2 rigid bodies attached in such way that they are in a constrained motion relative to each other. Throughout we use the Cartesian coordinates, (x, y, z), that are associated with the mutually orthogonal unit vectors $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$, where $\mathbf{i}$ points in the horizontal direction from left to right, $\mathbf{j}$ points in the positive vertical direction and $\mathbf{k}$ points horizontally out of the page. In terms of the global coordinates, a position vector, $\mathbf{x}$, relative to a fixed origin has the representation $\mathbf{x} = x\ \mathbf{i} + y\ \mathbf{j} + z\ \mathbf{k}$.

Rigid body dynamics :: The kinetic energy of a single rigid body of mass m is given by

$$T = \frac{1}{2} m \dot{\mathbf{X}} \cdot \dot{\mathbf{X}} + \frac{1}{2} \boldsymbol{\omega}^T \mathbf{I} \boldsymbol{\omega} \tag{2.1}$$

where $\mathbf{X}$ is the center of mass, $\boldsymbol{\omega}$ is the angular velocity vector and $\mathbf{I}$ is the inertia dyadic. If gravity is the only conservative force acting on the body then the potential energy of the body is given by

$$V = mg\mathbf{X} \cdot \mathbf{j} \tag{2.2}$$

Lagrangian formulation :: For a system of rigid bodies the Lagrangian, L, is defined by

$$L = T - V \tag{2.3}$$

where T and V, respectively, are now the kinetic and potential energy, respectively, of the multi-body system. The Lagrangian equations are

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = Q_j \tag{2.4}$$

where q_j represents the jth component of the vector $\mathbf{q}$ of generalized coordinates and Q_j is the jth component of the generalized forces given by

$$Q_j = \sum_i \mathbf{f}_i \cdot \frac{\partial \mathbf{x}_i}{\partial q_j} \tag{2.5}$$

where each $\mathbf{f}_i$ is a non-conservative force acting at $\mathbf{x} = \mathbf{x}_i$.

Vessel-float system :: The vessel-float system described in the first section is essentially a two body system. The variables associated with each body will be labeled by a bracketed superscript n, where $n = 1$ is the label associated with the main vessel and $n = 2$ is the label associated with the float. The float and hull of the main vessel are constrained to move radially about the revolute joints that are fixed to the sides of the deck of the main vessel. If $\boldsymbol{\xi}^{(n,k)}$, $k = 1, 2$, is the position vector of each of the two revolute joints on body n relative to the center of mass, $\mathbf{X}^{(n)}$, then the contraints of this system can be expressed as

$$\mathbf{X}^{(1)} + \boldsymbol{\xi}^{(1,k)} = \mathbf{X}^{(2)} + \boldsymbol{\xi}^{(2,k)} \tag{2.6}$$

where each $\mathbf{X}^{(n)}$ and $\boldsymbol{\xi}^{(n,k)}$ can be regarded as functions of $\mathbf{q}$.

The Lagrangian formulation (2.4) along with the constraints of motion (2.6) result in a system of differential-algebraic equations for the unknown variables $\mathbf{q}$ and $\dot{\mathbf{q}}$. In the interests of generality, computer models for applications of rigid body dynamics often employ Lagrangian multipliers to incorporate the constraints into the formulation.

An alternative approach is to choose a reduced dimensional vector of generalized coordinates that reflect the degrees of freedom imposed by the constraints. A system of ordinary differential equations can then be derived in closed form within which the constraints are implicitly embedded. While these formulations are less attractive because of their application specific nature they can often be more computationally efficient than those employing Lagrangian multipliers. In this respect the choice of the most suitable generalized coordinates is crucial.

Forces :: On each body the following forces are applied.

Gravity.

$$\mathbf{f}_g^{(n)} = -m^{(n)} g\, \mathbf{j}, \quad \mathbf{x} = \mathbf{X}^{(n)} \tag{2.7}$$

where $m^{(n)}$ is the mass of body n, g is the acceleration of gravity and $\mathbf{X}^{(n)}$ is the position vector of the center of mass of body n.

Buoyancy.

$$\mathbf{f}_b^{(n)} = \rho_w g V_b^{(n)}\ \mathbf{j}, \quad \mathbf{x} = \mathbf{x}_b^{(n)} \tag{2.8}$$

where $V_b^{(n)}$ and $\mathbf{x}_b^{(n)}$, respectively, is the volume and position vector of the centroid, respectively, of the submerged part of the body n and ρ_w is the density of water.

Drag. The drag forces, $\mathbf{f}_d^{(n)}$, come into effect when water flows across the surface of the submerged part of the hulls. We adopt an empirical expression for the drag forces that act at the centroids, $\mathbf{x} = \mathbf{x}_b^{(n)}$, of the submerged part of the hulls. The general form is given by

$$\mathbf{f}_d^{(n)} = -f_d^{(n)}\left(|\dot{\mathbf{X}}^{(n)}|, \frac{V_b^{(n)}}{V_{b,eq}^{(n)}}\right)\frac{\dot{\mathbf{X}}^{(n)}}{|\dot{\mathbf{X}}^{(n)}|}, \qquad \mathbf{x} = \mathbf{x}_b^{(n)} \tag{2.9}$$

where $V_{b,eq}^{(n)}$ is the volume of the submerged part of the vessel n in the equilibrium state.

Load. A torque load, τ, of magnitude $|\gamma\ \dot{\Theta}|$ is applied at the revolute joints that works against the angular displacement during the compression phase, $\dot{\Theta} < 0$. There is no load applied on the expansion phase, $\dot{\Theta} > 0$. Here $\gamma > 0$ is the load constant and Θ is the angular displacement of the vessel-float system defined by (3.6). The load constant, γ, is specific to the properties of the alternator that charges the batteries.

The load is transferred to each body at fixed points on brackets placed on either side of the revolute joints. If $\mathbf{x}_r^{(n,k)}$, $k = 1, 2$, are the position vectors of the two revolute joints on body n then the two load points, $\mathbf{x}_l^{(n,k,m)}$, $m = 1, 2$, are positioned either side of the revolute joints such that $\mathbf{x}_l^{(n,k,1)} - \mathbf{x}_r^{(n,k)} = -(\mathbf{x}_l^{(n,k,2)} - \mathbf{x}_r^{(n,k)})$.

Propulsion. The propulsion force $\mathbf{f}_p^{(1)}$ acting at $\mathbf{x} = \mathbf{x}_p^{(1)}$ on the main vessel hull is associated with the rotating propeller powered by the electric motor. There is no propulsion force acting on the float, i.e. $\mathbf{f}_p^{(2)} = \mathbf{0}$.

3 Computer model.

The behavior of the vessel-float system described in Section 1 is examined under various ocean conditions by computer modeling based on the Lagrangian formulation outlined in Section 2. We are particularly interested in the performance of the system in calm ocean conditions where we can expect the power generation to be minimal.

The ocean conditions are defined by the water waves that are described by

$$y_w = a \sin(2\pi(x/\lambda + \nu t)) \tag{3.1}$$

y_w	water surface height relative to the equilibrium, $y_w = 0$ (m)
a	amplitude of water wave (m)
λ	wavelength of water wave (m)
ν	frequency of water wave $(cycles/s)$
t	time (s)
$x = \mathbf{X}^{(1)} \cdot \mathbf{i}$	horizontal coordinate of the center of mass of the main vessel relative to the starting position $x = 0$ (m)

The wave height, h, is defined as the vertical distance between the highest point of the crest of the wave and the lowest point of the trough of the wave, i.e. $h = 2\ a$. We will present the results for the following case studies.

Case	a (m)	v_{cs} (m/s)
1	0.15	0
2	0.15	2.5 – 3.0
3	0.10	2.5 – 3.0
4	0.10	2.0 – 2.5

Here v_{cs} is the cruising speed. For all case studies presented here we will use the fixed parameters

$$\nu = 0.2\ cycles/s, \quad \lambda = 10\ m, \quad \gamma = 20000\ kg\ m^2/s \tag{3.2}$$

If $f_d(v)$ is the horizontal component of the drag force on the vessel-float system as a function of speed, v, and f_p is the horizontal component of the propulsion force induced by the propeller on the main vessel then the cruising speed can be estimated from the formula

$$f_p = -f_d(v_{cs}) \tag{3.3}$$

We consider cruising speeds of about 5 *knots* ($\approx 2.57\ m/s$) and always assume that the length of the vessel will be perpendicular to the oncoming wave fronts. Cruising speeds much higher than 5 *knots* are not considered realistic for simulation purposes since it can be expected that at these higher speeds the power of propulsion would be continuously varied by an intelligent agent to maintain some kind of vessel stability. While the computer model is capable of considering variations in propulsion power it would introduce complications in the analysis of the simulation results that would detract from the main objectives of this study.

The average power, $\bar{P}(t)$, over a time interval, $[t, t+T]$, is defined by

$$\bar{P}(t) = \frac{1}{T} \int_t^{t+T} P(s)\ ds \tag{3.4}$$

where $P(t)$ is the instantaneous power at time t. In typical ocean conditions all variables will be highly oscillatory due to the action of the water waves. It will be more useful to examine average powers rather than the instantaneous powers. In particular, we are interested in comparing the average power of propulsion, $\bar{P}_{prop}(t)$, with the average generated power, $\bar{P}_{gen}(t)$. We use the local time averages over time intervals, T, where

$$T = 1/\nu \tag{3.5}$$

is the the wave period

The vessel-float system is depicted in Figure 1. The main vessel hull has length 10 m, maximum width 3 m and maximum height 1.75 m. The weight of the main vessel is 4544 kg and the weight of the float is 756 kg. This rather large weight of the float relative to its size is found to be necessary to maintain some degree of stability. Simulation results indicate that for float weights significantly lower than this tend to introduce a higher degree of erratic motion. For brevity, simulation results demonstrating the behavior of the vessel-float system for various float weights will not be presented here.

It needs to be stressed that our objectives here are not focused on optimal vessel designs that address stability and maneuverability of a vessel in rough ocean conditions. As such it is more appropriate that we simplify the computer model as much as possible to avoid irrelevant details clouding the main objectives of this study. In the simulations we assume that the length of the main vessel is always pointing perpendicular to oncoming wave fronts. In this way we can assume planar kinematics where all motions are confined to the xy-plane. In planar kinematics the computation of the angular momentum is simplified by replacing the inertia dyadic, $\mathbf{I}$, with the moment of inertia, I. It follows that the angular momentum of each body, $\mathbf{L}^{(n)}$, can be written in the form $\mathbf{L}^{(n)} = I^{(n)}\dot{\theta}^{(n)}\ \mathbf{k}$, where $I^{(n)}$ is the moment of inertia of body n and $\theta^{(n)}$ is the angular displacement of body n about the center of mass $\mathbf{X}^{(n)}$. We define the angular displacement of the vessel-float system as

$$\Theta = \theta^{(1)} - \theta^{(2)} \tag{3.6}$$

In all cases we assume that the vessel-float system is initially in a stationary equilibrium state in calm ocean conditions, i.e. $y_w = 0$. The water waves described by (3.1) are activated from the equilibrium state by a short time envelope that maintains sufficient smoothness of the time dependent variables. Numerical solution of the differential equations of the system is performed using a fourth order Runge-Kutta method.

Along with the figures provided for each case study presented here are attached gif files showing animations of the vessel-float system.

Attached animation files.

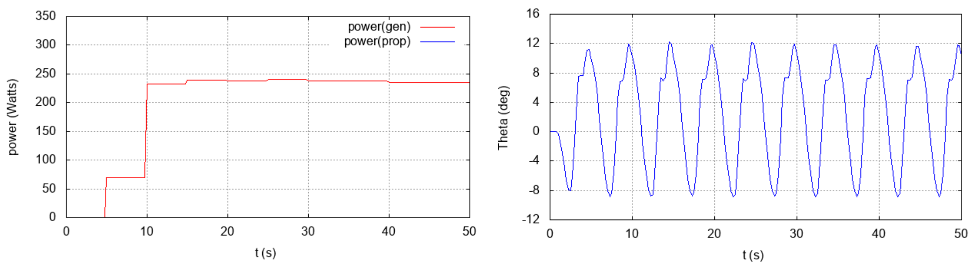
Case	Side view	3D view
1	2d1.gif	3d1.gif
2	2d2.gif	3d2.gif
3	2d3.gif	3d3.gif
4	2d4.gif	3d4.gif

4 Simulation results.

4.1 Case 1.

We first consider the case of an anchored vessel-float system in ocean conditions where the water wave amplitude $a = 0.15$ m. In such a case there is no charge being drained from the battery by the propulsion unit so that any power generated can be entirely dedicated to charging up the batteries. The vessel-float system is initially in the equilibrium state. The simulation is carried out over the time interval $0 \leq t \leq 50\ s$.

Figure 3 shows the power generated, $\bar{P}_{gen}$, averaged over time intervals of the water wave period, T, and the angular displacement, Θ, of the vessel-float system. The system quickly settles into a regular oscillatory pattern with the angular displacement varying from about -8 to 12 degrees. The power generated settles to an average output of about 240 $Watts$.



(a) (b)

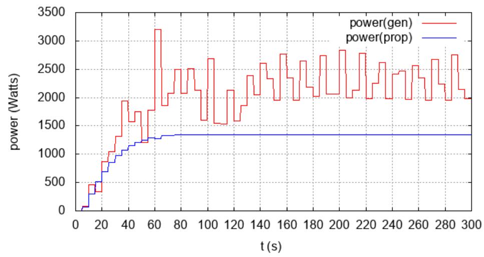
Figure 3: Case 1 :: (a) Power generated, $\bar{P}_{gen}$, averaged over time intervals of the water wave period, T, and (b) angular displacement, Θ. The vessel-float system is anchored in the ocean conditions where the water wave amplitude is $a = 0.15\ m$. The simulation is carried out over the time interval $0 \leq t \leq 50\ s$.

4.2 Case 2.

The vessel-float system is examined in the ocean conditions of $a = 0.15\ m$ with an applied power of propulsion to maintain a cruising speed, v_{cs}, of about 5 $knots$. The length of the main vessel is propelled in the positive x direction perpendicular to the oncoming water wave fronts.

Figure 4 shows the generated power and power of propulsion averaged over time intervals of the water wave period, T, the angular displacement, Θ, and the horizontal speed over a simulation time of $0 \leq t \leq 300\ s$. In the early stages of the simulation the vessel-float system oscillates in an erratic manner. This is most evident by the angular displacement, Θ, as shown in Figure 4(b). After about 150 s the system has settled down and the oscillations exhibit a more regular pattern with an angular displacement varying from about -4 to 24 degrees.

The power of propulsion is increased slowly until it begins to level off at about 1.35 kW. This results in a cruising speed, v_{cs}, of about 2.7 m/s ($\approx 5.25\ knots$). We see that under these conditions the generated power, averaged over time intervals of the water wave period, T, increases with the horizontal speed. After about 150 s it begins to rise and fall from about 2.0 to 2.8 kW. The generated power exceeds the power of propulsion during the cruising phase allowing for a net charge to the batteries.



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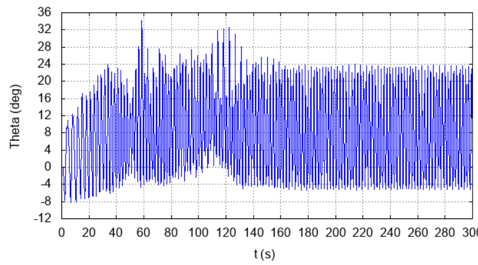

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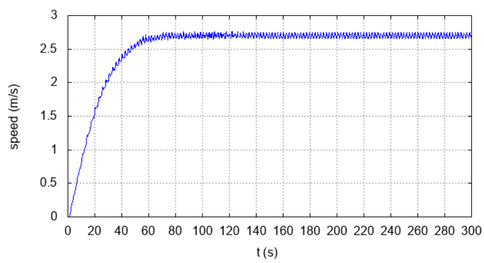


(c)

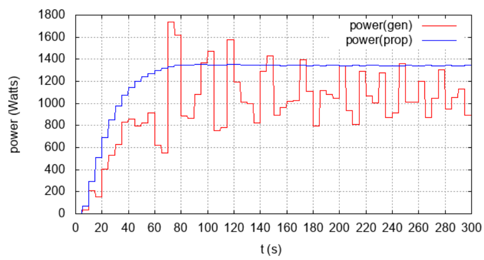
Figure 4: Case 2 :: (a) Power generated, $\bar{P}_{gen}$, (red) and power of propulsion, $\bar{P}_{prop}$, (blue) averaged over time intervals of the water wave period, T, (b) angular displacement, Θ, and (c) horizontal speed of the main vessel. A steady force of propulsion is applied to the main vessel initially at rest. The ocean conditions correspond to a water wave amplitude $a = 0.15\ m$. The simulation is carried out over the time interval $0 \leq t \leq 300\ s$.

4.3 Case 3.

In Case 2 we saw that under relatively moderate ocean conditions it is possible to generate enough power to exceed the power of propulsion while maintaining a cruising speed, v_{cs}, of about 5 $knots$. In Case 3 we examine the vessel-float system under even more moderate conditions where the ocean wave amplitude is only 10 cm. As in Case 2 the power of propulsion is increased slowly and then held roughly constant at about 1.35 kW to maintain a cruising speed, v_{cs}, of about 5.25 $knots$.

Figure 5 shows the generated power and power of propulsion averaged over time intervals of the water wave period, T, the angular displacement, Θ, and the horizontal speed over a simulation time of $0 \leq t \leq 300\ s$. As in Case 2 the vessel-float system oscillates in an erratic manner in the early stages of the simulation. After about 150 s the system begins to settle down into more regular pattern as is evident by the angular displacement shown in Figure 5(b). During this time the angular displacement varies from about -4 to 16 degrees.

We see that under these conditions the generated power, averaged over time intervals of the water wave period, T, increases with the horizontal speed until it begins to rise and fall from about 0.8 to 1.4 kW. The power generated is, for the most part, significantly lower than the power of propulsion resulting in a net loss of charge from the battery.



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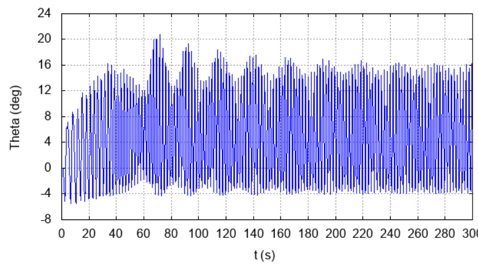

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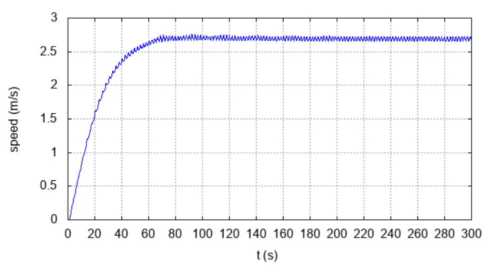


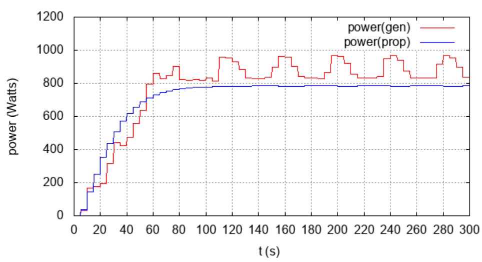
(c)

Figure 5: Case 3 :: (a) Power generated, $\bar{P}_{gen}$, (red) and power of propulsion, $\bar{P}_{prop}$, (blue) averaged over time intervals of the water wave period, T, (b) angular displacement, Θ, and (c) horizontal speed of the main vessel. A steady force of propulsion is applied to the main vessel initially at rest. The ocean conditions correspond to a water wave amplitude $a = 0.1\ m$. The simulation is carried out over the time interval $0 \leq t \leq 300\ s$.

4.4 Case 4.

In Case 3 we saw that under the ocean conditions $a = 0.1\ m$ the power generation is less than the power of propulsion required to maintain a cruising speed of about 5 $knots$. To prevent a net drainage of charge from the batteries one can reduce the power of propulsion. Here we apply a power of propulsion to accelerate the vessel from rest and then held approximately constant at just under $0.8\ kW$. This results in a cruising speed, v_{cs}, of about $2.2\ m/s$ ($\approx 4.3\ knots$).

Figure 6 shows the generated power and power of propulsion averaged over time intervals of the water wave period, T, the angular displacement, Θ, and the horizontal speed over a simulation time of $0 \leq t \leq 300\ s$. By the time $t \approx 100\ s$ the system appears to have settled into a regular cyclic pattern. During the cruising phase of the simulation the power of generation exceeds the power of propulsion, rising and falling within the range $0.8 - 1.0\ kW$.



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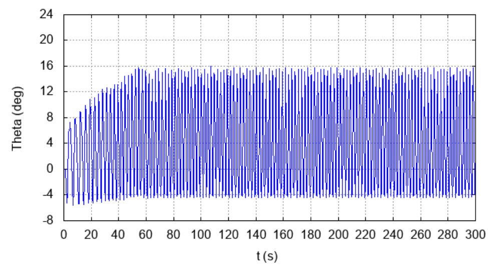

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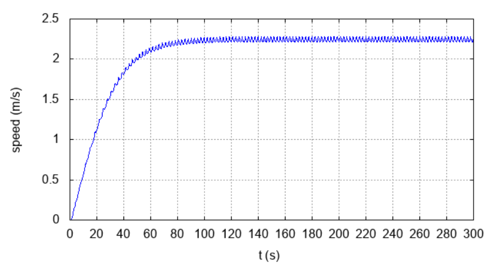

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Figure 6: Case 4 :: (a) Power generated, $\bar{P}_{gen}$, (red) and power of propulsion, $\bar{P}_{prop}$, (blue) averaged over time intervals of the water wave period, T, (b) angular displacement, Θ, and (c) horizontal speed of the main vessel. A steady force of propulsion is applied to the main vessel initially at rest. The ocean conditions correspond to a water wave amplitude $a = 0.1\ m$. The simulation is carried out over the time interval $0 \leq t \leq 300\ s$.

5 Conclusions.

The objective of this study was to examine power generation versus power of propulsion under moderate ocean conditions. A feasibility study based on the computer modeling of the vessel-float system depicted in Figure 1 demonstrates that under the moderate ocean conditions of water wave amplitudes in the range $10 - 15\ cm$ the vessel-float system is capable of generating enough power to maintain cruising speeds in the range of $4 - 6\ knots$.

It is important to note that the relatively modest speeds considered in the case studies are not meant to represent the speed limits of the vessel-float system. It can be expected that in ocean conditions associated with water wave amplitudes greater than 15 cm sufficient power can be generated to maintain higher speeds. This is fortuitous since it is in these rougher conditions that added power is needed to maintain vessel stability and maneuverability.

In order to simplify the simulations in Cases 2-4 the power of propulsion was increased slowly until the cruising speed is approached and then held approximately constant. However, this is perhaps not entirely realistic, especially under rougher ocean conditions, because it is more likely that the power of propulsion would be continuously varied by an intelligent agent to maintain some kind of vessel stability. It should be possible to design an automated system governing the propulsion unit that not only takes into account power generated versus power of propulsion but also includes vessel stability.

There are numerous design variants of the basic concept of the vessel-float system presented here. The introduction of a float increases the overall drag of the system although this increase is considerably less than the drag acting on the main vessel hull and is not a serious issue. Nevertheless, any design of a composite vessel-float system should aim to reduce drag as well as maintain stability and maneuverability of the system in a variety of ocean conditions. Although not discussed here in any detail, numerical simulations indicate that stability of the vessel-float system under an applied load is increased as the float weight is increased.

One advantage of the vessel-float design examined here is that the float, along with the beams and the added components that drive the alternator, can be constructed as a separate unit that can be attached to most vessels. As a side benefit, such units can also be installed at the end of a jetty or wharf for the purposes of local power generation.

In this study many other variants of float designs as well as their placement relative to the main vessel hull have been examined in some detail. These included placement of floats at the front of the main vessel. The vessel-float design presented here is in no way meant to represent an optimal design and its choice has been made on the basis of simplicity in conveying the main principles of the basic concept.

Another design variant that has been examined in this study but not presented here is a vessel that is separated lengthwise into two segments joined together by revolute joints in much the same way as the vessel-float system. Here the rear segment of the vessel is nothing more than an extended version of the attached float as depicted in Figure 1. Commuting between vessel segments should be easily accomplished by the introduction of a small flexible bridge between the two vessel decks. Such designs may be of interest for scaled up versions.