# CC BY-NC-SA 4.0, Creative Commons License David Nuno Quelhas @ 2021

### Multiple Sensor Interface by the same hardware to USB and serial connection

David Nuno Quelhas

• multiple sensor interface using same hardware to serial connection

• measurement of sensors based on resistance, capacitance, inductance, frequency

• versatile electronics for basic measurement requirements (or low-end usage)

• electronics design aiming for reuse, repurpose, repair, customization

### Multiple Sensor Interface by the same hardware to USB and serial connection

David Nuno Quelhas<sup>a,a</sup>

<sup>a</sup>Lisboa, Portugal

### Abstract

The Multiple Sensor Interface is a simplistic sensor interface to USB, RS485, GPIO, that allows to make measurements of a variety of sensors based on the variation of inductance, resistance, capacitance, frequency using exactly the same connector and same electronic interface circuit between the sensor and the microcontroler. The same device also provides some additional connectors for small voltage measurement. Any sensors for the measurement of distinct phenomena can be used as long the sensor output is based on inductance, resistance, capacitance, frequency within the measurement range of the device, obtaining a variable precision depending of used sensor. The device is not meant for precision/accuracy measurement, is meant to be a reusable hardware that can be reused for most distinct situations, providing to the user more freedom of sensor selection as well more options for device/system maintenance or reuse.

### Keywords:

multiple sensor interface, resistance capacitance inductance frequency, USB, serial UART RS485, GPIO

### 1. Introduction

The electronic waste (e-waste) is a modern problem that has been under increasing concern and awareness, there are various possible approaches to reduce and mitigate the effects of electronic waste. The most obvious are programs for collection and recycling of discarded devices, however the most ideal was just to design and produce technology that lasts because not only is physically fit by quality design, production and components, but because its design was intended to be most versatile possible ensuring the same device can be used and reused in various applications/contexts just by changing connections, jumpers and of course firmware/software configurations. Some obvious characteristics for making a device reusable is to design it using standard connectors and protocols, think of it as a module to be part of a larger system, minimize barriers for connecting/interfacing components and devices from distinct manufacturers.

-This article will focus on the design of a sensor interface I made as a hobby project, aimed at creating an electronic device that could interface to USB and serial(UART,RS-485), many distinct 2-wire sensors based on variation of inductance, resistance, capacitance, frequency and also small voltage; sensors that can be interchanged using the same hardware and using the same port on the device, thus meaning the electronics designed must also be versatile.

-Obviously providing a versatility device to the users will have its negative trade-offs, like: 1- probably significant lower precision/accuracy, 2- some sensor calibration must be performed by the end user, 3- the calibration function will not be linear or 'easy' as desired for sensors and its interfaces.

-However for some applications the mentioned trade-offs are not necessarily a deal-breaker, like when the end user is technical and is ok with using a device that requires more setup/configuration. Some users just like to have a device that is the more customizable even if that means more effort on how to use it.

-So the end goal here is to have technological applications/systems that even if they are harder to use, it will be easier for the user to solve a situation of damaged components or modules, and a technology less likely to be obsolete or without a purpose for the user. Also a device that even if no longer useful for a user, it might still be useful for another user on a different application/context.

### 2. License and context

- The hardware desing here disclosed is distributed under "CERN Open Hardware Licence Version 2 -Weakly Reciprocal" (CERN-OHL-W), its associated software/firmware under GNU licenses (GPL, LGPL).

- This article is published under the Creative Commons license CC BY-NC-SA 4.0 (Attribution-NonCommercial-ShareAlike 4.0 International).

*Email address:* david.n.quelhas@gmail.com (David Nuno Quelhas)

URL: https://pt.linkedin.com/in/dquelhas (David Nuno Quelhas), https://multiple-sensor-interface.blogspot.com (David Nuno Quelhas)

<sup>-</sup> This article is about a 'hobby' project done by the author (David Nuno Quelhas, MSc Electronics Eng, alumni of Instituto Superior Tecnico, Portugal) with occasional 'work' between years 2012 and 2021 on his 'free time'.



Figure 1: Diagram of the Multiple Sensor Interface device.

### 3. Sensor Interface Device

Here is presented the Multiple Sensor Interface, the interface main components/sub-circuits are: The connectors and sensor interface circuits for inductance, resistance, capacitance, frequency (CH.0, CH.1), the connectors and over-voltage protection(zener diode) for frequency measurement (CH.2, CH.3, CH.4, CH.5), the connectors and interface circuit for voltage measurement (ADC.0, ADC.1, ADC.2, ADC.3), analog multiplexer for the sensor channels, the microprocessor(PIC18F2550), I2C EEPROM for storing calibration tables, USB connector, serial connector and circuit for RS-485 and UART, digital outputs connector (OUT.1, OUT.2).

### 4. The sensor interface circuit (oscillator)

The sensor interface circuit is an oscillator, with its design based on the Pierce oscillator, however are some modifications to the Pierce oscillator, the 1st difference is that there isn't a quartz crystal and on the location of the crystal will be connected the sensor to be measured (variable inductance or resistance or capacitance), the 2nd difference is that instead of simple inverters ('NOT' gates) will be used Schmitt-trigger inverters(high-speed Si-gate CMOS, 74HC14) this is a very relevant difference that will allow the oscillator to work even with a resistive or capacitive sensor (in fact the oscillator circuit is two different oscillator types depending on the sensor connected by the user), also the use of Schmitt-Trigger inverters will minimize signal jitter of the oscillator output.

- The sensor interface circuit will have 2 pairs of series capacitors (C1 2.2nF,C1-A 22pF and C2 2.2nF,C2-B 22pF) instead of just 2 capacitors(C1,C2) so the value of C1 and C2 can be adjusted just by placing/removing a jumper; placing a jumper removes C1-A or C1-B from the circuit making 2.2nF the value of C1 or C2; removing the jumper lets the capacitors in series making 21.78pF the total value of (C1-C1-A) or (C2-C2-B). So whenever on the



Figure 2: Schematic of the sensor interface circuit (oscillator). rest of the article is mentioning C1 or C2 is meant the capacitor value ingoring the detail that its value is adjustable by the user.

- The sensors can be connected directly on the Multiple-Sensor Interface (on then screw terminals/connectors), or by using a cable, in case the cable is longer than 20cm is recommended the used of shielded twisted-pair (STP) cable to prevent cross-talk between sensor channels and/or external EMI.

### 5. Measurement steps

The Multiple-Sensor device has a microprocessor (PIC18F2550) that is able to make frequency and voltage measurements, so the device will make frequency measurements for sensor channels CH.0 to CH.5 ; and will make voltage measurements for sensor channels (ADC.0 to ADC.3); these frequency and voltage measurements made by the device will be designated as the RAW\_value of a sensor channel. To obtain the measurement of a sensor channel the device will then use a 2 column calibration table that is a long list of points (RAW\_value; measurement) relating the measurement value (obtained during calibration by an external reference device) to the corresponding RAW\_value obtained on the Multiple-Sensor device, these calibration tables will be stored on an I2C EEPROM mem-

### CC BY-NC-SA 4.0, Creative Commons License David Nuno Quelhas © 2021

ory on the Multiple-Sensor device.

-The Multiple-Sensor device can work in two modes: single-channel or multiple-channel, the CH.0 to CH.5 RAW\_value(frequency) will be calculated through a counter/timer of the PIC18F2550 by periodically reading its value and calculating the frequency f[Hz]=count[cycles]/period[s]. So in single-channel mode the frequency will be always calculated on the selected/enabled sensor channel, in multiple-sensor mode the frequency will be calculated for each sensor channel sequentially (time-division multiplexing) since there are 6 channels to measure but only on counter/timer of the microprocessor for that job. Thus in multiple-channel mode the measurement will take 6x more time to be updated/refreshed than in single-channel mode.

-For the sensor channels ADC.0 to ADC.3 the RAW\_value is the voltage of those channels that is measured by using the ADC (Analog to Digital Converter) of the microprocessor and for reading those channels and also reading a 2.5V voltage reference.

-The sensor measurements will be calculated by searching the RAW\_value on the correspondent calibration table, and will be used from the table 2 points (RAW\_value,measurement) referenced here as points A and B such that the measured RAW\_value is bigger than RAW\_value of A and is lower than RAW\_value of B; then will be calculated a linear equation: measurement=(a\*RAW\_value)+b defined by the points A and B; and so every-time the device calculates a sensor measurement it will calculate the current linear equation for the current RAW\_value and use it to obtain the current measurement (Fig. 3).



Figure 3: Plot experimental data with line, LDR light(brightness) sensor connected on Multiple-Sensor Interface; example of a calibration table exclusively from experimental data.

### 6. Device calibration for a sensor

-So device calibration is about obtaining calibration tables for each sensor channel, there are 2 ways to obtain a calibration table:

1- Do a full manual calibration using an external meter as reference where both the reference meter and the Multiple-sensor device(with sensor(s) connected) are exposed to same stimulus/environment that is controllable by the user to produce all adequate variations/intensities necessary to record an extensive calibration table, with all experimental pairs of (RAW\_value,measurement).

2- Using a know function that relates the measured phenomena to the obtained RAW-value on the Multiple-Sensor device (obtained by theoretical or experimental study), although a purely theoretical calibration could be used that is not recommended, and for obtaining a calibration table with useful/adequate precision probably is only possible by using that known function having its constants/parameters calculated by a data fitting to some few experimental data points (RAW\_value, measurement) obtained for the device calibration. So for example if the know function had 6 constants/parameters you would require at least 6 different experimental measurements to obtain the function for that sensor channel, then having the function is just a question calculating a long list of pairs (RAW\_value, measurement) on the desired measurement range. The (Fig. 4) is the result of fitting the model function Illuminance(f) = (a + (b/(c + d \* $(n * f^2) + m$  to the points (229Hz, 0LUX; 8500Hz, 10LUX; 14000Hz, 30LUX; 76500Hz, 300LUX; 130000Hz, 1072LUX; 194000Hz, 3950LUX).



Figure 4: Plot experimental data and fitted model, LDR light(brightness) sensor connected on Multiple-Sensor Interface; model fit by using 6 points of experimental data.

### 7. Multiple-Sensor Interface for Inductive sensors

When connecting an inductor or inductive sensor(having as output a variation of inductance) the Multiple-Sensor Interface (Fig.2) will work as a Pierce Oscillator(where the sensor is connected instead of a quartz crystal). The theoretical analysis used here for the oscillator will be based on a model of 2 circuit blocks named 'A' and ' $\beta$ ' connected for feedback by connecting the output of one to the input other. The 'A' is an electronic amplifier providing voltage gain, the ' $\beta$ ' is an electronic filter providing frequency selection (resonance), so whatever voltage signal amplified by 'A' is frequency selected by ' $\beta$ ' and feed back to the input of A for further amplification. As know this oscillator is start-up by whatever noise available at the input of 'A', figure 5 is a diagram depicting this concept. CC BY-NC-SA 4.0, Creative Commons License David Nuno Quelhas © 2021



Figure 5: Diagram of model for the oscillator with an inductive sensor (Pierce Oscillator, model of feedback linear oscillator).

- The analysis of the oscillator with an inductive sensor will be made using the Barkhausen stability criterion, that says  $A\beta = 1$  to be possible to occur sustained oscillations (oscillations on steady state analysis). This statement is basically saying that the transfer function of the closed loop is equal to 1 (the closed loop is viewing Vo port as the input and output of the circuit). This mathematical expression simply says that a circuit with a feedback loop after reaching the steady-state is expected that any voltage signal (for example Vo) will remain steady. So from  $A\beta = 1$  results:  $|A\beta| = 1$  and  $\angle A\beta = \pm n2\pi$ .

- Starting the analysis using a possible representation of the mentioned circuit having the 'feedback network' represented by its hybrid parameters (h-parameters of a 2-port network).[1] [2]



Figure 6: Representation of oscillator circuit by the h-parameters for feedback circuit (Pierce Oscillator).

- The Schmitt-Trigger Inverter(high-speed Si-gate CMOS) and resistors  $R_1$ ,  $R_2$  belong to circuit block 'A', the capacitors  $C_1$ ,  $C_2$  and inductive sensor  $L_{sensor}$  belong to circuit block  $\beta$ .

- Since the 'Basic Amplifier' has a very big input resistance the current  $I_1$  will be very small (the electric current on the input of the inverter (CMOS 'NOT' gate) is negligible), so by using the h-parameters to represent the feedback circuit block is possible to make the following simplifications/approximations: 1- The current source  $h_{21}I_1$  is also negligible (equal to zero, so removed from circuit); 2-The voltage drop across component  $h_{11}$  is negligible (since  $V_{11} = h_{11}I_1$ ,  $I_1 \rightarrow 0 \Rightarrow V_{11} \rightarrow 0$ ) and so  $h_{11}$  can be relocated to inside the circuit block 'A' keeping  $V_i$  as the name for the voltage drop on the input of 'A' block; 3-The  $h_{22}$  can be relocated to inside the circuit block 'A' since is connected in parallel to the input of ' $\beta$ ' block that is also the output of 'A' block. [2]

- The block 'Basic Amplifier' is then replaced by its



Figure 7: Simplified representation of oscillator circuit (Pierce Oscillator).



Figure 8: More simplified representation of oscillator circuit (Pierce Oscillator).

The venine equivalent circuit, obtaining the circuit on Fig-8. From this circuit you can write the transfer function (Laplace transform) of 'A' block ( $V_o = AV_i$ ,  $1/h_{22} = h_{22}^{-1}$ ) and ' $\beta$ ' block ( $V_1 = \beta V_2$ ):

$$A = \frac{h_{22}^{-1}}{h_{22}^{-1} + r_o} \mu \frac{r_i}{r_i + h_{11}} \qquad \qquad \beta = h_{12}$$

- The feedback network (the same network displayed on Fig-6 represented by h-parameters) is the frequencyselecting  $\pi$ (shaped)-network on Fig-9, where  $Z_1$ ,  $Z_2$ ,  $Z_s$ are respectively  $C_1, C_2, L_{sensor}$  of the Multiple Sensor Interface with an inductive sensor (Pierce oscillator).



Figure 9: Feedback network of oscillator (Pierce Oscillator, Multiple-Sensor Interface with inductive sensor).

- Based on the circuit of the feedback network and using the definitions of h-parameters of a 2-port network are obtained the values:

$$\begin{aligned} h_{11} &= Z_1 || Z_s \qquad h_{22}^{-1} &= Z_2 || (Z_1 + Z_s) \qquad h_{12} &= \frac{Z_1}{Z_1 + Z_s} \\ \text{(where } || \text{ is the impedance of 2 components in parallel,} \end{aligned}$$

 $Z_1||Z_2 = (Z_1Z_2)/(Z_1 + Z_2) ; h_{11} = (V_1/I_1)|_{V_2=0} ; h_{12} = (V_1/V_2)|_{I_1=0} ; h_{22} = (I_2/V_2)|_{I_1=0} ) .$ Finally:

Finally:  $A = \frac{Z_2 ||(Z_1 + Z_s)}{Z_2 ||(Z_1 + Z_s) + r_o} \mu \frac{r_i}{r_i + (Z_1 ||Z_s)} \qquad \beta = \frac{Z_1}{Z_1 + Z_s}$ 

## CC BY-NC-SA 4.0, Creative Commons License David Nuno Quelhas @ 2021

- In the circuit of Pierce oscillator (Fig-2) the amplifier was made by a gate inverter (CMOS 'NOT' gate) and a feedback resistor ( $R_1 = 2M\Omega$ ) biasing the inverter in its linear region of operation causing it to function as a highgain inverting amplifier. The input resistance of CMOS 'NOT' gate is very large and so making the approximation  $r_i = +\infty$  the expression of 'A' is significantly simplified, So obtaining: [2]

$$A\beta = \left(\frac{Z_2||(Z_1 + Z_s)}{Z_2||(Z_1 + Z_s) + r_o}\mu\right) \left(\frac{Z_1}{Z_1 + Z_s}\right)$$
$$= \frac{Z_1 Z_2 \mu}{Z_2 (Z_1 + Z_s) + r_o (Z_1 + Z_2 + Z_s)} \quad (1)$$

- If  $Z_1, Z_2, Z_s$  are purely reactive impedances given by  $Z_1 = jX_1, Z_2 = jX_2, Z_s = jX_s \ (j = \sqrt{-1})$ , then Eq-1 becomes:

$$A\beta = \frac{X_1 X_2 \mu}{X_2 (X_1 + X_s) - jr_o (X_1 + X_2 + X_s)}$$
(2)

- The Barkhausen criterion states  $A\beta = 1 \rightarrow \angle A\beta = \pm n2\pi$ , this means the phase shift of the loop ' $A\beta$ ' must be zero, and so that implies that the imaginary part of Eq-2 must be zero. That is, for stable oscillations on the circuit of Fig-6 with a Feedback Network of Fig-9, it must be assured: [2]

$$X_1(\omega_0) + X_2(\omega_0) + X_s(\omega_0) = 0$$
 (3)

- At the frequency  $\omega_0$  (frequency of oscillation at steady state), Eq-2 reduces to: [2]

$$A(\omega_0)\beta(\omega_0) = \frac{X_1(\omega_0)\mu}{X_1(\omega_0) + X_s(\omega_0)}$$
(4)

and using Eq-3 is obtained:

$$A(\omega_0)\beta(\omega_0) = -\mu \frac{X_1(\omega_0)}{X_2(\omega_0)}$$
(5)

- To start the oscillations the loop gain must be grater than unity (during Transient Response), but after achieving the Steady State Response on the Pierce oscillator for oscillations to occur the loop gain  $A(\omega_0)\beta(\omega_0)$  of Eq-5 must be equal to unity('1'). Since the amplifier is an inverter ('NOT' gate with R1 feedback resistor) then  $\mu$  is a negative number, and so  $X_1(\omega_0)$  and  $X_2(\omega_0)$  must have the same sign (both positive or negative).

- Thus, if  $Z_1(\omega_0)$  is capacitive $(X_1(\omega_0) = -1/(\omega_0 C_1))$ , then by Eq-5  $Z_2(\omega_0)$  must also be capacitive  $((X_2(\omega_0) = -1/(\omega_0 C_2)))$ . [2]

- Considering this and using Eq-3 its possible to conclude that  $Z_s(\omega_0)$  must be inductive since  $X_s(\omega_0) = -X_1(\omega_0) - X_2(\omega_0)$ , this is if  $X_1(\omega_0)$  and  $X_2(\omega_0)$  are negative numbers then  $X_s(\omega_0)$  must be a positive number, then  $X_s = \omega_0 L_s$ .

- So having  $C_1$ ,  $C_2$ ,  $L_s$  on the feedback network (where  $L_s$  is the component representing the inductive sensor on the Multipl-Sensor Interface) the Eq-3 becomes:

$$-\frac{1}{\omega_0 C_1} - \frac{1}{\omega_0 C_2} + \omega_0 L_s = 0 \tag{6}$$

So defining "load capacitance" 
$$C_L$$
 as: [2]  

$$\frac{1}{C_L} = \frac{1}{C_1} + \frac{1}{C_2}$$
(7)
The frequency of oscillation on the Pierce oscillator (using

The frequency of oscilation on the Pierce oscillator (using

$$C_1, C_2, L_s$$
) is:  
 $\omega_0 = \frac{1}{\sqrt{L_s C_L}} \quad \Leftrightarrow \quad f_0 = \frac{1}{2\pi\sqrt{L_s C_L}} \tag{8}$ 

- So the expression (theoretical) of measured inductance  $L_{sensor}$  as a function of frequency(f) is:

$$L_{sensor} = \frac{C_1 + C_2}{4\pi^2 C_1 C_2 f^2} \tag{9}$$

- The Fig-10 shows the experimental data for the Multiple-Sensor Interface with various inductance values connected as the sensor, and also shows the plot of  $L_{sensor}(f)$  using Eq-9 with  $C_1 = C_2 = 2.2nF$ :



Figure 10: Inductance [ $\mu H$ ] versus frequency [Hz] (Pierce Oscillator, Multiple-Sensor Interface with inductive sensor).

### 8. Multiple-Sensor Interface for Resistive sensors

- In case you connect a resistive sensor (or capacitive) to the Multiple-Sensor Interface you will not be able to satisfy the conditions for oscillation (Eq-3 and Eq-5) were  $A(\omega_0)\beta(\omega_0)=1$ , consequent of the Barkhausen criterion applied to the circuit of Multiple-Sensor Interface, as it was explained on the previous section that on the  $\pi$ (shaped)-network of Fig-9 if  $Z_1$  and  $Z_2$  are capacitive (corresponding to  $C_1$  and  $C_2$  of the Multiple-Sensor Interface) then  $Z_s$  must be inductive so that  $A(\omega_0)\beta(\omega_0)=1$ .

- So the conclusion is when you connect a resistive sensor (or capacitive) you no longer have a Pierce Oscillator, but that is not a problem because the Multiple-Sensor Interface is made using Schmitt-trigger inverters(high-speed Sigate CMOS, 74HC14), and the Schmitt trigger is a bistable multivibrator that can be used to implement another type of multivibrator, the relaxation oscillator. So in the case of a resistive (or capacitve) sensor the circuit to analyze will be a Schmitt-trigger inverter connected to a network of resistors and capacitors.

- So to analyze this circuit the Schmitt-trigger inverter will be replaced by a theoretical switch that will change the voltage of node  $v_O$  to VDDS (voltage of stabilized power supply for the sensor interface) when the voltage of  $v_I$ is lower than  $V_T^-$ , and will change  $v_O$  to GND when the voltage of  $v_I$  is higher than  $V_T^+$ .

- So the circuit of Fig-11 will be analyzed to obtain  $f(R_s)$ , and then its inverse function  $R_s(f) = R_{sensor}$  that is useful for using/configuring the Multiple-Sensor Interface. Notice that  $I_i \approx 0$  since  $V_i$  is the input of the Schmitt-trigger inverter(high-speed Si-gate CMOS) that has a very high input impedance and so  $I_i \approx 0$  is an appropriate approximation simplifying the circuit. So from



Figure 11: Multiple-Sensor Interface with resistive sensor (Schmitttrigger inverter oscillator).

the circuit are obtained the equations:

Nodes and loops:  $i_4 = i_s + i_1$ ,  $i_3 + i_s = i_2$ ,  $i_o = i_3 + i_4$ ,  $i_1 + i_2 = i_o$ ,  $i_1 + i_2 = i_3 + i_4$ ,  $v_1 - v_2 - v_s = 0$ ,  $v_4 + v_s + v_2 - v_o = 0$ ,  $v_4 + v_s - v_3 = 0$ ,  $v_3 = v_o - v_2$ ,  $v_4 = v_o - v_i$ ,  $v_2 = v_i - v_s$ .

Components:  $i_1 = C_1(dv_1/dt)$ ,  $i_2 = C_2(dv_2/dt)$ ,  $v_3 = R_2 i_3$ ,  $v_4 = R_1 i_4$ ,  $v_s = R_s i_s$ Solving:

$$\frac{v_o - v_i}{B_1} = \frac{v_s}{B_o} + C_1 \frac{dv_i}{dt} \tag{10}$$

$$\frac{v_o - v_i + v_s}{P} + \frac{v_s}{P} = C_2 \frac{dv_2}{\mu} \tag{11}$$

- Solving: 
$$v_2 = v_i - v_s \Rightarrow \frac{dt}{dv_2/dt} = \frac{dt}{d(v_i - v_s)/dt} \Rightarrow \frac{dv_2/dt}{dt} = \frac{d(v_i - v_s)}{dt}$$

- So using the previous result the Eq-11 can be changed to:

$$\frac{v_o - v_i}{R_2} + \left(\frac{1}{R_2} + \frac{1}{R_s}\right)v_s = C_2\left(\frac{dv_i}{dt} - \frac{v_s}{dt}\right)$$
(12)

$$v_s = \frac{R_s(v_o - v_i)}{R_1} - R_s C_1 \frac{dv_i}{dt}$$
(13)

- Calculating the derivative on both sides of Eq-13 is obtained (remember  $v_o$  is a constant equal to VDDS or GND depending on the position of the switch 'SW'):

$$\frac{dv_s}{dt} = \frac{-R_s}{R_1}\frac{dv_i}{dt} - R_sC_1\frac{d^2v_i}{dt^2}$$
(14)

- Now using Eq-13 and Eq-14 to remove the variables  $v_s$  and  $dv_s/dt$  from the Eq-12 is obtained an equation solvable for determining  $v_i(t)$ :

$$\begin{pmatrix} \frac{1}{R_2} + \frac{R_s}{R_2 R_1} + \frac{1}{R_1} \end{pmatrix} (v_o - v_i) = \\ \left( C_1 \left( 1 + \frac{R_s}{R_2} \right) + C_2 \left( 1 + \frac{R_s}{R_1} \right) \right) \frac{dv_i}{dt} + R_s C_1 C_2 \frac{d^2 v_i}{dt^2}$$
(1)

- The Eq-15 is of the type:  $c(v_o - v_i) = b(dv_i/dt) + a(d^2v_i/dt^2)$  that has the general solution:  $v_i(t) = v_o + k_1e^{\lambda_1t} + k_2e^{\lambda_2t}$ , where  $k_1,k_2$  are integration constants to be defined by 'initial conditions' and  $\lambda_1$ ,  $\lambda_2$  are defined by:  $a\lambda^2 + b\lambda + c = 0 \Leftrightarrow \lambda = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$  and e is the Euler-Napier constant  $e = \sum_{n=0}^{\infty} (1/(n!))$ .

- So for a solution to this circuit: 
$$a = R_s C_1 C_2$$
,  
 $b = (C_1(1 + (R_s/R_2))) + (C_2(1 + (R_s/R_1)))$ .

 $c = (1/R_2) + (R_s/(R_2R_1)) + (1/R_1)$ .

- Is selected the solution of  $\lambda_2 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$  by setting  $K_1 = 0$ , because is the one that will provide an adequate value for  $v_i(t)$  and  $f(R_s)$  consistent with experimental data, however for obtaining the function  $R_s(f)$  you may use any.

- For convenience of making  $v_i(t)$  more similar to typical RC circuits will be defined a new variable  $\tau = -1/\lambda$  and so  $v_i(t) = v_o + k_2 e^{-t/\tau_2}$ .

- Charging time of 
$$C_1$$
:  $v_o = VDDS$   
 $v_i(t=0)=V_T^- \rightarrow V_T^- = VDDS + k_2e^0 \rightarrow k_2 = V_T^- - VDDS$   
 $v_i(t=T_C)=V_T^+ \rightarrow V_T^+ = VDDS + k_2e^{-T_C/\tau_2} \rightarrow$   
 $\rightarrow T_C = -\tau_2 ln((V_T^+ - VDDS)/(V_T^- - VDDS))$ 

- Discharging time of  $C_1: v_o = 0$   $v_i(t=0)=V_T^+ \to V_T^+ = 0 + k_2 e^0 \to k_2 = V_T^+$   $v_i(t=T_D)=V_T^- \to V_T^- = 0 + k_2 e^{-T_D/\tau_2} \to$  $\to T_D = -\tau_2 ln(V_T^-/V_T^+)$ 

- The time for a complete cycle of charge and discharge of  $C_1$  is:  $T = T_C + T_D$ ; the frequency of  $v_i(t)$  is f = 1/T. - Solving:  $T = -\tau_2 \left( ln \left( \frac{V_T^+ - VDDS}{V_T^- - VDDS} \right) + ln \left( \frac{V_T^-}{V_T^+} \right) \right)$  $\Leftrightarrow T = \tau_2 ln \left( \frac{(V_T^- - VDDS)V_T^+}{(V_T^+ - VDDS)V_T^-} \right)$ 

- For convenience defining the constant 'H' by:

$$H = ln\left(\frac{(V_T^- - VDDS)V_T^+}{(V_T^+ - VDDS)V_T^-}\right),$$
  
$$H \Leftrightarrow f = -\lambda_0 / H$$

then  $f = 1/T \Leftrightarrow f = 1/(\tau_2 H) \Leftrightarrow f = -\lambda_2/H$ . - So the expression (theoretical) of measured resistance  $R_{\text{senser}}$  as a function of frequency(f) is:

$$R_{sensor} \text{ as a function of frequency(f) is:} R_{sensor} = R_s = \frac{(C_1 + C_2)R_2R_1Hf - R_2 - R_1}{(C_2R_2Hf - 1)(C_1R_1Hf - 1)}$$
(16)

- So using the values of  $C_1=C_2=2.2nF$ ,  $R_2=500\Omega$ ,  $R_1=2M\Omega$ ,  $V_T^-=1.2V$ ,  $V_T^+=2.2V$ , VDDS=4.18V, is obtained H=1.01496, and Fig-12 and Fig-13 shows experimental data for Multiple-Sensor Interface with various resistance values connected as the sensor and also shows the plot of  $R_{sensor}(f)$  using Eq-16 with the mentioned values of  $C_1$ ,  $C_2$ ,  $R_2$ ,  $R_1$ , H.



Figure 12: Resistance[ $\Omega$ ] versus frequency[0Hz, 440000Hz] (Schmitt-Trigger Oscillator, Multiple-Sensor Interface with resistive sensor).



Figure 13: Resistance[ $\Omega$ ] versus frequency[0Hz, 300000Hz] (Schmitt-Trigger Oscillator, Multiple-Sensor Interface with resistive sensor).

## CC BY-NC-SA 4.0, Creative Commons License David Nuno Quelhas © 2021

### 9. Multiple-Sensor Interface for capacitive sensors

- In case you connect a capacitive sensor (or resistive) to the Multiple-Sensor Interface you will not be able to satisfy the conditions for oscillation (Eq-3 and Eq-5) were  $A(\omega_0)\beta(\omega_0)=1$ , and so for capacitive sensors again you no longer have a Pierce Oscillator, but instead have a Schmitttrigger relaxation oscillator.

- So to analyze this circuit the Schmitt-trigger inverter will be replaced by a theoretical switch that will change the voltage of node  $v_O$  to VDDS (voltage of stabilized power supply for the sensor interface) when the voltage of  $v_I$ is lower than  $V_T^-$ , and will change  $v_O$  to GND when the voltage of  $v_I$  is higher than  $V_T^+$ .



Figure 14: Multiple-Sensor Interface with capacitive sensor (Schmitttrigger inverter oscillator).

- So the circuit of Fig-14 will be analyzed to obtain  $f(C_s)$ , and then its inverse function  $C_s(f) = C_{sensor}$  that is useful for using/configuring the Multiple-Sensor Interface. Notice that  $I_i \approx 0$  since  $V_i$  is the input of the Schmitt-trigger inverter(high-speed Si-gate CMOS) that has a very high input impedance and so  $I_i \approx 0$  is an appropriate approximation simplifying the circuit. So from the circuit are obtained the equations:

Nodes and loops:  $i_4 = i_s + i_1$ ,  $i_3 + i_s = i_2$ ,  $i_o = i_3 + i_4$ ,  $i_1 + i_2 = i_o$ ,  $i_1 + i_2 = i_3 + i_4$ ,  $v_1 - v_2 - v_s = 0$ ,  $v_4 + v_s + v_2 - v_o = 0$ ,  $v_4 + v_s - v_3 = 0$ ,  $v_3 = v_o - v_2$ ,  $v_4 = v_o - v_i$ ,  $v_2 = v_i - v_s$ .

Components:  $i_1 = C_1(dv_1/dt), \quad i_2 = C_2(dv_2/dt),$  $v_3 = R_2 i_3, \quad v_4 = R_1 i_4, \quad i_s = C_s(dv_s/dt)$ 

Solving:

$$\frac{v_o - v_i}{R_1} = C_s \frac{dv_s}{dt} + C_1 \frac{dv_i}{dt}$$
(17)

$$\frac{v_o - v_2}{R_2} + C_s \frac{dv_s}{dt} = C_2 \frac{dv_2}{dt} \tag{18}$$

$$\frac{v_o - v_2}{R_2} + \frac{v_o - v_i}{R_1} = C_1 \frac{dv_i}{dt} + C_2 \frac{dv_2}{dt}$$
(19)

- Since  $v_s = v_i - v_2$  then  $dv_s/dt = (dv_i/dt) - (dv_2/dt)$ , and so using it on Eq-17 is obtained:

$$\frac{dv_2}{dt} = \left(1 + \frac{C_1}{C_s}\right)\frac{dv_i}{dt} - \frac{v_o - v_i}{C_s R_1} \tag{20}$$

- Using  $dv_s/dt = (dv_i/dt) - (dv_2/dt)$  and Eq-20 on Eq-18 is obtained:

$$v_{2} = v_{o} + \frac{R_{2}(C_{2}+C_{s})(v_{o}-v_{i})}{C_{s}R_{1}} + R_{2}C_{s}\left(1 - \left(1 + \frac{C_{2}}{C_{s}}\right)\left(1 + \frac{C_{1}}{C_{s}}\right)\right)\frac{dv_{i}}{dt}$$
(21)

- Calculating the derivative of Eq-21 is obtained:  $\frac{d(v_2)}{dt} = -\frac{R_2(C_2+C_s)}{C_s R_1} \frac{dv_i}{dt} + R_2 C_s \left(1 - \left(1 + \frac{C_2}{C_s}\right) \left(1 + \frac{C_1}{C_s}\right)\right) \frac{d^2 v_i}{dt^2}$ (22)

- Now using Eq-21 and Eq-22 to remove the variables  $v_2$  and  $dv_2/dt$  from the Eq-19 is obtained an equation solvable

for determining  $v_i(t)$ :

 $v_o - v_i = (R_2(C_2 + C_s) + R_1(C_1 + C_s))\frac{dv_i}{dt} + R_2R_1(C_1C_2 + C_s(C_1 + C_2))\frac{d^2v_i}{dt^2}$ (23)

- The Eq-23 is of the type:  $c(v_o - v_i) = b(dv_i/dt) + a(d^2v_i/dt^2)$  that has the general solution:

 $v_i(t) = v_o + k_1 e^{\lambda_1 t} + k_2 e^{\lambda_2 t}$ , where  $k_1, k_2$  are integration constants to be defined by 'initial conditions' and  $\lambda_1$ ,  $\lambda_2$ are defined by:  $a\lambda^2 + b\lambda + c = 0 \Leftrightarrow \lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$  and eis the Euler-Napier constant  $e = \sum_{n=0}^{\infty} (1/(n!))$ .

- So for a solution to this circuit: c = 1,
- $b = (R_2(C_2 + C_s) + R_1(C_1 + C_s)),$

$$a = R_2 R_1 (C_1 C_2 + C_s (C_1 + C_2))$$
.

- Is selected the solution of  $\lambda_2 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$  by setting  $K_1 = 0$ , because is the one that will provide an adequate value for  $v_i(t)$  and  $f(C_s)$  consistent with experimental data, however for obtaining the function  $C_s(f)$  you may use any.

- For convenience of making  $v_i(t)$  more similar to typical RC circuits will be defined a new variable  $\tau = -1/\lambda$  and so  $v_i(t) = v_o + k_2 e^{-t/\tau_2}$ .

- So when is connected a capacitive sensor  $(C_s)$  the differential equation and solution  $v_i(t)$  are the same as when is connected a resistive sensor  $(R_s)$ , the only differences are on the values of a, b, c; and as such the equations of f(frequency) and T(period) are also the same and will be reused from the previous chapter.

- The constant 'H' defined by:

$$I = ln\left(\frac{(V_T^- - VDDS)V_T^+}{(V_T^+ - VDDS)V_T^-}\right),$$

and  $f = 1/T \Leftrightarrow f = 1/(\tau_2 H) \Leftrightarrow f = -\lambda_2/H$ . - So the expression (theoretical) of measured Capaci-

tance 
$$C_{sensor}$$
 as a function of frequency(f) is:  

$$C_{sensor} = C_s = \frac{(C_1R_1 + C_2R_2)Hf - 1 - C_1C_2R_1R_2H^2f^2}{Hf(C_1 + C_2R_2)Hf - 1 - C_1C_2R_1R_2H^2f^2}$$

$$Hf((C_1 + C_2)R_1R_2Hf - R_1 - R_2)$$
(24)

- So using the values of  $C_1=C_2=2.2nF$ ,  $R_2=500\Omega$ ,  $R_1=2M\Omega$ ,  $V_T^-=1.2V$ ,  $V_T^+=2.2V$ , VDDS=4.18V, is obtained H=1.01496, and Fig-15 shows the plot of  $C_{sensor}(f)$  using Eq-24 with the mentioned values of  $C_1$ ,  $C_2$ ,  $R_2$ ,  $R_1$ , H.



Figure 15:  $C_s(f)$  [F] versus frequency[-400kHz, 1MHz] (Schmitt-Trigger Oscillator, Multiple-Sensor Interface with capacitive sensor).

- Analyzing the plot on Fig-15 by firstly looking at plot regions with  $C_s > 0$ , its visible that the  $C_s(f)$  plot is over the  $C_s$  axis (this is the vertical axis f=0) and this would mean that for all values of  $C_s$  the frequency would be zero

## CC BY-NC-SA 4.0, Creative Commons License David Nuno Quelhas © 2021

(f=0), but also visible to the right is another curve that is also placed on the area of  $C_s>0$  (for frequency[447957Hz, 895689Hz]) and at first this curve would be expected to define the operating frequency of the oscillator depending on the capacitance of the connected sensor.

- However experimental data will show that the way the oscillator works using a capacitive sensor is very different from what would be expected from a first view on the plot of function  $C_s(f)$ , in order to compare the experimental data with the theoretical model is shown on Fig-16 and Fig-17 the experimental data for Multiple-Sensor Interface with various capacitance values connected as the sensor and also the plot of  $abs(C_s(f))$  (=  $|C_s(f)|$ ) using Eq-24 with the mentioned values of  $C_1$ ,  $C_2$ ,  $R_1$ ,  $R_2$ , H.



Figure 16: Capacitance[nF] versus frequency[0Hz, 440000Hz] by  $|C_s(f)|$  (Schmitt-Trigger Oscillator, Multiple-Sensor Interface with



Figure 17: Capacitance[nF] versus frequency[0Hz, 170000Hz] by  $|C_s(f)|$  (Schmitt-Trigger Oscillator, Multiple-Sensor Interface with capacitive sensor).

- So it seems that the obtained function of  $C_s(f)$  although strangely indicates negative values for the sensor capacitance it can provide a theoretical curve/plot very similar to what was obtained on the experimental data for  $C_{sensor}$ . On the following chapters will be given a better insight on why  $C_s(f)$  has a negative value.

### 10. Multiple-Sensor Interface circuit simplified (for $C_s$ or $R_s$ )

- The Multiple-Sensor Interface circuit when using a capacitive or resistive sensor can be studied and understood in a more intuitive way by making some simplification/approximation that may be inaccurate for quantitative purposes but still captures the essence of how the circuit works with the advantage of exposing how it works and resulting in much simpler differential equations. So when using a capacitive or resistive sensor the circuit is a Schmitt-Trigger oscillator that uses a more complex circuit but its essence is the same, it is just some capacitors being charged by currents that pass trough some resistors, and the voltage on a capacitor( $v_i$ ) will trigger(at  $V_T^-$  or  $V_T^+$ ) a switch(electronic inverter) to change the voltage( $v_o$ ).[3]

- So the proposed simplification/approximation is to admit the Multiple-Sensor Interface circuit (for  $C_s$  or  $R_s$  as sensor) can be described approximately by using the circuit of a basic/normal Schmitt-Trigger Oscillator that only has one capacitor and one resistor (that determine the frequency of oscillation), and so will be used as alternative the circuit of Fig-18 where  $C_{approx}$  is a capacitor and  $R_{approx}$  is a resistor that approximate in general the capacitance and resistance respectively of the Multiple-Sensor Interface.



Figure 18: Schematic of a basic/normal Schmitt-Trigger Oscillator to be used as an approximation of the circuit of Multiple-Sensor Interface with capacitive or resisitve sensor).

- To build expressions of  $C_{approx}$  and  $R_{approx}$  as a function of  $R_1, R_2, C_1; C_2$  will be considered initially 2 extreme cases of the sensor impedance  $(Z_s)$ : 1st  $|Z_s| = 0$  the sensor can be replaced by a wire, and 2nd  $|Z_s| = \infty$  the sensor can be removed (open circuit), these 2 extreme cases possible for the sensor impedance are represented in Fig-.



Figure 19: Schematic of the RC network of the Schmitt-Trigger Oscillator for the 2 extreme cases of the sensor impedance  $(Z_s)$ .

- Now the sensor can be described as an electric connection that can be weekned or intensified depending on the sensor impedance, so when  $|Z_s|$  changes progressively from 0 to  $+\infty$  the circuit changes progressively and smoothly from the left circuit to the right circuit on Fig-19. So to obtain equations for  $R_{approx}$  and  $C_{approx}$  will be selected an expression that allows to change smothly the resistance and capacitance of the RC circuit on the left circuit (Fig-19) to the resistance and capacitance of the RC circuit on the right circuit (Fig-19).

- So from the 2 circuits on Fig-19 is obtained the resistance and capacitance on the 2 extreme values of  $|Z_s|=0$  and  $|Z_s|=+\infty$ :

$$\begin{array}{l} C_{approx}(Z_{s}=0)=C_{1}+C_{2}; & C_{approx}(Z_{s}=\infty)=C_{1}; \\ R_{approx}(Z_{s}=0)=(R_{1}R_{2})/(R_{1}+R_{2}); & R_{approx}(Z_{s}=\infty)=R_{1}; \end{array}$$

### 10.1. $R_{approx}$ and $C_{approx}$ for a resisitve sensor $(R_s)$

Here are functions modeled to describe  $C_{approx}$  and  $R_{approx}$  (with resistive sensor) with a smooth transition

### CC BY-NC-SA 4.0, Creative Commons License David Nuno Quelhas © 2021

from its values at  $|Z_s|=0$  and  $|Z_s|=+\infty$ , where  $|Z_s|=R_s$ :

$$C_{approx} = (C_1 + C_2) \frac{R_1}{|Z_s| + R_1} + C_1 \frac{|Z_s|}{|Z_s| + R_1}$$
(25)

$$R_{approx} = \frac{R_1 R_2}{R_1 + R_2} \frac{2R_1}{|Z_s| + 2R_1} + R_1 \frac{|Z_s|}{|Z_s| + 2R_1}$$
(26)

Using the equations:  $f = 1/T \Leftrightarrow f = 1/(\tau_2 H) \Leftrightarrow f = -\lambda_2/H$ ; and using  $\tau_2 = R_{approx}C_{approx}$  can be obtained  $f(R_s)$  and/or  $R_s(f)$ .

- So using the values of  $C_1=C_2=2.2nF$ ,  $R_2=500\Omega$ ,  $R_1=2M\Omega$ ,  $V_T^-=1.2V$ ,  $V_T^+=2.2V$ , VDDS=4.18V, is obtained H=1.01496, and Fig-20 shows the plot of  $R_s(f)$  using the mentioned approximate model  $(C_{approx}, R_{approx})$  with the mentioned values of  $C_1$ ,  $C_2$ ,  $R_2$ ,  $R_1$ , H.



Figure 20: Resistance[ $\Omega$ ] versus frequency[0Hz, 300000Hz] by approximate model ( $C_{approx}$ ,  $R_{approx}$ ; Multiple-Sensor Interface with resistive sensor).

### 10.2. $R_{approx}$ and $C_{approx}$ for a capacitive sensor $(C_s)$

Here are functions modeled to describe  $C_{approx}$  and  $R_{approx}$  (with capacitive sensor) with a smooth transition from its values at  $|Z_s|=0$  and  $|Z_s|=+\infty$ , where  $|Z_s|=1/(2\pi f C_s)$ :

$$C_{approx} = (C_1 + C_2) \frac{R_1}{|Z_s| + R_1} + C_1 \frac{|Z_s|}{|Z_s| + R_1}$$
(27)

$$R_{approx} = \frac{R_1 R_2}{R_1 + R_2} \frac{R_1}{|Z_s| + R_1} + R_1 \frac{|Z_s|}{|Z_s| + R_1}$$
(28)

- So using the values of  $C_1=C_2=2.2nF$ ,  $R_2=500\Omega$ ,  $R_1=2M\Omega$ ,  $V_T^-=1.2V$ ,  $V_T^+=2.2V$ , VDDS=4.18V, is obtained H=1.01496, and Fig-21 and Fig-22 shows the plot of  $C_s(f)$  using the mentioned approximate model  $(C_{approx}, R_{approx})$  with the mentioned values of  $C_1$ ,  $C_2$ ,  $R_2$ ,  $R_1$ , H.



Figure 21: Capacitance [nF] versus frequency [0Hz, 450000 Hz] by approximate model ( $C_{approx}, R_{approx}$ ; Multiple-Sensor Interface with capacitive sensor).



Figure 22: Capacitance[nF] versus frequency[0Hz, 170000Hz] by approximate model ( $C_{approx}$ ,  $R_{approx}$ ; Multiple-Sensor Interface with capacitive sensor).

### 11. Why $C_s(f) < 0$ on Multiple-Sensor Interface with capacitive sensor

- About  $C_s(f) < 0$  have in mind the Multiple-Sensor with a capacitive sensor is studied on transient behavior (relaxation oscillator), where 'frequency' is a measure of the speed of charge and discharge on C1; and also of how fast the transient circuit analysis alternates between 2 different schematics ( $v_o = VDDS$  and  $v_o = 0$ ).

- To understand why a normal capacitor behaves as negative capacitance when connected as the sensor of the Multiple-Sensor Interface (this is, why  $C_s(f) < 0$ ), is important to highlight some things already explored on the previous chapters: 1)  $C_1=C_2$ ,  $R_1 \gg R_2$ ;

2) The primary path (always available) to charge  $C_1$  is through  $R_1$ , the primary path (always available) to charge  $C_2$  is through  $R_2$ , since  $R_1 \gg R_2$  and  $C_1=C_2$  this implies that capacitor  $C_2$  will charge/discharge much faster(takes less time) than capacitor  $C_1$ .

3) The purpose of sensor  $C_s$  on this circuit is to act as a variable impedance that can establish an alternative path on the circuit  $(V_o \rightarrow R_2 \rightarrow C_s \rightarrow C_1)$  to charge/discharge capacitor  $C_1$ ; so  $C_s \nearrow \Rightarrow |Z_s| \searrow \Rightarrow R_{approx} \searrow \Rightarrow \tau_2 \searrow \Rightarrow C_1$ charges faster;

4) No matter how small  $|Z_s|$  may be the capacitor  $C_2$  will always charge/discharge faster than capacitor  $C_1$ , and on the limit where  $|Z_s|=0$  the capacitors  $C_1$  and  $C_2$  will be charged/discharged simultaneously.

- For the following discussion will be used as definition of capacitance the formula  $C_s = i_s/(dv_s/dt)$ , where the  $|C_s| = |Q_s|/|v_s|$  ( $C_s$ : [F] farad;  $Q_s$ : [C] coulomb;  $v_s$  [V] volt), and since the only purpose is to show how  $C_s$  can be a negative number it will be used the approximate expression  $C_s \approx \overline{i_s}/(\Delta v_s/\Delta t)$  that will provide exactly the same sign as the exact formula.

- To show is possible  $C_s<0$  will be considered qualitative relations of the circuit electrical parameters on the RC network of the oscillator, the relevant electrical parameters and their variation between  $t{=}t_1$  and  $t{=}t_2$  is represented on Fig-23 .

- It will be assumed symbolic values for the voltages on the circuit, used as specimen values to determine how fast a voltage is changing between  $t_1$  and  $t_2$  time moments. So for representing a small amount of electrical charge will be used the symbols: [+] for positive charge and [-] for negative charge, since already stated  $C_1 = C_2$  for each additional amount of [+] and [-] charge stored on each plate (of  $C_1$  or  $C_2$ ) will cause an increase of capacitor voltage that



Figure 23: Schematic of RC network of the Schmitt-Trigger Oscillator with a representation of electrical charge on  $C_1$  and  $C_2$  on  $t=t_1$  and  $t=t_2$ .

will be represented as  $[+\underline{v}]$ , where  $C_1 = C_2 = [+]/[+\underline{v}]$ .

- As visible on Fig-23  $V_o = VDDS \approx +4.18V$ , and so VDDS will eventually be the voltage on  $C_1$  and  $C_2$  when  $t \rightarrow \infty$ . For making visual on the schematic the charging process, the charge accumulated in  $C_1$ ,  $C_2$  was divided in 20 sets, each represented by [+], [-]; and for each set of accumulated charge is associated a corresponding increase in voltage of  $[+\underline{v}]$ , and so  $[+\underline{v}] = VDDS/20$ .

- Accordingly on Fig-23 is represented that  $C_2$  is charged to near the final value (*VDDS*) during the interval  $[0; t_1]$ while  $C_1$  charges much slower. During interval  $[t_1; t_2]$ is visible that  $C_2$  increased its charge only by 1[+] becoming charged to approximately(or practically) its final value( $v_2 \approx VDDS$ ), whether  $C_1$  is still charging and  $v_1$  is far from its final value(*VDDS*), but interestingly  $v_1$  is now increasing faster than  $v_2$ , because  $v_2$  already reached its final value, this is  $dv_1/dt > dv_2/dt$ ,  $\forall t \in [t_1; t_2]$ . The specimen values here mentioned are in line with the exponential function typical of capacitors charging through a resistor, where lets say a capacitor initially charges very fast, when has some charge stored it charges more slowly, and when close to being full it charges very slowly (where full means the capacitor voltage is close to power supply voltage).

### 11.1. Voltage and current specimens for $t=t_1$

- So looking at the schematic on left side of Fig-23 is visible  $C_1$  and  $C_2$  are charging and for  $t=t_1$  the charge on  $C_1$  is 19[+] and on  $C_2$  is 5[+], so capacitor  $C_2$  is almost charged while  $C_1$  is still charging. The capacitor  $C_1$  is charging through the path  $V_o \rightarrow R_1 \rightarrow C_1$  but mainly is charging through path  $V_o \rightarrow R_2 \rightarrow C_s \rightarrow C_1$  and since  $v_2 > v_1$  then  $i_s(t=t_1) < 0$ .

- For  $t=t_1$  since  $Q_1=5[+]$  and  $Q_2=19[+]$  then  $v_1=5[+\underline{v}]$ and  $v_2=19[+\underline{v}]$ , since  $v_s = v_1 - v_2$  then  $v_s(t=t_1) = 5[+\underline{v}] - 19[+\underline{v}] = -14[+\underline{v}]$ .

#### 11.2. Voltage and current specimens for $t=t_2$

- So looking at the schematic on right side of Fig-23 is visible  $C_1$  and  $C_2$  are charging and for  $t=t_2$  the charge on  $C_1$  is 20[+] and on  $C_2$  is 9[+], so capacitor  $C_2$  is fully charged while  $C_1$  is still charging. The capacitor  $C_1$  is charging through the path  $V_o \to R_1 \to C_1$  but mainly is charging through path  $V_o \to R_2 \to C_s \to C_1$  and since  $v_2 > v_1$  then  $i_s(t=t_2) < 0$ .

- For  $t=t_2$  since  $Q_1=9[+]$  and  $Q_2=20[+]$  then  $v_1=9[+\underline{v}]$ and  $v_2=20[+\underline{v}]$ , since  $v_s = v_1 - v_2$  then  $v_s(t=t_2) = 9[+\underline{v}] - 20[+\underline{v}] = -11[+\underline{v}]$ . 11.3. Sign of  $C_s$  as calculated from  $v_s$  and  $i_s$  during  $[t_1; t_2]$ - The schematics on Fig-23 refer to a charging cycle of the Schmitt Trigger Oscillator. Also  $t_2 > t_1 \rightarrow \Delta t > 0$ .

- For  $t \in [t_1; t_2]$  the capacitor  $C_1$  is being charged through the path  $V_o \to R_2 \to C_s \to C_1$  and so  $i_s(t) < 0, \forall t \in [t_1; t_2] \Rightarrow \overline{i_s} < 0$ .

- Also  $\Delta v_s$  between  $t_1$  and  $t_2$  is  $\Delta v_s = v_s(t=t_2) - v_s(t=t_1) = -11[+\underline{v}] - (-14[+\underline{v}]) = 3[+\underline{v}]$ , and so  $\Delta v_s > 0$  between  $t_1$  and  $t_2$ .

- So concluding between  $t_1$  and  $t_2$ ,  $\Delta t > 0$ ,  $\Delta v_s > 0$ ,  $\overline{i_s} < 0 \Rightarrow C_s < 0$  accordingly with  $C_s \approx \overline{i_s} / (\Delta v_s / \Delta t)$ .

### 12. Multiple-Sensor Int. for measuring frequency

- For measuring frequency of an external voltage signal (between 0V and VDDS, so preferentially a digital signal or in case of analog signal it should be limited/trimmed before) is possible to use the mentioned Multiple-Sensor Interface and so using the same port/connector of the device. For this the user should remove/open the jumpers "JP.A", "JP.B" making the capacitors C1-A, C2-B active on the circuit, this will make  $C_1 = C_2 = 21.7 pF$  that is a quite low capacitance that will have an insignificant effect on the external voltage signal. The external voltage signal should be connected to the 1st pin of the sensor channel that is the one connected directly to input of the Schmitt-Trigger Inverter, so making the inverter directly driven by the external voltage signal, then the Multiple-Sensor Interface is just a converter of the voltage signal to a square wave signal where its frequency will be measured through the counter/timer of the PIC18F2550.

- The external voltage signal would preferentially be from a sensor with a square wave output, and the sensor have its power supplied by one of the VDD,GND ports/connectors of the Multiple-Sensor Interface device or by an external connection to the same power supply used to power the device.

### Declaration of competing interest

None.

#### References

- [1] Jaeger, R.C.; Blalock, T.N. (2011). Microelectronic Circuit Design (4rd ed.). McGraw-Hill. ISBN 978-0-07-338045-2. "C.2 THE HYBRID OR h-PARAMETERS Appendix C Two-Port Review" Pag.1310-1311. Alternative Source: "Hybrid parameters (h-parameters) Two-port Network Wikipedia (2020)" https://en.wikipedia.org/wiki/Two-port\_network
- [2] Peter McLean (2020) Topic 4 Crystal Oscillators (Analysis of the Pierce oscillator) - Digital Electronics (UTS-AU) - https: //pmcl.net.au/de/
- [3] Eduardo Corpeno (2018) Exactly How Schmitt Trigger Oscillators Work - AllAboutCircuits https://www.allaboutcircuits.com/technical-articles/ exactly-how-schmitt-trigger-oscillators-work
- [4] Thomas Williams, Colin Kelley (2007) GNUplot software For drawing the plots/graph - http://gnuplot.sourceforge.net