
Towards understanding earthquake triggering due to enhanced geothermal systems

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ABSTRACT

The effect of fluid pulse driven fractures (FPDF) propagating in poroelastic media on fault slip in the presence of natural fractures is a complicated interplay between fracture propagation, fracture-fracture interaction, fracture-fault interaction, friction model governing fault slip and wave propagation associated with pulsing injection. Furthermore, the problem is stochastic due to the uncertainty associated with the existing fracture-fault topology.

1 Introduction



Figure 1: States where hydrofracturing in ultra-low permeability rocks is currently ongoing. Source: [1]

Hydraulic fracturing (HF) in porous media is a complex multi-physics problem [2–6] which involves nucleation and propagation of cracks by injecting a combination of padding

fluid and additives in the form of proppant at high pressure down a wellbore. HF is omnipresent in the United States (Fig. 1) due to its historical use for stimulation of oil and gas reservoirs [3,7]. In the recent past, techniques like pulse or fatigue fracturing [8–12] have been deployed in which a controlled dynamic release of fluid energy pressurizes a wellbore to initiate multiple fractures, which are then propagated by static or dynamic pressurization. Pulsing been proposed as a strategy to enhance the productivity and mitigate the environmental impact of HF hydrocarbon projects due to lower quantities of fracturing fluid required [11,13]. Pulse fracturing in hot dry rock has been documented to have long lasting implications for enhanced geothermal energy (EGS) production [4,14–17].

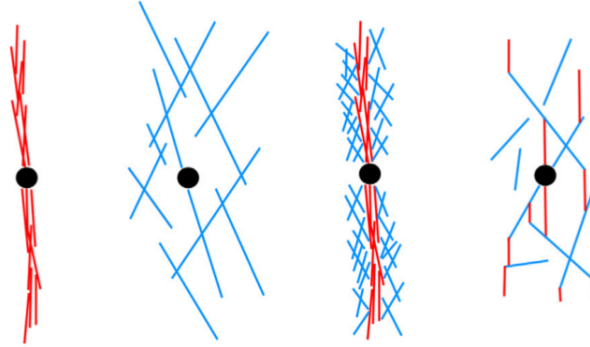


Figure 2: Four conceptual models for EGS stimulation. The black dot represents the wellbore. New fractures are represented with red lines, and preexisting fractures are represented with blue lines. Source: [14]

The conceptual models that have been proposed in literature as stimulation mechanisms in EGS [14] include (1) Only new, propagating fractures contribute to permeability enhancement, (2) Stimulation occurs only through induced slip on preexisting fractures, (3) Continuous new fractures propagate away from the wellbore, but fluid leaks off into natural fractures, which slip and experience enhanced transmissivity, and (4) Continuous pathways for flow involve both new and preexisting fractures. The fourth mechanism has been widely accepted to model hydraulic stimulation of EGS [18–21].

The fracture initiated from a sheared preexisting natural fracture is called a wing crack or a splay fracture. Wing cracks found in rock are tensile fractures but they are different from hydraulic fractures because (1) wing cracks are initiated from the tension field induced by shear slip of a preexisting natural fracture while hydraulic fractures are initiated from an injection well by fluid pressure; (2) wing cracks are curved cracks while hydraulic fractures propagate straight and perpendicularly to the least principal stress; and (3) tension forces to open wing cracks are supported both by fluid pressure and shear slip of a preexisting natural fracture while tension forces to open hydraulic fractures are supported only by fluid pressure [21].

Hitherto, HF has been associated with microseismic events due to their small magnitudes. However, there is now growing evidence to suggest that HF induced fault activity in EGS projects across the world has caused larger earthquakes [18,20,22]. Fig. 3 shows the distribution of seismicity in the Pohang, South Korea EGS site. Postulates suggest the actual amount of injected fluid was only 0.2% of what is theoretically expected for the largest

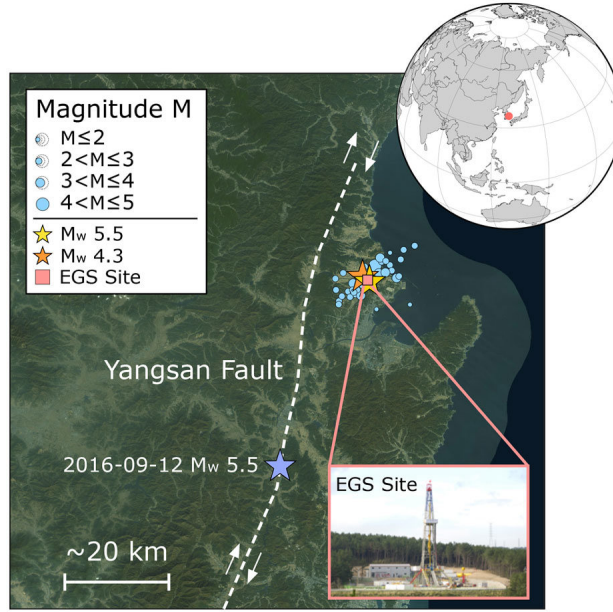


Figure 3: Seismicity in the Pohang EGS site. Source: [22]

recorded earthquake at the site [22, 23]. The causation is established via spatio-temporal correlation; the events occur close (< 10 km) to the injection well and shortly (minutes to hours) after injection. Most of the sites are in geographical regions with little to no history of prior natural seismicity, which further supports the causative link between fracturing and the observed seismic events. The HF-induced earthquake triggering mechanisms (see Fig. 4) that have been proposed are: (1) propagation of the fluid pressure diffusion front through the newly created fracture network, followed by pressurization/lubrication of a basement fault that slips seismically [24], (2) poroelastic stress transfer from slipped fractures on to the fault causing an increase in shear and/or tension on the fault followed by seismic slip, and (3) fluid injection-induced aseismic rupture front which propagates faster and to larger distances than the fluid pressure diffusion front, thereby transmitting stresses to unstable portions of the fault beyond the fluid-pressurized region [25]. All three mechanisms can activate a critically stressed fault (initial stress state close to frictional failure), which can culminate into an induced seismic event [1, 23, 26–32].

In quasi-static simulations, which neglect elastodynamics and wave propagation effects, the induced seismicity mechanisms are often quantified using Coulomb Failure Function (CFF) [33–35], which includes contributions from the effective stress and fault’s friction coefficient. The effective stress is defined as the difference between the total stress (from tectonics and expansion/contraction of surrounding rock) and the pore pressure (from injection) scaled by the Biot coefficient. The friction coefficient evolution often requires a fault rheological model e.g., the Slip Weakening or the Rate- and State-dependent Friction (RSF) model. RSF is the gold standard for modeling earthquake cycles (inter-seismic loading followed by co-seismic relaxation) on faults [36–38], whereas slip weakening provides a reasonable approximation for modeling the drop in friction during a single seismic/aseismic event. CFF is computed using the fault traction vector which is given by the product of the

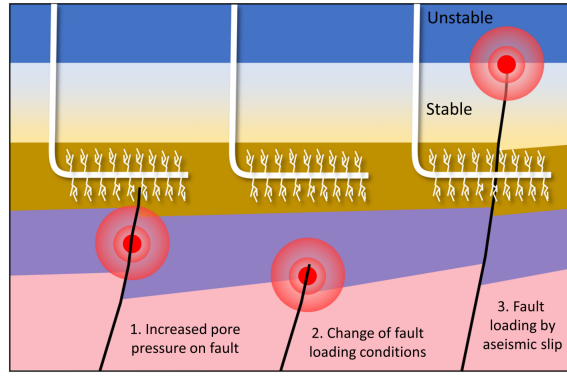


Figure 4: Three proposed HF earthquake triggering mechanisms in literature. Source: [23]

effective stress tensor and the fault normal vector. Initially, when the fault is locked and under equilibrium with the tectonic and hydrostatic stresses, $CFF < 0$. As injection proceeds, the pore pressure increases and the near-wellbore region expands volumetrically, which applies compression on the region farther from the wellbore. The spatial distribution of the total stress tensor and pressure around the fault change. An increase in CFF, i.e. $\Delta CFF > 0$, indicates fluid-induced destabilization of the fault. When CFF exceeds fault's intrinsic cohesion, the fault may slip. The slip releases the host rock strain energy, accumulated from natural and/or anthropogenic loading, over multiple years. The slip can be seismic or aseismic depending on fault's frictional properties (e.g., velocity-weakening vs. velocity-strengthening) and the fault loading rate. Similarly, a decrease in fault's CFF, e.g., due to stresses transferred from a newly created HF, can induce mechanical stabilization of the fault.

2 Literature Review

The modeling and simulation of fluid driven fracture is important for a variety of applications in geotechnical engineering [39,40], biomedical engineering [41,42], and glaciology [43].

2.1 Pulse/fatigue loading:

In a typical fracturing job, a padding fluid (mostly water) is first pumped into the formation to generate fractures. Few hours later, a slurry with proppant (granular material like beach sand or ceramic particles) is injected into the fracture. The pad volume is usually around 10 to 50 percent of the total injected fluid volume. In a non-pulse fracturing job, the proppant concentration monotonically increases to ensure the near-wellbore region maintains high fracture conductivity as the treatment continues. After 2012, the concentration of the proppant has increased in the industry from 0.5 lb/gal to 5 lb/gal [44,45] to create fractures with larger aperture and permeability. In the pulsed proppant fracturing technique, proppant-laden and proppant-free fluid is intermittently pumped into the fracture to promote heterogeneous proppant placement inside the fracture [11,12]. Such intermittent pumping is expected to create wave-like features in both fracture propagation and proppant transport with a hope of enhanced productivity.

2.2 Elastodynamics and fluid inertia:

The HF-induced seismicity problem is expected to be marked with multiple loading and unloading episodes such as dynamic HF propagation, fluid diffusion, and fault-fracture interaction. High-rate transient phenomena such as fracture/fatigue pulsing is also expected. These processes will likely generate dynamic stress wave fronts [46,47] triggering stick-slip behavior of the fault [48]. Modeling these processes require a poro-elastodynamics framework with solid and fluid inertia and the RSF fault rheology. Quasi-static simulations are limited to aseismic slip velocities ($< 1 \mu\text{m/s}$) and cannot be applied to faults that follow the RSF rheology, e.g. due to non-monotonic friction vs. slip behavior or due to multiple loading-unloading episodes [49,50]. The quasi-dynamics approach [51], which approximates solid inertia via a seismic radiation damping term containing a tunable viscosity parameter, has been used as a middle ground between quasi-static and full elastodynamics for modeling induced seismicity [52,53]. However, the approach cannot model dynamic stress transfer and wave propagation effects [54], which are vital for modeling HF-fault interactions in the problem considered here, and suffer from poor accuracy in the simulated seismic magnitudes. Some of the prior studies are also limited by their assumption that the host rock is impermeable [52] or fluid flow is decoupled from rock mechanics [55,56], both of which are not justified for the reservoir rocks we consider here.

Although Darcy's law is the de facto standard for the fluid momentum equation for flow in porous media in the limit of slow flow [57–60], there are multiple reasons why the slow flow assumption does not work for HF under pulse or fatigue loading. Firstly, dimensional analyses [61,62] and experimental studies [63,64] reveal that hydraulic fractures propagate in essentially three regimes, out of which the first and third regimes may violate the slow flow assumption. The first regime is the initiation regime, and the initial fracture velocity can be as high as 100 m/s. Since the fracture velocity is dependent on the pumping rate in the third regime, the inertial effects on fluid flow cannot be ignored for pulse/fatigue loading. Secondly, recent studies [65–68] suggest that including the solid acceleration causes oscillations in the fluid pressure in the high permeability fracture zone. These oscillations are potentially numerical artifacts which can only be removed by incorporating fluid inertia in the mathematical formulation.

Although theoretical frameworks have been proposed [69,70], current dynamic analyses of fractured porous media are often based on the inconsistent extension of Biot's quasi-static theory that neglects fluid inertia. For this reason, numerical methods and tools to solve dynamic hydraulic fracture problems do not yet exist, although some attempts have recently been made employing the extended finite element method [10,71], cohesive zone models [72] and discontinuous pressure model [65].

2.3 Coupling of proppant transport to dynamic HF:

Proppants play a critical role in creating and maintaining the shape and size of hydraulic fractures by controlling the rheology of the fluid inside HF. As a result, we hypothesize proppant transport exerts important control on HF-induced stresses. Yet most modeling studies, which account for HF propagation and flow inside HF, ignore the dynamics of proppant flow due to multiphysics modeling challenges. They approximate the effect of proppants on fluid mobility via empirical functions [73]. Proppant modeling has been recognized as one of the pressing issues in HF modeling [74]. Proppant is usually modeled by

representing the slurry as a two-component, interpenetrating continuum in which the flow equations are solved for the mixture [44, 75–77]. However, none of these studies investigate HF-induced seismicity and sometimes suffer from idealized assumed HF geometry [76].

The four equations that are solved in proppant transport coupled to HF are deformation, fluid flow, fracture propagation and proppant transport. The existing solution algorithms in literature do not iterate between the deformation/flow/propagation and the proppant concentration, thereby yielding a loosely coupled scheme which lead to inaccuracies in the final solution [78–83]. The fixed stress split iterative scheme [34, 84–109] is leveraged in this realm.

2.4 Quantification of induced seismicity:

Elastodynamics [54, 110] and quasi-dynamic [52, 53] approaches allow calculation of the seismic magnitude from dynamic fault rupture simulations. In case of quasi-static simulations (infinite wave speed assumption), with or without aseismic fault slip [34, 35, 111], it is common to use the CFF approach to quantify the likelihood of an induced seismic event. The CFF approach is based on the Mohr-Coulomb frictional failure theory, which defines $CFF = |\tau_s| + \mu_f \sigma'_n$, where $|\tau_s|$ is the shear traction magnitude, $\sigma'_n = \sigma_n + \alpha p$ is the effective normal traction (positive under tension), σ_n is the total normal traction, p is the pore pressure, α is the Biot coefficient, and μ_f is the friction coefficient. All quantities are defined on the fault and σ'_n and τ_s are computed from the fault traction vector which is the product of the effective stress tensor and the fault normal vector. In quasi-dynamics, the radiation damping term, $G\dot{d}/(2V_s)$, is added to the right hand side of the CFF equation, where G is the shear modulus, V_s is the shear wave speed, and $\dot{d}$ is the fault slip velocity along the rake direction.

In case of post-mortem studies of induced seismicity [112–115], the change in CFF, computed over a loading period, is often compared against the recorded seismic magnitude and shear stress released, to confirm or deny the anthropogenic causality. It has been a popular approach to monitor and forecast induced seismicity because solving a quasi-static poro-elastic problem in absence of dynamic slip is mathematically and numerically simpler than solving a poro-elastodynamics problem with RSF slip and possibly transient flow effects. However, the CFF approach neglects dynamic stress transfer processes that we hypothesize play an important role in modulating the interaction between HF and faults in the induced seismicity problem considered in this proposal. Recent work on modeling injection-induced fault slip in confirms that wave propagation is different between a quasi-static poroelastic system and a poro-elastodynamic system [110].

Recent studies [48, 116] has proposed that stable fault creep in hydraulically fractured shales loads distant unstable parts of the fault capable of hosting an earthquake, even hundreds of metres above the hydraulic fracturing zone. They suggest that injection of fracturing fluids can lead to slow slip on a fault, and over time, this creates sufficient strain on another section of the fault to cause it to slip seismically. These studies postulate that following questions arise during elastodynamic extension of induced seismicity models: How far do these wave fronts traverse before they die out? Do these wave fronts get amplified or mitigated by natural fractures and faults? How do the reflected waves affect the influence of the incident waves? Do the seismic waves emanating from hypocenters of induced slip on a fault interact with hydraulic fractures?

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