

# Determination of the Upper Limit Up To Which the Linear Flow Law (Darcy's Law) Can Be Applied

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**Abstract**— In the practice of hydrodynamic calculations the linear flow law, commonly called Darcy's law, is now widely used. It is well known that it is violated at large pressure gradients. This means that there is a certain limit value of the pressure gradient  $\Delta p^*$  above which a deviation from the linear character of the flow law begins. This value of the pressure gradient is the upper limit of applicability.

A method is presented for the direct determination of the upper limit of the validity of the linear flow law (Darcy's law) for any porous media. The method is based on the principles of percolation modelling of fluid flows in porous media. The influence of the structure of the pore space on the value of the boundary gradient is analysed. A qualitative comparison with the experimental data is performed.

**Keywords**— Darcy's law, Upper limit, Linear flow, Pressure gradient, Fluid flow.

## I. INTRODUCTION

The upper limit of applicability of Darcy's law arises from the manifestation of inertial forces resulting from continuous changes in the direction and cross-section of pore channels [1]-[5].

Traditionally, to determine the upper limit of applicability of Darcy's law, a comparison of the flow Reynolds number ( $Re$ ) with its critical value ( $Re_c$ ) has been proposed. This issue has been addressed in many literatures, beginning in the first half of the last century [6]-[10]. However, due to the diversity of the proposed expressions for calculating the flow numbers  $Re$  and the large dispersion of their critical values, the application of this method in practice proves to be impossible.

In this work, a different approach to solve this problem is proposed, based on the analysis of the fluid flow in a porous medium at the micro level. In this case, the calculation of the flow in the model of a porous medium - the capillary grid - allows to take into account both linear hydraulic pressure losses due to the forces of viscous friction and pressure losses at local resistances associated with the sudden narrowing and expansion of capillaries.

## II. METHODOLOGY

Fluids move in productive formations along conductive channels formed by a system of interconnected pore channels (capillaries) between rock grains or a system of interconnected fractures, or a combination of such systems [11]-[14]. The nature of the formation of capillary channels, which are the narrowest elements in the structure of the pore space, varies in reservoirs of different types [15]-[18]. For the sake of certainty, we will make further considerations for the case of a granular medium (Fig. 1), when the pore sizes are significantly larger than the transverse dimensions of the capillaries.

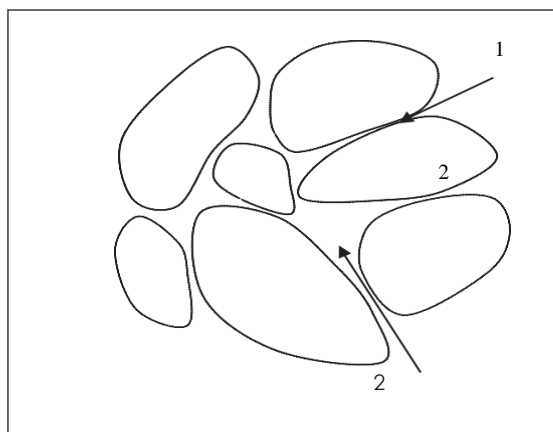


Fig. 1 Schematic representation of a granular porous medium:  
1- capillary; 2 — pore

The problem of the flow of a viscous Newtonian fluid of density  $\rho$  and viscosity  $\mu$  in a model of a porous medium - a spatial lattice of capillaries - is considered. Such a model porous structure is characterized by the mean capillary length  $l$ , the function of the distribution density of capillaries along the radii  $f(r)$  (porosimetric curve) and the coordination number of the lattice  $z$  [19],[20].

Since the cross-sectional radii of capillaries in granular porous media are small compared to the characteristic pore size, the hydraulic resistance in capillaries is much larger than in pores, and therefore the hydraulic losses in pores can be neglected.

Based on the concept of the structure of an infinite conducting cluster (IC) [19], we come to the problem of determining the conductivity of its skeleton, which is responsible for the transport properties of both the infinite cluster itself and, accordingly, the porous medium as a whole.

The IC skeleton consists of chains of hydraulically interconnected pore channels (capillaries) with different radii (Fig. 2).

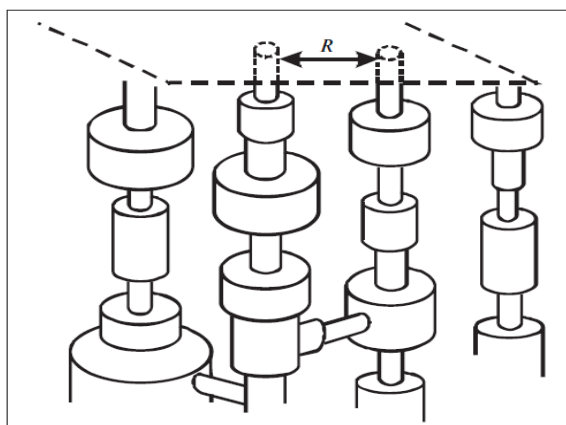


Fig. 2 Schematic representation of the formation of conducting r-chains

The conductivity of each such chain is determined by the thinnest capillary contained in its structure. It is therefore natural to consider its radius as the main characteristic of chains and to call chains in which the minimum radius of the capillaries forming them lays in the range  $r \dots r + dr$  "r-chains".

The pressure loss along the length, due to the forces of viscous friction, in each individual capillary of radius  $r$  is determined by Poiseuille's formula [21]-[24],  $q$  is the fluid flow in the capillary

$$\Delta p_c = \frac{8\mu l q}{\pi r^4}$$

Pressure losses due to local resistances are associated with sudden contractions and expansions at the junction of capillaries and pores and depend on their radii  $r_1$  and  $r_2$ , the fluid density  $\rho$ , their viscosity  $\mu$ , the flow rate  $q$ , and the characteristic value of shear stresses on the capillary walls  $\tau_0$ :

$$\Delta p_s = F(r_1, r_2, \mu, \rho, \tau_0, q) \quad (1)$$

This does not take into account that the area where the capillaries meet is not an ideal transition of the type of sudden expansion or contraction (Fig. 3).

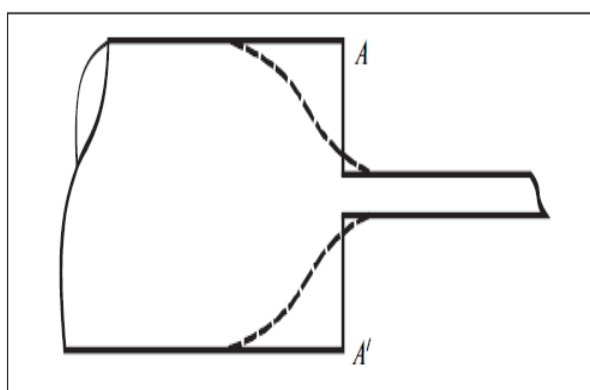


Fig. 3 Schematization of the connecting element of capillaries of different radii in the chain.

To determine the value of  $\Delta p_s$ , we use dimensional analysis. The functional dependence (1) contains six defining parameters, three of which ( $r_1$ ,  $p$ ,  $q$ ) have independent dimensions.

By the  $\pi$ -theorem, (1) can be represented in a dimensionless form

$$\pi = f^*(\pi_1, \pi_2, \pi_3),$$

Where

$$\pi = \frac{\Delta p_s}{\rho q^2 r_1^{-4}}; \quad \pi_1 = Re^{-1} = \frac{\mu}{\rho q r_1^{-1}};$$

$$\pi_2 = He = \frac{\tau_0}{\rho^{-1} \mu^2 r_1^{-2}}; \quad \pi_3 = \frac{r_2}{r_1}$$

Since the radius of the capillary  $r_1$  is small relative to the radius of the pore  $r_2$ , the Reynolds numbers are quite large ( $Re > 500$ ), while the Hedstrom numbers ( $He$ ) are small (shear stresses  $\tau_0$  are usually neglected, i.e.  $He \ll 1$ ). Assuming self-similarity in the parameters  $Re$  and  $He$ , we obtain

$$\Delta p_s = \rho q^2 r_1^{-4} f^* (r_2/r_1) = q^2 \zeta(r_1, r_2),$$

Here  $\zeta$  is the coefficient of local resistance expressed by the formulas Borda-Carnot and Zhukovsky for the expansion and contraction of the channel, respectively:

$$\zeta(r_1, r_2) = \rho r_1^{-4} (1 - r_1^2 r_2^{-2})^2, r_1 \leq r_2;$$

$$\zeta(r_1, r_2) = \rho r_1^{-4} (1 - r_2^2 r_1^{-2}), r_1 > r_2$$

The total pressure losses are composed of the hydraulic losses and the local resistance losses:

$$\Delta P = \sum_i \Delta P_{ki} + \sum_i \Delta P_{sij} \quad (2)$$

Where  $i$  is the number of the capillary and  $j$  is the number of the next node after it.

Next, consider a chain consisting of  $N$  capillaries of a given lattice with radii  $r$  greater than a given value  $r_1$ .

After dividing (2) by the length of the chain  $Nl$ , we obtain

$$\frac{\Delta P}{Nl} = \frac{8\mu}{\pi} \frac{\sum_{i=1}^N r_i^{-4}}{N} q + \frac{\sum_{i=1}^N \zeta(r_i, r_j)}{Nl} q^2$$

It is assumed that the length of the conduction chain  $Nl = L$  is large enough so that a transition from summation to integration is possible.

The pore radii are independently distributed with distribution density  $f(r)$ , and then the pressure gradient acting in the chain can be written as [20].

$$\frac{\Delta p}{L} = I_1(r)q + I_2(r)q^2, \quad I_1(r) = \frac{\frac{8\mu}{\pi} \int_r^\infty f(\rho) \rho^{-4} d\rho}{\int_r^\infty f(\rho) d\rho},$$

$$I_2(r) = l^{-1} \frac{\int_r^\infty \int_r^\infty \zeta(\rho_1, \rho_2) f(\rho_1) f(\rho_2) d\rho_1 d\rho_2}{\left[ \int_r^\infty f(\rho) d\rho \right]^2}$$

Where  $r$  is the minimum radius of the capillaries contained in this chain.

The values  $I_1$  and  $I_2$  are analogues of the hydraulic and local resistance coefficients for the porous structure.

If we express  $q$  in terms of  $\Delta P / L$ , we obtain the relationship between the fluid flow in the  $r$  chain and the external pressure gradient acting on the macroscopic region under consideration:

$$q = \frac{2\Delta P / L}{I_1(r) + (I_1^2(r) + 4I_2(r)\Delta P / L)^{1/2}}$$

Using the Shklovsky-de Genet model of the IC structure [19] and the hierarchical model for the summation of the conductivities of  $r$ -chains [19],[25], we obtain Newton's fluid flow law in a granular porous medium

$$w = \int_0^{r_c} \frac{2G(r)|\Delta P| dr}{I_1(r) + (I_1^2(r) + 4I_2(r)|\Delta P|)^{1/2}} \quad (3)$$

Here,  $w$  is the modulus of the fluid flow rate, defined as the flow rate through a unit area of the flow cross section;  $|\Delta P|$  is the modulus of the pressure gradient, and  $G(r)$  is the distribution density of the  $r$ -chains.

Obviously, the behaviour of the function  $w(|\Delta P|)$  depends essentially on the ratio of the parameters contained in (3).

For the case where  $I_1^2(r) \gg 4 I_2(r) |\Delta P|$ , the second term in parentheses in the denominator of the integrand of equation 3 can be neglected.

As a result, expression (3) takes the form

$$w \cong |\Delta P| \int_0^{r_c} \frac{G(r)}{I_1(r)} dr \quad (4)$$

Consequently, the flow law (3) is a linear dependence (Darcy's law) at low pressure gradients.

If  $I_1^2(r) \ll 4 I_2(r) |\Delta P|$ , we can neglect the first term in the indicated parenthesis and write (3) in the form

$$w \cong |\Delta P|^{\frac{1}{2}} \int_0^{r_c} \frac{G(r) dr}{(I_2(r))^{1/2}} \quad (5)$$

The relation (5) is a quadratic flow law valid for large gradients.

The linear law (4) starts to be violated (Fig. 4) when the inertial losses increase to the level of hydraulic losses. In this case, the terms in brackets become quantities of the same order:

$$I_1^2(r) \approx 4I_2(r)|\Delta P|$$

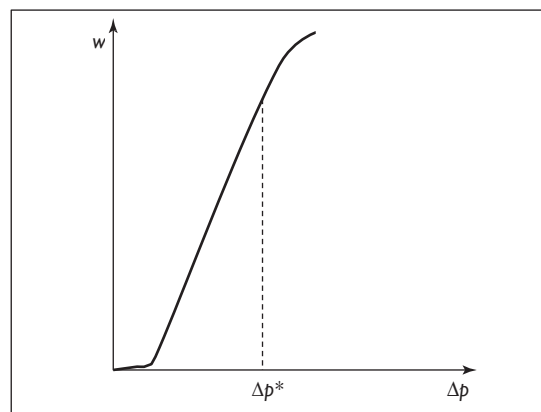


Fig. 4 Dependence of the flow rate on the pressure gradient

The limiting pressure gradient, above which the fluid flow is no longer described by the linear law, is thus

$$\Delta p^*(r) \cong \frac{I_1^2(r)}{4I_2(r)} \quad (6)$$

### III. NUMERICAL SOLUTION

The solution to the problem of determining the upper bound on the applicability of Darcy's linear law is reduced to the calculation of the coefficients  $I_1$  and  $I_2$  in expression (6). Having established the parameters of the fluid (density and viscosity), let us analyse how the structure of the pore space affects the location of this limit.

The most important properties of the pore space are the density function of the distribution of the pore channels along the radii (porosimetric curve), as well as the lattice coordination number and the mean capillary length [26]-[30]. The porosimetric curve, and to some extent the lattice coordination number, characterize the degree of homogeneity (or heterogeneity) of the reservoir pore space.

Let us first consider the variants of a homogeneous porous rock with capillary radii of the order of  $10^{-6}$ ,  $10^{-5}$ ,  $10^{-4}$  m. The distribution functions considering the normalization condition

$$\int_0^{\infty} f(r) = 1$$

In this case they have the form shown in Figure 5.

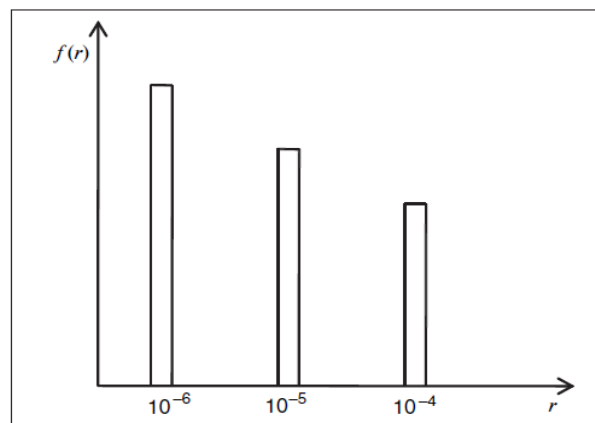


Fig. 5 Model distribution functions for homogeneous porous media:  $r = 1.25 \cdot 10^{-6} \pm 0.25 \cdot 10^{-6}$  m;  
 $r = 1.25 \cdot 10^{-5} \pm 0.25 \cdot 10^{-5}$  m;  $r = 1.25 \cdot 10^{-4} \pm 0.25 \cdot 10^{-4}$  m.

The calculations were made for a fluid with a coefficient of viscosity  $\mu = 5 \cdot 10^{-3}$  Pa.s and a density  $\rho = 850$  kg / m<sup>3</sup>. These calculations give the following values of the limiting pressure gradients for the corresponding distribution functions:

1.  $\Delta p^* = 1,5 \cdot 10^{12}$  Pa/m;
2.  $\Delta p^* = 1,4 \cdot 10^8$  Pa/m;
3.  $\Delta p^* = 0,2 \cdot 10^5$  Pa/m.

A qualitative picture of the influence of the characteristic radius of pore channels in homogeneous porous media on the boundary pressure gradient is shown in Figure 6.

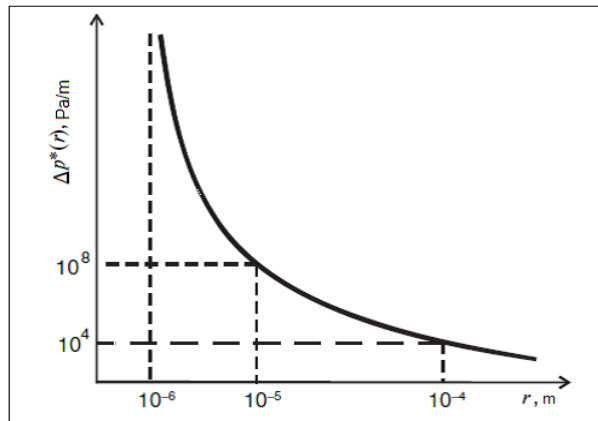


Fig. 6 Qualitative curve of the dependence of the limiting pressure gradient on the radius of the capillaries for homogeneous porous media

Consider next a real pore reservoir whose porosimetric curve is determined by the standard method on a core sample (Fig. 7) [31]-[34].

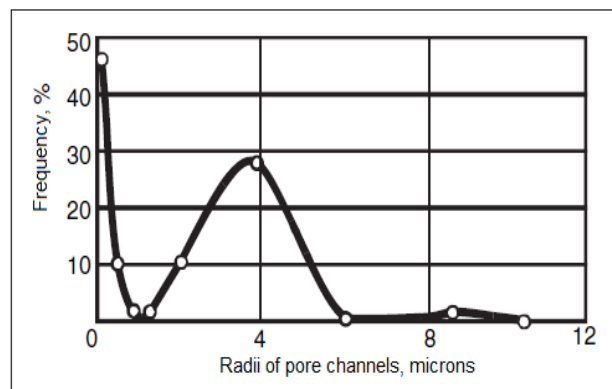


Fig. 7 Porosimetric curve of a real rock sample

To make this distribution function more convenient for the calculations, we take its representation in the form of a linear spline (Fig. 8). The results of the calculations are shown graphically in Fig.9.

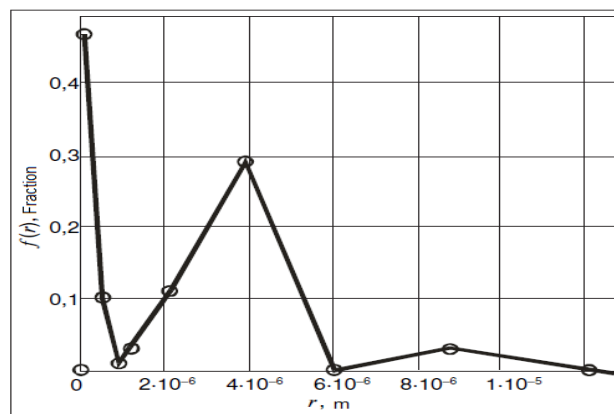


Fig. 8 Approximation of a porosimetric curve by a linear spline

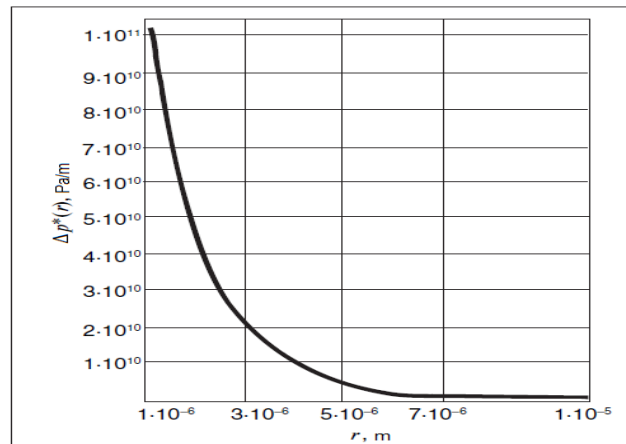


Fig. 9 Curve of the dependence of the limiting pressure gradient on the radius of the capillaries for the porosimetric curve of the core sample

Since the limiting pressure gradient is a function of the capillary radius, each  $r$  chain will have its own local limiting gradient. A deviation from the linearity of the macroscopic flow law occurs when the applied external pressure gradient exceeds the smallest of the local boundary gradients. It is this smallest local boundary gradient that is naturally defined as the global value of the boundary gradient for a given porous medium.

It is obvious that the smallest local boundary gradient occurs for the thickest  $r$ -chains of IC: the so-called  $r_c$ -chains. In [19], [20] it is shown that, with an accuracy of up to a few percent, the radius  $r_c$  is given by the ratio

$$\int_0^{\infty} f(r) dr = \frac{D}{z(D-1)}$$

The right-hand side of equation (7) is the limit of flow in a porous medium ( $D$  is the dimension of the problem), and the lower limit of integration is the critical radius  $r_c$ . Accordingly, the global boundary gradient is determined by the  $r_c$  value (Fig. 10).

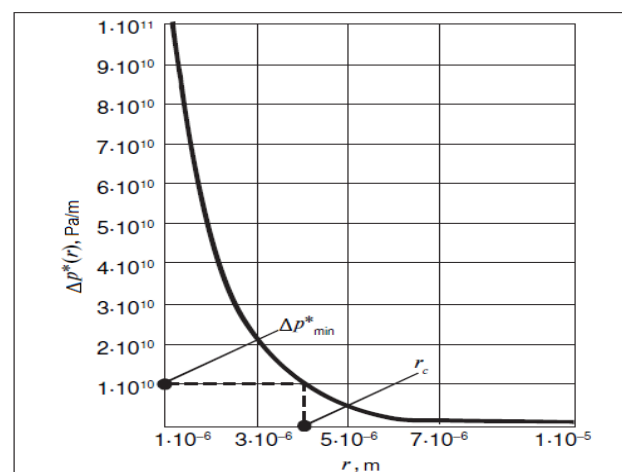


Fig. 10 Determination of the global limiting gradient along the critical radius  $r_c$



#### IV. CONCLUSIONS

An investigation has been made to find a method for the direct determination of the upper limit of the validity of the linear flow law (Darcy's law) for any porous media.

- The proposed approach provides a sound methodology for directly determining the upper bound of the validity of the linear flow law (Darcy's law) for any porous media. To apply the technique, it is necessary to know the porosimetric curve of the rock sample, the average capillary length and the coordination number of the grid simulating the pore space.
- Calculations have shown that Darcy's law works best in shallow- and medium-porosity reservoirs when the pore-channel radii do not exceed  $(2 \dots 3) \times 10^{-5}$  m. In such media, the limiting gradient is about 108 Pa / m, which is well above the typical values of pressure gradients in the field.
- It was found that for large-pore rocks ( $r \geq 10^{-4}$  m), Darcy's law is violated in almost the entire range of real gradients, since in this case the limiting gradient does not exceed 104 Pa / m.
- Paragraphs 2 and 3 of these conclusions are in agreement with the experimental data.

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