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Highlights

## Multiple Sensor Interface by the same hardware to USB and serial connection

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- multiple sensor interface using same hardware to serial connection
- measurement of sensors based on resistance, capacitance, inductance, frequency
- versatile electronics for basic measurement requirements (or low-end usage)
- electronics design aiming for reuse, repurpose, repair, customization


# Multiple Sensor Interface by the same hardware to USB and serial connection 

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#### Abstract

The Multiple Sensor Interface is a simple sensor interface that works with USB, RS485 and GPIO. It allows one to make measurements using a variety of sensors based on the change of inductance, resistance, capacitance, and frequency using the same connector and same electronic interface circuit between the sensor and the microcontroller. The same device also provides some additional connectors for small voltage measurement. Any sensors used for the measurement of distinct phenomena can be used if the sensor output is based on inductance, resistance, capacitance or frequency within the measurement range of the device, obtaining a variable precision depending on the used sensor. The device presented is not meant for precise or accurate measurements. It is meant to be a reusable hardware that can be adapted/configured to a varied number of distinct situations, providing, to the user, more freedom in sensor selection as well as more options for device/system maintenance or reuse.


## Keywords:

Computer peripherals, Oscillators, Sensor systems and applications, Signal processing

## 1. Introduction

The electronic waste (e-waste) is a modern problem under increasing concern and awareness, there are various possible approaches to reduce and mitigate it, most obvious is the collection and recycling of discarded devices, however the most ideal was just to design and produce technology that lasts because not only is physically fit by quality design, production, and components; but because its design was intended to be most versatile ensuring the same device can be used and reused in various applications/contexts just by changing connections, jumpers, and firmware configurations. Some design aspects for making a device more reusable are: use of standard connectors and protocols, think of it as a module to be part of a larger system, minimize barriers for connecting/interfacing components and devices from distinct manufacturers.

### 1.1. Project objectives and trade-offs

-This article is focused on the design of a sensor interface device with USB and serial(UART,RS-485), aimed to allow interface to many distinct 2 -wire sensors based on the change of inductance, resistance, capacitance, frequency, and also small voltage; sensors that can be interchanged using the same hardware and same port of the device, thus meaning the electronics designed must also be versatile.
-Obviously providing a versatile device to the users will have its negative trade-offs, like:
1- probably significant lower precision/accuracy;
2 - some sensor calibration must be provided/done by the end user after replacing a sensor;

[^0]3- the calibration function will not be linear or 'easy' as desired for sensors and its interfaces.
-However for some applications the mentioned trade-offs are not necessarily a deal-breaker, like when the user is technical and is ok with using a device that requires more setup/configuration, some users like devices that are more customizable or repairable. Also a device that if no longer useful for a user, it might still be useful for another user on a different application/context.

### 1.2. License and context

- The hardware design here disclosed is distributed under "CERN Open Hardware Licence Version 2 - Weakly Reciprocal" (CERN-OHL-W), its associated software/firmware under GNU licenses (GPL, LGPL).
- This article is published under the Creative Commons license CC BY-NC-SA 4.0 (Attribution-NonCommercialShareAlike 4.0 International).
- This article is about a 'hobby' project done by the author (David Nuno Quelhas, MSc Electronics Eng, alumni of Instituto Superior Tecnico, Portugal) with occasional 'work' between years 2012 and 2021 on his 'free time'.


### 1.3. State of the art

- The purpose of this article is to disclose a design aimed to be most versatile possible, and so not focused on a specific application type or use; and since the author doesn't know of other articles focused on a most versatile/reusable/repairable design to same type of device; the author did not find meaningful to mention other articles or designs as state of the art or to make comparisons.


Figure 1: Diagram of the Multiple Sensor Interface device.

## 2. Sensor Interface Device

Here is presented the Multiple Sensor Interface, the interface main components/sub-circuits are: The connectors and sensor interface circuits (oscillators) for inductance, resistance, capacitance and frequency ( $\mathrm{CH} .0, \mathrm{CH} .1$ ); the connectors and over-voltage protection(zener diode) for frequency measurement (CH.2, CH.3, CH.4, CH.5); the connectors and interface circuit for voltage measurement (ADC.0, ADC.1, ADC.2, ADC.3); analog multiplexer for the sensor channels, the microprocessor(PIC18F2550); I2C EEPROM for storing calibration tables; USB connector; connector and circuit for RS-485 and UART; digital outputs connector (OUT.1, OUT.2).

The digital outputs have the value of a boolean function defined by the user, boolean functions with logic variables that are the result of a comparison ('bigger' or 'smaller' than), between the value/measurement of a sensor channel and a configurable threshold value. The connectors used for frequency measurement may be connected to external single sensor interface circuits (oscillators).

### 2.1. The sensor interface circuit (oscillator)

The sensor interface is an oscillator with a circuit design based on the Pierce oscillator with some modifications. The 1st difference is that there is no quartz crystal, and on the location of the crystal will be connected the sensor to be measured (variable inductance or resistance or capacitance), the 2nd difference is that instead of simple inverters ('NOT' gates) will be used Schmitt-trigger inverters(high-speed Si-gate CMOS, 74HC14), this is a very relevant difference that will allow the oscillator to work even with a resistive or capacitive sensor, in fact the interface circuit works with sensors mostly as a Schmmit-Trigger oscillator. Also the Schmitt-Trigger inverters may minimize signal jitter of the oscillator output.

- The sensor interface circuit has 2 pairs of series capacitors (C1 2.2nF,C1-A 22pF and C2 2.2nF,C2-B 22pF) instead of just 2 capacitors( $\mathrm{C} 1, \mathrm{C} 2$ ) so the value of C 1 and C 2 can be adjusted just by placing/removing a jumper; placing a jumper removes C1-A or C1-B from the circuit making 2.2 nF the value of C 1


Figure 2: Schematic of the sensor interface circuit (oscillator). or C 2 ; removing the jumper lets the capacitors in series making 21.78 pF the total value of $(\mathrm{C} 1, \mathrm{C} 1-\mathrm{A})$ or $(\mathrm{C} 2, \mathrm{C} 2-\mathrm{B})$. So on the rest of the article, whenever is mentioned C 1 or C 2 is meant the resulting capacitor value that can be 2.2 nF (jumper on) or 21.78 pF (jumper off), accordingly with mentioned jumper configuration.

- The sensors can be connected directly on the MultipleSensor Interface(on then screw terminals/connectors), or by using a cable; for a cable longer than 20 cm is recommended the use of shielded twisted-pair(STP) cable to prevent cross-talk between sensor channels or external EMI.


## 3. Measurement process

The Multiple-Sensor device has a microprocessor (PIC18F2550) that is able to make frequency and voltage measurements, so the device makes frequency measurements for sensor channels CH .0 to CH .5 ; and makes voltage measurements for sensor channels ADC. 0 to ADC.3; these frequency and voltage measurements made by the device are designated as the RAW_value of a sensor channel. To obtain the measurement of a sensor channel the device uses a 2 column calibration table that is a long list of points (RAW_value; measurement) relating the measurement value (obtained during calibration by an external reference device) to the corresponding RAW_value obtained on the Multiple-Sensor
device, these calibration tables are stored on an I2C EEPROM memory on the Multiple-Sensor device.

- The Multiple-Sensor device can work in two modes: single-channel or multiple-channel, the CH. 0 to CH. 5 RAW_value(frequency) are calculated through a counter/timer of the PIC18F2550 by periodically reading its value and calculating the frequency $\mathrm{f}=$ count/period ([Hz]=[cycles $] /[\mathrm{s}]$ ). So in single-channel mode the frequency is always calculated on the selected/enabled sensor channel, in multiple-sensor mode the frequency is calculated for each sensor channel sequentially (time-division multiplexing), since there are 6 channels to measure but only on counter/timer of the microprocessor for that job. Thus in multiple-channel mode a measurement will take 6x more time to be updated/refreshed than in single-channel mode.
-For the sensor channels ADC. 0 to ADC. 3 the RAW_value is the voltage of those channels measured by using the ADC (Analog to Digital Converter) of the microprocessor and also reading a 2.5 V voltage reference.
-The sensor measurements are calculated by searching the RAW_value on the correspondent calibration table, and by using from the table 2 points (RAW_value, measurement) referenced here as points $A$ and $B$ such that the measured RAW_value is bigger than RAW_value of A and is lower than RAW_value of B; then is calculated a linear equation: measurement $=a \cdot($ RAW_value $)+b$, defined by the points A and B. So every-time the device calculates a sensor measurement, it will calculate the correspondent linear equation for the current RAW_value and use it to obtain the current measurement (Fig 3) .


### 3.1. Device calibration for a sensor

-Device calibration is about obtaining calibration tables for each sensor channel, here are 2 ways to obtain a calibration table:

1- Do a full manual calibration using an external meter as reference where both the reference meter and the Multiplesensor device(with a sensor connected) are exposed to same stimulus/environment that is controllable by the user to produce all adequate variations/intensities necessary to record


Figure 3: Plot experimental data with line, LDR light(brightness) sensor connected on Multiple-Sensor Interface; example of a calibration table exclusively from experimental data.


Figure 4: Plot experimental data and fitted model(by using 6 points), LDR light(brightness) sensor connected on Multiple-Sensor Interface.
an extensive calibration table, with all experimental pairs of (RAW_value, measurement).

2- Using a know function that relates the measured phenomena to the obtained RAW_value on the Multiple-Sensor device (obtained by theoretical or experimental study), although a purely theoretical calibration could be used, probably is better or easier to obtain a calibration table by using a known function and have its constants/parameters calculated by a data fitting to some few experimental data points (RAW_value, measurement) obtained for the device calibration. So for example if the know function had 6 constants/parameters you would require at least 6 different experimental measurements to obtain the function for that sensor channel, then having the function is just a question calculating a longer list of pairs (RAW_value, measurement) on the desired measurement range. The $(\operatorname{Fig} 4)$ is the result of fitting the model function Illuminance $(f)=(a+(b /(c+d \cdot f))) \cdot\left(\left(n \cdot f^{2}\right)+m\right)$, to the points $(229 \mathrm{~Hz}, 0 \mathrm{LUX} ; 8500 \mathrm{~Hz}, 10 \mathrm{LUX} ; 14000 \mathrm{~Hz}, 30 \mathrm{LUX} ; 76500 \mathrm{~Hz}$, 300LUX; 130000Hz, 1072LUX; 194000Hz, 3950LUX).

## 4. Sensor Interface Circuit Analysis

### 4.1. Multiple-Sensor Interface for Inductive sensors

When is connected an inductor or inductive sensor the Multiple-Sensor Interface (Fig 2]) may work as a Pierce Oscillator(where the sensor is connected instead of a quartz crystal). The theoretical analysis used here for the oscillator was based on a model of 2 circuit blocks named ' $A$ ' and ' $\beta$ ' connected for feedback by connecting the output of one to the input other. The ' A ' is an electronic amplifier providing voltage gain, the ' $\beta$ ' is an electronic filter providing frequency selection (resonance), so whatever voltage signal amplified by ' A ' is frequency selected by ' $\beta$ ' and feed back to the input of A for further amplification. As know this oscillator is start-up by whatever noise $\left(v_{s}\right)$ available at the input of ' A ', Fig 5 is a diagram depicting this concept.

- The analysis of the circuit as Pierce oscillator was made using the Barkhausen stability criterion, that says $A \beta=1$ to be possible to occur sustained oscillations (oscillations on steady state analysis). The purpose/focus is to obtain the inductance(of sensor) as function of oscillation frequency, although most of the circuit analysis strategy used here is similar as for typical


Figure 5: Diagram of model for the oscillator with an inductive sensor (Pierce Oscillator, model of feedback linear oscillator).


Figure 6: Representation of oscillator circuit by the h-parameters for feedback circuit (Pierce oscillator).
Pierce Oscillator with piezoelectric crystal, as available on the bibliography list "Crystal Oscillators for Digital Electronics" class notes by Peter McLean [2]. So from $A \beta=1$ results: $|A \beta|=1$ and $\angle A \beta= \pm n 2 \pi$. This mathematical expression simply says that a circuit with a feedback loop after reaching the steadystate is expected that any voltage signal (for example $V_{o}$ ) will remain steady.

- Starting the analysis on Fig, 6, using a possible representation of Fig 5 diagram with the 'feedback network' represented by its hybrid parameters (2-port network h-parameters).[1] [2]
-The Schmitt-Trigger inverter and resistors $R_{1}, R_{2}$ belong to block ' $A$ ', the capacitors $C_{1}, C_{2}$, and inductive sensor $L_{s}$ belong to block ' $\beta$ '.
- Since the 'Basic Amplifier' has a very big input resistance the current $I_{1}$ will be very small (the electric current on the input of the inverter, CMOS 'NOT' gate, is negligible), so by using the h-parameters to represent the feedback circuit block is possible to make the following simplifications/approximations: 1The current source $h_{21} I_{1}$ is also negligible (equal to zero, so removed from circuit); 2-The voltage drop across component $h_{11}$ is negligible (since $V_{11}=h_{11} I_{1}, I_{1} \rightarrow 0 \Rightarrow V_{11} \rightarrow 0$ ) and so $h_{11}$ can be relocated to inside the circuit block ' $A$ ' keeping $V_{i}$ as the name for the voltage drop on the input of ' $A$ ' block; 3The $1 / h_{22}$ can be relocated to inside the circuit block ' $A$ ' since is connected in parallel to the input of ' $\beta^{\prime}$ block that is also the output of ' $A$ ' block. [2]
- The block 'Basic Amplifier' is then replaced by its Thevenin equivalent circuit, obtaining the Fig 8 circuit. From this you can write the transfer function (Laplace transform) of ' $A$ ' block $\left(V_{o}=A V_{i}, 1 / h_{22}=h_{22}^{-1}\right)$ and ' $\beta$ ' block $\left(V_{1}=\beta V_{2}\right)$ :

$$
A=\frac{h_{22}^{-1}}{h_{22}^{-1}+r_{o}} \mu \frac{r_{i}}{r_{i}+h_{11}} \quad \beta=h_{12}
$$

- The feedback network (the same network displayed on Fig 6 represented by h-parameters) is the frequency-selecting


Figure 7: Simplified representation of oscillator circuit (Pierce osc.)


Figure 8: More simplified representation of the circuit (Pierce osc.) $\pi$ (shaped)-network on Fig 9 , where $Z_{1}, Z_{2}, Z_{s}$ are respectively $C_{1}, C_{2}, L_{\text {sensor }}$ of the Multiple Sensor Interface with an inductive sensor (Pierce oscillator).

- Based on the circuit of the feedback network and using the definitions of h-parameters of a 2-port network are obtained the values:

$$
h_{11}=Z_{1}\left\|Z_{s} \quad h_{22}^{-1}=Z_{2}\right\|\left(Z_{1}+Z_{s}\right) \quad h_{12}=\frac{Z_{1}}{Z_{1}+Z_{s}}
$$

(where $\|$ is the impedance of 2 components in parallel, $Z_{1} \| Z_{s}=\left(Z_{1} Z_{s}\right) /\left(Z_{1}+Z_{s}\right) ; h_{11}=\left.\left(V_{1} / I_{1}\right)\right|_{V_{2}=0} ; h_{12}=$ $\left.\left.\left(V_{1} / V_{2}\right)\right|_{I_{1}=0} ; h_{22}=\left.\left(I_{2} / V_{2}\right)\right|_{I_{1}=0}\right) .[1]$

Finally:

$$
A=\frac{Z_{2} \|\left(Z_{1}+Z_{s}\right)}{Z_{2} \|\left(Z_{1}+Z_{s}\right)+r_{o}} \mu \frac{r_{i}}{r_{i}+\left(Z_{1} \| Z_{s}\right)} \quad \beta=\frac{Z_{1}}{Z_{1}+Z_{s}}
$$

- In the circuit of Pierce oscillator (Fig 2) the amplifier was made by a gate inverter (CMOS 'NOT' gate) and a feedback resistor $\left(R_{1}=2 M \Omega\right)$, biasing the inverter and allowing it to function as a high-gain inverting amplifier. The input resistance of CMOS 'NOT' gate is very large and so making the approximation $r_{i}=+\infty$ the expression of ' $A$ ' is significantly simplified,


Figure 9: Feedback network of oscillator (Pierce Oscillator, Multiple-Sensor Interface with inductive sensor).

So obtaining: [2]

$$
\begin{align*}
& A \beta=\left(\frac{Z_{2} \|\left(Z_{1}+Z_{s}\right)}{Z_{2} \|\left(Z_{1}+Z_{s}\right)+r_{o}} \mu\right)\left(\frac{Z_{1}}{Z_{1}+Z_{s}}\right) \\
&=\frac{Z_{1} Z_{2} \mu}{Z_{2}\left(Z_{1}+Z_{s}\right)+r_{o}\left(Z_{1}+Z_{2}+Z_{s}\right)} \tag{1}
\end{align*}
$$

- If $Z_{1}, Z_{2}, Z_{s}$ are purely reactive impedances given by $Z_{1}=j X_{1}, Z_{2}=j X_{2}, Z_{s}=j X_{s}(j=\sqrt{-1})$, then 11 becomes:

$$
\begin{equation*}
A \beta=\frac{X_{1} X_{2} \mu}{X_{2}\left(X_{1}+X_{s}\right)-j r_{o}\left(X_{1}+X_{2}+X_{s}\right)} \tag{2}
\end{equation*}
$$

- The Barkhausen criterion states $A \beta=1 \rightarrow \angle A \beta= \pm n 2 \pi$, this means the phase shift of the loop ' $A \beta$ ' must be zero, and so that implies that the imaginary part of $2 \sqrt{2}$ must be zero. That is, for stable oscillations on the circuit of Fig 6 with a Feedback Network of Fig 9 it must be assured: [2]

$$
\begin{equation*}
\bar{X}_{1}\left(\omega_{0}\right)+X_{2}\left(\omega_{0}\right)+X_{s}\left(\omega_{0}\right)=0 \tag{3}
\end{equation*}
$$

- At the frequency $\omega_{0}$ (frequency of oscillation at steady state), using (3) with (2), is obtained: [2]

$$
\begin{equation*}
A\left(\omega_{0}\right) \beta\left(\omega_{0}\right)=-\mu \frac{X_{1}\left(\omega_{0}\right)}{X_{2}\left(\omega_{0}\right)} \tag{4}
\end{equation*}
$$

- To start the oscillations the loop gain must be grater than unity (during Transient Response), but after achieving the Steady State Response on the Pierce oscillator for oscillations to occur the loop gain $A\left(\omega_{0}\right) \beta\left(\omega_{0}\right)$ of (4) must be equal to ' 1 ' (unity). Since the amplifier is an inverter ('NOT' gate with $R_{1}$ feedback resistor) then $\mu$ is a negative number, and so $X_{1}\left(\omega_{0}\right)$ and $X_{2}\left(\omega_{0}\right)$ must have the same sign (both positive or negative).
- Thus, if $Z_{1}\left(\omega_{0}\right)$ is capacitive $\left(X_{1}\left(\omega_{0}\right)=-1 /\left(\omega_{0} C_{1}\right)\right)$, then by (4] $Z_{2}\left(\omega_{0}\right)$ must also be capacitive $\left(\left(X_{2}\left(\omega_{0}\right)=-1 /\left(\omega_{0} C_{2}\right)\right)\right.$. [2]
- Considering this and using (3) its possible to conclude that $Z_{s}\left(\omega_{0}\right)$ must be inductive since $X_{s}\left(\omega_{0}\right)=-X_{1}\left(\omega_{0}\right)-X_{2}\left(\omega_{0}\right)$, this is if $X_{1}\left(\omega_{0}\right)$ and $X_{2}\left(\omega_{0}\right)$ are negative numbers then $X_{s}\left(\omega_{0}\right)$ must be a positive number, then $X_{s}=\omega_{0} L_{s}$.
- So having $C_{1}, C_{2}, L_{s}$ on the feedback network (where $L_{s}$ is the component representing the inductive sensor on the MultiplSensor Interface) the (3) becomes:

$$
\begin{equation*}
-\frac{1}{\omega_{0} C_{1}}-\frac{1}{\omega_{0} C_{2}}+\omega_{0} L_{s}=0 \tag{5}
\end{equation*}
$$

- So defining "load capacitance" $C_{L}$ as: [2]

$$
\begin{equation*}
\frac{1}{C_{L}}=\frac{1}{C_{1}}+\frac{1}{C_{2}} \tag{6}
\end{equation*}
$$

The frequency of oscilation on the Pierce oscillator (using $C_{1}$, $C_{2}, L_{s}$ ) is:

$$
\begin{equation*}
\omega_{0}=\frac{1}{\sqrt{L_{s} C_{L}}} \quad \Leftrightarrow \quad f_{0}=\frac{1}{2 \pi \sqrt{L_{s} C_{L}}} \tag{7}
\end{equation*}
$$

- So the expression (theoretical) of measured inductance $L_{\text {sensor }}$ as a function of frequency( f ) is:

$$
\begin{equation*}
L_{\text {sensor }}=L_{s}=\frac{1}{4 \pi^{2} C_{L} f^{2}}=\frac{C_{1}+C_{2}}{4 \pi^{2} C_{1} C_{2} f^{2}} \tag{8}
\end{equation*}
$$

- On experimental tests done was observed that when using $C_{1}=C_{2}=2.2 \mathrm{nF}$ (JP.A and JP.B closed) or when using $C_{1}=21.78 \mathrm{pF}$ (JP.A open), $C_{2}=2.2 \mathrm{nF}$ (JP.B closed), with decreasing values of $L_{s}$ connected, the oscillation frequency exhibited a sudden change, at some small value of L (around $10 \mu H$ for $C_{1}=C_{2}=2.2 \mathrm{nF}$ ), not coherent with theoretical model of Pierce oscillator. This may be related to the fact the same circuit also implements a Schmitt-Trigger oscillator(next section), that oscillates under different criteria, so the author opin-


Figure 10: $L_{s}(f)[\mathrm{mH}]([\mathrm{Hz}])$ with $C_{1}=C_{2}=2.2 \mathrm{nF}$ and $C_{1}=21.78 \mathrm{pF}, C_{2}=2.2 \mathrm{nF}$ (Pierce Osc., Multi-Sensor Int. with inductive sensor)


Figure 11: Frequency jump of $L_{s}(f)[\mu \mathrm{H}]([\mathrm{Hz}])$ with $C_{1}=C_{2}=2.2 \mathrm{nF}$ (Pierce Osc., Multi-Sensor Int. with inductive sensor).
ion is when $L_{s}$ approaches some small value it may change from Pierce oscillator to Schmitt-Trigger oscillator. Fig 10 and Fig 11 shows the experimental data for various inductance values connected as the sensor and the plot $L_{s}(f)$ using (8) with $C_{1}=C_{2}=2.2 \mathrm{nF}$ and $C_{1}=21.78 \mathrm{pF}, C_{2}=2.2 \mathrm{nF}$.

- Since the mentioned sudden change of oscillator mode and frequency is not adequate on a $L_{s}(f)$ function usable for sensor interfacing; then on experimental tests with jumper configuration: JP.A on and JP.B off ( $C_{1}=2.2 n F, C_{2}=21.78 \mathrm{pF}$ ), it was observed a continuous and progressive $f\left(L_{s}\right)$ function. With $C_{1}=2.2 \mathrm{nF}, C_{2}=21.78 \mathrm{pF}$, the experimental $L_{s, \text { expr }}(f)$ followed a straight line for $L_{s}$ in $[0 \mu H ; 100 \mu H]$, then for $L_{s}>100 \mu H$ the


Figure 12: $L_{s}(f)[\mathrm{mH}]([\mathrm{Hz}])$ with $C_{1}=2.2 \mathrm{nF}, C_{2}=21.78 \mathrm{pF}$ (Multi-Sensor Int. with inductive sensor).


Figure 13: Experimental data of $L_{s}$ in $[0 \mu H ; 100 \mu H]$, with $C_{1}=2.2 \mathrm{nF}$, $C_{2}=21.78 \mathrm{pF}$ (Multi-Sensor Int. with inductive sensor).
experimental $L_{s, \text { expr }}(f)$ continued with a shape that has some visual similarity to theoretical (as Pierce oscillator), but with significantly different $L_{s}$ values. It was noticed that for larger values of $L_{s}$ the theoretical (Pierce oscillator, JP.A on, JP.B off) could be approximated to the experimental data by a constant multiplicative factor $\left(L_{s, \text { expr }}(f) \approx 0.03 \cdot L_{s, t h e o}(f)\right.$, for $\left.L_{s}>1 \mathrm{mH}\right)$.

- For modeling(data fitting) purposes, the author knows that a function $L_{s}(f)=(a+(b /(c+d \cdot f))) \cdot(n \cdot f+m)$, where 'a, b, c, d, m, n ' are constants to fit, can be fitted to experimental data on both low and high values of $L_{s}$.
- On following section 5 was used an approximated model (for Schmitt-Trigger oscillator) applied to Multiple-Sensor Interface with inductive sensor (JP.A on, JP.B off), that exhibited a theoretical $L_{s}(f)$ plot much closer to the experimental data, corroborating the hypothesis that with jumper configuration JP.A on, JP.B off ( $C_{1}=2.2 \mathrm{nF}, C_{2}=21.78 \mathrm{pF}$ ), it operates as a SchmittTrigger oscillator, where $L_{s}$ acts as an impedance influencing $C_{1}$ charge and discharge speed.


### 4.2. Multiple-Sensor Interface for Resistive sensors

- In case you connect a resistive sensor (or capacitive) to the Multiple-Sensor Interface it will not be able to satisfy the conditions for oscillation of equations (3) and (4), consequent of the Barkhausen criterion applied to the circuit, as it was explained on the previous section that on the $\pi$ (shaped)-network of Fig 9 if $Z_{1}$ and $Z_{2}$ are capacitive (corresponding to $C_{1}$ and $C_{2}$ ) then $Z_{s}$ must be inductive so that $A\left(\omega_{0}\right) \beta\left(\omega_{0}\right)=1$. So the conclusion is when you connect a resistive sensor (or capacitive) you no longer have a Pierce Oscillator. The Multiple-Sensor Interface is made using Schmitt-trigger inverters(high-speed Si-gate CMOS, 74 HC 14 ), and the Schmitt trigger is a bistable multivibrator that can be used to implement another type of multivibrator, the relaxation oscillator. So in the case of a resistive sensor the circuit to analyze is a Schmitt-trigger inverter connected to a network of resistors and capacitors.
- To analyze this circuit the Schmitt-Trigger inverter was replaced by a theoretical switch that changes the voltage of node $v_{o}$ to VDDS (power supply stabilized voltage for the sensor interface) when voltage $v_{i}$ is lower than $V_{T}^{-}$, and changes $v_{o}$ to GND when voltage $v_{i}$ is higher than $V_{T}^{+}$.
- So the circuit of Fig 14 was analyzed to obtain $f\left(R_{s}\right)$, and then its inverse function $R_{s}(f)=R_{\text {sensor }}$ that may be useful for using/configuring the Multiple-Sensor Interface. Notice that $i_{i} \approx 0$ since $v_{i}$ is the input of a Schmitt-trigger inverter (high-speed Sigate CMOS) that has a very high input impedance and so $i_{i} \approx 0$ is an appropriate approximation simplifying the circuit. So from the circuit are obtained the equations:

$$
\text { Nodes and loops: } i_{4}=i_{s}+i_{1}, \quad i_{3}+i_{s}=i_{2}, \quad i_{o}=
$$

$$
i_{3}+i_{4}, \quad i_{1}+i_{2}=i_{o}, \quad i_{1}+i_{2}=i_{1+0}+i_{4}, \quad v_{1}-v_{2}-v_{s}=0,
$$

Figure 14: Multi-Sensor Int., resistive sensor(Schmitt-trigger osc.)
$v_{4}+v_{s}+v_{2}-v_{o}=0, \quad v_{4}+v_{s}-v_{3}=0, \quad v_{3}=v_{o}-v_{2}$, $v_{4}=v_{o}-v_{i}, \quad v_{2}=v_{i}-v_{s}$.

Components: $\quad i_{1}=C_{1}\left(d v_{1} / d t\right), \quad i_{2}=C_{2}\left(d v_{2} / d t\right)$, $v_{3}=R_{2} i_{3}, \quad v_{4}=R_{1} i_{4}, \quad v_{s}=R_{s} i_{s}$

Solving:

$$
\begin{gather*}
\frac{v_{o}-v_{i}}{R_{1}}=\frac{v_{s}}{R_{s}}+C_{1} \frac{d v_{i}}{d t}  \tag{9}\\
\frac{v_{o}-v_{i}+v_{s}}{R_{2}}+\frac{v_{s}}{R_{s}}=C_{2} \frac{d v_{2}}{d t} \tag{10}
\end{gather*}
$$

- Solving: $v_{2}=v_{i}-v_{s} \Rightarrow d v_{2} / d t=d\left(v_{i}-v_{s}\right) / d t \Rightarrow d v_{2} / d t=$ $\left(d v_{i} / d t\right)-\left(d v_{s} / d t\right)$
- So using the previous result the 10p can be changed to:

$$
\begin{equation*}
\frac{v_{o}-v_{i}}{R_{2}}+\left(\frac{1}{R_{2}}+\frac{1}{R_{s}}\right) v_{s}=C_{2}\left(\frac{d v_{i}}{d t}-\frac{v_{s}}{d t}\right) \tag{11}
\end{equation*}
$$

- Solving (9) for $v_{s}$ is obtained:

$$
\begin{equation*}
v_{s}=\frac{R_{s}\left(v_{o}-v_{i}\right)}{R_{1}}-R_{s} C_{1} \frac{d v_{i}}{d t} \tag{12}
\end{equation*}
$$

- Calculating the derivative on both sides of (12) is obtained (remember $v_{o}$ is a constant equal to VDDS or GND depending on the position of the switch 'SW'):

$$
\begin{equation*}
\frac{d v_{s}}{d t}=\frac{-R_{s}}{R_{1}} \frac{d v_{i}}{d t}-R_{s} C_{1} \frac{d^{2} v_{i}}{d t^{2}} \tag{13}
\end{equation*}
$$

- Now using (12) and (13) to remove the variables $v_{s}$ and $d v_{s} / d t$ from equation 11 is obtained an equation solvable for determining $v_{i}(t)$ :

$$
\begin{align*}
& \left(\frac{1}{R_{2}}+\frac{R_{s}}{R_{2} R_{1}}+\frac{1}{R_{1}}\right)\left(v_{o}-v_{i}\right)= \\
& \quad\left(C_{1}\left(1+\frac{R_{s}}{R_{2}}\right)+C_{2}\left(1+\frac{R_{s}}{R_{1}}\right)\right) \frac{d v_{i}}{d t}+R_{s} C_{1} C_{2} \frac{d^{2} v_{i}}{d t^{2}} \tag{14}
\end{align*}
$$

- The equation (14) is of the type: $c\left(v_{o}-v_{i}\right)=b\left(d v_{i} / d t\right)+$ $a\left(d^{2} v_{i} / d t^{2}\right)$ that has the general solution: $v_{i}(t)=v_{o}+k_{1} e^{\lambda_{1} t}+$ $k_{2} e^{\lambda_{2} t}$, where $k_{1}, k_{2}$ are integration constants to be defined by 'initial conditions' and $\lambda_{1}, \lambda_{2}$ are defined by: $a \lambda^{2}+b \lambda+c=$ $0 \Leftrightarrow \lambda=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}$ and $e$ is the Euler-Napier constant $e=$ $\sum_{n=0}^{\infty}(1 /(n!))$.
- So for a solution to this circuit: $a=R_{s} C_{1} C_{2}$,

$$
b=\left(C_{1}\left(1+\left(R_{s} / R_{2}\right)\right)\right)+\left(C_{2}\left(1+\left(R_{s} / R_{1}\right)\right)\right),
$$

$c=\left(1 / R_{2}\right)+\left(R_{s} /\left(R_{2} R_{1}\right)\right)+\left(1 / R_{1}\right)$.

- Is selected the solution $\lambda_{2}=\frac{-b+\sqrt{b^{2}-4 a c}}{2 a}$ by setting $k_{1}=0$, because is the one that provides an adequate value for $v_{i}(t), f\left(R_{s}\right)$, consistent with experimental data, however for obtaining the function $R_{s}(f)$ you may use any.
- For convenience of making $v_{i}(t)$ more similar to typical RC circuits is defined $\tau=-1 / \lambda$, and so $v_{i}(t)=v_{o}+k_{2} e^{-t / \tau_{2}}$.
- Charging time of $C_{1}: v_{o}=V D D S$

$$
\begin{aligned}
& v_{i}(t=0)=V_{T}^{-} \rightarrow V_{T}^{-}=V D D S+k_{2} e^{0} \rightarrow k_{2}=V_{T}^{-}-V D D S \\
& v_{i}\left(t=T_{C}\right)=V_{T}^{+} \rightarrow V_{T}^{+}=V D D S+k_{2} e^{-T_{C} / \tau_{2}} \rightarrow
\end{aligned}
$$

- Discharging time of $C_{1}: v_{o}=0$
$v_{i}(t=0)=V_{T}^{+} \rightarrow V_{T}^{+}=0+k_{2} e^{0} \rightarrow k_{2}=V_{T}^{+}$
$v_{i}\left(t=T_{D}\right)=V_{T}^{-} \rightarrow V_{T}^{-}=0+k_{2} e^{-T_{D} / \tau_{2}} \rightarrow$

$$
\rightarrow T_{D}=-\tau_{2} \ln \left(V_{T}^{-} / V_{T}^{+}\right)
$$

- The time for a complete cycle of charge and discharge of $C_{1}$ is: $T=T_{C}+T_{D}$; the frequency of $v_{i}(t)$ is $f=1 / T$.


Figure 15: $R_{S}[\mathrm{k} \Omega]$ versus frequency $[0 \mathrm{~Hz}, 300 \mathrm{kHz}]$ (Schmitt-Trigger Oscillator, Multiple-Sensor Interface with resistive sensor).

$$
\left.\begin{array}{rl}
\text { - Solving: } T=-\tau_{2}\left(\ln \left(\frac{V_{T}^{+}-V D D S}{V_{T}^{-}-V D D S}\right)+\right. & \left.\ln \left(\frac{V_{T}^{-}}{V_{T}^{+}}\right)\right) \\
& \Leftrightarrow T=\tau_{2} \ln \left(\frac{\left(V_{T}^{-}-V D D S\right) V_{T}^{+}}{\left(V_{T}^{+}-V D D S\right.}\right) V_{T}^{-}
\end{array}\right)
$$

- For convenience defining the constant ' H ' by:

$$
H=\ln \left(\frac{\left(V_{T}^{-}-V D D S\right) V_{T}^{+}}{\left(V_{T}^{+}-V D D S\right) V_{T}^{-}}\right)
$$

then $f=1 / T \Leftrightarrow f=1 /\left(\tau_{2} H\right) \Leftrightarrow f=-\lambda_{2} / H$.

- So the expression (theoretical) of measured resistance $R_{\text {sensor }}$ as a function of frequency $(\mathrm{f})$ is:

$$
\begin{equation*}
R_{\text {sensor }}=R_{s}=\frac{\left(C_{1}+C_{2}\right) R_{2} R_{1} H f-R_{2}-R_{1}}{\left(C_{2} R_{2} H f-1\right)\left(C_{1} R_{1} H f-1\right)} \tag{15}
\end{equation*}
$$

- Using the values $C_{1}=C_{2}=2.2 n F, R_{2}=500 \Omega, R_{1}=2 M \Omega$, $V_{T}^{-}=1.2 \mathrm{~V}, V_{T}^{+}=2.2 V, V D D S=4.18 \mathrm{~V}$, is obtained $H=1.01496$, Fig 15 shows experimental data for Multiple-Sensor Interface with various resistance values connected as the sensor and also shows the plot of $R_{\text {sensor }}(f)$ using (15) with the mentioned values of $C_{1}, C_{2}, R_{2}, R_{1}, H$.


### 4.3. Multiple-Sensor Interface for capacitive sensors

- In case you connect a capacitive sensor (or resistive) to the Multiple-Sensor Interface it will not be able to satisfy the conditions for oscillation of the Pierce oscillator, equations (3) and (4], by $A\left(\omega_{0}\right) \beta\left(\omega_{0}\right)=1$, and so it is again a Schmitt-Trigger relaxation oscillator. So to analyze this circuit the Schmitt-trigger inverter was replaced by a theoretical switch (Schmitt-Trigger), just like previously with resistive sensors.
- So the circuit of Fig 16 was analyzed to obtain $f\left(C_{s}\right)$, and then its inverse function $C_{s}(f)=C_{\text {sensor }}$ that is useful for using/configuring the Multiple-Sensor Interface. Notice that $i_{i} \approx 0$ since $v_{i}$ is the input of the Schmitt-trigger inverter(high-speed Si-gate CMOS) that has a very high input impedance and so $i_{i} \approx 0$ is an appropriate approximation simplifying the circuit. So from the circuit are obtained the equations:

Nodes and loops: $i_{4}=i_{s}+i_{1}, \quad i_{3}+i_{s}=i_{2}, \quad i_{o}=$ $i_{3}+i_{4}, \quad i_{1}+i_{2}=i_{o}, \quad i_{1}+i_{2}=i_{3}+i_{4}, \quad v_{1}-v_{2}-v_{s}=0$, $v_{4}+v_{s}+v_{2}-v_{o}=0, \quad v_{4}+v_{s}-v_{3}=0, \quad v_{3}=v_{o}-v_{2}$, $v_{4}=v_{o}-v_{i}, \quad v_{2}=v_{i}-v_{s}$.

Components: $\quad i_{1}=C_{1}\left(d v_{1} / d t\right), \quad i_{2}=C_{2}\left(d v_{2} / d t\right)$,


Figure 16: Multi-Sensor Int., capacitive sensor (Schmitt-trigger osc.)
$v_{3}=R_{2} i_{3}, \quad v_{4}=R_{1} i_{4}, \quad i_{s}=C_{s}\left(d v_{s} / d t\right)$
Solving:

$$
\begin{gather*}
\frac{v_{o}-v_{i}}{R_{1}}=C_{s} \frac{d v_{s}}{d t}+C_{1} \frac{d v_{i}}{d t}  \tag{16}\\
\frac{v_{o}-v_{2}}{R_{2}}+C_{s} \frac{d v_{s}}{d t}=C_{2} \frac{d v_{2}}{d t}  \tag{17}\\
\frac{v_{o}-v_{2}}{R_{2}}+\frac{v_{o}-v_{i}}{R_{1}}=C_{1} \frac{d v_{i}}{d t}+C_{2} \frac{d v_{2}}{d t} \tag{18}
\end{gather*}
$$

- Since $v_{s}=v_{i}-v_{2}$ then $d v_{s} / d t=\left(d v_{i} / d t\right)-\left(d v_{2} / d t\right)$, and so using it on equation (16), is obtained:

$$
\begin{equation*}
\frac{d v_{2}}{d t}=\left(1+\frac{C_{1}}{C_{s}}\right) \frac{d v_{i}}{d t}-\frac{v_{o}-v_{i}}{C_{s} R_{1}} \tag{19}
\end{equation*}
$$

- Using $d v_{s} / d t=\left(d v_{i} / d t\right)-\left(d v_{2} / d t\right)$ and (19) on equation (17), is obtained:
$v_{2}=v_{o}+\frac{R_{2}\left(C_{2}+C_{s}\right)\left(v_{o}-v_{i}\right)}{C_{s} R_{1}}+R_{2} C_{s}\left(1-\left(1+\frac{C_{2}}{C_{s}}\right)\left(1+\frac{C_{1}}{C_{s}}\right)\right) \frac{d v_{i}}{d t}$
- Calculating the derivative of (20) is obtained:
$\frac{d\left(v_{2}\right)}{d t}=-\frac{R_{2}\left(C_{2}+C_{s}\right)}{C_{s} R_{1}} \frac{d v_{i}}{d t}+R_{2} C_{s}\left(1-\left(1+\frac{C_{2}}{C_{s}}\right)\left(1+\frac{C_{1}}{C_{s}}\right)\right) \frac{d^{2} v_{i}}{d t^{2}}$
- Now using 20p and (21) to remove the variables $v_{2}$ and $d v_{2} / d t$ from the equation (18), is obtained an equation solvable for determining $v_{i}(t)$ :
$v_{o}-v_{i}=\left(R_{2}\left(C_{2}+C_{s}\right)+R_{1}\left(C_{1}+C_{s}\right)\right) \frac{d v_{i}}{d t}+R_{2} R_{1}\left(C_{1} C_{2}+C_{s}\left(C_{1}+C_{2}\right)\right) \frac{d^{2} v_{i}}{d t^{2}}$
- The equation $\sqrt{22}$ is of the type: $c\left(v_{o}-v_{i}\right)=b\left(d v_{i} / d t\right)+$ $a\left(d^{2} v_{i} / d t^{2}\right)$, that has the general solution:
$v_{i}(t)=v_{o}+k_{1} e^{\lambda_{1} t}+k_{2} e^{\lambda_{2} t}$, where $k_{1}, k_{2}$ are integration constants to be defined by 'initial conditions' and $\lambda_{1}, \lambda_{2}$ are defined by: $a \lambda^{2}+b \lambda+c=0 \Leftrightarrow \lambda=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}$ and $e$ is the Euler-Napier constant $e=\sum_{n=0}^{\infty}(1 /(n!))$.
- So for a solution to this circuit: $c=1$,
$b=\left(R_{2}\left(C_{2}+C_{s}\right)+R_{1}\left(C_{1}+C_{s}\right)\right)$,
$a=R_{2} R_{1}\left(C_{1} C_{2}+C_{s}\left(C_{1}+C_{2}\right)\right)$.
- Is selected the solution $\lambda_{2}=\frac{-b+\sqrt{b^{2}-4 a c}}{2 a}$ by setting $k_{1}=0$, because is the one that provides an adequate value for $v_{i}(t), f\left(C_{s}\right)$, consistent with experimental data, however for obtaining the function $C_{s}(f)$ you may use any.
- For convenience of making $v_{i}(t)$ more similar to typical RC circuits is defined $\tau=-1 / \lambda$, and so $v_{i}(t)=v_{o}+k_{2} e^{-t / \tau_{2}}$.
- So when is connected a capacitive sensor $\left(C_{s}\right)$ the differential equation and solution $v_{i}(t)$ are the same as when is connected a resistive sensor $\left(R_{s}\right)$, the only differences are on the values of $a, b, c$; and as such the equations of $f$ (frequency) and $T$ (period) are also the same and are reused from the previous section.
- The constant 'H' defined by:

$$
H=\ln \left(\frac{\left(V_{T}^{-}-V D D S\right) V_{T}^{+}}{\left(V_{T}^{+}-V D D S\right) V_{T}^{-}}\right),
$$

and $f=1 / T \Leftrightarrow f=1 /\left(\tau_{2} H\right) \Leftrightarrow f=-\lambda_{2} / H$.

- So the expression (theoretical) of measured capacitance $C_{\text {sensor }}$ as a function of frequency(f) is:

$$
\begin{equation*}
C_{\text {sensor }}=C_{s}=\frac{\left(C_{1} R_{1}+C_{2} R_{2}\right) H f-1-C_{1} C_{2} R_{1} R_{2} H^{2} f^{2}}{H f\left(\left(C_{1}+C_{2}\right) R_{1} R_{2} H f-R_{1}-R_{2}\right)} \tag{23}
\end{equation*}
$$

- Using the values $C_{1}=C_{2}=2.2 n F, R_{2}=500 \Omega, R_{1}=2 M \Omega$, $V_{T}^{-}=1.2 V, V_{T}^{+}=2.2 V, V D D S=4.18 V$, is obtained $H=1.01496$,


Figure 17: $C_{s}(f)[\mathrm{nF}]$ versus frequency $[-400 \mathrm{kHz}, 1 \mathrm{MHz}]$ (Schmitt-Trigger Oscillator, Multiple-Sensor Interface with capacitive sensor).


Figure 18: $C_{s}[\mathrm{nF}]$ versus frequency $[0 \mathrm{~Hz}, 450 \mathrm{kHz}]$ by $\left|C_{s}(f)\right|$ (Schmitt-Trigger osc., Multi-Sensor Int. with capacitive sensor)


Figure 19: $C_{s}[\mathrm{nF}]$ versus frequency $[0 \mathrm{~Hz}, 170 \mathrm{kHz}]$ by $\left|C_{s}(f)\right|$ (Schmitt-Trigger osc., Multi-Sensor Int. with capacitive sensor)
and Fig 17 shows the plot of $C_{s}(f)$ using 23 with the mentioned values of $C_{1}, C_{2}, R_{2}, R_{1}, H$.

- Analyzing the plot on Fig 17 by firstly looking at plot regions with $C_{s}>0$, its visible that the $C_{s}(f)$ plot is over the vertical axis $(\mathrm{f}=0)$ and this would mean that for all values of $C_{S}$ the frequency would be zero $(\mathrm{f}=0)$, but visible to the right is another curve that is also placed on the area of $C_{s}>0$ (for frequency $[447957 \mathrm{~Hz}, 895689 \mathrm{~Hz}]$ ) and at a first view this curve would seem appropriate. But strangely on $C_{s}>0, \mathrm{f}>0$ for each value of $C_{s}$ are 2 values of frequency, while for $C_{s}<0, \mathrm{f}>0$ each value of $C_{s}$ has only one possible value of frequency ( $\mathrm{f}<0$ is considered meaningless/ignored).
- The experimental data shows that the way the oscillator works using a capacitive sensor is different from what some would expect on a first view of the plot $C_{s}(f)$, in order to compare the experimental data with the theoretical model is shown on Fig 18 and Fig 19 the experimental data for Multiple-Sensor Interface with various capacitance values connected as the sensor and also the plot of $a b s\left(C_{s}(f)\right)\left(=\left|C_{s}(f)\right|\right)$ using 23) with the mentioned values of $C_{1}, C_{2}, R_{1}, R_{2}, H$.
- So it seems that the obtained function of $C_{s}(f)$ although
strangely indicates negative values for the sensor capacitance it can provide a theoretical curve/plot similar to what was obtained on the experimental data for $C_{\text {sensor }}$. On the following sections is given a better insight on why $C_{s}(f)$ has a negative value.


### 4.4. Multiple-Sensor Int. for measuring frequency

- For measuring frequency of an external voltage signal (between 0 V and VDDS, so preferentially a digital signal or in case of analog signal it should be limited/trimmed before) is possible to use the mentioned Multiple-Sensor Interface and so using the same port/connector of the device. For this the user should remove/open the jumpers "JP.A", "JP.B" making the capacitors $\mathrm{C} 1-\mathrm{A}, \mathrm{C} 2-\mathrm{B}$ active on the circuit, this will make $C_{1}=C_{2}=21.8 p F$ that is a quite low capacitance that will have an insignificant effect on the external voltage signal. The external voltage signal should be connected to the 1 st pin of the sensor channel that is the one connected directly to input of the Schmitt-Trigger Inverter, so making the inverter directly driven by the external voltage signal, then the Multiple-Sensor Interface is just a converter of the voltage signal to a square wave signal where its frequency will be measured through the counter/timer of the PIC18F2550.
- The external voltage signal would preferentially be from a sensor with a square wave output, and the sensor have its power supplied by one of the VDDS,GND ports/connectors of the Multiple-Sensor Interface device or by an external connection to the same power supply used to power the device.


## 5. Alternative Approximate Circuit Analysis

### 5.1. Sensor interface circuit simplified

- The Multiple-Sensor Interface circuit when working as Schmitt-Trigger oscillator (using $R_{S}, C_{S}$, or $L_{s}$ with specific $C_{1}, C_{2}$ values) can be studied and understood in a more intuitive way by making some simplification/approximation that may be inaccurate for quantitative purposes but still captures its essence, with the benefit of exposing how it works and resulting in much simpler differential equations. So the interface circuit is a more complex Schmitt-Trigger oscillator, but its essence is the same, it is just some capacitors being charged by currents that pass trough some resistors, and the voltage on


Figure 20: Schematic of a basic Schmitt-Trigger Oscillator to be used as an approximation of the circuit of Multiple-Sensor Interface.


Figure 21: Schematic of the RC network of the Schmitt-Trigger Oscillator for the 2 extreme cases of the sensor impedance $\left(Z_{s}\right)$.
a capacitor $\left(v_{i}\right)$ will trigger $\left(\right.$ at $V_{T}^{-}$or $\left.V_{T}^{+}\right)$a switch(electronic inverter) to change the voltage $\left(v_{o}\right)$.[3]

- So the sensor and interface circuit can be described approximately as a basic Schmitt-Trigger oscillator that only has one capacitor and one resistor (that determine the frequency of oscillation), and so was used the simplified circuit on Fig 20 where $C_{\text {approx }}$ is a capacitor and $R_{\text {approx }}$ is a resistor that approximate in overall the capacitance and resistance of the sensor interface oscillator.
- To build expressions of $C_{\text {approx }}$ and $R_{\text {approx }}$ that include $R_{1}$, $R_{2}, C_{1}, C_{2}$ are considered initially 2 extreme cases of the sensor impedance $\left(Z_{s}\right): 1$ st $\left|Z_{s}\right|=0$ the sensor can be replaced by a wire, and 2 nd $\left|Z_{s}\right|=\infty$ the sensor can be removed(open circuit), these 2 extreme cases possible for the sensor impedance are represented on Fig 21.
- Now the sensor can be described as an electric connection that can be weakened or intensified depending on the sensor impedance, so when $\left|Z_{s}\right|$ changes progressively from 0 to $+\infty$ the circuit behavior changes progressively and smoothly from the behavior of the left circuit to the behavior of right circuit of Fig 21 So to obtain equations for $R_{\text {approx }}$ and $C_{\text {approx }}$ was selected an expression that allows to change smoothly the resistance and capacitance of the left side circuit to the resistance and capacitance of the right side circuit of Fig 21.
- So as on Fig 21 , here are the values of $C_{\text {approx }}$ and $R_{\text {approx }}$ for the 2 extreme values of $\left|Z_{s}\right|=0$ and $\left|Z_{s}\right|=+\infty$ :
$C_{\text {approx }}\left(Z_{s}=0\right)=C_{1}+C_{2}$;
$C_{\text {approx }}\left(Z_{s}=\infty\right)=C_{1} ;$
$R_{\text {approx }}\left(Z_{s}=0\right)=\left(R_{1} R_{2}\right) /\left(R_{1}+R_{2}\right) ; \quad R_{\text {approx }}\left(Z_{s}=\infty\right)=R_{1}$;


### 5.1.1. $R_{\text {approx }}$ and $C_{\text {approx }}$ for a resisitve sensor $\left(R_{s}\right)$

Here are functions modeled to describe $C_{\text {approx }}$ and $R_{\text {approx }}$ (with resistive sensor) with a smooth transition from its values at $\left|Z_{s}\right|=0$ and $\left|Z_{s}\right|=+\infty$, where $\left|Z_{s}\right|=R_{s}$ :

$$
\begin{align*}
& C_{\text {approx }}=\left(C_{1}+C_{2}\right) \frac{R_{1}}{\left|Z_{s}\right|+R_{1}}+C_{1} \frac{\left|Z_{s}\right|}{\left|Z_{s}\right|+R_{1}}  \tag{24}\\
& R_{\text {approx }}=\frac{R_{1} R_{2}}{R_{1}+R_{2}} \frac{2 R_{1}}{\left|Z_{s}\right|+2 R_{1}}+R_{1} \frac{\left|Z_{s}\right|}{\left|Z_{s}\right|+2 R_{1}} \tag{25}
\end{align*}
$$

Using the equations: $f=1 / T \Leftrightarrow f=1 /(\tau H) \Leftrightarrow f=-\lambda / H$, and using $\tau=R_{\text {approx }} C_{\text {approx }}$, where $\left|Z_{s}\right|$ was removed using $\left|Z_{s}\right|=R_{s}$, it can be obtained $R_{s}(f)$.

- Using values $V_{T}^{-}=1.2 V, V_{T}^{+}=2.2 V, V D D S=4.18 \mathrm{~V}$, is obtained $H=1.01496$ (valid for any type of sensor).
- Using the values $C_{1}=C_{2}=2.2 n F, R_{2}=500 \Omega, R_{1}=2 M \Omega$, $H=1.01496$ with the approximate model ( $C_{\text {approx }}, R_{\text {approx }}$ ), is obtained the plot of $R_{s}(f)$ on Fig 22.


Figure 22: $R_{s}(f)[\mathrm{k} \Omega]([\mathrm{Hz}])$ by approximate model ( $C_{\text {approx }}, R_{\text {approx }}$; MultipleSensor Int. with resistive sensor).


Figure 23: $L_{s}[\mathrm{mH}]$ versus frequency $[0 \mathrm{~Hz}, 850 \mathrm{kHz}]$ by approximate model ( $C_{\text {approx }}, R_{\text {approx }} ;$ Multi-Sensor Int. with inductive sensor)


Figure 24: $L_{s}[\mu \mathrm{H}]$ versus frequency[ $\left.400 \mathrm{kHz}, 900 \mathrm{kHz}\right]$ by approximate model ( $C_{\text {approx }}, R_{\text {approx }} ;$ Multi-Sensor Int. with inductive sensor)


Figure 25: $C_{s}[\mathrm{nF}]$ versus frequency $[0 \mathrm{~Hz}, 450 \mathrm{kHz}]$ by approximate model ( $C_{\text {approx }}, R_{\text {approx }}$; Multi-Sensor Int. with capacitive sensor)


Figure 26: $C_{s}[\mathrm{nF}]$ versus frequency $[0 \mathrm{~Hz}, 170 \mathrm{kHz}]$ by approximate model ( $C_{\text {approx }}, R_{\text {approx }}$; Multi-Sensor Int. with capacitive sensor)
5.1.2. $R_{\text {approx }}$ and $C_{\text {approx }}$ for an inductive sensor $\left(L_{s}\right)$

This approximate model for the interface circuit with inductive sensor is only valid for jumper configuration(capacitor values) that make it work as Schmitt-Trigger oscillator, as is expected for JP.A closed, JP.B open ( $C_{1}=2.2 \mathrm{nF}, C_{2}=21.78 \mathrm{pF}$ ). Here are functions modeled to describe $C_{\text {approx }}$ and $R_{\text {approx }}$ (with inductive sensor) with a smooth transition from its values at $\left|Z_{s}\right|=0$ and $\left|Z_{s}\right|=+\infty$, where $\left|Z_{s}\right|=2 \pi f L_{s}$ :

$$
\begin{align*}
C_{\text {approx }} & =\left(C_{1}+C_{2}\right) \frac{R_{1}}{\left|Z_{s}\right|+R_{1}}+C_{1} \frac{\left|Z_{s}\right|}{\left|Z_{s}\right|+R_{1}}  \tag{26}\\
R_{\text {approx }} & =\frac{R_{1} R_{2}}{R_{1}+R_{2}} \frac{R_{1}}{\left|Z_{s}\right|+R_{1}}+R_{1} \frac{\left|Z_{s}\right|}{\left|Z_{s}\right|+R_{1}} \tag{27}
\end{align*}
$$

- Using the values $C_{1}=2.2 \mathrm{nF}, C_{2}=21.78 \mathrm{pF}, R_{2}=500 \Omega$, $R_{1}=2 M \Omega, H=1.01496$ with the approximate model ( $C_{\text {approx }}$, $\left.R_{\text {approx }}\right)$, is obtained the plot of $L_{s}(f)$ on Fig 23 and Fig 24 .


### 5.1.3. $R_{\text {approx }}$ and $C_{\text {approx }}$ for a capacitive sensor $\left(C_{s}\right)$

Here are functions modeled to describe $C_{\text {approx }}$ and $R_{\text {approx }}$ (with capacitive sensor) with a smooth transition from its values at $\left|Z_{s}\right|=0$ and $\left|Z_{s}\right|=+\infty$, where $\left|Z_{s}\right|=1 /\left(2 \pi f C_{s}\right)$ :

$$
\begin{align*}
C_{\text {approx }} & =\left(C_{1}+C_{2}\right) \frac{R_{1}}{\left|Z_{s}\right|+R_{1}}+C_{1} \frac{\left|Z_{s}\right|}{\left|Z_{s}\right|+R_{1}}  \tag{28}\\
R_{\text {approx }} & =\frac{R_{1} R_{2}}{R_{1}+R_{2}} \frac{R_{1}}{\left|Z_{s}\right|+R_{1}}+R_{1} \frac{\left|Z_{s}\right|}{\left|Z_{s}\right|+R_{1}} \tag{29}
\end{align*}
$$

- Using the values $C_{1}=C_{2}=2.2 n F, R_{2}=500 \Omega, R_{1}=2 M \Omega$, $H=1.01496$, with the approximate model ( $C_{\text {approx }}, R_{\text {approx }}$ ), is obtained the plot $C_{s}(f)$ on Fig 25 and Fig 26.


### 5.2. Why $C_{s}(f)<0$ on Multi-Sensor with capacitive sensor

- About $C_{s}(\mathrm{f})<0$ have in mind the Multiple-Sensor with capacitive sensor is studied on transient behavior (relaxation oscillator), where 'frequency' is a measure of the speed of charge and discharge on C 1 ; and also of how fast the transient circuit analysis alternates between $v_{o}=V D D S$ and $v_{o}=0$.
- To understand why a normal capacitor behaves as negative capacitance when connected as the sensor of the MultipleSensor Interface (this is, why $C_{s}(f)<0$ ), is important to highlight some things already explored on the previous sections:

1) $C_{1}=C_{2}, R_{1} \gg R_{2}$;
2) The primary path (always available) to charge $C_{1}$ is through $R_{1}$, the primary path (always available) to charge $C_{2}$ is through $R_{2}$, since $R_{1} \gg R_{2}$ and $C_{1}=C_{2}$ this implies that capacitor $C_{2}$ will charge/discharge much faster(takes less time) than capacitor $C_{1}$.
3) The purpose of sensor $C_{s}$ on this circuit is to act as a variable impedance that can establish an alternative path on the circuit ( $V_{o} \rightarrow R_{2} \rightarrow C_{s} \rightarrow C_{1}$ ) to charge/discharge capacitor $C_{1}$; so $C_{s} \nearrow \Rightarrow\left|Z_{s}\right| \searrow \Rightarrow R_{\text {approx }} \searrow \Rightarrow \tau_{2} \searrow \Rightarrow C_{1}$ charges faster;
4) No matter how small $\left|Z_{s}\right|$ may be the capacitor $C_{2}$ will always charge/discharge faster than capacitor $C_{1}$, and on the limit where $\left|Z_{s}\right|=0$ the capacitors $C_{1}$ and $C_{2}$ will be charged/discharged simultaneously.

- For the following discussion was used as definition of capacitance the formula $C_{s}=i_{s} /\left(d v_{s} / d t\right)$, where the $\left|C_{s}\right|=$ $\left|Q_{s}\right| /\left|v_{s}\right|\left(C_{s}:[\mathrm{F}]\right.$ farad; $Q_{s}:[\mathrm{C}]$ coulomb; $v_{s}:[\mathrm{V}]$ volt), and since the only purpose is to show how $C_{s}$ can be a negative number it was used the approximate expression $C_{s} \approx \overline{\bar{i}_{s}} /\left(\Delta v_{s} / \Delta t\right)$ that provides exactly the same sign as the exact formula. To show is possible $C_{s}<0$ were considered qualitative relations of the circuit electrical parameters on the RC network of the oscillator, the relevant electrical parameters and their variation between $t=t_{1}$ and $t=t_{2}$ is represented on Fig 27
- It were assumed symbolic values for the voltages on the circuit, used as specimen values to determine how fast a voltage is changing between $t_{1}$ and $t_{2}$ time moments. So for representing a small amount of electrical charge are used the symbols: [+] for positive charge and [-] for negative charge, since already stated $C_{1}=C_{2}$ for each additional amount of [ + ] and [-] charge stored on each plate (of $C_{1}$ or $C_{2}$ ) will cause an


Figure 27: Schematic of RC network of the Schmitt-trigger osc. with a representation of electrical charge on $C_{1}, C_{2}$ on $t=t_{1}$ and $t=t_{2}$.
increase of capacitor voltage that will be represented as $[+\underline{v}]$, where $C_{1}=C_{2}=[+] /[+\underline{v}]$.

- As visible on $\operatorname{Fig} 27 \bar{V}_{o}=V D D S \approx+4.18 \mathrm{~V}$, and so $V D D S$ will eventually be the voltage on $C_{1}$ and $C_{2}$ when $\mathrm{t} \rightarrow \infty$. For making visual on the schematic the charging process, the charge accumulated in $C_{1}, C_{2}$ was divided in 20 sets, each represented by [+], [-]; and for each set of accumulated charge is associated a corresponding increase in voltage of $[+\underline{v}]$, and so $[+\underline{v}]=V D D S / 20$.
- Accordingly on Fig 27 is represented that $C_{2}$ is charged to near the final value (VDDS) during the interval [ $0 ; t_{1}$ ] while $C_{1}$ charges much slower. During interval $\left[t_{1} ; t_{2}\right]$ is visible that $C_{2}$ increased its charge only by $1[+]$ becoming charged to approximately(or practically) its final value ( $v_{2} \approx V D D S$ ), whether $C_{1}$ is still charging and $v_{1}$ is far from its final value (VDDS), but interestingly $v_{1}$ is now increasing faster than $v_{2}$, because $v_{2}$ already reached its final value, this is $d v_{1} / d t>d v_{2} / d t, \forall t \in\left[t_{1} ; t_{2}\right]$. The specimen values here mentioned are in line with the exponential function typical of capacitors charging through a resistor, where lets say a capacitor initially charges very fast, when has some charge stored it charges more slowly, and when close to being full it charges very slowly (where full means the capacitor voltage is close to power supply voltage).


### 5.2.1. Voltage and current specimens for $t=t_{1}$

- So looking at the schematic on left side of Fig 27 is visible $C_{1}$ and $C_{2}$ are charging and for $t=t_{1}$ the charge on $C_{1}$ is $5[+]$ and on $C_{2}$ is $19[+]$, so capacitor $C_{2}$ is almost charged while $C_{1}$ is still charging. The capacitor $C_{1}$ is charging through the path $V_{o} \rightarrow R_{1} \rightarrow C_{1}$ but mainly is charging through path $V_{o} \rightarrow R_{2} \rightarrow C_{s} \rightarrow C_{1}$ and since $v_{2}>v_{1}$ then $i_{s}\left(t=t_{1}\right)<0$.
For $t=t_{1}, Q_{1}=5[+], Q_{2}=19[+]$, then $v_{1}=5[+\underline{v}]$ and $v_{2}=19[+\underline{v}]$, since then $v_{s}=v_{1}-v_{2}$ then $v_{s}\left(t=t_{1}\right)=5[+\underline{v}]-19[+\underline{v}]=-14[+\underline{v}]$.


### 5.2.2. Voltage and current specimens for $t=t_{2}$

- So looking at the schematic on right side of Fig 27 is visible $C_{1}$ and $C_{2}$ are charging and for $t=t_{2}$ the charge on $C_{1}$ is $9[+]$ and on $C_{2}$ is $20[+]$, so capacitor $C_{2}$ is fully charged while $C_{1}$ is still charging. The capacitor $C_{1}$ is charging through the path $V_{o} \rightarrow R_{1} \rightarrow C_{1}$ but mainly is charging through path $V_{o} \rightarrow R_{2} \rightarrow C_{s} \rightarrow C_{1}$ and since $v_{2}>v_{1}$ then $i_{s}\left(t=t_{2}\right)<0$.

For $t=t_{2}, Q_{1}=9[+], Q_{2}=20[+]$, then $v_{1}=9[+\underline{v}]$ and $v_{2}=20[+\underline{v}]$, since $v_{s}=v_{1}-v_{2}$ then $v_{s}\left(t=t_{2}\right)=9[+\underline{v}]-20[+\underline{v}]=-11[+\underline{v}]$.

### 5.2.3. Sign of $C_{s}$ as calculated from $v_{s}$ and $i_{s}$ during $\left[t_{1} ; t_{2}\right]$

- The schematics on Fig 27 refer to a charging cycle of the Schmitt Trigger Oscillator. Also $t_{2}>t_{1} \rightarrow \Delta \mathrm{t}>0$.

For $t \in\left[t_{1} ; t_{2}\right]$ the capacitor $C_{1}$ is being charged through the path $V_{o} \rightarrow R_{2} \rightarrow C_{s} \rightarrow C_{1}$ and so $i_{s}(t)<0, \forall t \in\left[t_{1} ; t_{2}\right] \Rightarrow \overline{i_{s}}<0$.

Also $\Delta v_{s}$ between $t_{1}$ and $t_{2}$ is $\Delta v_{s}=v_{s}\left(t=t_{2}\right)-v_{s}\left(t=t_{1}\right)=$ $-11[+\underline{v}]-(-14[+\underline{v}])=3[+\underline{v}]$, and so $\Delta v_{s}>0$ between $t_{1}$ and $t_{2}$.

- So concluding between $t_{1}$ and $t_{2}, \Delta t>0, \Delta v_{s}>0, \overline{i_{s}}<0$ $\Rightarrow C_{s}<0$ accordingly with $C_{s} \approx \overline{i_{s}} /\left(\Delta v_{s} / \Delta t\right)$.


### 5.3. Comparison to known cases of negative capacitance

- Aspects of Multiple Sensor Interface circuit possibly related to negative capacitance phenomenon:
1- Use of Schmitt-Trigger 'NOT' gate which exhibits hysteresis on its $v_{o}\left(v_{i}\right)$ graph.
2- Multiple Sensor Interface with a capacitive sensor operates under transient(time domain) step voltage changes, caused by its 'NOT' gate(Schmitt-Trigger) alternating between 0 V and + VDDS (relaxation oscillator).
- Negative capacitance phenomenon is reported on some scientific articles/texts, and interestingly with some coincidence to the 2 aspects mentioned above. Quotes:
1- "Effective negative capacitance has been postulated in ferroelectrics because there is hysteresis in plots of polarizationelectric field.", article "Towards steep slope MOSFETs using ferroelectric negative capacitance", year 2014 [4].
2- "The phenomenon of negative capacitance, which has been reported in a variety of situations involving electrolytic as well as electronic systems, ... . It is suggested that the physically correct approach lies in the analysis of the corresponding timedomain behavior under step function bias, which involves a current initially falling and then rising gradually over a period of time before finally decaying to zero.", article "The physical origin of negative capacitance", year 1986 [5].


## 6. Conclusion

The author demonstrated theoretically a more versatile design for use with sensor applications, also was provided experimental data that corroborates the presented theory. The motivation of the author was to make available an electronics design that could be more sustainable in terms of life-cycle duration, by making a design more customizable by the user and also not closed/locked to a specific application/purpose. No warranty is given that the design can provide accuracy or convenience to a specific application/use; as the article is focused on showing how a versatile design can be achieved.

## Declaration of competing interest

None.

## Appendix A. Experimental Datasets

Here is made available subsets (small list) of experimental data with measured values of Inductance, Resistance, Capacitance paired with measured frequency on the Multiple Sensor Interface device. Is made available only a limited subset of the experimental data that was used for drawing the plots of $L_{s}(f), R_{s}(f), C_{s}(f)$ for reference purposes, since placing here the full dataset would make the article exceedingly long. On the tests were used arrays(PCBs) with inductor, resistor, capacitor that allow to obtain various different values just by changing a jumper/switch, also were used single components (including in series or parallel association); these fixed value components
were connected as the sensor on the device.

## Appendix A.1. Frequency measurement by Multi-Sensor

- The Multiple-Sensor device measures frequency using a counter inside the microcontroller and has some accuracy and range limitations, the device can measure up to 3 MHz (higher frequency causes counter overflow). The Multiple-Sensor device was tested with a square wave signal from signal generator JDS6600 (by Joy-IT, frequency accuracy: $\pm 20 \mathrm{ppm}$ ).
- The Multiple-Sensor device measurement accuracy (percentage error) of frequency, is worst at low frequencies with $9 \%$ error at 100 Hz and $0.7 \%$ error at 1 kHz , above 5 kHz the error was always smaller than $0.2 \%$ (ignoring any accuracy error by JDS6600 used as reference). The Multiple-Sensor device measurement precision (variation) for frequency was worst at low frequencies with $5 \%$ variation at 300 Hz , above 1500 Hz was always smaller than $1 \%$, and above 15 kHz was always smaller than $0.1 \%$.


## Appendix A.2. Experimental data on Multi-Sensor Int.

## - Reference instruments:

The measurements of inductance $\left(L_{s}\right)$ and capacitance $\left(C_{s}\right)$ were obtained using the LCR meter TH2821A (by Tonghui, basic accuracy $0.3 \%$ ), configured to 10 Khz test signal.

The measurements of resistance $\left(R_{s}\right)$ were obtained using the meter UT603 (by UNI-T, accuracy: $0.8 \%$ for $\mathrm{R} \leq 2 \mathrm{M} \Omega ; 2 \%$ for $\mathrm{R}>2 \mathrm{M} \Omega$;).

Here are subset (some pairs) of measured experimental data for $L_{s}(f), R_{S}(f), C_{s}(f)$.

- Jumper Configurations:
${ }^{\mathrm{a}}$ (JPA on, JPB off): $C_{1}=2.2 \mathrm{nF} ; C_{2}=21.8 \mathrm{pF}$.
${ }^{\mathrm{b}}$ (JPA off, JPB on): $C_{1}=21.8 \mathrm{pF} ; C_{2}=2.2 \mathrm{nF}$.
${ }^{\mathrm{c}}$ (JPA on, JPB on): $C_{1}=2.2 \mathrm{nF} ; C_{2}=2.2 \mathrm{nF}$.
- Units: $\mathrm{Hz}=$ hertz, $\mathrm{H}=$ henry, $\Omega=\mathrm{ohm}, \mathrm{F}=$ farad.

| $C_{s}[\mathrm{nF}]$ <br> Capacitance | $\mathrm{f}[\mathrm{Hz}]^{\mathrm{c}}$ <br> JPA on, JPB on | $C_{s}[\mathrm{nF}]$ <br> Capacitance | $\mathrm{f}[\mathrm{Hz}]^{\mathrm{c}}$ <br> JPA on, JPB on |
| :---: | :---: | :---: | :---: |
| 0 | 229 | 3.98 | 356012 |
| 0.152 | 321 | 4.97 | 373045 |
| 0.31 | 458 | 5.97 | 385032 |
| 0.568 | 2614 | 6.95 | 394283 |
| 0.615 | 54570 | 7.94 | 400750 |
| 0.689 | 110286 | 8.98 | 405766 |
| 0.776 | 140178 | 10.07 | 411246 |
| 1.015 | 186522 | 12.04 | 418028 |
| 1.34 | 231460 | 15.02 | 426040 |
| 1.58 | 255526 | 20.02 | 433089 |
| 1.79 | 271015 | 24.97 | 437446 |
| 2 | 285020 | 29.68 | 440382 |
| 2.56 | 314530 | 34.62 | 442584 |
| 2.99 | 329820 | 39.43 | 444235 |

Precision error(maximum frequency variation): $\pm 3 \mathrm{kHz}\left(600 \mathrm{pF}<C_{s}<1.6 \mathrm{nF}\right) ; \pm 2 \mathrm{kHz}\left(1.6 \mathrm{nF}<C_{s}<21 \mathrm{nF}\right)$; $\pm 1 \mathrm{kHz}\left(C_{s}>21 \mathrm{nF}\right) ; \pm 50 \mathrm{~Hz}\left(C_{s}<600 \mathrm{pF}\right)$.

Table A.2: Subset of experimental data for $L_{s}(f)$

| $L_{s}[\mu H]$ <br> Inductance | Table A.2: Subset of experimental data for $L_{s}(f)$ <br> JPA on, JPB off | $\mathrm{f}[\mathrm{Hz}]^{\mathrm{b}}$ <br> JPA off, $J P B$ on | $\mathrm{f}[\mathrm{Hz}]^{\mathrm{c}}$ <br> JPA on, JPB on |
| ---: | ---: | ---: | ---: |
| 1.21 | 834161 | 879205 | 446590 |
| 1.85 | 833014 | 887049 | 458577 |
| 4.7 | 826011 | 881239 | 496405 |
| 9.7 | 813397 | 858548 | 615743 |
| 11.77 | 808321 | 936894 | 1432920 |
| 15.76 | 799009 | 895932 | 1251070 |
| 21.39 | 785691 | 931940 | 1090760 |
| 31.8 | 760632 | 1056660 | 889633 |
| 46.7 | 726580 | 1369450 | 736014 |
| 53.44 | 712070 | 2853370 | 690970 |
| 95.34 | 620697 | 2103380 | 509845 |
| 173.5 | 502398 | 1575330 | 382571 |
| 341.8 | 368641 | 1112820 | 271168 |
| 558.1 | 287880 | 868105 | 212362 |
| 777.6 | 241031 | 737803 | 178480 |
| 921.2 | 218738 | 677591 | 163587 |
| 1491 | 161768 | 606844 | 124338 |
| 2171 | 130897 | 499264 | 102978 |
| 3170 | 105439 | 406011 | 84614 |
| 3640 | 97779 | 383305 | 79110 |
| 4646 | 84110 | 319423 | 69783 |
| 6880 | 68162 | 282360 | 57750 |
| 10140 | 54034 | 224090 | 47230 |
| 15040 | 43377 | 192088 | 38790 |
| 20375 | 36588 | 171981 | 33683 |

Precision error(maximum frequency variation):
$\pm 2 \mathrm{kHz}$ (at high 'f[Hz]'); $\pm 300 \mathrm{~Hz}$ (at low 'f[Hz]'); $\pm 5 \mathrm{kHz}\left(2.85 \mathrm{MHz} \leftrightarrow 1.36 \mathrm{MHz} ; \mathrm{at}^{\mathrm{b}}\right.$ JPA off, JPB on)

Table A.3: Subset of experimental data for $R_{s}(f)$

| $R_{S}[\Omega]$ <br> Resistance | f $[\mathrm{Hz}]^{\mathrm{c}}$ <br> JPA on, JPB on | $R_{s}[\Omega]$ <br> Resistance | $\mathrm{f}[\mathrm{Hz}]^{\mathrm{c}}$ <br> JPA on, JPB on |
| ---: | ---: | ---: | ---: |
| 1.2 | 456284 | 6970 | 55288 |
| 5.2 | 454678 | 9960 | 41879 |
| 10.2 | 447385 | 14950 | 29861 |
| 20.1 | 439144 | 19940 | 23271 |
| 50 | 416407 | 24900 | 19066 |
| 99.9 | 383396 | 39800 | 16176 |
| 139.7 | 361700 | 49800 | 12430 |
| 199.5 | 333306 | 69700 | 7385 |
| 299 | 296289 | 99400 | 5320 |
| 398 | 268278 | 149300 | 3623 |
| 498 | 246214 | 199100 | 2798 |
| 697 | 213647 | 299000 | 1972 |
| 996 | 180865 | 498000 | 1284 |
| 1494 | 147150 | 697000 | 978 |
| 1992 | 125699 | 995000 | 779 |
| 2990 | 98819 | 1993000 | 504 |
| 3980 | 82153 | 5080000 | 351 |
| 4980 | 70593 | 9040000 | 305 |

Precision error(maximum frequency variation):
$\pm 1 \mathrm{kHz}\left(\right.$ at low $\left.R_{s}\right) ; \pm 300 \mathrm{~Hz}($ at $30 \mathrm{k} \Omega) ; \pm 100 \mathrm{~Hz}\left(\right.$ at high $\left.R_{s}\right)$

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