Impact of node count on energy-optimal control of stratified vertical water heaters in smart grid applications

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Abstract

The energy requirements for an Electric Water Heater (EWH) accounts for 40% of a household's total energy consumption and 30 % of greenhouse gases emissions. The flexibility of the device to store thermal energy for long periods highlights how the intensity of the grid demand can be alleviated by implementing demand-side management (DSM) strategies. In this paper, we evaluate energy savings that can be achieved by modelling the EWH as a variable number of multiple nodes and providing it with optimal control with perfect foreknowledge of the hot water usage profile. We simulated 77 household's for all four seasons and determined that an average daily energy saving of 6.24% for temperature-matching and 16.3% for energy-matching can be achieved for a 20-node EWH. We also evaluated how increasing the number of nodes of the EWH when determining the optimal planning affects energy savings. It was concluded that using more than four nodes produced diminishing returns.

Keywords: Smart grid; optimal control; electric water heaters

1. Introduction

Demand-side management (DSM) focuses on reducing the demand on the energy grid by implementing strategies that shift energy consumption during peak to off-peak hours or by decreasing the overall energy usage. The EWH accounts for as much 40 % of a household's total energy consumption and 30 % of its greenhouse gases emissions (Skinner et al., 2012; Nejat et al., 2015; Hohne et al., 2019; Mabina et al., 2021). This makes the thermal storage device convenient for reducing a household's energy demand because it provides the flexibility of storing energy for long periods with small energy losses (Ericson, 2009). The implementation of advanced control strategies can greatly improve the energy efficiency of EWHs by minimising energy loss, enhancing user comfort, and preventing the risk of *Legionella* (Pomianowski et al., 2020). *Legionella pneumophila* is a bacteria that is potentially harmful to humans and grows inside water tanks if the temperature remains low for long periods. The bacteria can be sterilised if the tank is sufficiently heated at least once a day (Armstrong et al., 2014; Stone et al., 2019; Stout et al., 1986).

Taking South Africa as an example, the country has suffered from *loadshedding*, a term that refers to scheduled regional power interruptions, because of the incapability of the electricity supply to meet the country's growing demand, and is expected to continue for another five years (ESKOM, 2021; Jamie McKane, 2021). Since the country's primary energy source is coal, contributing to approximately 88 % of the electricity generation, reducing the overall energy demand can lessen the impact of carbon emissions (Mathu, 2017).

Determining the effective energy savings of control strategies requires accurate modelling of tanked water heaters and is significantly influenced by the accuracy of the hot water usage data (Dongellini et al., 2015; Spur et al., 2006). Fernández-Seara et al. (2007) performed an experimental analysis of a vertically-orientated EWH with 11 temperature sensors to determine the thermal behaviour during static operation. The results indicated that stratification occurs inside the tank due to varying water temperature at each layer and the temperature differences were most significant at the lower parts of the tank.

Most related studies have been found to utilise one of the following four EWH models: A *one-node* model assumes that the temperature of the tank is at a uniform temperature. A *two-mass composite* model

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Nomenclature				
Δt	Sampling period		T_{hys}	Hysteresis
c	Specific heat capacity of water		T_{inlet}	Inlet water temperature
E_{tank}	Thermal energy of EWH		$T_{\text{Legionella}}$	Legionella prevention temperature
$E_{\text{tank,n}}$	Thermal energy of node n		T_{max}	Maximum EWH temperature
J	Cost function		T_{min}	Minimum EWH temperature
ρ	Density of water		T_{outlet}	Outlet water temperature
P_{elec}	Power supplied by heating element		T_{set}	Set-point temperature
P_{rated}	Rated power of heating element		T_{start}	Initial temperature boundary condition
Q_{draw}	Outlet volumetric flow rate		T_{tank}	Water temperature of EWH
R_{T}	Thermal resistance of EWH		$T_{\text{tank, n}}$	Water temperature of node n
R_{TH}	Thermal resistance of thermocline		V_{tank}	Tank volume of EWH
t_f	Final time instant		$V_{\text{tank,n}}$	Tank volume of node n
t_i	Initial time instant		T_{usage}	Minimum usage temperature
T_{amb}	Ambient temperature		u	Control input
T_{end}	Final temperature boundary condition		$\mathbf{x}$	System state vector

represents the body of water inside the tank as an upper and lower layer. The upper node is either at a higher or equal temperature to the lower node and water stratification in the layers is not accounted for. A *two-node* model, similar to the previous model, does not neglect the transfer of heat that occurs at the thermocline of the two bodies of water. The volume of each node also varies: hot water drawing depletes the volume of the upper node and cold water entering the tank is mixed with the lower node. The *multi-node* model models the tank as a finite number of nodes stacked on top of each other. Each node has a constant volume and distinct temperature and stratification occurs at the boundary of adjacent nodes. More information on these models can be found in (Kepplinger et al., 2019).

This paper determines the energy savings achieved by optimal control of an EWH with *perfect foreknowledge* of the hot water usage profile, by evaluating the impact of the number of nodes by which the natural stratificatio is modelled. The EWH simulator and optimisation algorithm were developed in *Jupyter Notebook* and using the *Python* programming language. The paper is structured as follows: Section II reviews work related to the optimal control of EWHs. Section III describes the hot water data and simulation setup for obtaining the results. Section IV presents the development of the multi-node EWH model and optimisation algorithm. Section V and VI show the results and conclusion of this study.

1.1. Contributions

A control strategy is developed for the EWH optimal control problem which is formulated to determine the optimal switching schedule of the heating element to minimise the electrical energy usage while satisfying the comfort of the user. The novelty of the developed optimisation algorithm is contributed by its capability to model an EWH with N nodes in the optimal planning. The novel results produced in this study determines the theoretical optimal energy savings that are achieved by a multi-node EWH with perfect foreknowledge of the hot water usage profile. This study also determines the maximum number of EWH nodes required for optimal planning before the results show diminishing returns.

2. Related Work

Gholizadeh and Aravinthan (2016) evaluated the cost benefits of managing the set-point temperature and heating scheduling of residential EWH's. They reported that the savings were 5.9 % to 6.4 %. However, their study used random variations of the ASHRAE standard to represent typical water profiles instead of using real data. The shortcoming of using this water usage model is that the hot water profiles are generalised and predicted.

Booysen and Cloete (2016) determined energy savings using a two-node EWH model developed by Nel et al. (2016). Their aim was to understand the energy impact of non-optimal manual schedule control on EWH's and they performed field trials of four water heaters and a controlled laboratory experiment of a single

water heater to validate their results. They confirmed that the simulated results accurately described the measured results. They obtained energy savings of 29 % but did not consider temperature or energy-matching the EWH to a baseline scenario during water usages.

Booysen et al. (2019a) determine how much energy can theoretically be saved by providing the optimal control of an EWH for known water usage profiles, using only a single node to model the water as a lumped mass. Their study used hot water draw data that was measured over 20 days for 30 EWH's installed in South African household's. They used dynamic programming to determine the optimal heating schedule for the heating element to achieve the best savings. They performed simulations for cases where the temperature and energy are matched to the baseline condition during water usages, and the average energy savings were 8 % and 18 %. They also modified the energy-matched strategy to prevent the growth of *Legionella* and achieved 13 % energy savings.

Engelbrecht et al. (2021) follow on this research by using the two-node EWH model developed by Nel et al. (2016) to improve the accuracy of the results. They also expanded the database of water usage profiles to one month for 77 households. The increased complexity of the two-node EWH thermodynamics motivated the development of an A* search algorithm to determine the optimal control. They obtained energy savings of 6.3 % for temperature-matching, 21.9 % for energy-matching, and 16.2 % for energy-matching with *Legionella* prevention.

Ritchie et al. (2021a) developed a hot water usage model that predicted the hot water usage demand of a household based on historical data. This was used in a more recent study to determine the practical energy savings when water usages are forecasted. The energy savings achieved were 2.2 % for temperature-matching and 9.6 % for energy-matching with *Legionella* prevention (Ritchie et al., 2021b).

Kepplinger et al. (2015) built an auto-scheduling method that provides hourly control of EWHs to minimise the overall cost and energy usage. The optimisation problem was formulated using a binary integer program for a one-node EWH model and simulated for a multi-node EWH. They predicted future water demand by using a nearest-neighbour data-mining algorithm on historically measured data. Their strategy ensured that the delivered energy was matched to the baseline method but didn't consider temperature matching. They achieved cost and energy savings of 12 % and 4 %.

Xiang et al. (2019) designed a novel direct load control of EWH's strategy that produced a Customer Satisfaction Prediction Index (CSPI) based on a weight matrix. This matrix was calculated from hot water usage patterns and determined the EWH's comfort levels. The proposed strategy produced a peak shifting service that ensured user comfort.

Siegel et al. (2020) developed a system that is capable of predicting hot water usage demand and achieved safe energy savings while minimising *Legionella* growth. They accomplished their goal by combining an auto-aggressive convolutional neural network with a Cognitive Supervisor. They suggested that the results of their model would perform similarly to the energy matching strategy developed by Booysen et al. (2019a).

3. Experimental Setup

This section presents the various components that describe the experimental setup. The acquisition of data is first described, followed by the setup of the simulator and the implementation of a temperature feedback controller and water mixer.

3.1. Hot Water Usage Profiles

The hot water usage data that is used to determine the optimal control schedule is obtained from 77 household's in South Africa using Smart EWH Controller devices Bridgiot. The hot water profile for each household consists of minutely recorded water flow-rate data that spans over four weeks, one week for each season. Since a household's hot water usage behaviour varies with season, the results should reflect simulations of the water profiles for all seasons (Zhou et al., 2002; Gato et al., 2007; Gerin et al., 2014; Roux and Booysen, 2017; Booysen et al., 2019b).

3.2. Simulator Setup

The results are obtained by simulating EWHs for all of the hot water usage profiles. Each EWH's heating element is controlled by an optimal control schedule, predetermined with perfect foreknowledge of water usages, to determine the results. The energy savings are calculated by comparing the results of optimal

control with a baseline strategy (defined in Section IV). The thermodynamics of a multi-node EWH is defined in the following section and is used for simulations as well as for solving the optimal control problem.

The optimal planning and simulation execution uses a M and N-node EWH model, respectively. We use 20 nodes for N when simulating EWH's for all of the optimal schedules. We vary the number of nodes for M, where $M \in 1, 2...20$, and simulate the optimal planning on the N-node EWH. EWH feedback which handles discrepancies that can occur between planning and execution is described in the following subsection. Table 1 lists the parameters and constraints used for the EWH model and optimisation algorithm.

Table 1: Table of EWH parameters and optimisation constraints.

Parameter	Value	Unit
c	4184	$\frac{J}{kg \cdot K}$
ρ	1000	$\frac{kg}{m^3}$
P_{rated}	3	kW
R_{T}	0.4807	$\frac{K \cdot day}{kWh}$
T_{amb}	20	°C
T_{hys}	±1.5	°C
T_{inlet}	20	°C
T_{set}	68.5	°C
T_{tank}	150	L
Constraint	Value	Unit
Δt	1	min
T_{end}	68.5	°C
$T_{\text{Legionella}}$	60	°C
T_{max}	70	°C
T_{min}	20	°C
T_{start}	68.5	°C
T_{usage}	40	°C

3.3. Temperature feedback controller and user water mixer

The simulator uses a temperature feedback mechanism to override the optimal input for the EWH when the measured temperature strays from the desired temperature. This ensures that the EWH temperature does not deviate from the optimal plan as a result of unexpected water usages or model inaccuracies.

The user and water mixer is simulated to account for the user's behaviour when they react to an initial water temperature that is lower than expected. The user will adjust the mixing of hot and cold water to achieve the same delivery of thermal energy as the baseline strategy, despite the lower temperature. This is a model of reality and is used to perform and evaluate simulation tests.

4. Development

4.1. EWH Model Development

We use a lumped-mass model to describe the dynamics of the EWH model. We first describe the modelling of an EWH that consists of a single node, and then further expand it into the multi-node EWH model. The modelling of the one-node was originally presented by Nel (2015). Figure 1a shows a diagram of a vertical one-node EWH model.

The temperature of the water inside the tank is assumed to be a single, uniform temperature. When a positive flow rate occurs at the outlet pipe at the top of the tank, Q_{draw}, the outlet water temperature, T_{outlet}, is equal to that of the whole tank, T_{tank}. The water leaving the tank is replaced with water at a cold temperature, T_{inlet}, through the inlet pipe at the bottom of the tank. A heating element is also situated at the bottom of the tank and provides electrical energy, E_{elec}, to increase the overall energy of the tank, E_{tank}. Thermal energy E_{loss} is lost from the tank to the surrounding environment due to standing losses. The rate of energy loss is dependant on the ambient temperature, T_{amb}, and the thermal resistance of the tank wall, R_{T}.

Figure 1b shows a diagram of the multi-node EWH model. The significant change from the one-node model is that the temperature of the tank is modelled as a vertical gradient. The body of water is split into N nodes stacked on top of each other. At node n, the temperature of the water is always higher or equal to
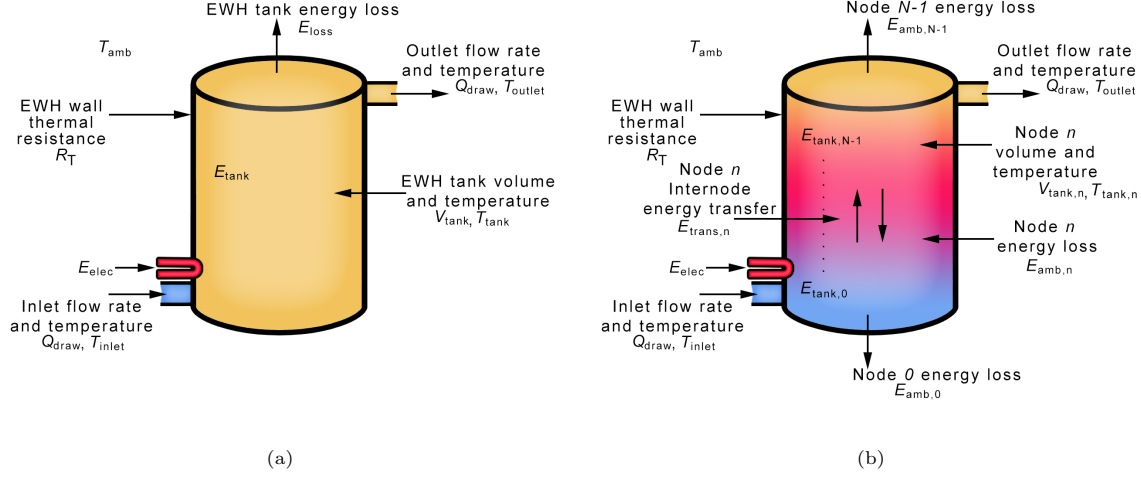
Figure 1: Energy flow, thermal resistance, flow rate, temperature and volume in a) a single node and b) multi-node EWH.

that of the lower node at $n-1$. Water drawn from the tank is at the temperature of the top node, $N-1$ and cold water enters the bottom node 0. The heating element also supplies electrical energy to this node. For each node, the standing losses occur at the segment of the tank that surrounds that node and is dependent on the water temperature of the node, $T_{\text{tank, n}}$. Water stratification occurs at every boundary of two adjacent nodes due to the temperature difference. The internode energy transfer at a node, $E_{\text{trans,n}}$, occurs at a rate determined by the thermal resistance of the thermocline.

The thermal dynamics for each node of the multi-node EWH model is described as follows:

$$\dot{E}_{\text{tank,n}}(t) = \left\{ \begin{array}{ll} P_{\text{elec}}(t) + P_{\text{inlet}}(t) - P_{\text{flow,n-1}}(t) + P_{\text{inv,n+1}}(t) - P_{\text{loss,n}}(t) & if\, n = 0 \\ -P_{\text{flow,n-1}}(t) - P_{\text{loss,n}}(t) - P_{\text{inv,n-1}}(t) & if\, n = N-1 \\ -P_{\text{flow,n-1}}(t) - P_{\text{loss,n}}(t) - P_{\text{inv,n-1}}(t) - P_{\text{inv,n+1}}(t) & \text{otherwise} \end{array} \right\} \quad (1)$$

where $E_{\text{tank,n}}(t)$ is the thermal energy in node n, P_{elec} is the electrical energy supplied by the heating element, $P_{\text{flow,n-1}}$ is the power transfer of mass flow due to buoyancy, $P_{\text{inv,n-1}}$ and $P_{\text{inv,n+1}}$ is the water stratification that occurs between the adjacent nodes, $P_{\text{loss,n}}(t)$ is the power leaving node n due to losses to the environment, and $n = 0$ and $n = N-1$ refers to the top and bottom nodes.

The thermal energy of the tank is calculated as the sum of energy in each node, and is calculated as follows:

$$E_{\text{tank}}(t) = \sum_{n=0}^{N-1} E_{\text{tank,n}}(t) \quad (2)$$

When water is drawn from the top of the tank and replaced with cold water at the bottom, a mass flow occurs due to buoyancy and the transfer of thermal energy in each node is calculated as follows:

$$P_{\text{flow,n}}(t) \approx c\rho Q_{\text{draw}}(t) T_{\text{tank,n-1}}(t) \quad (3)$$

where c is the constant volume-specific heat capacity, ρ is the density of water, and $T_{\text{tank,n-1}}$ is the temperature of the water inside the lower adjacent node. The power loss from each node to the environment is calculated as follows:

$$P_{\text{loss,n}}(t) = \frac{1}{R_{\text{T,n}}}[T_{\text{tank,n}}(t) - T_{\text{amb}}(t)] \quad (4)$$

where $R_{\text{T,n}}$ is the thermal resistance of the wall that is occupied by node n, and $T_{\text{tank,n}}$ is the temperature of the water in node n. If there is a temperature difference between node n and any adjacent node, the power transfer from node n to the adjacent nodes is calculated as follows:

$$P_{\text{inv,n-1}}(t) \quad = \quad \frac{1}{R_{\text{TH}}}[T_{\text{tank,n}}(t) - T_{\text{tank,n-1}}(t)] \tag{5}$$

$$P_{\text{inv,n+1}}(t) \quad = \quad \frac{1}{R_{\text{TH}}}[T_{\text{tank,n}}(t) - T_{\text{tank,n+1}}(t)] \tag{6}$$

The relationship between the node energy, $E_{\text{tank,n}}$, and the node water temperature relative to a reference temperature, where we define energy to be zero, is given as follows:

$$T_{\text{tank,n}}(t) \approx \frac{E_{\text{tank,n}}(t)}{c\rho V_{\text{tank,n}}} \tag{7}$$

where $V_{\text{tank,n}}$ is the volume of node n.

4.2. Optimisation Algorithm

The optimal temperature and heating planning for the multi-node EWH are produced by an A* search algorithm. The algorithm was previously formulated for a stratified two-node EWH model by Engelbrecht et al. (2021) and is modified in this study to produce the optimal planning for a multi-node EWH.

Optimal control problem

The optimal control problem is to determine the optimal switching sequence of the heating element to minimise the total electrical energy supplied to the EWH while satisfying a given hot water usage profile. The profile is satisfied if the user never experiences a temperature below the minimum usage temperature, T_{usage}, when hot water is drawn from the tank.

The system dynamics are defined by the differential equations that describe the thermodynamics of the multi-node EWH, specified by equations (1), (2), (3), (4), (6) and (7). The system states is represented by $\mathbf{x}(t)$ and is defined by the thermal energy in each node. The state vector is defined as follows:

$$\mathbf{x}(t) = \begin{bmatrix} E_{\text{tank,0}}(t) & E_{\text{tank,1}}(t) & ... & E_{\text{tank,N-1}}(t) \end{bmatrix}^T \tag{8}$$

The state constraints are defined by the physical limitations of the thermal energy of the tank. For node n, the lower and upper bounds are represented by $E_{\text{min,n}}$ and $E_{\text{max,n}}$. These correspond to the minimum and maximum admissible temperatures, $T_{\text{min,n}}$ and $T_{\text{max,n}}$, with a constant volume of $V_{\text{tank,n}}$.

The control input is represented by $u(t)$ and is defined as P_{elec}, the electrical power supplied by the heating element. The control input is constrained to supply power equal to either zero or P_{rated}, the power rating of the heating element.

The cost function has the objective of minimising the electrical energy and is defined as follows:

$$J = \int_{t_i}^{t_f} P_{\text{elec}}(t)dt \tag{9}$$

where t_i and t_f is the initial and final time instant. Temperature profile constraints are defined for each node to ensure that the hot water usage profile is satisfied. The following inequality represents the objective of the temperature of each node to satisfy the hot water usage profile:

$$T_{\text{tank,n}}(t) \geq T_{\text{profile,n}}(t) \tag{10}$$

where $T_{\text{profile,n}}$ are the constraints imposed on node n and is expressed as follows:

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$$T_{\text{profile,n}}(t) = \left\{ \begin{array}{ll} T_{\text{usage}} & \text{if } Q_{\text{draw}}(t) > 0 \\ T_{\text{Legionella}} & \text{once per day} \\ T_{\text{min,n}} & \text{otherwise} \end{array} \right\} \tag{11}$$

where $T_{\text{Legionella}}$ is the temperature required to prevent the growth of *Legionella*. These constraints show that the outlet water temperature cannot fall below T_{usage} during water usage, the entire tank must be sufficiently heated to ensure *Legionella* prevention, and the temperature of the tank cannot exceed the minimum EWH temperature, T_{min}.

A * *search algorithm*

The A* search algorithm is a shortest path search algorithm that is popular and widely used. The algorithm can solve non-linear optimal control problems by creating multiple node-based paths that originate at a starting position, and each one navigates towards a desired final position. The algorithm performs efficiently as heuristics are introduced to help optimise the decision making of the process.

Before the A* search algorithm can be applied to the optimal control problem, it is required that the problem is discretised into discrete time instants and states to represent the decision stages and choices, respectively. Since the hot water usage data is presented with one-minute resolution, equations (1), (2), (3), (4), (6) and (7) are discretised with a minutely sampling period Δt.

The purpose of the algorithm is to find the shortest path from an initial state at time instant t_i to a goal state at time instant t_f. At the initial state, the temperature of the entire tank is assumed to be at the starting temperature T_{start}. At the goal state, the temperature of the entire tank must greater than or equal to the final temperature T_{end}. T_{start} and T_{end} are the boundary conditions for the optimal path and are specified for the algorithm.

A binary search tree (BST) data structure is defined to aid the navigation of the search process (more information on the BST data structure can be found in Nagaraj (1997)). The BST comprises multiple search paths that extend from the initial state and navigate to the final state. The paths are made up of nodes connecting states in the previous time instants to the calculated state in the next time instant as a result of the scenarios where the control input $u = 0$ and $u = 1$.

Each search path ending calculates a cost that is the sum of two components: the amount of electrical energy that was required to reach its current position and the estimated amount of electrical energy that is still required to reach the final state at the last time instant. A priority queue keeps track of the cost of every path ending, prioritises them from lowest to highest cost, determines which path end is currently the closest (and most optimal path) from the desired final state, and instructs that path to extend further.

The first path that reaches the desired state at the last time instant is also the optimal path and the algorithm finishes executing. With the optimal path acquired, the optimal temperature trajectory and heating schedule is produced for the given hot water usage profile.

4.3. Heating control strategies

Three heating control strategies are presented in this study:

Thermostat Control (TC): This is the baseline strategy that is used to evaluate the energy savings achieved by the following two methods. This strategy represents how the EWH typically operates and is vastly inefficient. The water temperature is maintained at the set-point temperature of the thermostat. Therefore, high standing losses are expected as the tank is always hot, even when water is not drawn for long periods.

Temperature-matched optimal control (TM): This strategy is produced by the A* search algorithm and determines the optimal control of the EWH for a given hot water usage profile. However, the temperature profile constraints are modified such that the temperature of the tank at the start of water usage is exactly matched to the temperature that is expected in the scenario where the same water usage occurs for TC. This strategy ensures that the temperature and energy of the water drawn from the EWH is not compromised by the reduction in electrical energy that is supplied.

Energy-matched optimal control with Legionella prevention (EML): This strategy is similar to the previous strategy, except that the temperature is only constrained to T_{usage} during water usages. However, the hot water flow rate is increased during water usages to ensure that an equivalent amount of energy is delivered, despite the lower temperature experienced by the user. A lower temperature profile can cause the growth of *Legionella* inside the tank and introduce a health risk. This problem is mitigated by ensuring that the EWH is heated to $T_{\text{Legionella}}$ at least once a day to prevent the growth of *Legionella*.

5. Simulation Results

The simulations results for the TM and EML heating control strategies are summarised in Table 2 and 3, respectively.

Figure 2 shows the 4-node EWH simulation results for the temperature-matching and energy-matching with *Legionella* prevention heating control strategies for the same 24-hour hot water profile (shown in blue). The plot shows the temperature profile for all four nodes, the state of the heating element P_{elec} (shown in red), and the minimum usage temperature T_{usage} of 40°C (shown in black). The temperature profile of the highest node for TC is also shown. All simulations started with the temperature of the whole tank at the set-point temperature (68.5°C) at the beginning of the first day. However, the observed day in the figures is not the first day of the simulation and the state of the EWH at the start of day may vary.

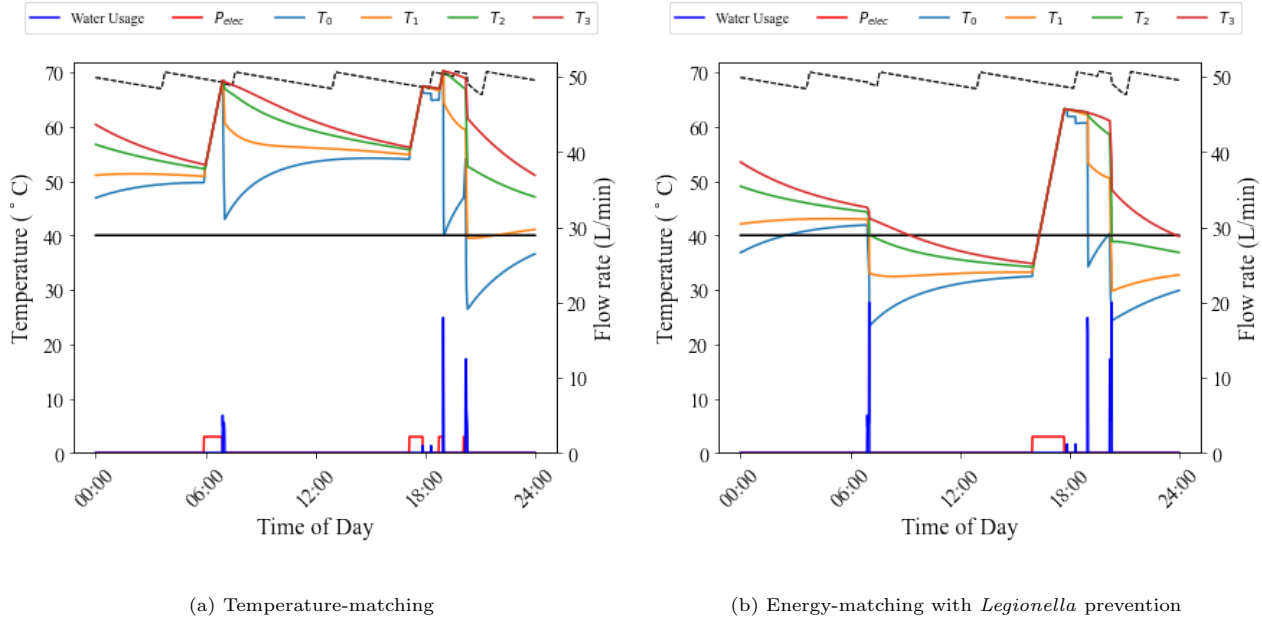(a) Temperature-matching   (b) Energy-matching with *Legionella* prevention

Figure 2: Simulation results of a 4-node EWH for optimal temperature matching and energy matching with *Legionella* prevention. The plots show the temperature of each node, the outlet flow rate, and the heating element state (either a value of zero or the rated power of 3 kW). The temperature profile of the highest node for thermostat control is repeated on each plot with a segmented black line. The minimum usage temperature of (40°C) is shown by with a solid black line on each plot.

Thermostat Control (TC): The thermostat keeps the temperature of the tank at the set-point temperature of 68.5°C and allowing for 1.5°C hysteresis. The heating element switches on when the temperature falls below 67°C and switches off again once the temperature has risen to 70°C. The electrical energy used for TC was 8.7 kWh.

Optimal temperature matching (TM): As shown in Figure 2a, this strategy ensures that the temperature of the whole tank is equal to that of TC at the start of each water usage. This means that the heating element only switches on just before the start of a water usage and won't turn on again until the next usage. This results in the temperature being lower than TC during the period between usages. By observing the water usage at 7am, the heating element increases the water temperature until the temperature reaches that of TC right before the usage occurred. During the usage, cold water mixes with the lowest node and the dropped to 44°C. Following this, the temperature of all the nodes exponentially converge to a single temperature until

the heating element is switched on again in preparation for the next water usage that occurs at 6pm. The electrical energy used for TM was 6.25 kWh.

Optimal energy matching with Legionella prevention (EML): As shown in Figure 2b, this strategy ensures that the temperature of the whole tank remains above 40°C during water usage. Despite a outlet temperature lower than TC, the flow rate is increased during usage to ensure that the equivalent amount of thermal energy is delivered. This simulates the fact that the user will adjust the mixing of hot and cold water to allow for more hot water to leave the tank. By observing the water usage at 7am, it can be seen that the maximum flow rate increased form 5 to 19 L/min. This strategy also ensures the prevention of *Legionella* by heating the whole tank to 60°C for at least 11 minutes. By observing the water usage at 6pm, it can be seen that this occurred right before the usage. The electrical energy used for EML was 5.25 kWh.

Figure 3 shows the temperature planning for TC of the bottom node for a 24-hour hot water usage profile (shown in dark blue) for $M = 2, 4, 10$ and 20. The inlet water temperature, T_{inlet}, is marked with a black segmented line on each figure.
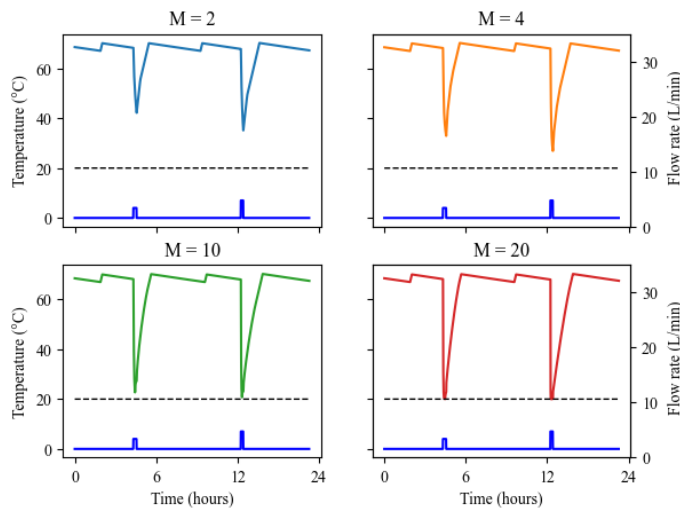

Figure 3: Plots of the temperature profile for the bottom node for TC when the EWH is modelled with $M = 2, 4, 10$ and 20. The 24-hour hot water usage profile (dark blue) is repeated on each plot and T_{inlet} is marked with a black segmented line.

Comparing these sub-figures, the water temperature of the node decreases to a lower temperature during water usage when the number of nodes M is increased. This is because a bigger portion of the lower nodes is mixed with the cold inlet water because the volume of each node is smaller. For $M = 20$, it can be seen that the bottom nodes temperature does not drop below T_{inlet}. This figure is important for understanding the optimal planning of the EWH for more nodes and aids the analysis of the simulation results presented next.

Looking at the results for TM in Table 2, the control strategy for all values of M was successful in matching the temperature during water usages and delivering the same amount of energy as the baseline strategy. The key difference in the results for the different number of nodes is observed in the daily energy losses. The daily standing losses decreased as M increased. As seen from Figure 3, the optimal temperature plan for M nodes will expect the temperature of the bottom node to decrease less when M is lower than N. Therefore, the temperature feedback controller will ensure that the 20-node EWH follows a temperature plan that is higher between water usages and resulting in higher standing losses. For $M = 20$, the achieved energy savings, given as [25th *percentile*, **median**, 75th *percentile*], of [*3.14*, **6.24**, *11.51*] % (or [*0.30*, **0.44**, *0.60*] kWh/day) was obtained for TM. The results for $M = 20$ showed the best performance because the temperature feedback controller is not required for the simulated EWH temperature to follow the optimal trajectory.

Looking at the results for EML in Table 3, the same trend occurs as M increases. The difference is that the daily average usage temperature decreased to 56°C. This is because the temperature does not need to match the temperature of the baseline condition during water usages, and must only remain about T_{usage}. The best energy savings of [*11.21*, **16.30**, *21.02*] % (or [*0.82*, **1.05**, *1.34*] kWh/day) was achieved by the
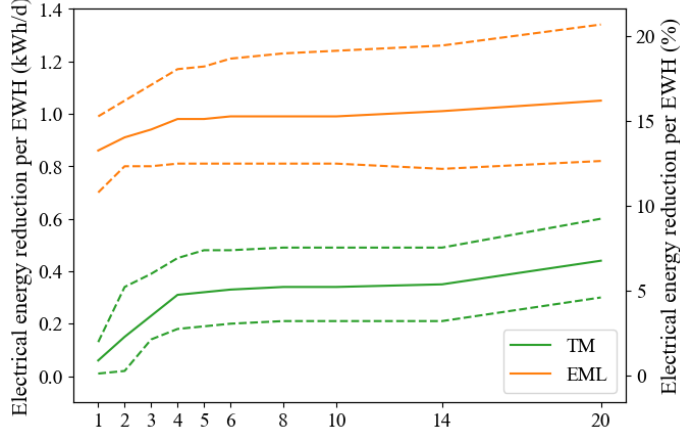
Figure 4: Daily energy reduction reduction in electrical energy for various values of M for TM (green) and EML (orange). The solid lines represent the median savings achieved by an EWH for a day. The segmented lines represent the corresponding 25th and 75th percentiles for each strategy.

20-node EWH model for EML.

Figure 4 shows the daily reduction in electrical energy for the median energy savings (solid line) and the 25th and 75th percentiles (segmented lines) for using different values of M for TM (green) and EML (orange). The energy-saving plots for both TM and EML show that the energy savings significantly decrease for using lower values of M, and the gradient flattens after $M = 4$. This suggests that determining the optimal control of a multi-node EWH can obtain sufficiently accurate results by using four nodes and anything more than this produces diminishing returns.

Table 2: Daily temperature-matched simulation results for increasing the nodes of the EWH during the planning and simulating on a 20-node EWH.

No. of nodes (M)	T_{usage}	Electricity Used (kWh/day)	Energy Used (kWh/day)	Energy Loss (kWh/day)	Energy Savings (kWh/day)	Energy Savings (%)
1	68.6	7.12	4.68	2.18	_0.01_, **0.06**, _0.13_	_0.15_, **0.93**, _2.01_
2	68.3	6.98	4.68	2.03	_0.02_, **0.15**, _0.34_	_0.31_, **2.31**, _5.25_
3	68.1	6.89	4.68	1.97	_0.14_, **0.23**, _0.39_	_2.16_, **3.55**, _6.02_
4	68.2	6.85	4.68	1.96	_0.18_, **0.31**, _0.45_	_2.78_, **4.78**, _6.94_
5	68.2	6.80	4.68	1.94	_0.19_, **0.32**, _0.48_	_2.93_, **4.94**, _7.41_
6	68.2	6.75	4.68	1.94	_0.20_, **0.33**, _0.48_	_3.09_, **5.09**, _7.41_
8	68.2	6.75	4.68	1.93	_0.21_, **0.34**, _0.49_	_3.24_, **5.25**, _7.56_
10	68.2	6.75	4.68	1.93	_0.21_, **0.34**, _0.49_	_3.24_, **5.25**, _7.56_
14	68.1	6.80	4.68	1.92	_0.21_, **0.35**, _0.49_	_3.24_, **5.40**, _7.56_
20	68.7	6.65	4.68	1.83	_0.30_, **0.44**, _0.60_	_4.63_, **6.79**, _9.26_

Note: The distributions are reported as 25th percentile, **median**, 75th percentile

6. Conclusion

In this paper, we determine the best energy savings that can be achieved by modelling the EWH as multi-nodal and providing it with optimal control with perfect foreknowledge of the hot water usage profile. The average daily energy savings achieved for 77 household's over one month was 6.24 % (0.44 kWh/day) for temperature-matched and 16.30 % (1.05 kWh/day) for energy-matched optimal control of the heating element.

Furthermore, we also evaluated how increasing the number of nodes of the EWH when calculating the optimal heating schedule affected the energy savings. It was determined that increasing the nodes resulted

Table 3: Daily energy-matched simulation results for increasing the nodes of the EWH during the planning and simulating on a 20-node EWH.

No. of nodes (M)	T_{usage}	Electricity Used (kWh/day)	Energy Used (kWh/day)	Energy Loss (kWh/day)	Energy Savings (kWh/day)	Energy Savings (%)
1	56.5	6.20	4.65	1.43	*0.74*, **0.86**, *0.99*	*11.42*, **13.27**, *15.27*
2	56.1	6.15	4.64	1.38	*0.80*, **0.91**, *1.05*	*12.34*, **14.04**, *16.20*
3	56.0	6.10	4.64	1.34	*0.80*, **0.94**, *1.11*	*12.34*, **14.50**, *17.13*
4	56.0	6.05	4.64	1.33	*0.81*, **0.98**, *1.17*	*12.50*, **15.12**, *18.05*
5	56.0	6.05	4.64	1.32	*0.81*, **0.98**, *1.18*	*12.50*, **15.12**, *18.21*
6	55.9	6.05	4.64	1.32	*0.81*, **0.99**, *1.21*	*12.50*, **15.27**, *18.67*
8	56.0	5.95	4.62	1.32	*0.81*, **0.99**, *1.23*	*12.50*, **15.27**, *18.98*
10	56.0	5.93	4.63	1.32	*0.81*, **0.99**, *1.24*	*12.50*, **15.27**, *19.13*
14	56.3	5.95	4.63	1.32	*0.79*, **1.01**, *1.26*	*12.19*, **15.58**, *19.44*
20	56.0	5.78	4.64	1.26	*0.82*, **1.05**, *1.34*	*12.65*, **16.20**, *20.67*

*Note: The distributions are reported as 25th percentile, **median**, 75th percentile*

in smaller standing losses between water usages and improving the savings. We concluded that the extent of improvement is negligible if more than four nodes are used.

In future work, the computational complexity of simulations and algorithms for producing accurate outcomes can be greatly reduced if a 4-node EWH model is used.

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