

Flexure wheels for spacecraft attitude control

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Abstract

This paper presents innovative mechanisms capable of advantageously providing attitude control for spacecrafts. These new mechanisms, which we have named *flexure wheels*, are the dynamic equivalent of a rotating wheel and can be entirely implemented with flexures.

A *reaction wheel* is a well known device for controlling the orientation of spacecrafts [5]. It consists in a motorised fly-wheel which is placed within the spacecraft. To set the wheel into angular rotation, a torque is applied to the wheel which in response applies the opposite torque back to the spacecraft, according to Newton's third law. This reaction torque is how the spacecraft rotates in order to control its orientation. In order to enable this wheel to rotate around a fixed axis, several methods have been implemented such as ball bearings, which suffer from frictional losses and imperfections which lead to vibrations and failure, as well as magnetic bearings which do not suffer from these issues but have an increased power consumption and complexity [5].

The subject of this paper is to introduce alternative mechanisms that are able to produce the same constant angular momentum as a rotating wheel, but which do not suffer from the above defects.

In order to reach this goal, our inventions use flexure mechanisms to produce the required constant angular momentum. Note that the term *flexure mechanism* is exactly equivalent to *compliant mechanism* [18] [19]. The difficulty in this task is that flexures only have a limited stroke making it virtually impossible for a flexure bearing wheel to rotate around a fixed axis with constant angular momentum. We therefore found alternate methods for generating angular momentum by using flexure mechanisms.

Two methods are presented in this paper. The first consists of a rigid body whose centre of mass has a circular trajectory around a fixed point, but the body does not rotate around its centre of mass. The body moves in translation and acts dynamically as a point mass, and thereby generates angular momentum in a constant direction. The second consists of two bodies rotating around their centres of mass, but whose total angular momentum lies in a fixed direction.

The first method was successfully exploited in the *IsoSpring* project whose goal was to introduce new two degree of freedom oscillators in mechanical clocks and watches, in order to remove their traditional escapement mechanism [16]. The second mechanism is also inspired from the *IsoSpring* project where a sphere oscillating around its centre of mass provided a two degree of freedom oscillator less sensitive to the direction of gravity [15].

The paper presents flexure wheel designs along with their implementations. Moreover, methods to control the uniform circular motion are presented, among which a novel flexure bearing which restricts the motion of a body to translation on a circular orbit. Two prototypes were successfully built and tested. Finally, qualitative results from this proof of concept are presented.

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1 Introduction and statement of results

1.1 Attitude control for spacecrafts

A key aspect of sending a satellite into space is being able to control its orientation. This capability is defined as *attitude control* [2]. A number of techniques have already been developed in this goal. These include:

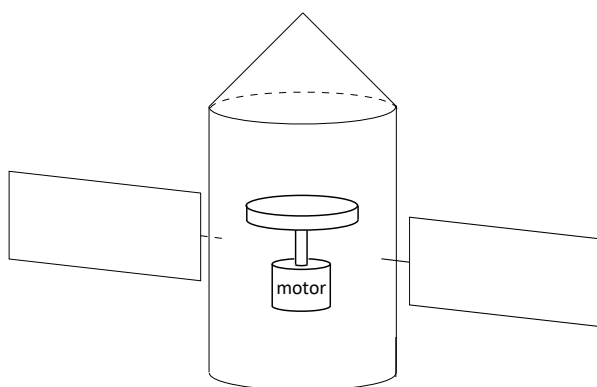
- Magnetic torquers which take advantage of Earth's magnetic field to generate Lorentz forces [2]
- Thrusters which expel matter and use Newton's third law of motion [2]
- Reaction wheels which also use the same action-reaction principle, but in rotation by means of accelerating or decelerating a set of wheels. [2]

The latter are the focus of this paper as they are based on mechanical engineering, do not expel any matter and do not require any external influence.

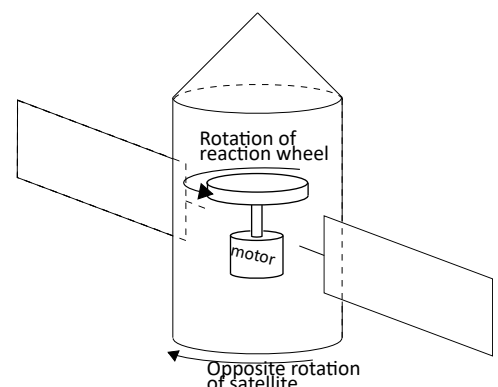
1.2 State of the art for reaction wheels

A reaction wheel consists in a fly-wheel whose angular rotation is motorised and which is placed within the spacecraft [3]. To set the wheel into angular rotation, the spacecraft applies a torque to the wheel which in response applies the opposite torque back to the spacecraft, according to Newton's third law. This reaction torque rotates the spacecraft in order to control its orientation (Fig. 1).

The principle underlying the reaction wheel is that the spacecraft is an isolated system having constant total angular momentum and when torque is applied to the reaction wheel, this increases its angular momentum so the rest of the spacecraft is forced to rotate in the opposite direction in order to keep the total angular momentum constant. This alternative view of a reaction wheel highlights one of its fundamental requirements: that it must be able to provide constant direction angular momentum in the frame of the spacecraft.



(a) Satellite and reaction wheel at rest



(b) Satellite with its reaction wheel set into motion

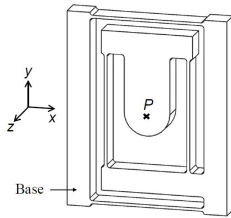
Figure 1: The principle of a reaction wheel.

During its use, the wheel naturally performs complete turns around its centre of mass and it remains in the same qualitative state. This property of being able to turn indefinitely in the same direction while retaining the same qualitative state is defined as *unlimited rotation*. This unlimited angular

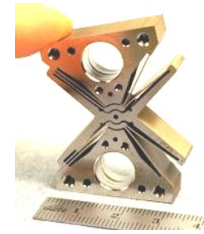
rotation requires bearings at the interface between the rotating mass and its central axis. Currently, the most common bearing implementation is ball bearings [13] [4]. However, these suffer from a number of issues, such as wear, failure and the need for lubrication. Due to these issues, we investigated the use of *flexures*, since these do not suffer from the above defects [18]. The application of flexures, also known as *compliant mechanisms* [19], to reaction wheels has, to our knowledge, not previously been explored.

1.3 Innovative designs of flexure wheels

Flexures are a well studied category of mechanical components which rely on elastic deformation to enable motion [18] [19]. They are analogous to standard mechanical building blocks such as a pin or slider (Fig. 2), but are free of friction (since no parts are sliding or rolling on each other). Their drawbacks include their limited range of motion and the fact that they act as springs and therefore require force to be kept away from their neutral position.



(a) XY translation stage made of two parallel leaf spring stages mounted in series [16]



(b) Butterfly hinge which acts as a pivot [17]

Figure 2: Examples of flexures

Since the goal of this paper is to find a dynamical equivalent of a reaction wheel using flexures, we have named our new mechanisms *flexure wheels*.

A key feature of reaction wheels is unlimited angular rotation. This aspect can evidently not be directly translated into flexures as these have a limited stroke.¹ The conceptual contribution of this article is the design of flexure mechanisms which behave like fly wheels even though each individual joint has a limited stroke.

Generating angular momentum using joints with limited stroke As unlimited angular rotation seems out of reach with flexures, other types of motion need to be considered in order to obtain constant angular momentum. Two types of motion have been found to satisfy this condition.

The first is a rigid body in translation on a circular trajectory (Fig. 3). As the body does not rotate around its centre of mass, i.e. keeps the same orientation, it can be considered as a point mass. This type of motion is of interest since, when evaluated at the centre of the orbit, the angular momentum is constant. This circular motion is implementable with flexures by using planar two-degree-of-freedom (2-DOF) mechanisms such as the XY-stage shown in Fig.2a. The issue with this single point mass is that the centre of mass is not fixed, which results in shaking forces exported to the spacecraft. The solution is to combine point masses in order to keep the centre of mass fixed while still conserving the overall angular momentum (Fig. 4).

¹Some progress in this direction seems to have been made using the Dirac belt trick [20] [21].

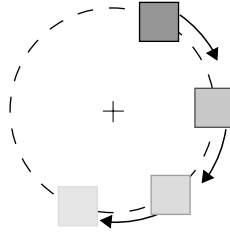


Figure 3: The different steps of a rigid body in translation on a circular orbit

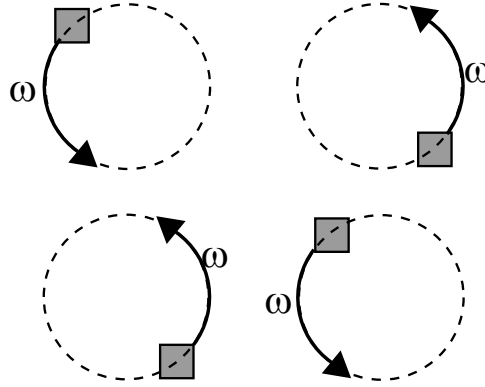


Figure 4: Flexure Wheel concept consisting in combining circular motions which results in an overall constant angular momentum.

The second type of motion that can also replace unlimited angular rotation is a rigid body rotating in circles around its centre of mass. An example of such a motion is a human eye tracking the edge of a circle: the eye is always in motion even though its stroke is limited (Fig. 5). Note that eye movements are essentially 2-DOF spherical rotations, see [15]. In this case, several of the requirements are already satisfied: the centre of mass remains fixed and the angular momentum has a constant amplitude. However, the angular momentum points in different directions throughout the movement. Similarly as to the previously discussed planar mechanisms, combining these spherical mechanisms properly enables angular momentum to be constant (Fig. 6).

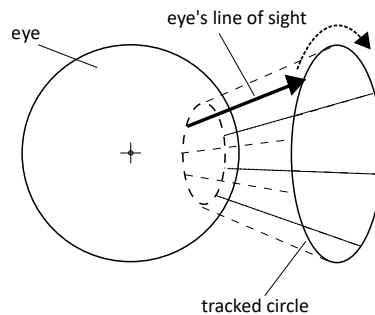


Figure 5: The different steps of an eye tracking the edge of a circle.

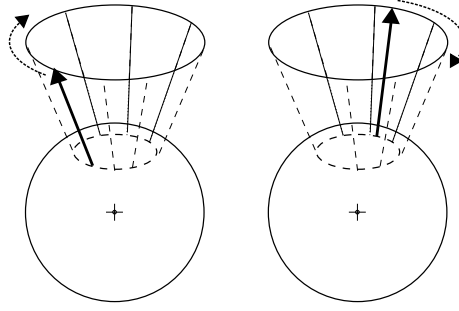


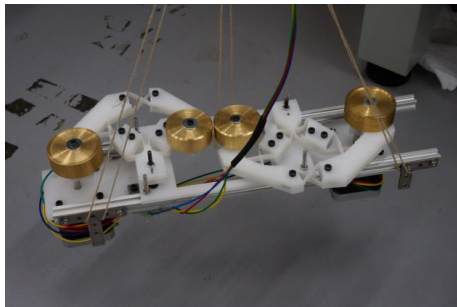
Figure 6: Flexure Wheel concept consisting in combining spherical mechanisms which results in an overall constant angular momentum.

1.4 Description of demonstrators and results

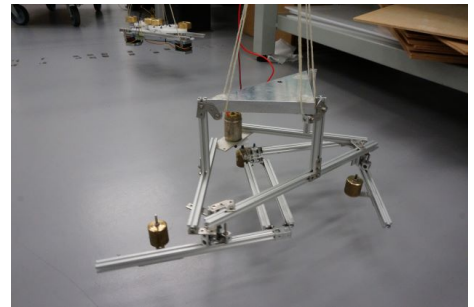
Two demonstrators were fabricated. They both use masses translating on circular orbits and rely on different combinations of mass trajectories. The first demonstrator uses exclusively masses on circular orbits which all contribute to the angular momentum while their combination keeps the centre of mass fixed. The second demonstrator mixes two classes of motion: a mass on a circular orbit which generates the angular momentum and a pair of masses oscillating linearly. The latter do not contribute to the angular momentum, but correct the centre of mass so that it remains fixed for the entire device.

The prototypes were tested in a proof-of-concept experiment which mimics the absence of gravity of outer space. The setup consists in suspending the flexure wheel on the end of a string and the frame holding the flexure wheel can be seen as the satellite which the flexure wheel must set into rotation (Fig. 7).

The system was initially at rest and as the flexure wheel was put into motion, the entire system rotated successfully.



(a) Demonstrator prototype 1



(b) Demonstrator prototype 2

Figure 7: Overview of the two demonstrators

2 Flexure Wheels for spacecraft attitude control

2.1 Attitude control for spacecrafts

Attitude control can be defined as the process of controlling the orientation of an aerospace vehicle in space [2]. This is an essential aspect of a space mission as it is required for example for correct

orientation of instruments such as a telescope, antennas and solar panels as well as manoeuvring two spacecrafts for docking.

Attitude control is separated into two main categories: spin stabilisation and three-axis stabilisation. Spin stabilisation consists in “setting the spacecraft spinning, using the gyroscopic action of the rotating spacecraft mass as the stabilizing mechanism.” Three-axis stabilisation consists in holding the spacecraft “in the desired orientation without any rotation.” [2]

We will focus on the three-axis stabilisation as it is more relevant for the presented mechanisms.

Types of actuators The main types of actuators for attitude control are:

- Thrusters: basically small rockets which expel matter and use the principle of action/reaction to orient the spacecraft. This is the same principle as in a jet engine or in Hero’s aeolipile (a radial steam turbine that dates back to approx. 100BC).
If we consider the system consisting of the spacecraft and the matter to be expelled as isolated (i.e no external forces act upon the system), the momentum is conserved. Once the matter is expelled in one direction, to conserve momentum, the spacecraft moves in the opposite direction. A single actuator therefore delivers a force. To generate a pure torque, a pair of thrusters must be used. These actuators can deliver a large amount of torque but cannot work indefinitely as fuel runs out and they have limited precision of stability.
- Reaction wheels/momentum wheels: these electrically powered wheels which are put into rotation at a controlled speed also use the principle of action/reaction, but for rotation. Indeed, by again considering the spacecraft and reaction wheel system to be isolated, the angular momentum is conserved. Therefore if the reaction wheel is accelerated (with respect to the spacecraft) in one direction, the spacecraft will rotate in the opposite direction. [2]
- Magnetic torquers: “Coils or (on very small satellites) permanent magnets exert a torque against [Earth’s] local magnetic field” [2]
- Control moment gyroscopes: These are similar to the reaction wheel, but instead of varying the speed of the wheel, the wheel is spun at a constant speed and direction of its axis is changed. This provides a gyroscopic effect. The maximum torque is higher than for reaction wheels, but the design is more complex, more expensive and heavier. These device are therefore usually used on larger satellites, such as the International Space Station. [2]
- Solar sails: produce thrust by the reaction force of reflecting incident light. [2]
- Gravity-gradient stabilisation: for a long enough satellite, the gravitational field is not assumed to be constant, but a gradient. This allows the satellite to stabilise itself with its long axis pointing towards Earth. Indeed, its lower end feels a stronger attraction than the upper end. [2]

The spacecraft is not an entirely closed system: it receives some external forces due to solar radiation pressure, gravity gradients, geo-magnetic torque and aerodynamic drag [9] [6]. Excess momentum can build up in the spacecraft. In the case of reaction wheels, the consequence is that the wheels will eventually reach their maximum speed and will no longer be able to control the attitude. This calls for a *moment desaturation* manoeuvre in which thrusters are turned on to bring the reaction wheels back to a normal speed [2].

Numerical values for these external torques are given in [9] and [6] for a nano-satellite resp. pico-satellite. Their total contribution are in the order of 10^{-5} [Nm].

2.2 State of the art for reaction wheels

2.2.1 Bearing types

The current reaction wheels use an actual wheel (i.e. a mass in angular rotation around one of its principal axes of inertia). This unlimited angular rotation requires bearings for which two main solutions currently exist: the standard ball bearings and the more elaborate and complex magnetic bearings.

Ball bearing reaction wheels The great majority of today's reaction wheels use ball bearings. This solution is relatively simple and cheap, but has the drawbacks of friction, lubrication and wear which limit its lifetime and reliability. Moreover, if the wheel speed passes by zero, stiction affects the spacecraft stability [4] [5].

As further explained in Sect. 2.2.2.2, ball bearing reaction wheels are also responsible for microvibration in satellites, either due to an imbalance of the wheel (i.e. the axis of rotation is not aligned with the centre of mass) or due to uneven raceways [4] [5].

Magnetic bearing reaction wheels The alternative solution is to use magnetic bearings in which the wheel axis is kept in place with magnetic levitation. For this reason, they have no matter-on-matter friction and overcome the issue of stiction when speed passes by zero. They are also capable of reaching much higher speeds than the ball bearing solution. [5]

These types of bearing normally have active control to maintain the wheel axis in the correct position. This requires sensing and continuous power input. This active control does have its advantages, it can for example adjust the axis of rotation to eliminate the imbalance. This involves ensuring that the axis of rotation passes by the wheel's centre of gravity and that it is parallel to the wheel's principal axis [5]. Moreover, gimbaling of the wheel is even possible which leads to gyroscopic forces. This means the "all three spacecraft axes may be controlled by just one wheel" [11].

2.2.2 Issues with ball-bearing reaction wheels

2.2.2.1 Failures and mission impact In the case of ball bearings, reaction wheels have a limited mechanical lifetime. Indeed, they are subject to wear and to electrostatic discharge (due to space weather) [7].

In the past, several reaction wheel failures have been reported in missions among which Hayabusa, Kepler and Dawn. These can significantly compromise the mission as the satellite loses its ability to orient itself [2].

2.2.2.2 Micro-vibrations in reaction wheels The satellite pointing stability is a critical aspect of a mission for earth observation, space astronomy and advanced scientific missions as it directly reduces the capabilities of optical and imaging systems [4].

One of the main drawbacks of ball-bearing reaction wheels is the induced micro-vibrations due to manufacturing and assembly tolerances. Indeed, reaction wheels are a major source of disturbance and have been studied and described in detail by Le [4]. Le lists three approaches to reduce these micro-vibrations: deal with the initial physical cause, add a damper and use active control. (The latter is more adapted for actively controlled magnetic bearings).

The physical causes of the micro-vibrations are separated in three categories: rotor unbalances, ball bearing vibration and torque ripple. Rotor unbalances are caused by the misalignment between the wheel's axis of rotation and its principal axis of inertia. Ball bearing vibration is caused by imperfections of the bearings' balls, raceways or cages. Torque ripple is caused by the switching of the motor's coils (we will no longer consider this effect as it is smaller than the others).

One last issue with reaction wheels is the need for magnetic cleanliness.

2.2.2.2.1 Rotor unbalance Rotor unbalances is known as one of the major disturbing factors of micro-vibrations. The rotor unbalances come under two forms:

- Static unbalance is due to the axis of rotation not passing by the wheel's centre of mass. This induces a centrifugal force on the bearing, $F_s = mr_g\omega^2$, where m is the wheel's mass, r_g the distance between the centre of mass and the axis of rotation and ω the wheel's angular velocity. We define as *static unbalance* $U_s = mr_g$ which depends only on the wheel's dimensions on not on its speed. The units of the static unbalance are therefore[kgm], but can also be expressed in [gcm].

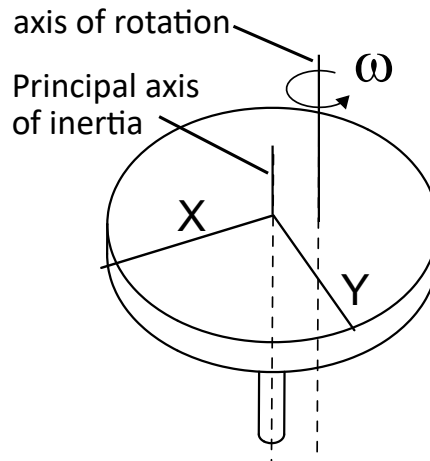


Figure 8: Static unbalance

- Dynamic unbalance is due to a misalignment between the rotation axis and the principal axis of inertia. This results in gyroscopic forces described by Eq. 5. In this case, even if the wheel rotates at a constant rate, the system is continuously exporting a torque which is perpendicular to the effective axis of rotation and whose direction turns with the wheel.

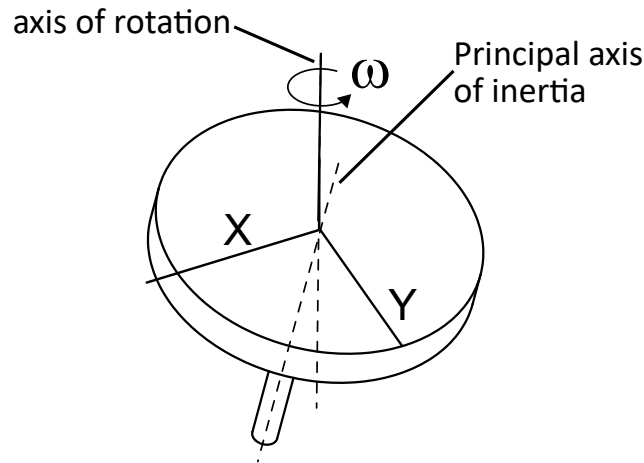


Figure 9: Dynamic unbalance

2.2.2.2.2 Ball bearing vibration Ball bearing vibrations are caused by imperfections in the bearings' balls, cages and raceways. This is a well-studied field as it is a general mechanics question. As explained in [14], ball-bearing vibrations can be split into three groups depending on their origin: manufacturing, structure and handling. Structure imperfections consist in “imperfections and waviness which excite the resonances of the bearing and surrounding structures. The result is structural vibrations” [4].

- Manufacturing: the parts constituting the ball bearings are never perfectly manufactured. This means that neither the balls nor the raceways are perfectly round and smooth, but present defects described by waviness.
- Structure: “imperfections and waviness excite the resonances of the bearing and surrounding structures. The result is structural vibrations” [4].
- Handling: “contaminations and imperfections created during the assembly process or verification tests are vibration sources due to handling” . For example, “mechanical impact during verification tests such as random vibration and shock tests” can create such imperfections. [4]

2.2.2.3 Magnetic cleanliness in reaction wheels Satellites can be equipped with highly sensitive measurement instruments which are disturbed by the magnetic fields of the satellite itself. “Reaction wheels, often accommodated to meet pointing requirements, are a known source of disturbance to both the DC and AC magnetic fields” [12]. The strategies to mitigate these disturbances include keeping a distance between reaction wheels and instruments, adding mu-metal shielding and selecting the most similar magnets to build the motor.

2.3 Innovative designs of flexure wheels

Why wheels?

The new mechanisms that will be presented have a similar use and properties as reaction wheels and control moment gyroscopes (CMGs) for the fact that they rely on rigid mechanical parts in movement, do not expel any matter remain fixed to the spacecraft and do not require any forces external to the spacecraft. Indeed, the mechanisms that will be presented differ from magnetic torquers as these rely on

electro-magnetic forces. They also differ from thrusters as they do not expel matter. Last but not least, they differ from solar sails as they function without external forces (such as solar radiation pressure). We therefore focus on reaction wheels and CMGs in the rest of this work. Moreover, for simplicity we will limit ourselves to reaction wheels for the rest of the state of the art as CMGs also contain rotating wheels and therefore suffer from similar problems. The issues discussed for reaction wheels can thus be extended to CMGs.

We have however not yet addressed a central aspect of reaction wheels and CMGs: why are wheels used in particular? The answer comes from the fact that the spacecraft's orientation must be stable. This means that the spacecraft must not receive any exported torque or forces from the attitude control system. If a constant torque is exported from the attitude control system, the spacecraft will accelerate in rotation. If an alternating torque is exported, the spacecraft will shake in rotation. In the case of forces, we only consider alternating forces which will induce a shaking in translation as well as in rotation of the spacecraft (if the spacecraft's attitude control system's centres of gravity are not coincident). Moreover, the mechanism must be able to provide a constant angular momentum to compensate for the built up angular momentum due to external forces such as solar wind or gravity gradient.

In order to translate these requirements into physical terms, a brief introduction to systems mechanics is required.

Introduction to systems mechanics: We wish analyse the forces and torque that an attitude control mechanism exports to the rest of the spacecraft. In order to achieve this, we assume the spacecraft to be an inertial frame. This means that the spacecraft's orientation and position are not affected by the attitude control system.

We consider the attitude control mechanism to be a system of rigid bodies. It consists in a set of N masses m_i . The system's total mass is M and its centre of mass is located by vector $\mathbf{X}$.

The values we are interested in are the exported force $\mathbf{F}_{\text{exp}}$ and torque $\mathbf{M}_{\text{exp}}$ that the system applies to the spacecraft.

In an arbitrary reference frame, the angular momentum $L_{\text{sys}}^{\rightarrow}$ the entire system is given by:

$$\mathbf{L} = M\mathbf{X} \times \dot{\mathbf{X}} + \mathbf{L}_{\text{g}}. \quad (1)$$

This equation has two terms. The first is dependent on the frame's position with respect to the solid's centre of mass $\mathbf{X}$, its time derivative $\dot{\mathbf{X}}$ and the solid's mass M . The second term of the equation is $\mathbf{L}_{\text{g}}$, the angular momentum evaluated at the solid's centre of mass $\mathbf{X}$. It is defined in terms of ξ which is the position of each mass element dm in the frame of the centre of gravity:

$$\mathbf{L}_{\text{g}} = \int \xi \times \dot{\xi} dm. \quad (2)$$

We do not wish a reaction wheel to export forces and therefore its centre of mass must remain fixed. Indeed, the exported force $\mathbf{F}_{\text{exp}}$ is given by:

$$\mathbf{F}_{\text{exp}} = M\ddot{\mathbf{X}}. \quad (3)$$

To have no exported force, we therefore have $\ddot{\mathbf{X}} = 0$. Moreover, it would make no sense that $\ddot{\mathbf{X}} = 0$ and $\dot{\mathbf{X}} \neq 0$ as this would lead to the wheel forever drifting away from the spacecraft at a constant velocity. We therefore also have $\dot{\mathbf{X}} = 0$, which proves that the centre of mass must remain fixed in order to export no forces to the spacecraft.

In the case we are interested in where $\dot{\mathbf{X}} = 0$, we have

$$\mathbf{L} = \mathbf{L}_g. \quad (4)$$

In the case of a standard reaction wheel, the system consists in a single solid part in angular rotation around its centre of mass. This enables $\mathbf{L}_g$ to be written as $\mathbf{L}_g = \Theta\omega$, where Θ is the three by three inertia tensor. As explained in [1] the difficulty in this equation is that, in the inertial frame, Θ changes as the solid rotates. For this reason it is easier to place ourselves in the non-inertial frame of the solid itself, in which Θ is constant. The consequence is that the exported torque $\mathbf{M}_{\text{exp}}$ is slightly more elaborate:

$$\mathbf{M}_{\text{exp}} = \dot{\mathbf{L}} = \dot{\mathbf{L}}_g = \Theta\dot{\omega} + \omega \times \Theta\omega \quad (5)$$

The last step is introducing the fact that the wheel rotates around one of its principal axes of inertia. This means that ω is an eigenvector of Θ and therefore $\omega \times \Theta\omega = \omega \times \lambda\omega = \mathbf{0}$. This leads to the simple and well known equation:

$$\mathbf{M}_g = \dot{\mathbf{L}}_g = \Theta\dot{\omega}. \quad (6)$$

If the rotation were not along a principal axis of inertia, the wheel would export an additional torque. Moreover, this torque lies in the plane perpendicular to the axis of rotation and turns with the solid. Stability and accuracy of orientation as well as lifetime are key aspects of an attitude control system. The stability and accuracy of orientation are impacted by the micro-vibrations and misalignment in ball bearings. The lifetime is also determined by the ball bearings. Moreover, assembling a reaction wheel is elaborate as the system must be encapsulated in a low pressure atmosphere prevent degassing of lubricant. It can therefore be understood that these bearings are a critical and limiting factor in reaction wheels.

For these reasons, the goal of this work is to provide new flexure mechanism designs capable of providing attitude control. The main challenge lies in the fact that the essence of a wheel is unlimited angular rotation. This behaviour obviously cannot be directly translated into flexures as they have a limited range of motion. We therefore need an articulated structure in which each joint has a limited range of motion and which, for an external observer, behaves like a wheel.

To be able to provide attitude control, the mechanism cannot export any forces to the spacecraft and must be able to export a controlled torque to the spacecraft. Moreover, the mechanism must be able to have a constant angular momentum in order to keep the spacecraft in a fixed orientation even in presence of external forces. Indeed, these external forces cause a build-up of angular momentum to the whole spacecraft which must be compensated by the attitude control system. To export torque to the rest of the spacecraft, the mechanism's angular momentum must vary. This can be achieved by changing its amplitude, its direction, or both.

The actuation must also be frictionless in order to avoid wear, the release of particles and the need for lubrication. The use of a standard rotating motor is therefore prohibited, as such a motor is in itself a wheel (with ball bearings) and therefore has the issues we are in fact trying to solve. We can now summarize the mechanisms' specifications.

Summary of mechanism specifications

1. No exported forces
2. Capable of providing angular momentum that is constant in direction and in amplitude
3. Capable of modifying the amplitude of angular momentum, either in amplitude or direction
4. The actuation must be frictionless

Outline and overview of found mechanisms and methods of use: We start by examining mechanisms that satisfy conditions 1 and 2 (which is equivalent to a wheel rotating at constant speed). We then discuss condition 3 with different strategies to modify the angular momentum. Detailed designs are then presented which allows the actuation of condition 4 to be dealt with.

Two main classes of mechanisms have been invented: planar and spherical mechanisms. The planar mechanisms consist in a set of point masses each that collectively have a controllable angular momentum. The spherical mechanisms are based on a set of solids that rotate around their centre of gravity in an alternate manner. By combining these solids, a total angular momentum with a constant direction is obtainable.

The invented mechanisms have two methods of use. A first in which the mechanism is considered as a two-degree-of-freedom (2-DOF) oscillator. In this case the mechanism moves at its natural frequency and the angular momentum is changed by adjusting the oscillation amplitude (which is equivalent to modifying the moment of inertia of the system). The second method of use consists in constraining the mechanism to circular orbits of fixed amplitude. This reduces its degrees of freedom to one. In this case, the mechanism does not oscillate at a fixed frequency, but is free to follow its orbit at any arbitrary speed.

2.3.1 Abstract designs of new mechanisms

2.3.1.1 Planar mechanisms

2.3.1.1.1 Introductory example To introduce the principle of these mechanisms, we will first give a specific design as an example. We will start from a usual wheel in rotation and gradually bring modifications that do not change its behaviour from a external point of view (i.e. that do not change its exported forces and torque).

The starting point is a standard rotating wheel as shown in Fig. 10.

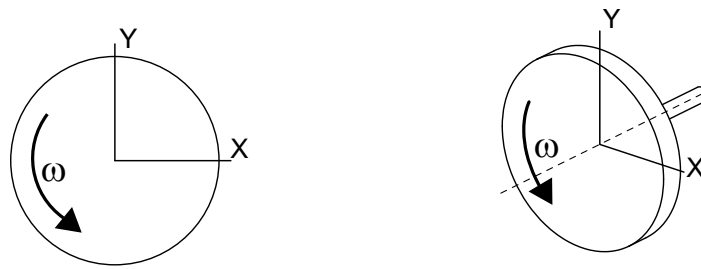


Figure 10: Front and isometric views of a wheel in rotation.

We now consider the mass of the wheel to be concentrated at two opposite sides of the wheel, represented by the dark circles in Fig. 11.



Figure 11

Since the mass is only contained the two dark circles, the rest of the wheel can be removed and replaced with a simple lever linking the two masses, as in Fig. 12.



Figure 12

We can even remove this lever and we are left with two independent masses at opposite ends of a circular trajectory, as in Fig. 13.



Figure 13

In this configuration, we removed the requirement for continuous angular rotation. This is the key step that enables flexures to be introduced.

A reaction wheel can therefore be replaced by a mechanism in which two point masses are at opposite ends of a same circular trajectory.

2.3.1.1.2 Formalisation of planar mechanisms This first example intuitively brought to light how a rotating wheel can be replaced by a set of point masses. To proceed in a more systematic and analytical manner, it is necessary to take step back to understand how point masses affect a system's centre of mass and angular momentum through their respective movements. These consideration will eventually provide building blocks which, when put together, are able to provide mechanisms such as the previous example which can replace a reaction wheel.

Intrinsic angular momentum: Depending on the trajectory that a point mass has, its angular momentum may or may not be zero depending at the point at which it is evaluated. In other words, for

some trajectories it is possible to find a point of observation at which the angular momentum is zero at all times while for other trajectories it is impossible to find such points of observation. In the case where it is impossible to observe zero angular momentum, we define the point mass as having *intrinsic angular momentum*.

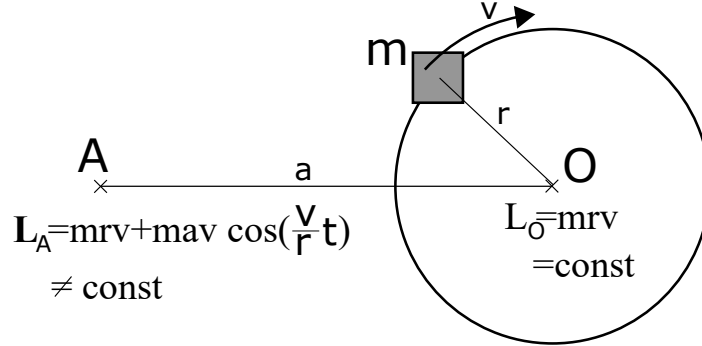


Figure 14: System with intrinsic angular momentum consisting of a mass m on a uniform circular motion.

An example of trajectory that has intrinsic angular momentum is a mass with a clockwise uniform circular motion as shown in Fig. 14. In this system, a mass m on a uniform circular motion of radius r at velocity v . When evaluating the angular momentum L_A from an arbitrary point A at a distance a from the orbit's centre O , L_A is non-zero, though not constant (see appendix for complete calculation). However, when evaluating from O , the angular momentum L_O is constant and non-zero. An important remark is that this mass on a circular orbit cannot, on its own, replace a reaction wheel since its centre of mass (the mass itself) is not fixed; it follows the same circular orbit.

An example of trajectory that does not have intrinsic angular momentum is that of a mass oscillating back and forth on a straight line as shown in Fig. 15. Indeed, even if the angular momentum L_A observed at point A is equal to $mr v$, when observed from points B or C , the angular momenta L_B and L_C are both zero at all times.

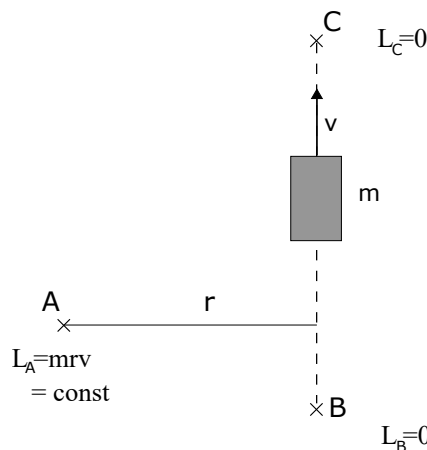


Figure 15: System without intrinsic angular momentum consisting of a mass m moving in a straight line at velocity v .

A last example is two masses that are each oscillating on their respective linear trajectories which are at 90° as shown in Fig. 16. The two masses m are located at points A and B and their oscillation amplitudes R are equal. Moreover these oscillations have a relative phase shift of 90° . The centre of

mass is therefore located at point C which follows a uniform circular motion of radius $R/2$ (anticlockwise in this illustration). Concerning the angular momenta, when observing at points D and E , their angular momenta are non-zero due to masses A respectively B . However, when observing from point O , the angular momentum is always zero and therefore the system has no intrinsic angular momentum

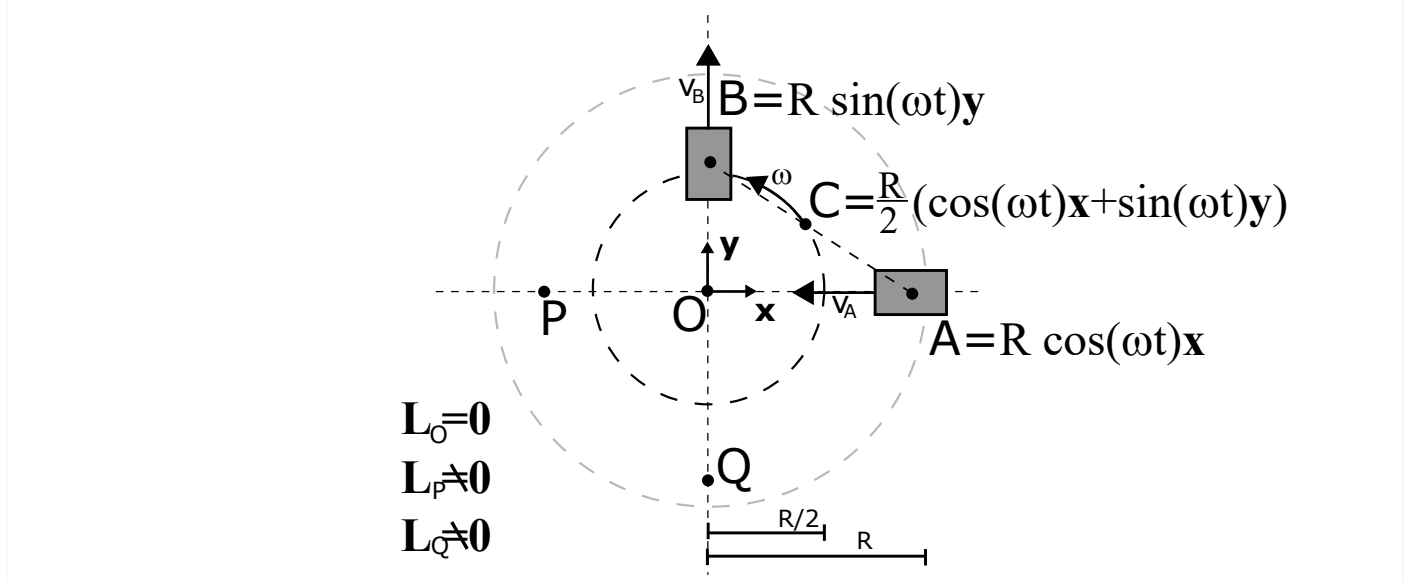


Figure 16: System without intrinsic angular momentum consisting of two identical masses oscillating linearly around a same point.

We have now provided the necessary building blocks to create several equivalents of a reaction wheel using point masses. We have indeed shown the circular orbit in which the angular momentum can be constant, but which suffers from a moving centre of mass. We have also shown the last example which has a moving centre of mass but with no angular momentum. It is then easy to understand that several of these mechanisms can be combined to reach a fixed centre of mass while still having angular momentum. Several combinations exist. A first example is shown in Fig. 17 in which the point mass from Fig. 14 (with intrinsic angular momentum) is combined with the system of Fig. 16 (without intrinsic angular momentum). The mass with uniform circular motion provides the angular momentum but suffers from a moving centre of mass. The two masses compensate the centre of mass displacement without adding angular momentum. When these two subsystem are combined, the total system therefore has a fixed centre of mass and angular momentum L that is constant in both amplitude and direction. Another possible combination is taking twice the point mass of Fig. 14 so that the displacements of their respective centres of mass compensate each other while their angular momenta are added. This is in fact the underlying approach that leads to the introductory example (Fig.13).

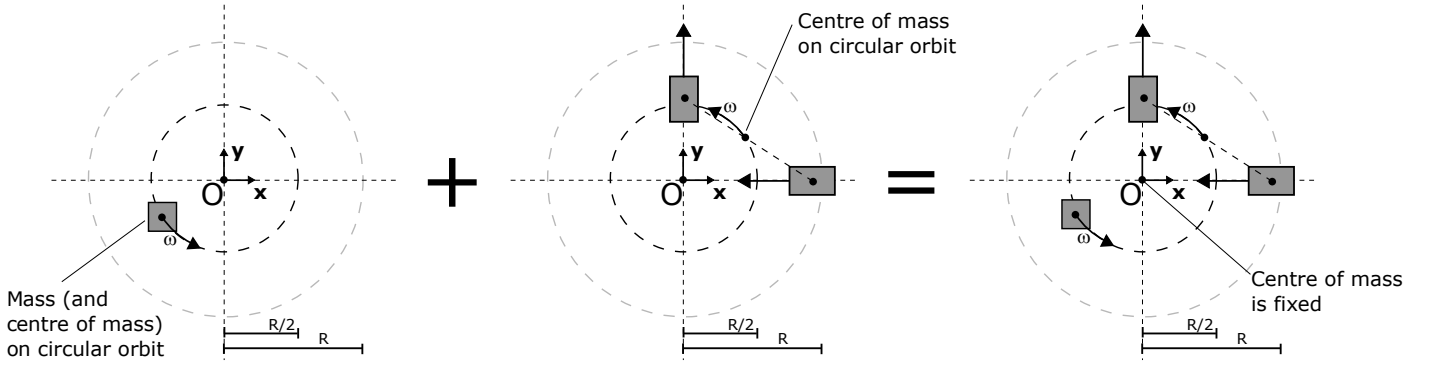


Figure 17: Equivalent of a reaction wheel (right) consisting in a mass on a circular orbit (left) combined with a set of two masses oscillating around the centre of this orbit (centre).

2.3.1.2 Spherical mechanisms A critical design criterion in spatial applications is optimising the amount of angular momentum can be stored with respect to the mechanism's weight and volume. In the planar mechanisms, this requires the circular orbits of the masses to be as large as possible or the mass speed to be increased. There are no ways of optimising the shape or arrangement of these point masses as their effect relies directly on their effective mass.

To overcome this limitation of using point masses in translation, solid masses are considered. In this case, a rigid body whose movement is not restricted to translation within a plane, but to rotation around its centre of gravity. The advantage of this approach is that it is not the solid's mass that is responsible for the angular momentum, but its moment of inertia. By using a solid whose mass is distributed as far as possible from the centre of rotation, it is possible to obtain a large moment of inertia (and therefore a large angular momentum) for a light mechanism.

Circular orbits for spherical mechanisms Since these mechanisms will be implemented with flexures, the solid in rotation has a limited range of motion. It is however possible for such an object to come close to continuous motion by following a closed circular orbit. We will use the analogy of a planet to describe the orientation of the solid. We assume that, at rest, the 'north pole' points upwards along the z -axis. The circular orbit previously mentioned involves the north pole moving on a circular orbit around the z -axis (the planet's axis between its north and south poles therefore cuts the z -axis with a constant angle). The different steps of such a circular motion are shown in Fig. 18 in which a cardan joint provides the motion of the solid part. In this figure, one end of the cardan joint is the ground and the other, which corresponds to the north pole, follows the said circular motion.

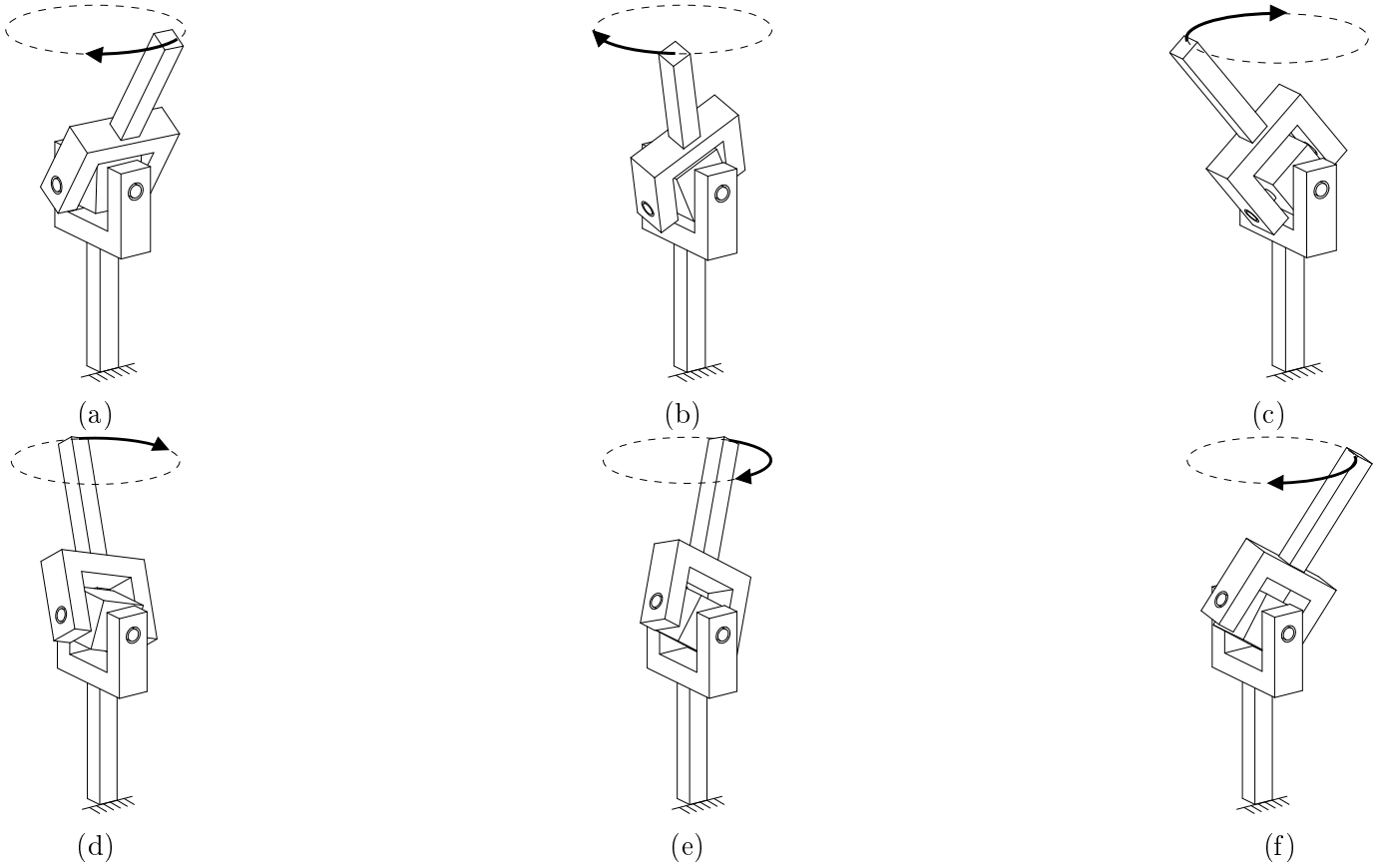


Figure 18: Cardan joint following a circular orbit.

Angular momentum of a spherical mechanism on a circular orbit At any moment of the circular orbit, the solid's instantaneous angular velocity is a vector that lies within the plane described by the z -axis and the north pole. Indeed, since the tilt angle of the planet from the z -axis does not change, there is no component of the rotation that exits the said plane. Moreover, as shown in Sect. 4.3, if the solid's kinematics are governed by a constant-velocity joint (CV joint), the angular velocity will remain the same throughout the orbit (it will naturally rotate with the solid). The trace of the angular velocity will therefore describe a cone whose apex is the solid's centre of rotation. Indeed, the angular momentum vector, when departing from the solid's centre of rotation, points onto a circle that lies in a plane parallel to the xy -plane and centred on the z -axis.

Also, since the angular momentum is proportional (and colinear) to the angular velocity, the angular momentum also describes a similar cone.

Conclusion: The angular momentum of a spherical mechanism (with CV joint) on circular motion has a constant amplitude, a constant z -component, but changes direction in the xy -plane.

The evolution of angular momentum with respect to time is shown in Fig. 19 which shows the cardan in different configurations and the corresponding angular momentum. The plane containing the angular momentum, the north pole and the z -axis is also shown. Note: a cardan joint is not a CV-joint, but it has nevertheless been used for illustrative purposes as it is more visually understandable than other joints.

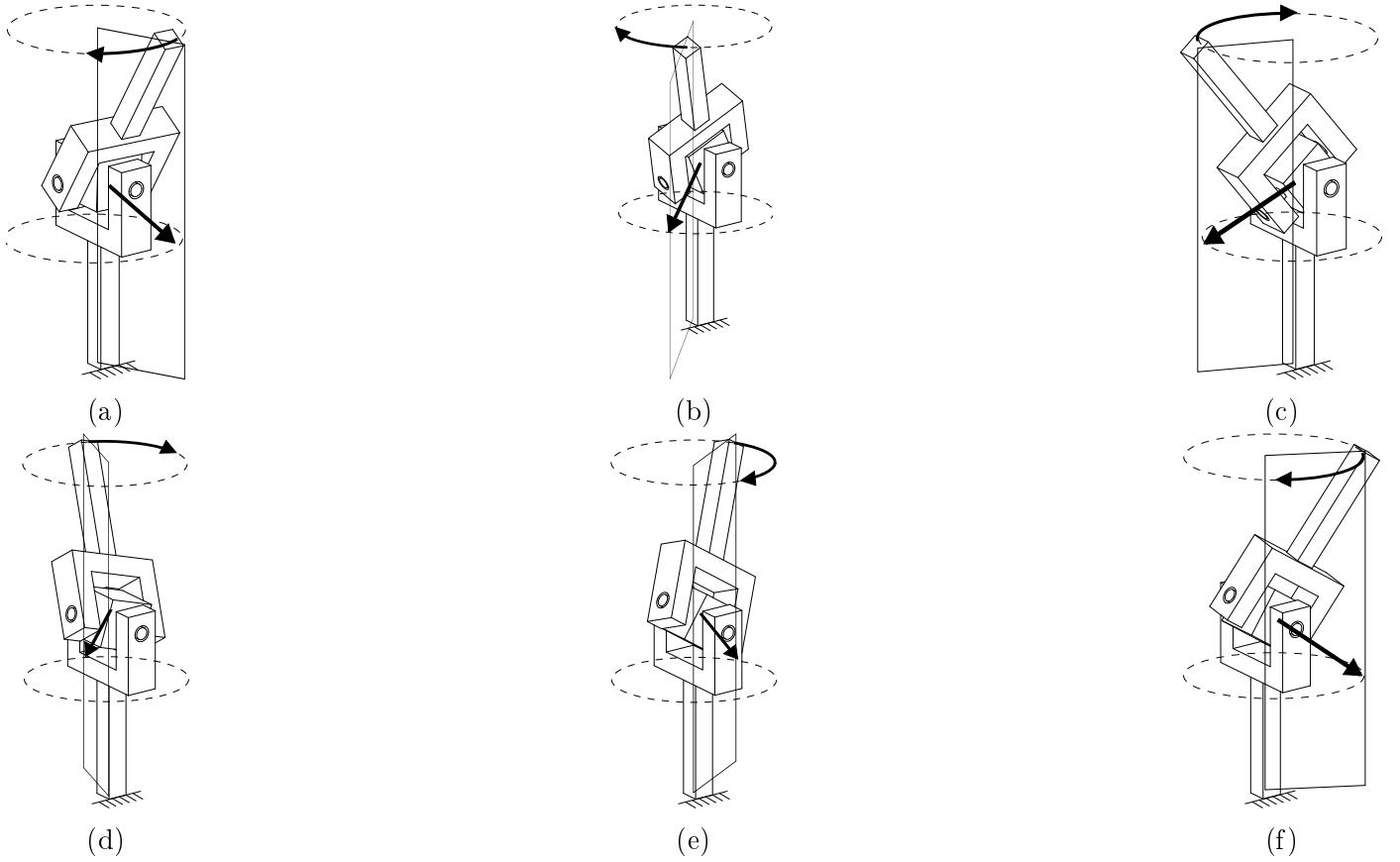


Figure 19: Same cardan as Fig. 18 following a circular orbit. The angular momentum is shown along with the circle onto which it points. The plane containing the angular momentum and the solid's north pole is also shown.

Combining spherical mechanisms to obtain a constant angular momentum We will now restrict ourselves to the case of CV joints. To make the drawings clearer, we also only represent the different solids in rotation by their respective angular momenta. An example of this representation is Fig. 20 which shows the angular momentum for each step as well as the cone that is swept.

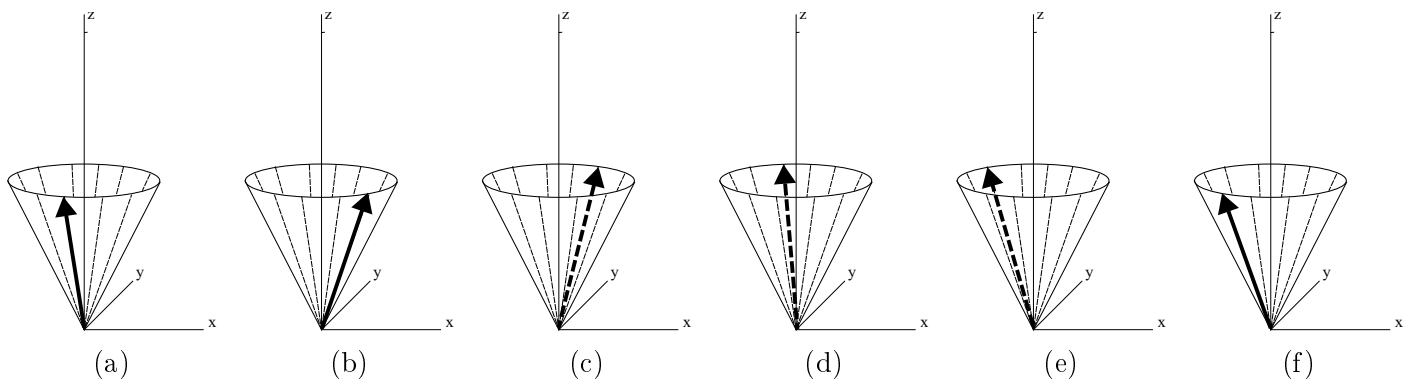


Figure 20: The different steps of angular momentum for a spherical mechanism on a counter-clockwise circular orbit. The angular momentum is dotted when at the rear side of the cone.

The goal of these flexure wheel being to generate a constant angular velocity, we ask ourselves the following question: How can we isolate the angular momentum's constant z -component while removing

the alternating xy -components? A solution is to combine spherical mechanisms so that their respective angular momenta will (vectorially) summed.

The simplest combination consists in two identical spherical mechanisms that are at opposite ends of their respective orbits. Since these two mechanisms are in anti-phase, their angular momenta have the same z -component, but opposite x - and y -components. Since these individual angular momenta are vectorially summed, the total angular is therefore constant in direction and amplitude along the z -axis and a flexure wheel has thus been created. Fig. 21 illustrates this case in which the first spherical mechanism's angular momentum and cone is shown along with angular momentum of the second spherical mechanism. It can be seen that throughout the different steps the vectorial sum of the angular momenta remains constant along the z -axis.

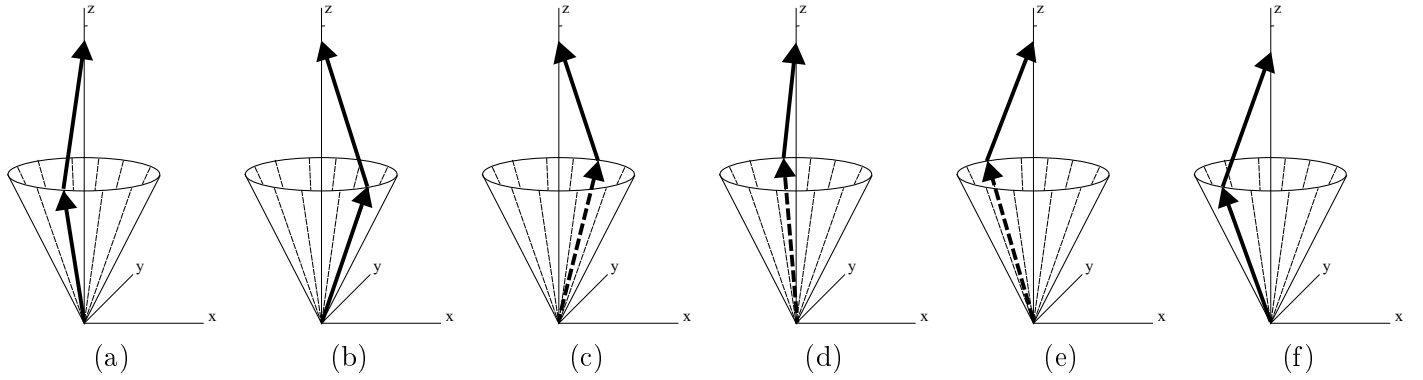


Figure 21: The different steps of angular momenta of two spherical mechanisms combined in order to achieve an overall constant angular momentum along the z -axis.

Conclusion: A flexure wheel can be created with two identical spherical mechanisms at opposite ends of their circular orbit

Some additional considerations may be noted concerning the combination of spherical mechanisms:

- It is possible to combine any number of spherical mechanisms, as long as the x - and y -components of their angular momenta cancel each other out. For example, three spherical mechanisms which have a 120° with respect to each other
- It is not strictly necessary to have identical spherical mechanisms on circular orbits of identical amplitude. The only requirement is that summed xy -components of the angular momentum is zero. It then becomes imaginable to use spherical mechanisms with different moments of inertia (which changes the cone amplitude) and different orbit amplitudes (which changes the cone angle).
- The spherical mechanisms can be arbitrarily placed with respect to one another. Indeed, as a spherical mechanism's centre of mass is fixed, the exported forces (which are zero) and exported torque do not depend on its location. This is similar to a standard reaction wheel which can be placed anywhere on the spacecraft.

2.3.2 Methods of use

It will now be discussed how these abstract designs can be used to provide attitude control. This will be done in two steps: a first which presents how a constant angular momentum can be obtained and a

second step which discusses how this angular momentum can be changed in order to export the desired torque.

2.3.2.1 Obtaining constant angular momentum A key aspect of all presented mechanisms is that they follow circular motion, which is not normally achieved with flexures. Indeed, the type of motion obtained with flexures is usually back-and-forth, whether it be rotational or rectilinear.

Two main methods to enable such circular motion have been found. The first consists in creating a new type of flexures that restrict motion to a circle, which are defined as *Circular Motion Flexures* (CMFs). The second option consists in using mechanisms that have two degrees-of-freedom, but only using them as one-degree-of-freedom.

Mechanically restricting the motion to circular orbits with Circular Motion Flexures

This method consists in design a novel type of flexures that restrict the motion to a circle. Such a flexure is therefore defined as *Circular Motion Flexure* (CMF).

The design is based on the buckling of a beam with a circular cross-section, as shown in Fig. 22. Due to the circularity of the cross-section, the buckling can take place in any direction. The centre of the beam, which deflects to the side, is therefore restricted to a circle.

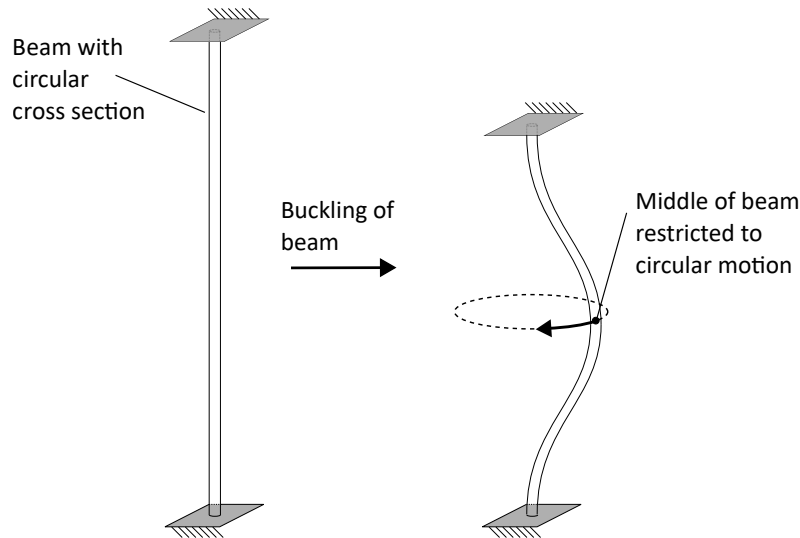


Figure 22

By observing this system in more detail, it can be seen that if a mass is at the middle of the buckled beam, this mass has an additional three degrees-of-freedom in rotation since the two half-beams linking the mass to the ground have internal degrees of freedom. Moreover, this system lacks radial stiffness. Indeed, if a radial force is applied, such as the centrifugal force if the mass is in circular motion, the size of the orbit increases. To solve this issue, a design that uses rigid parts and localized flexible elements is proposed. The design is shown in Fig. 23 in which two rigid segments are linked together by a spherical joint (three degrees-of-freedom) and are linked to the ground by CV-joints (two degrees-of-freedom).

Note: Other combinations of joints are possible, as long as no hyperstatics are induced.

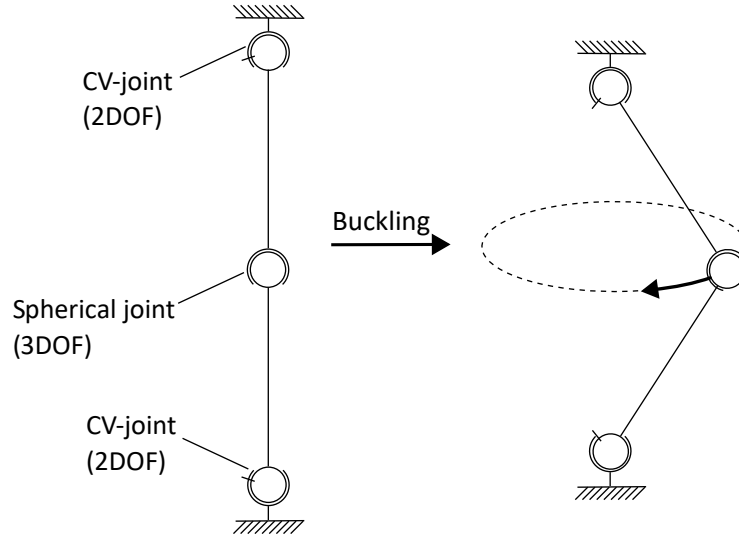


Figure 23

Using the mechanisms as isotropic 2-DOF oscillators A mass needn't be physically restricted to a fixed circular orbit in order to follow a circular motion. Indeed, the mechanism has two degrees-of-freedom and is implemented with flexures which provide a restoring force. If this restoring force is isotropic, the mechanism can be seen as an isotropic 2-DOF harmonic oscillator. In this case, when oscillating freely, the mass's trajectory is an ellipse and its period is independent of its amplitude (Fig. 24). A circular motion is then nothing more than an ellipse with the right proportions.

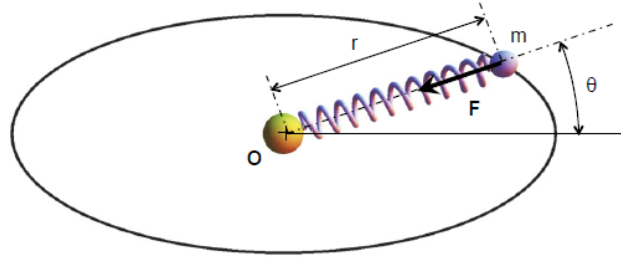


Figure 24: Illustration of an isotropic 2-DOF harmonic oscillator. [16]

These properties have been advantageously used for their time-keeping applications in the IsoSpring project (see [16]). Indeed, as the mass follows a circular motion at a constant rate, it can be directly linked to the rest of the gear train by means of a crank in a mechanical clock or watch (Fig. 25).

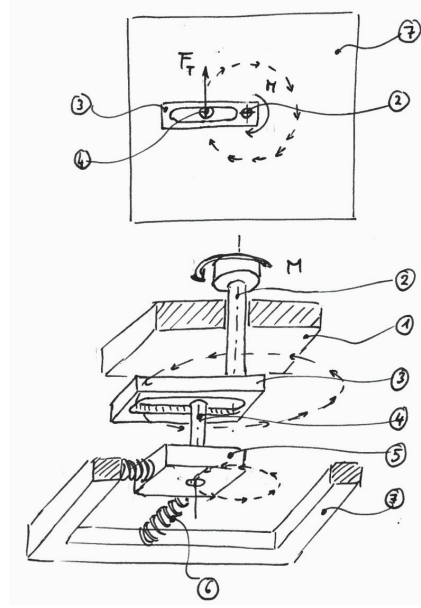


Figure 25: Illustration of the crank mechanism used to drive the 2-DOF oscillator. [16]

In this case, increasing the angular momentum it is not done by modifying the angular frequency (as it is given by the oscillator's natural frequency), but only the orbit's amplitude. This is equivalent to increasing the system's moment of inertia.

Several practical issues arise when using the mechanisms as 2-DOF oscillators due to asymmetries in the mechanism. Indeed, if the restoring force is not perfectly isotropic (inherently present in manufacturing and assembly), the orbit will not be perfectly circular nor elliptic. If left to itself, the trajectory will degenerate into patterns such as Lissajous figures. Moreover, the restoring force is no longer central and therefore creates a torque (when evaluated at the neutral position) which is exported to the spacecraft.

Forcing circular orbits with an active control This approach is similar to the previous method, but by having the mass go at a rate different from its natural frequency by means of actuation. This case can also be seen as a harmonic oscillator, but where the actuation provides additional stiffness which changes the frequency. This means that throughout the motion the actuation is always applying force or torque.

This is a serious disadvantage as it means that the system is continuously consuming power, which is scarce in spatial applications. However, if the actuation guides the mass at its natural frequency, it will only have to make up for the small departures from the ideal circular trajectory due to the mechanism's asymmetries, which would not require a large consumption.

2.3.2.2 Exporting torque To remain as general as possible, a vector, in this case the angular momentum, can see itself change in amplitude, in direction, or a combination of both. Both scenarios will be separately discussed with a focus on the change of amplitude, for which two sub-methods exist.

2.3.2.2.1 Exporting a torque by changing the angular momentum's amplitude Two mechanism categories capable of providing constant angular momentum have been shown: planar and

spherical mechanisms. In both cases, the angular momentum depends on the orbit amplitude and velocity. There are therefore two levers with which to adjust the angular momentum.

In the case of planar mechanisms, we can consider a single mass on a circular orbit without losing generality. The angular momentum L is given by:

$$L = mrv \quad (7)$$

$$= mr^2\omega \quad (8)$$

$$= I\omega, \text{ with } I = mr^2, \quad (9)$$

where m is the mass, r its orbit radius, v its velocity and ω its angular velocity.

In the case of spherical mechanisms, we assume the solid's mass distribution is spherically symmetric (i.e. that the moment of inertia is the same for all rotation axes and can therefore be given by a scalar I). Moreover, since spherical masses are combined to remove the angular momentum's alternating xy -components, only the vertical component L_z is considered and is given by:

$$L_z = I \sin(\theta_L)\omega, \quad (10)$$

where ω is the sphere's angular velocity and θ_L being the half angle of the cone formed by the angular momentum (its sine therefore being the vertical projection).

Increasing angular momentum by increasing velocity Eq. 8 and 10 have strong similarities in their form. They both contain a first factor that depends solely on the mass or inertia, a second factor relating to the orbit amplitude and a last on velocity. This last factor provides the first lever with which to adjust the angular momentum's amplitude. Indeed, in the case of planar mechanisms, Eq. 7 and 8 show that angular momentum is proportional to both velocity and angular velocity. For spherical mechanisms, the conclusion is identical.

Increasing angular momentum by increasing moment of inertia A second way to adjust the angular momentum's amplitude is to play with the first parts of Eq. 8 and 10. These depend on the motion amplitude, which relates to the system's moment of inertia. Indeed, for planar mechanisms, Eq. 8 shows that the angular momentum L increases quadratically with the orbit amplitude r . However, as highlighted in Eq. 9, mr^2 simplifies to the moment of inertia I of the point mass. The amplitude of angular momentum can therefore be adjusted by modifying the system's moment of inertia.

In the case of spherical mechanisms, the reasoning is more subtle. Increasing the amplitude of the circular orbit results in a smaller angle of the cone swept by the angular momentum. This smaller angle leads to a more important vertical component of the angular momentum (described by $\sin(\theta_L)$). This can be seen as an increase of the system's moment of inertia around the z -axis.

2.3.2.2.2 Exporting a torque by changing the angular momentum's direction The previous section discussed the case where the angular momentum's amplitude was varied in order to export a torque. We will now examine the second manner to vary this vector: by changing its direction. This option to export a torque has already been implemented in traditional attitude control systems, more specifically in control moment gyroscopes (CMGs). These systems consist in a wheel rotating at a constant rate (usually by means of ball bearings) whose direction is changed in order to export the gyroscopic torque to the spacecraft.

This option of exporting torque has not been further explored for several reasons. First, at first glance this would simply consist in one of the previously described systems (with constant angular

momentum) on gimbals. As designing these gimbals with flexures is relatively simple, this option does not bring any extra novelty. Second, if the gimbals are used, the actuation will be mobile as they are on the end of the gimbals, which complicates the implementation.

2.3.3 Detailed designs of new mechanisms

2.3.3.1 Planar Mechanisms Planar mechanisms design has been extensively studied, particularly in the field of flexure because of its simplicity.

As discussed in Sect. 2.3.2, these planar mechanisms can follow circular orbits by being used as 2-DOF oscillators or by active control. Adding a Circular Motion Flexure is another option, but this requires an additional intermediate linkage to avoid over-constraining the mechanism.

Without losing generality, these mechanisms will be presented without specifying which method (oscillations, active control or CMF) is used as all can be implemented.

2.3.3.1.1 Inverter pantographs A crucial aspect of the abstract design is that masses are at opposite ends of identical trajectories. This behaviour can be understood as the two masses being subject to an inversion along both the x - and y -axes. A mechanism capable of providing such property is the inverting pantograph.

This mechanism is based on the well known pantograph, mainly used for copying by drawing or engraving. The mechanism has two degrees-of-freedom and consists in an end effector whose motion is restricted to the xy -plane and an intermediate end effector whose motion is a perfect scaled-down version of the end effector's as shown in Fig. 26. The scale can be adjusted, but only the ratio $1/2$ is of interest in this case. The transformation from this standard pantograph to the inverting pantograph is achieved by using the intermediate end effector as ground and the initial end effector and ground become two coupled end effectors with inverted motion, as shown in Fig. 27. Masses can then be placed on the end effectors to generate the angular momentum.

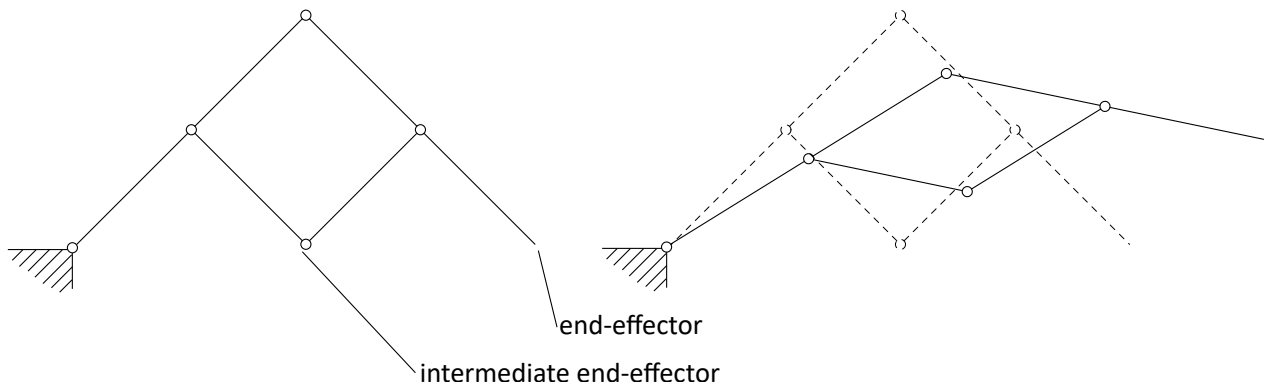


Figure 26: Standard pantograph mechanism in its neutral position (left) and displaced configuration (right). It can be seen that the two end effectors have the same motion, but with a ratio of $1/2$

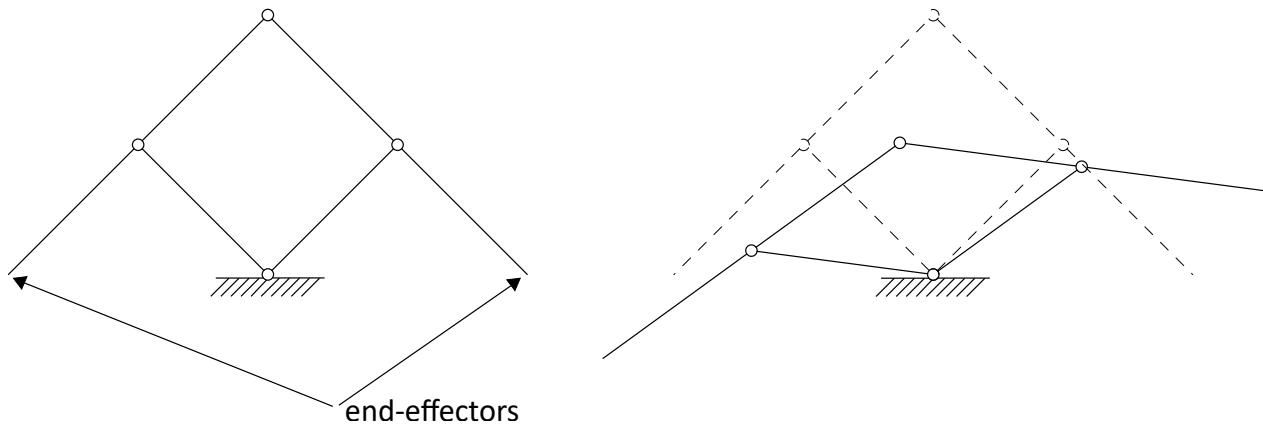


Figure 27: Inverter pantograph mechanism in its neutral position (left) and displaced configuration (right). It can be seen that the two end effector have perfectly inverted movement.

If point masses are placed on the end effectors, this mechanism is not a Flexure Wheel as the orbits of the two masses do not coincide. Indeed, this is the same configuration as detailed in Sect. 4.2 and leads to an angular momentum that is not constant, but oscillates around a fixed value (see Sect. 4.2 for complete calculation). To solve this issue, two solutions have been found: a first in which solids are used instead of point masses and a second in which inverted pantographs are combined.

Croissants, donuts and cupcakes The fact that the orbits of the two masses do not coincide can be solved by no longer using point masses, but solids with particular shapes which have their centres of mass placed outside of themselves. This enables the two centres of mass on circular motion to be arbitrarily placed while avoiding collision, since there is no matter at the location of the centre of mass.

Such a design is shown in Fig. 28 in which two 'croissant' shaped solids provide the coinciding circular orbits. These croissants have additional weights (the black circles) to bring the centre of mass to the desired position. Another design is shown in Fig. 29 in which a 'donut' and -following the pastry theme- 'cupcake' shaped solids are used. Both of these are circular and are arranged concentrically when the pantograph is in its neutral position, making their centres of mass coincide.

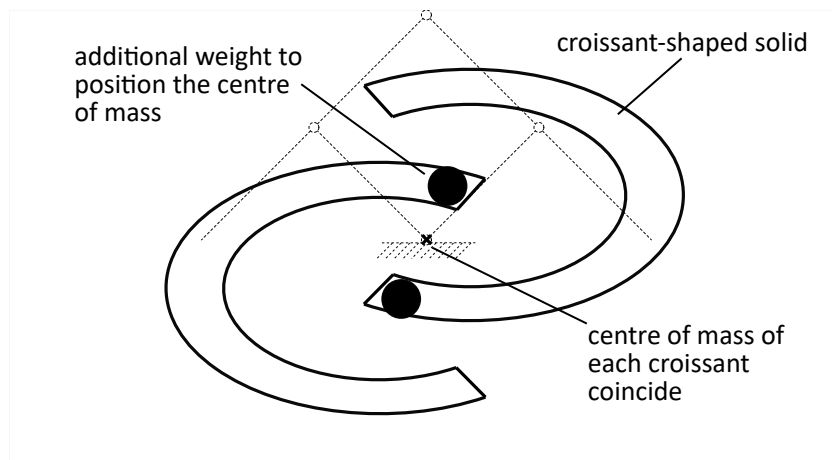


Figure 28: Inverter pantograph mechanism (dotted line) in which the point masses at the end effectors have been replaced by croissant-shaped solids whose centres of mass are coincident.

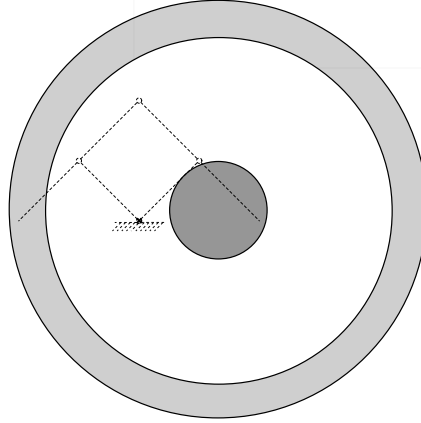


Figure 29: Inverter pantograph with donut and cupcake shaped masses. The donut (light grey) is fixed to the pantograph's left end effector while the cupcake (dark grey) is fixed to the right end effector. These two solids being concentric, their centres of mass coincide.

Using solids does however bring up another issue. Up to now, the masses placed and the inverting pantograph's end effector were assumed to be punctual. For this reason, the orientation of these masses was irrelevant as point masses have no size and therefore no inertia when rotating around themselves. When considering solids fixed to the pantograph's end effectors, their orientations will change throughout the motion, which leads to additional angular momentum and possible collisions between the masses (if not dimensioned properly). To solve this issue, a second inverting pantograph needs to be added as well as additional branches of arms to maintain the masses' orientation in a given direction. This branch consists in arms that run parallel to those of the pantograph which blocks the rotation of the solids, as shown in Fig. 30 and 31.

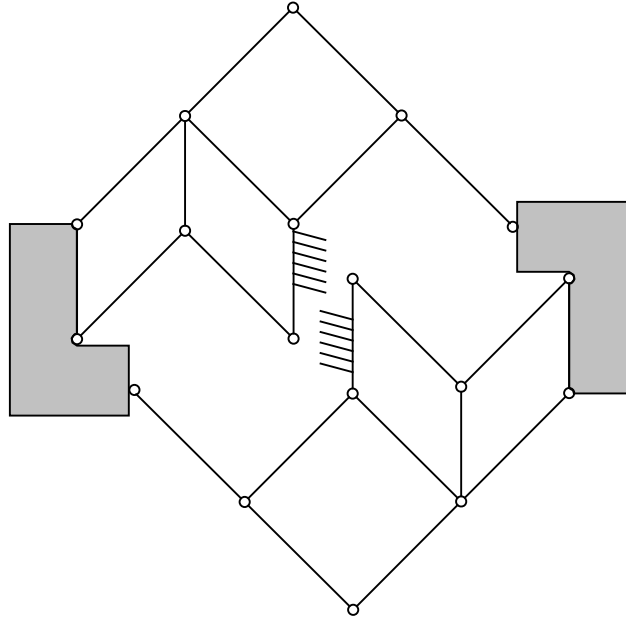


Figure 30: Double inverting pantograph mounted in parallel. The dashed part is the ground and the two masses are in gray.

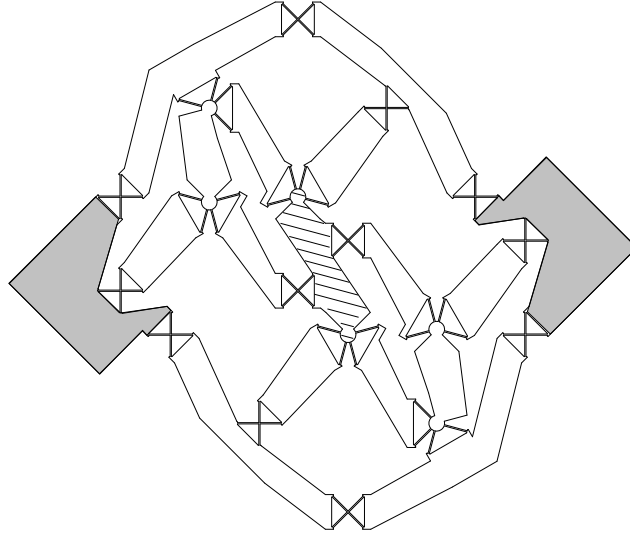


Figure 31: Top view of a flexure implementation of the previous double inverting pantograph. The dashed part is the ground and the two end effectors are in gray.

Combining inverting pantographs As explained in Sect. 4.2, if two masses are at opposite ends of circular orbits that do not coincide, the total angular momentum is not constant, but oscillates around a fixed value. To eliminate this oscillation, a second pair of masses is added so that the alternating components of their angular momenta cancel out while their averages are added.

These two pairs can be arbitrarily placed with respect to one another. However, they can be advantageously arranged side by side. When looking from left to right, the first two masses are inverted, the second and third follow identical orbits and the third and fourth masses are again inverted. These motions can again be implemented with two inverting pantographs mounted in series, as shown in Fig. 32.

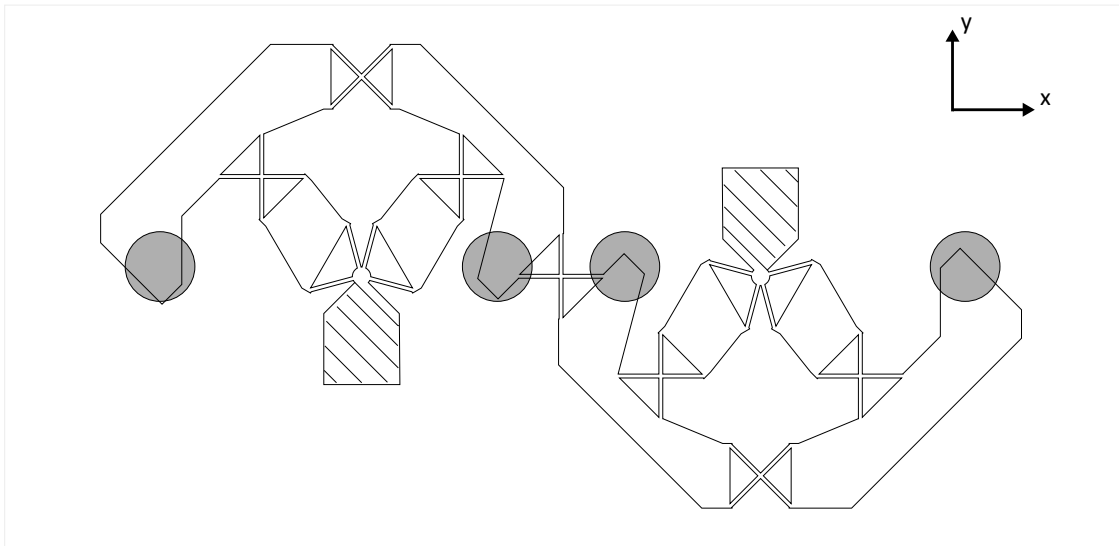


Figure 32: Flexure implementation of two inverting pantographs mounted in series. The masses on circular orbits are the four dark circles. The dashed parts are the ground

2.3.3.1.2 Combining masses on circular and linear motions This type of design is based on that of Fig. 17 in the sense that a mass on a circular motion has its centre of gravity compensated by a set of masses on linear motion. In this design, the mass on circular motion crosses the path of those in translation, which would in practice result in collisions.

To bypass this issue, the mass on circular motion is connected to a set of inverting levers, each holding a counter-mass (Fig. 33). A counter-mass moves in opposition to the mass on circular motion only when the latter moves in the direction of the corresponding inverting lever. By having these inverting levers displayed symmetrically around the mass on circular orbit, the centre of gravity remain fixed and the centre of the circular orbit. Such an arrangement with 4 inverting levers and counter-masses is shown in Fig. 34.

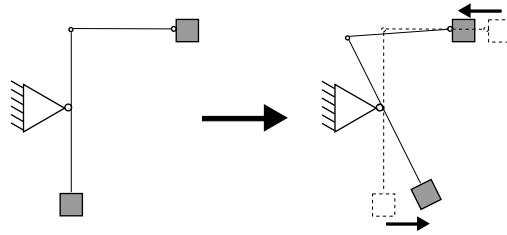


Figure 33: Inverting lever with a counter-mass.

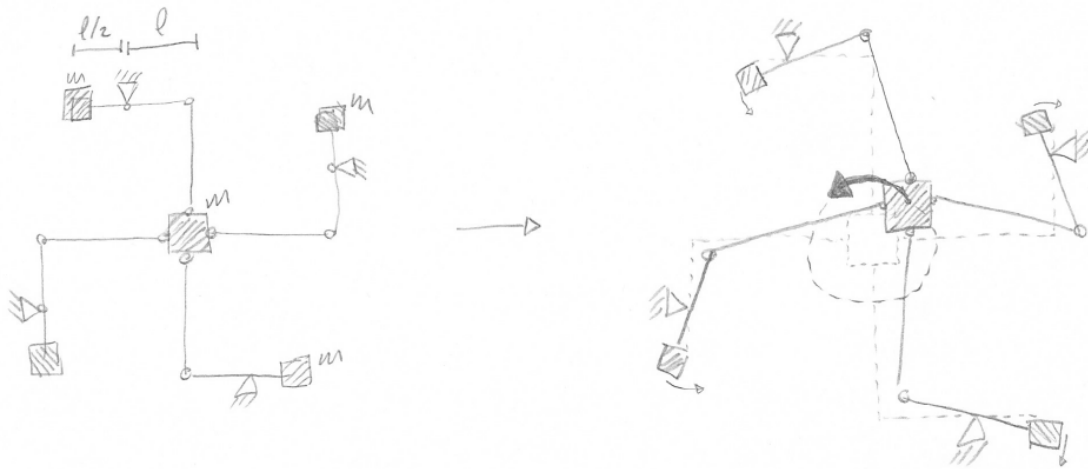


Figure 34: Other configuration equivalent to a reaction wheel. Left: mechanism in its neutral position. Right: mechanism on a circular orbit.

2.3.3.2 Spherical Mechanisms The abstract design presented in Sect. 2.3.1.2 require two aspects which will be dealt with separately: a flexure mechanism providing the kinematics of a CV-joint and combining such mechanisms to obtain the constant angular momentum.

Creating a single spherical mechanism CV-joints have been extensively studied in general mechanical design and a number of designs have already been discovered, such as the Rzeppa joint (Fig. 35). However, their implementation with flexures has only been recently achieved in the IsoSpring

project (see [15]). Again following the analogy of a planet, this design consists in a first set of three equatorial beams which fixes the xy -position of the sphere as well as blocs its rotation around the z -axis (Fig. 36a). The last degree-of-freedom to remove is translation in the z -direction and is taken care of by a polar beam placed either beneath or above the sphere (Fig. 36b).

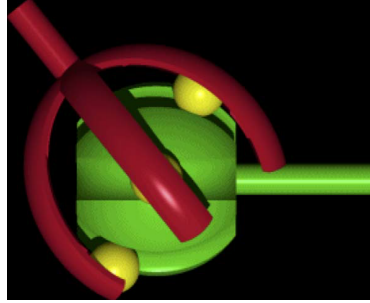
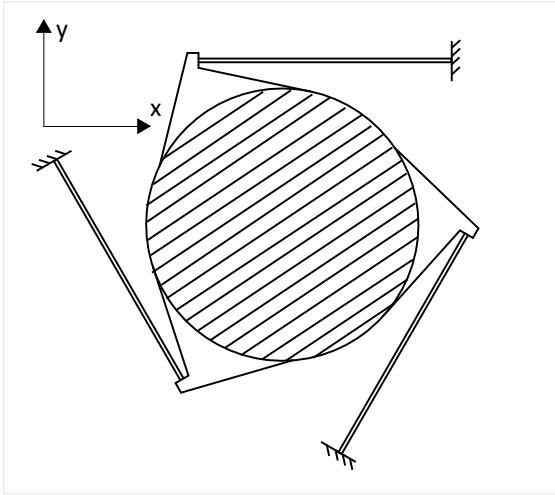
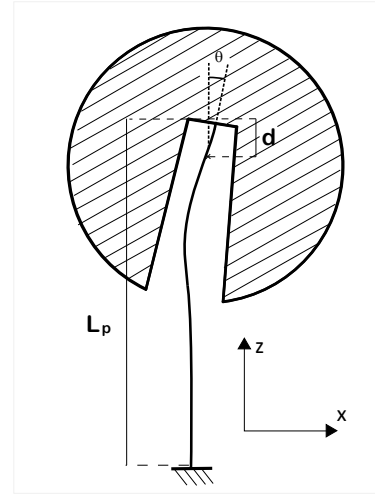


Figure 35: A Rzeppa CV-joint [15]



(a) Equatorial beams



(b) Polar beam, in a displaced state

Figure 36: Flexure CV-joint of the Spherical IsoSpring.

The fabrication of this design is however relatively complex. Indeed, even though the three equatorial beams are easy to manufacture or assemble (as they lie in the same plane), the polar beam adds complexity. It would be highly desirable to have a design in which all the flexures lie in the same plane. Luckily, such a design exists and uses the equivalence between a beam and a corner blade (Fig. 37). In this design, the polar beam is replaced by its equivalent corner blade, which then lies in the same plane as the equatorial beams. This coplanarity of equatorial beams and polar corner blade means that the flexure can be manufactured in a single part (monolithic) as shown in Fig. 38.

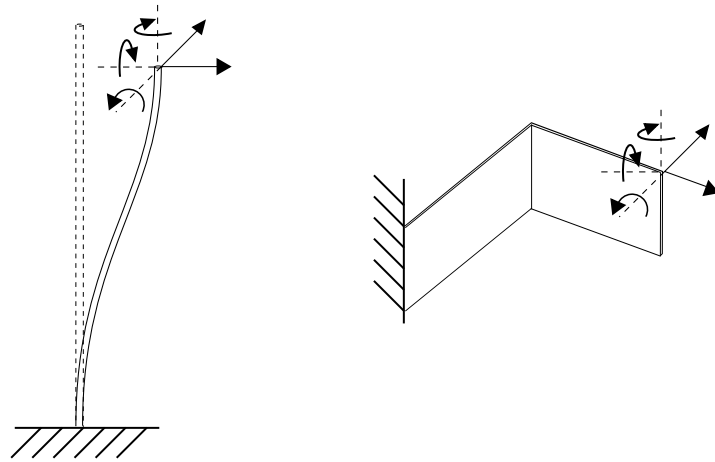


Figure 37: Equivalence between a beam and a corner blade. Left: flexible beam of circular cross section with its degrees-of-freedom (two translations and three rotations). Right: corner blade with its degrees-of-freedom, which are the same as the beam.

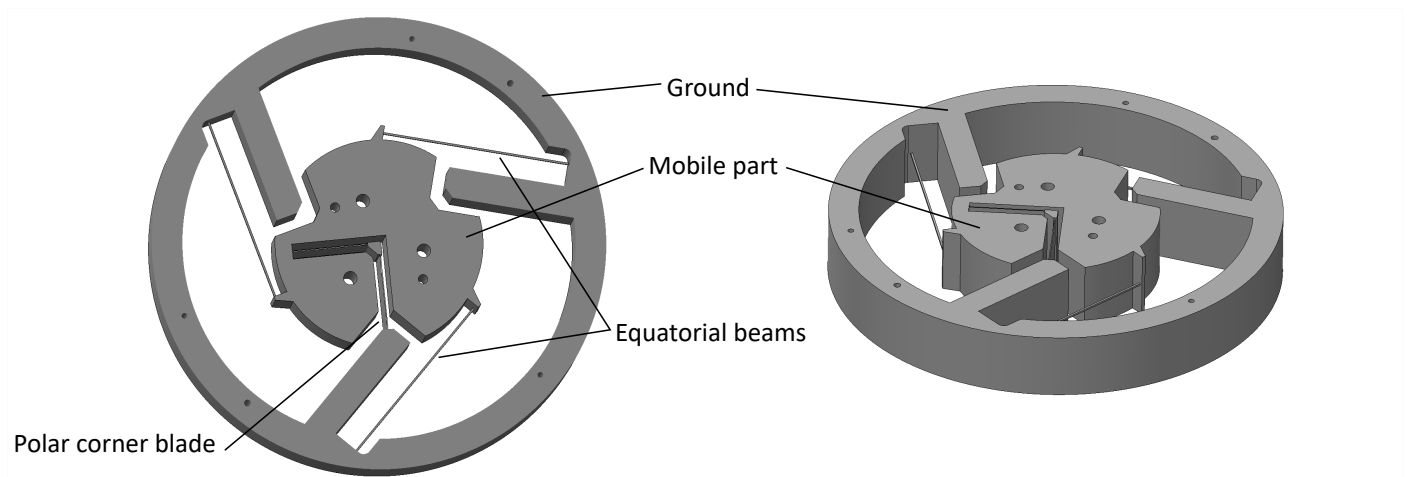


Figure 38: Monolithic flexure CV-joint consisting of three equatorial beams and one polar corner-blade.

Concerning the oscillating mass, spherical masses are to be used. Such masses have the property that their inertia are identical along each of their principal axes. Note: this is equivalent to their inertia matrix being diagonal. Such a spherical mass can be obtained in a variety of manners. The trivial approach is to actually use a sphere (either hollow or filled), but other shapes are equivalent from the perspective of their moments of inertia. For example, a system of two discs can achieve this property if they are separated by the right amount as shown in Fig. 39. Other possibilities include having point masses placed at the apexes of regular polyhedrons such as a tetrahedron.

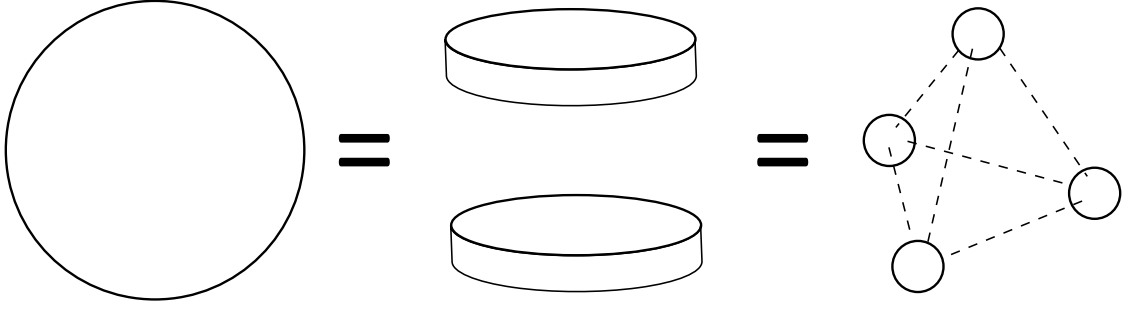


Figure 39: Examples of spherical masses. Left to right: a sphere, two discs, a tetrahedron.

Combining and coupling spherical mechanisms As in the planar mechanisms, all three methods of use (CMF, 2-DOF oscillator, active control) are implementable, but spherical mechanisms are particularly suited for the Circular motion Flexure. Indeed, referring to the CMF with rigid segments and localized flexures of Fig. 23, it can be seen that this mechanism already contains two CV-joints in anti-phase and on circular motion. To convert this CMF into a Flexure Wheel, one only needs to add a spherical mass to each of these CV-joints as shown in Fig. 40. The entire Flexure Wheel therefore consists in the two CV-joints, both with their respective spherical mass, linked together by a spherical joint.

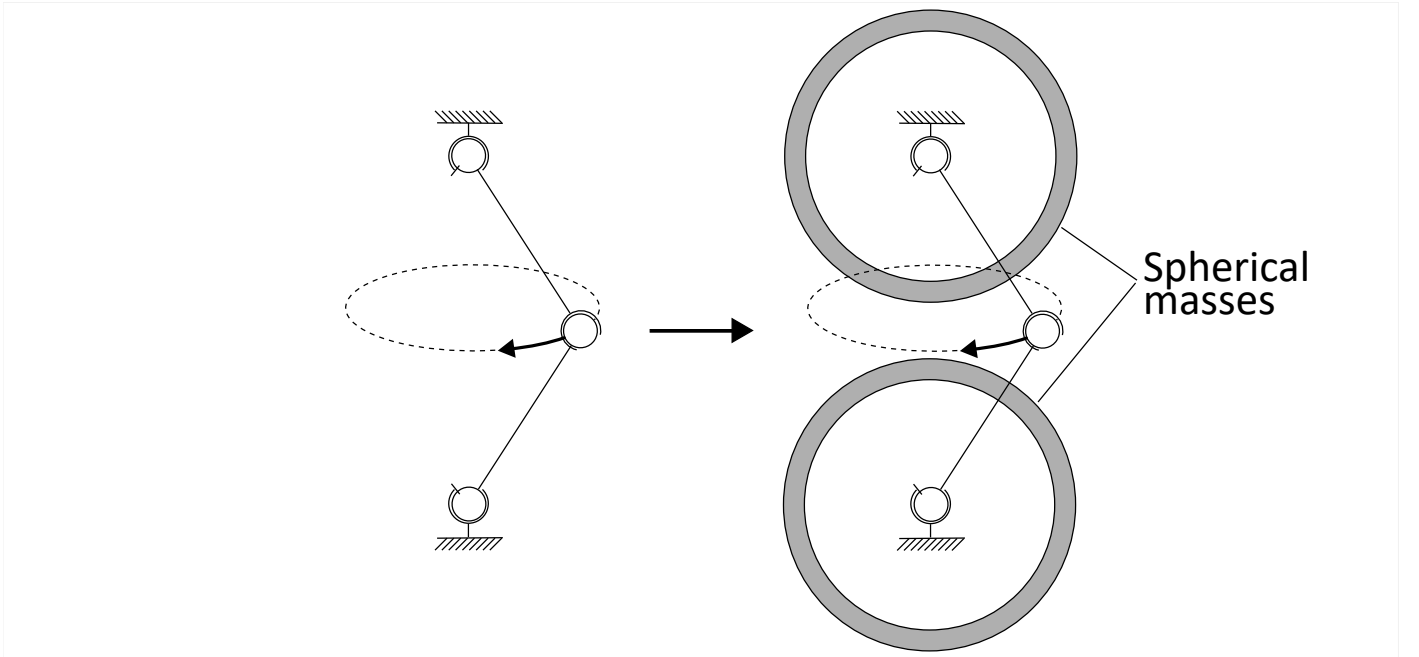
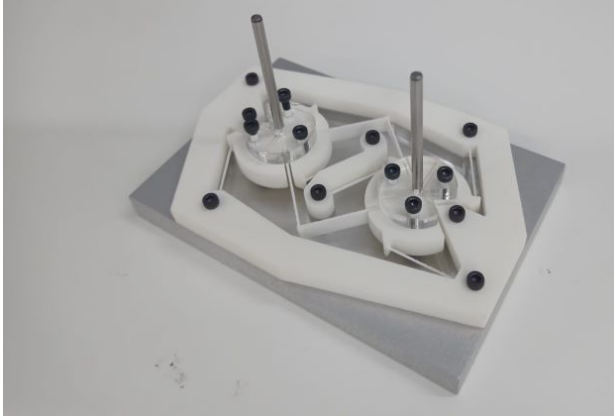


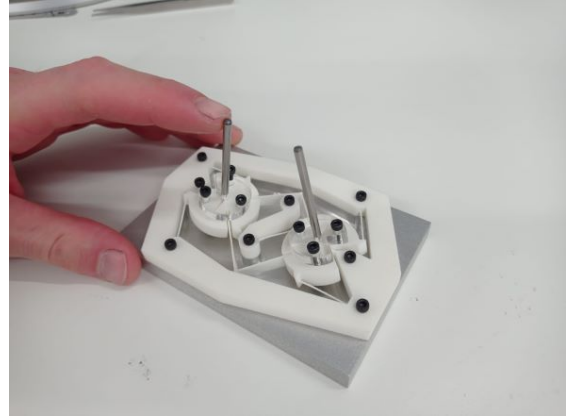
Figure 40: Left: initial Circular Motion Flexure of Fig. 23. Right: resulting Flexure Wheel after adding the two spherical masses

Another noteworthy design, which this time can also be used as a 2-DOF oscillator or active control, is to invert the motion of two spherical mechanisms with corner blades (or their equivalent rods). The inversion is done in two steps and requires two additional intermediate spherical mechanisms, but whose spherical masses are negligible. The first consists in linking a spherical mass to one of these intermediate spherical mechanisms by means of two corner blades, as shown in Fig.41a. This arrangement inverts the motion of these two mechanisms only in one direction (x -axis for example) and they therefore behave as if they were the reflection of one another in a mirror, as shown in Fig.41b. The second step consists

in repeating this mirroring effect in order to invert the motion along the other direction (y -axis). The final result is shown in Fig. 42 which consists in four spherical mechanisms, where two neighbouring spherical mechanisms mirror each other while two diagonally opposed spherical mechanisms are entirely inverted.

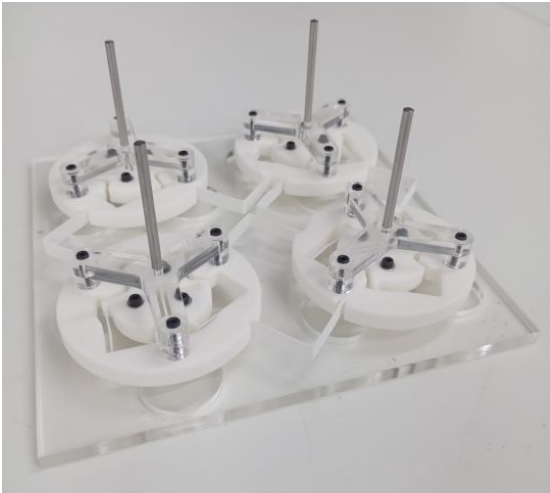


(a) neutral position

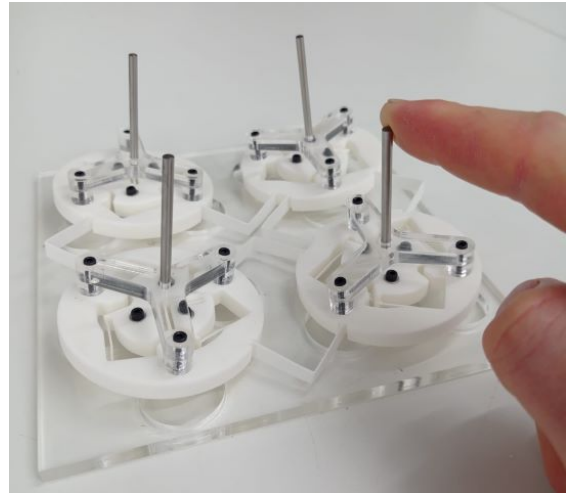


(b) displaced

Figure 41



(a) neutral position



(b) displaced

Figure 42

2.4 Demonstrator prototypes and results

2.4.1 Obtaining the circular motion

To keep the prototypes as simple as possible for these first experiments, aspects such as using a circular motion flexure and frictionless actuation have been left aside. The actuation and circular motion were implemented with a standard rotating motor. To provide the circular orbit of fixed amplitude, the mass on circular orbit has a pin which lodges into a lever via a hole eccentric to the motor shaft. As the motor turns, the pin is held in this hole and therefore follows circular orbits of fixed amplitude.

It can be noted that this standard rotating motor contributes to the total angular momentum, since its shaft is on angular rotation (i.e. the motor by itself can be seen as a standard reaction wheel).

In order to remove this undesired contribution, a second motor and crank are added. They are not connected to anything and turn at the same speed as the first motor, but in opposite direction. Their only role is to counter the angular momentum generated by the first driving motor so that the total angular momentum is solely produced by the Flexure Wheel. Note: this method of suppressing the motor's angular momentum was only implemented in the first prototype.

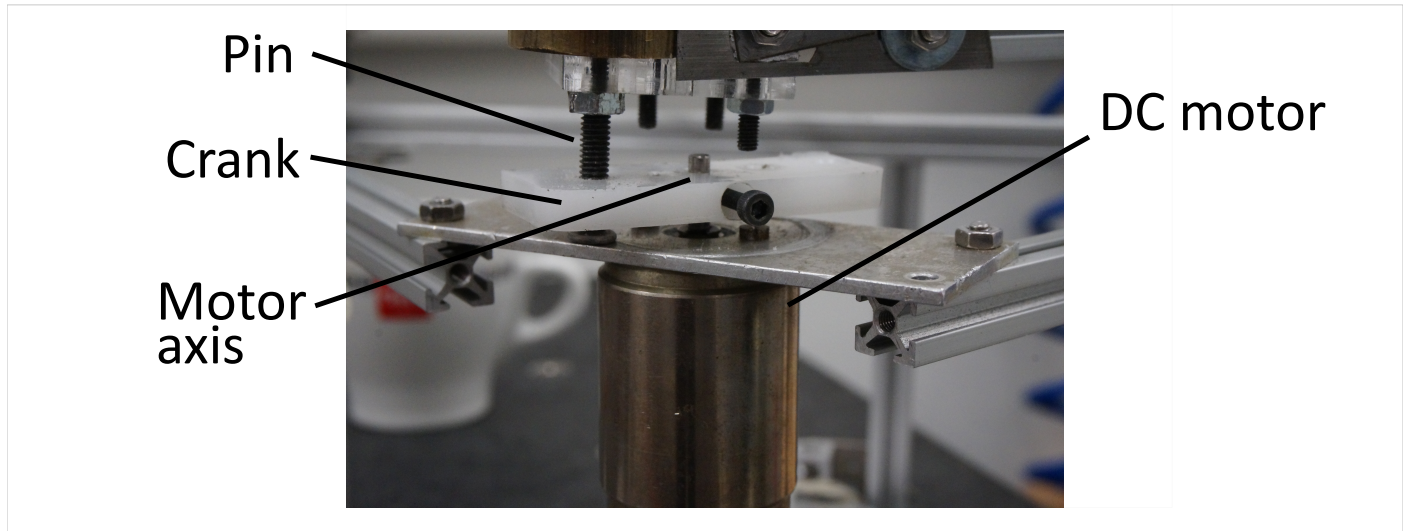
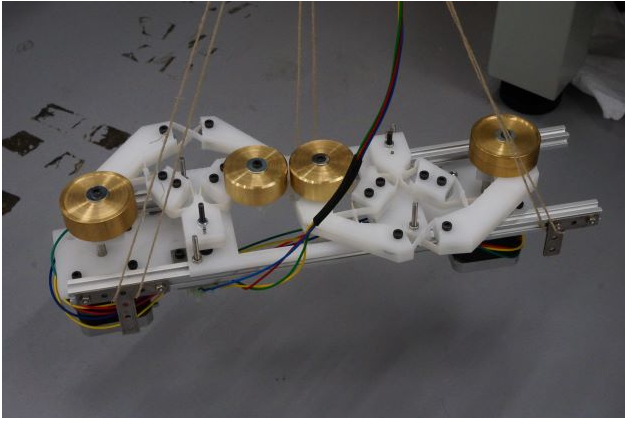


Figure 43: Pin and hole mechanism that restricts to circular motion. Note: in this construction a screw was used as the pin

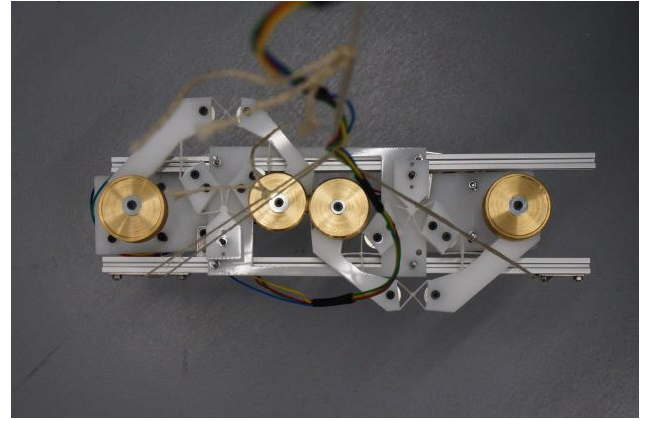
2.4.2 Prototype 1: two serial inverting pantographs

This prototype is based on the design presented in Sect. 2.3.3.1.1 consisting in two serially arranged inverting pantographs. Its flexure structure is made of two 5mm sheets of POM that have been laser cut and screwed together to form the rigid segments as well as crossed leaf-spring pivots. The masses consist in four brass cylinders of approx. 35[mm] in diameter and 15[mm] high. Note: these masses are naturally not punctual, but this aspect can be neglected as it is only a demonstrator prototype.

A stepper motor forms the actuation by means of the lever and pin mechanism presented in the previous section. The diameter of the orbits has been set to approx. 12[mm] by the hole in which the masses' pin lodges and the mechanism runs at 240[rpm]. A second motor has been added in order to suppress the additional unwanted angular momentum brought by the first motor, as shown in Fig. 45b.

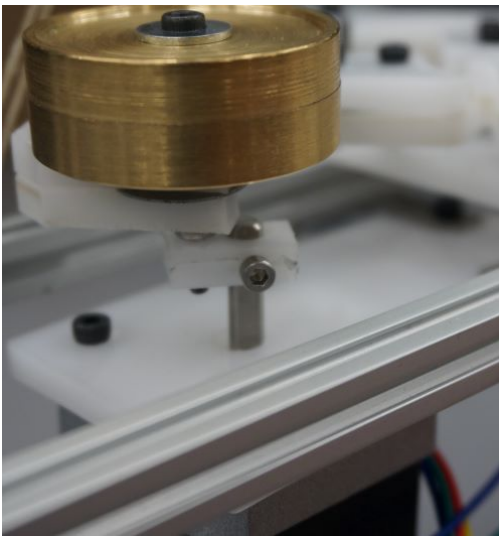


(a) Isometric view

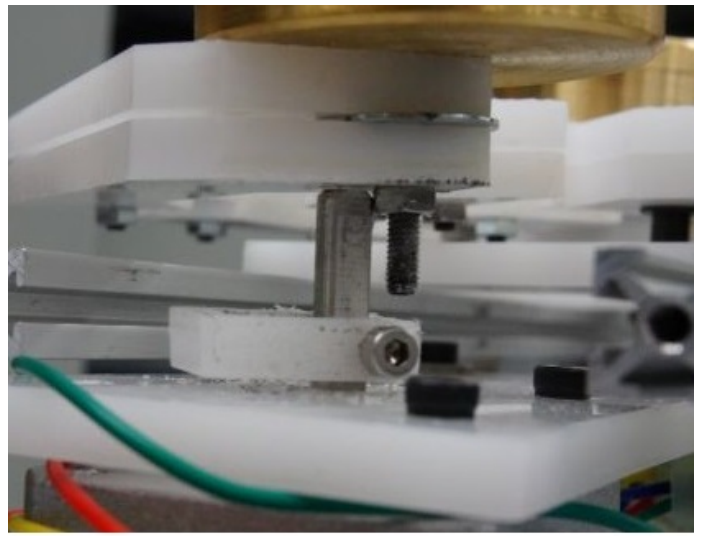


(b) Top view

Figure 44: Overview of the first prototype



(a) The driving crank with the pin lodged in a hole



(b) The extra motor and crank which suppress the angular momentum of the first motor. They are not linked to the Flexure Wheel.

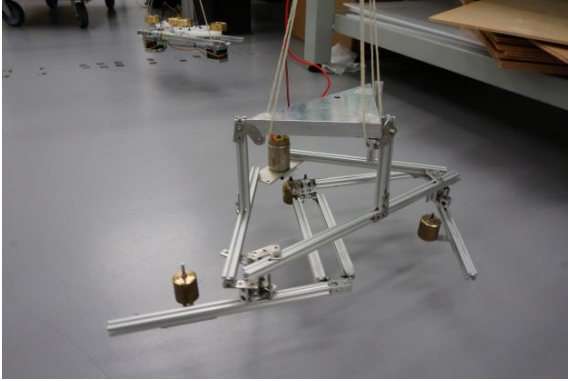
Figure 45: Pin and lever actuation of the first prototype.

2.4.3 Prototype 2: one mass on circular orbit and two masses to stabilise the centre of mass

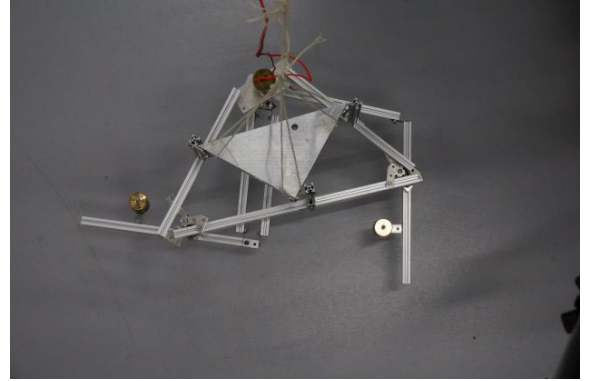
This design is similar to that of Fig. 34 with the difference that there are only two masses to keep the centre of mass fixed. This design can also be seen as that of Fig. 17, but where the two masses in translation do not oscillate around the centre of the circular orbit, but have an offset.

The consequence is that the centre of mass is not located at the centre of the circular orbit, but has an offset. This leads to the fault of this mechanism, which is an alternating angular momentum, though on average positive. This mechanism is therefore not rigorously a Flexure Wheel, but is still capable of putting a spacecraft in rotation, though not at a constant rate (i.e. if this mechanism is suddenly put into motion, the spacecraft's orientation will be the superposition of rotation at a constant rate and shaking in rotation).

Concerning its fabrication, this prototype uses aluminium profiles as rigid segments which are then linked together by short spring steel blades. The actuation has the same pin and hole arrangement as the first prototype, but uses a DC motor.

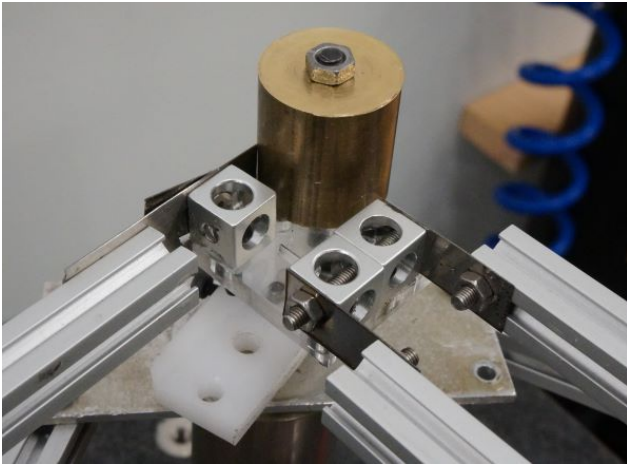


(a) Isometric view

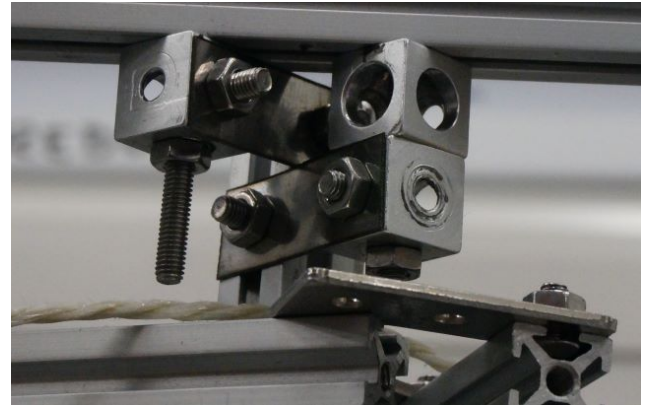


(b) Top view

Figure 46: Overview of the second prototype



(a) Links to the mass on circular orbit



(b) Crossed leaf spring pivots

Figure 47: Construction details of the second prototype

2.4.4 Experimental setup

The main goal of the setup was to validate whether the flexure prototypes were able to put themselves into rotation without any external influence. The second goal was to evaluate whether the system was statically balanced (i.e. no shaking in translation).

The usual test bench for such experiments consists in having the system float on a flat surface via an air cushion as it almost entirely removes friction while still allowing rotation and translation in the plane. For simplicity and availability reasons, the setup consisted in suspending the prototypes on the end of a string. Indeed, such a setup still allows for rotation as well as translation to some extent.



Figure 48: Experimental setup consisting in the two demonstrator prototypes suspended by strings.

The fact of suspending on a string does however bring more complexity to the experiment than with an air bearing. Indeed, as the prototype rotates, torsion in the string appears and creates a restoring torque, which affects the prototype's rotation.

It has been chosen that the actuation suddenly puts the flexure mechanism into motion at a constant rate. This results in an instantaneous increase of the angular momentum, which means that the exported torque that prototype receives is a single impulse.

The expected experimental procedure is as follows:

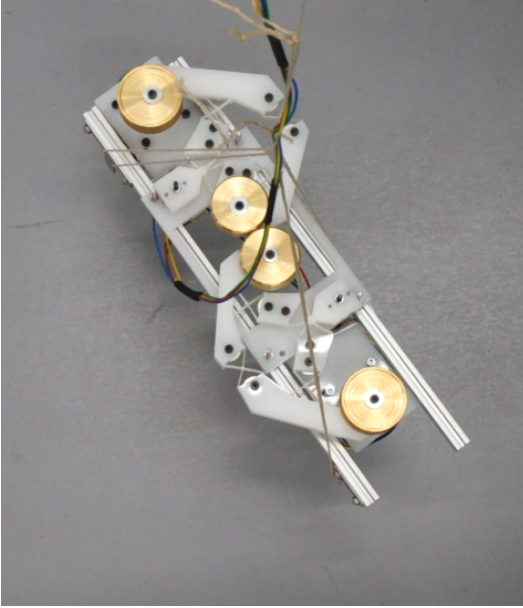
- The entire system is at rest. This means that neither the flexure mechanism is in motion and neither the prototype is rotating on the end of the string.
- The actuation is turned on, which sets the flexure mechanism into motion.
- The prototype starts to rotate around the string.
- As the prototype rotates, the string's restoring torque increases
- At some point the prototype slows down, stops and starts to rotate the other way.

2.4.5 Results

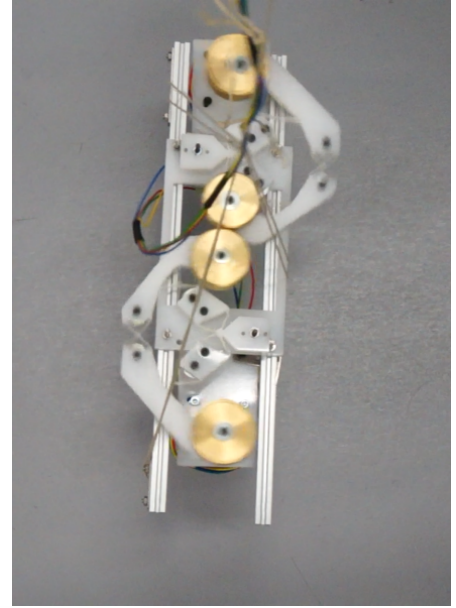
The results of the qualitative experimental setup consist in the following figures and in a video recording of the experiment. The video recording gives a better understanding of the experiment and performances and it is available at:

<https://doi.org/10.5281/zenodo.5148075>

The following figures illustrate the main results of the video. Indeed, it can be seen that the prototypes have turned clockwise after having turned on the motors.

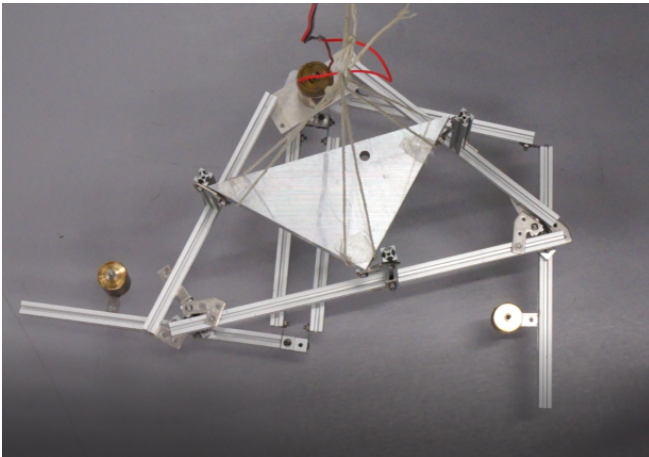


(a) Before turning the motors on

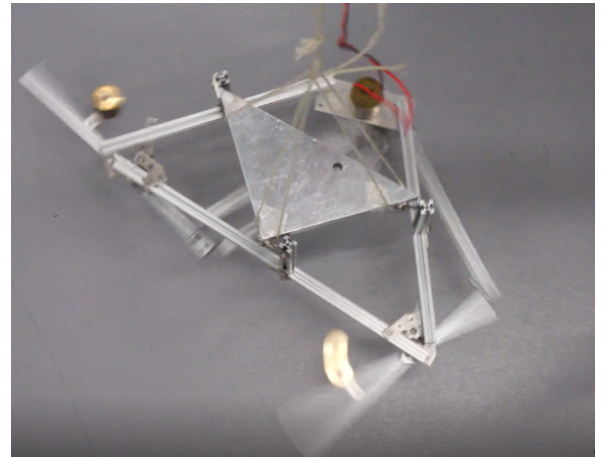


(b) After having turned the motors on, still in motion

Figure 49: Top view of prototype 1 before and after having turned on the motors



(a) Before turning the motor on



(b) After having turned the motor on, still in motion

Figure 50: Top view of prototype 1 before and after having turned on the motor

3 Conclusion and future work

This paper started with an overview of ball bearing reaction wheels and their issues. The fundamental properties of reaction wheels were then highlighted using systems mechanics. This step enabled other types of motion to be considered as a replacement. These invented alternative types of motion consisted in a point mass on circular motion and spherical mechanisms on circular orbits around their centre of

mass. To fully match the behaviour of a reaction wheel, several masses or spherical mechanisms are combined. It was then shown how these types of motion could be implemented with flexures as well as how circular motion could be reached. Last, the proof-of-concept prototypes were presented along with their results.

One theoretical point must be noted: the presented mechanisms have only been studied in a fixed frame (therefore that is not subject to rotations) and the possibility of them disturbing an already rotating spacecraft has not been studied. This is the case for the first prototype. Indeed, when all the masses are aligned (as in Fig. 32) the moment of inertia along the x -axis is at its minimum. However, during motion, this moment of inertia will change. It can then be understood that if the spacecraft is already rotating around the x -axis, the spacecraft will see its moment of inertia change, which disturbs its rotation. Future work therefore involves further study of these balancing aspects.

Future work also includes the fabrication of a Circular Motion Flexure and its integration into a proof-of-concept prototype for the spherical mechanisms, similar to the two that were presented. The actuation methods will also have to be studied.

Last, it would be very interesting to study the scaling laws at play in these Flexure Wheels. Indeed, their monolithic fabrication allows for miniaturisation. This could be a potential advantage over magnetic bearings for which miniaturisation proves to be challenging [13].

4 Appendices

4.1 Appendix: Angular momentum of a mass on uniform circular motion

We consider a particles of mass m on a circular orbit of radius r within the xy -plane. The mass follows this orbit with a velocity v , which results in an angular velocity of $\omega = v/r$.

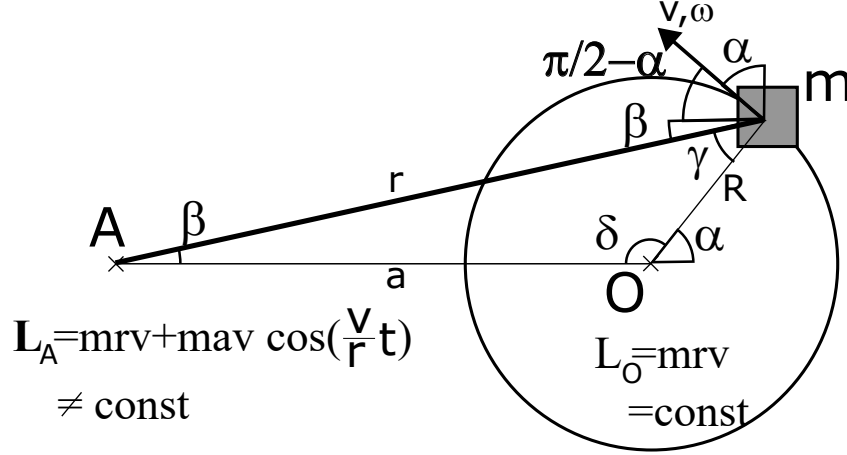


Figure 51: Particle of mass m on a uniform circular motion of radius R .

When evaluated from the orbit's centre O , the angular momentum L_O is simply given by $L_O = mrv = m\omega r^2$. However, when evaluating from an arbitrary point A at a distance a from the centre, the general definition of angular momentum is required:

$$\mathbf{L}_A = m\mathbf{r} \times \mathbf{v}, \quad (11)$$

where $\mathbf{r}$ is the particle's position and $\mathbf{v}$ its velocity. As $\mathbf{r}$ and $\mathbf{v}$ are both within the xy -plane, their cross-product is exclusively along the z -axis. We will therefore only consider the z -component of the angular momentum $L_{A,z}$.

We therefore have:

$$L_{A,z} = mrv \sin(\pi/2 - \alpha + \beta) \quad (12)$$

$$= mrv \cos(\alpha - \beta) \quad (13)$$

$$= mrv(\cos(\alpha) \cos(\beta) + \sin(\alpha) \sin(\beta)) \quad (14)$$

Moreover, since $v = \omega r$ and:

$$\begin{cases} r \cos(\beta) &= a + R \cos(\alpha) \\ r \sin(\beta) &= R \sin(\alpha) \\ \alpha &= \omega t \end{cases}, \quad (15)$$

we have:

$$L_{A,z} = m\omega R(a \cos(\alpha) + R \cos^2(\alpha) + R \sin^2(\alpha)) \quad (16)$$

$$(17)$$

which leads to:

$$L_{A,z} = m\omega R(a \cos(\omega t) + R) \quad (18)$$

Conclusion: The angular momentum of a mass with uniform circular motion is not constant when evaluated at an arbitry point, but oscillates around a fixed value.

4.2 Appendix: Angular momentum of two masses on uniform circular motion

We now consider two particles A and B of equal mass m each on circular orbits of same amplitude within the xy -plane. These orbits are centred on the x -axis at equal distance from the origin, as shown in Fig. 52. The distance between the orbit centres is $2a$.

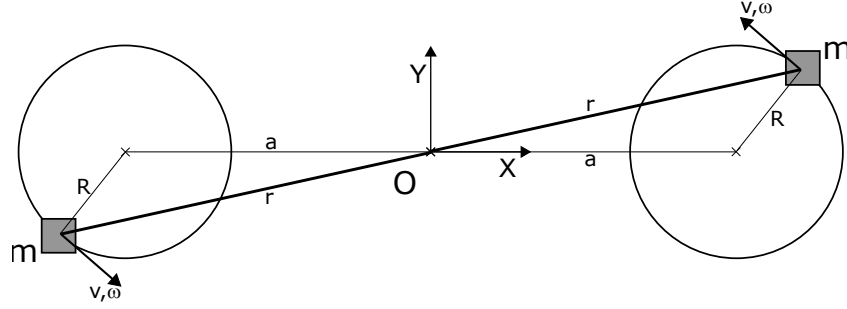


Figure 52: System consisting in two masses at opposite ends of identical circular orbits.

Moreover, the masses are at opposite ends of their respective orbits (i.e. when particle A is at the top-left end of its orbit, particle B is at the bottom-right.) The fact that their displacements are opposed means that the centre of mass of the whole system does not move.

As the centre of mass is immobile, the total angular momentum does not depend on the position of the frame from which it is evaluated (see Eq. 1). We chose to evaluate the angular momentum at the origin (halfway between the two orbits). To calculate the total angular momentum, we use the previous result of Eq. 18 and apply it to both particles A and B .

The z -component $L_{A,z}$ of mass A 's angular momentum is by definition:

$$L_{A,z} = m\omega R(a \cos(\omega t) + R). \quad (19)$$

For particle B , $L_{B,z}$ of mass B is then easily deducted by replacing a by $-a$ and α by $\pi + \alpha$ (since the masses are at opposite ends of their respective orbit). The angular momenta of each mass is therefore identical:

$$L_{A,z} = L_{B,z} = m\omega R(a \cos(\omega t) + R). \quad (20)$$

The z -component L_z of the total angular momentum is therefore the sum of $L_{A,z}$ and $L_{B,z}$:

$$\Rightarrow L_z = 2m\omega R(a \cos(\omega t) + R) \quad (21)$$

Conclusion: The angular momentum of a two-mass system on circular orbit is not constant, but oscillates around a fixed value.

Note: This oscillation of angular momentum can be removed by adding a second set of masses so that the angular momentum oscillations will cancel each other out.

4.3 Appendix: Angular velocity of a CV-joint on circular orbits

In this section, the angular velocity of a spherical mechanism on circular motion will be studied. As these mechanisms have already been presented in [15], the same notation describing the sphere's position will be used. These consist in angles θ and φ which describe respectively how much the sphere is tilted and in which direction.

The approach that will be developed is gradual. Indeed, the starting point is a CV-joint in the configuration of a shaft coupling (where both extremities of the joint rotate). The next step involves placing this system in a rotating frame. As a verification, the results shall be compared to [15] which uses a more direct approach. In this case, the kinematics of the moving mass are described with Euler angles and the resulting angular velocity is calculated.

Starting from the shaft coupling The starting point is a CV-joint in the configuration in which it is most commonly found: as a shaft coupling. Indeed, having a rotating motor drive another shaft is a typical situation in mechanics. These shafts may not be perfectly aligned and have an angle between them. It is however still desirable for the driven output shaft to have a constant angular velocity (for an input shaft with constant angular velocity) and this property is the definition of a CV-joint. This situation is depicted in Fig. 53 in which the CV-joint is schematically shown by a Cardan joint. In this figure, the input and output shafts form an angle θ . Moreover, as it is a CV-joint and as the shaft axes are not aligned, both parts have an angular velocities with the same amplitude ω_0 , but do not point in the same direction. The input shaft has an angular velocity vertically upwards along the z -axis $\omega_1 = \omega_0 \hat{z}$ while the output shaft has its angular velocity ω_2 forming an angle θ with the z -axis. Note: Even though a Cardan joint is not a CV-joint, it is still used for visual purposes as known CV-joints are more elaborate and difficult to visualise.

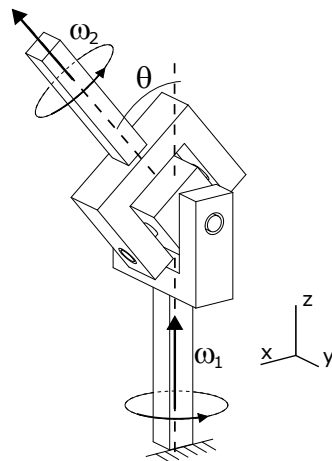


Figure 53: CV-joint in the configuration of a shaft coupling

The next step consists in placing this rotating shaft coupling on a vertically rotating frame in order to cancel out the rotation of the input shaft. Indeed, the spherical mechanism on circular motion assumes the base (here the input shaft) to be fixed. The result of adding such a rotating frame is that the input shaft is now fixed and the output shaft now follows the circular motion of a spherical mechanism. Kinematically speaking this consists in adding an angular velocity of amplitude $-\omega_1$ downwards along the z -axis to both the input and output shafts. Fig. 54 shows the shaft coupling placed on the rotating frame and Fig. 55 isolates each shaft while highlighting their respective total angular velocities.

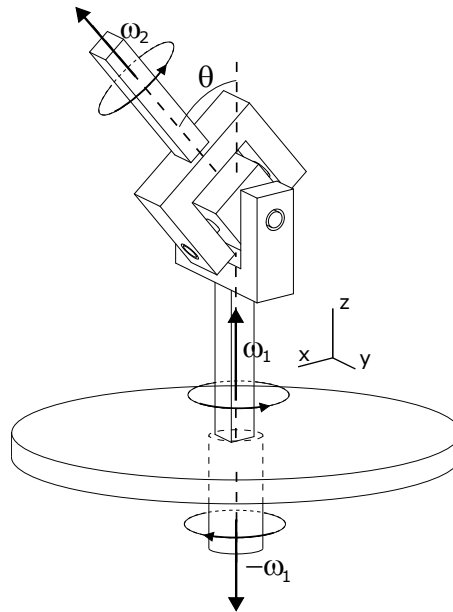


Figure 54: CV-joint in the configuration of a shaft coupling, mounted on a rotating frame to cancel the rotation ω_1 of the input shaft, therefore creating a spherical mechanism on circular motion

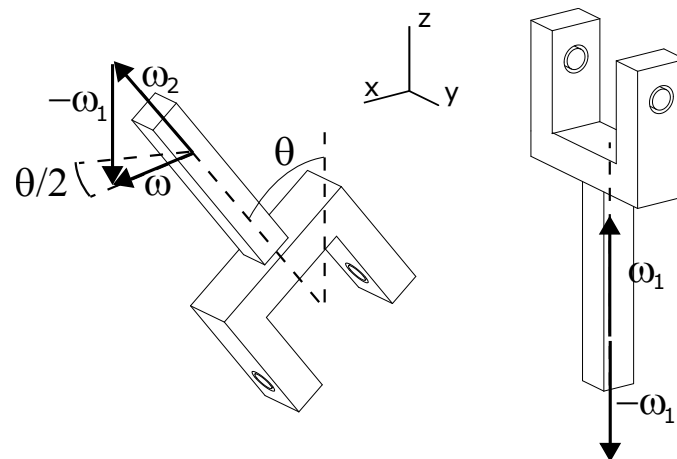


Figure 55: Output shaft (left) and input shaft (right) of Fig. 54 with their respective overall angular velocities.

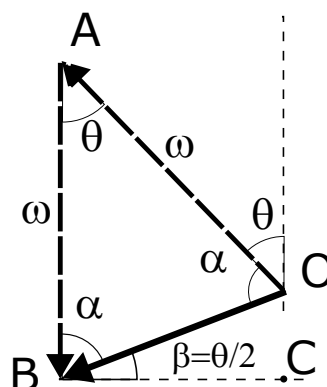


Figure 56: Detailed view of the angular velocities of the output shaft of Fig. 55.

The input shaft therefore has overall no angular velocity (as intended) while for the output shaft, now on circular motion, the angular velocity is the sum of two vectors of same amplitude ω_0 : one pointing vertically downwards and the other diagonally upwards forming an angle θ with the z -axis. As highlighted in Fig. 56, the resulting angular velocity ω points at an angle $\theta/2$ from the xy -plane. Indeed, applying the property that the sum of the angles of a triangle is π to the OAB triangle, we obtain:

$$2\alpha + \theta = \pi \quad (22)$$

$$\Rightarrow \alpha = \frac{\pi}{2} - \frac{\theta}{2} . \quad (23)$$

To find the angle β of the overall angular velocity, we use the fact that angle $\hat{ABC}$ is at a right angle and is formed by angles α and β . This means:

$$\alpha + \beta = \frac{\pi}{2} \quad (24)$$

$$\Rightarrow \beta = \frac{\theta}{2} . \quad (25)$$

Concerning the amplitude of overall angular velocity ω :

$$\omega = 2\omega_0 \sin(\theta/2) \quad (26)$$

which results in the vertical component ω_z :

$$\omega_z = 2\omega_0 \sin(\theta/2)^2 . \quad (27)$$

The vertical component ω_z is the most relevant parameter as it is the component that contributes to the constant angular momentum of spherical mechanisms.

Validation through direct calculation The work presented in [15] gives a direct formula of the angular velocity ω in terms of θ , φ and the Euler angles' rotation axes $\mathbf{i}$, $\mathbf{n}$, $\mathbf{v}$:

$$\omega = \dot{\varphi}(\mathbf{i} - \mathbf{v}) + \dot{\theta}\mathbf{n} . \quad (28)$$

As the spherical mechanism is on a circular motion, the tilt angle θ does not change and therefore $\dot{\theta} = 0$. The only term left contains $\dot{\varphi}$ which corresponds to ω_1 from the previous approach. Moreover, vectors $\mathbf{i}$ and $\mathbf{v}$ are unit vectors, the first pointing vertically upwards along the z -axis and the second diagonally upwards with an angle θ from the z -axis. The rest of the development is identical to above which leads to the same results as Eq. 25-27.

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