LOCALLY ORTHOTROPIC LAY-UP AS AN OPTIMAL SOLUTION FOR VAT POST-BUCKLED COMPOSITE PLATES EXPERIENCING LARGE DEFLECTIONS ABOVE VON KARMAN LIMITS

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Abstract

The present paper deals with the optimization of post-buckled VAT (variable angle tow) composite plates with large deflections. The Kirchhoff assumptions are used. The plates have a symmetric lay-up. The large deflection geometrically nonlinear theory above the von Karman limits is employed. The structural potential energy is treated as a measure of structural stiffness. For the plate stiffness maximization problem, the first-order necessary conditions of the local optimality are derived. The mathematical analysis of the conditions is performed. The conditions contain two terms. One of them corresponds to the mid-plane strains; another one corresponds to the generalized plate curvatures. A locally orthotropic lay-up is identified as an optimal solution. A particular solution of the linear combination of the ply optimality conditions is indicated. For the solution two pairs of the structural tensors are co-axial: the force and the strain tensors, as well as the moment and the generalized curvature tensors.

Keywords: composite plate, variable angle tow, postbuckling, large deflections, lay-up, optimality, co-axiality.

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1. INTRODUCTION

Postbuckling of thin composite plates is under consideration of numerous researchers, working in various modern application areas. It is known that the account of the load-carrying capability in postbuckling gives an opportunity of extra weight saving, as compared to existing traditional “up-to-buckling” design.

In the paper of (Selyugin, 2019) the lay-up optimality conditions for thin VAT plates were derived, basing on the Kirchhoff assumptions and on the von Karman approach.

The present paper extends the results of (Selyugin, 2019) beyond the von Karman limits, using the geometrically nonlinear elasticity theory (Novozhilov 1961, 2011).

The main goal of the present paper is theoretical; the task is to reveal the features of the optimal lay-ups for the VAT composite plates in postbuckling.

In the present paper, the first order necessary local optimality conditions are derived for the stiffness optimization formulation, with the objective function being the structural potential energy.

Note that the optimality conditions of other types (Legendre, Weierstrass, Jacobi, etc.) are not considered in the paper.

2. THEORETICAL CONSIDERATION

2.1. ASSUMPTIONS

The consideration of the paper deals with a post-buckled thin, flat VAT composite plate. The Classical Laminated Plate Theory (CLPT) (Gibson 1994, Reddy 2007) is employed. The plate is of symmetric lay-up and contains $2N$ tape layers. For simplicity, the laminates with the odd number of layers are outside the discussion; the generalization to the case is straightforward.

We consider a linear-elastic plate of constant thickness h. The plate thickness is much smaller than the linear plate dimensions. Fig. 1 demonstrates the coordinate system and some used notations. The X-Y plane coincides with the mid-plane Γ of the plate. The plate lateral surface S consists of the statically loaded part S_1 (the X, Y forces per unit area are f_x, f_y, respectively) and the part with the prescribed displacements S_2. The mid-plane Γ intersects the lateral surface S at the smooth contour C, with the portion C_1 belonging to S_1 and the portion C_2 belonging to S_2. The normal to C is the vector $\vec{n}$ with the components l, m. The tangential direction to C is s. The n, s, z directions create the right-hand triplet of vectors.

The plate experiences large deflections generated in postbuckling due to static loading (the deflections are much larger than the plate thickness).

The geometrically nonlinear elasticity theory and the Kirchhoff assumptions are employed (Novozhilov 1961, 2011). The latter assumptions are written as follows:

$$\varepsilon_{zz} = \varepsilon_{xz} = \varepsilon_{yz} = 0 \tag{1}$$

where ε is the Green strain tensor.

The deformed plate is assumed to be stable.

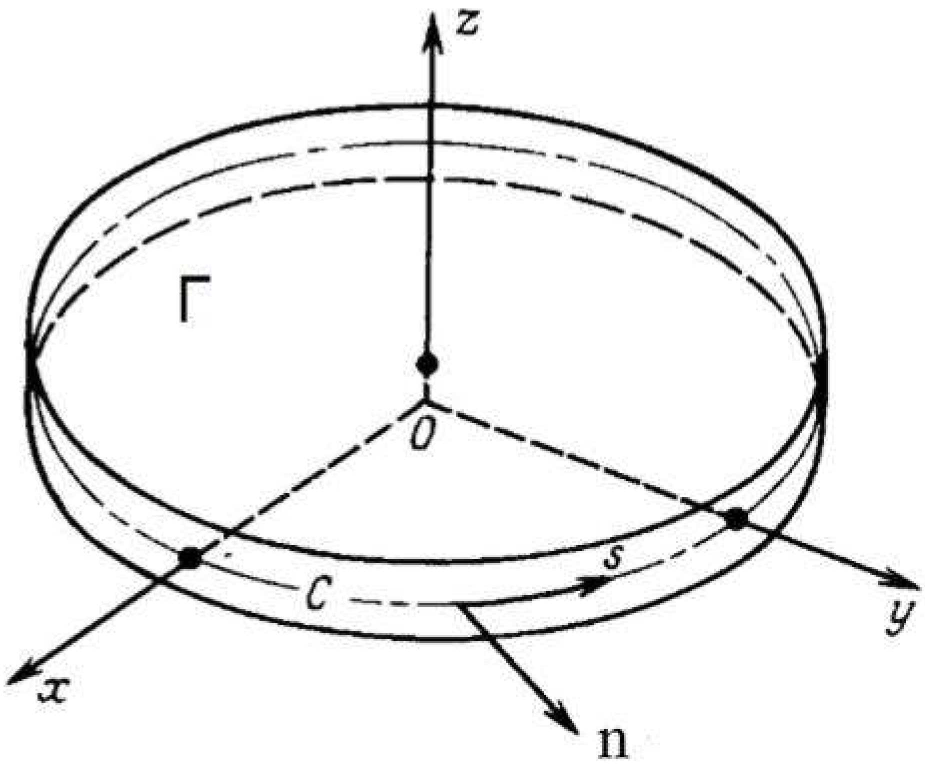

Fig. 1.

The ordinary notations are used hereinafter

$$\frac{\partial}{\partial x}(...) = (...),_x$$
$$\frac{\partial}{\partial y}(...) = (...),_y \tag{2}$$

Small elongations, small shears and small squared in-plane rotations (negligible as compared to unity) are assumed. Following (Novozhilov, 1961, 2011), the displacements *u, v, w* within the plate (respectively along X, Y, Z) are supposed to be

$$\begin{aligned} u(x,y) &= \hat{u}(x,y) + z\cdot\theta(x,y) \\ \mathrm{v}(x,y) &= \hat{\mathrm{v}}(x,y) + z\cdot\psi(x,y) \\ w(x,y) &= \hat{w}(x,y) + z\cdot\chi(x,y) \end{aligned} \tag{3}$$

where upper 'tilde' means quantities at the mid-plane.

After using (3) and the definition of the Green tensor components it is obtained in (Novozhilov, 1961, 2011) that

$$\begin{aligned} \theta &= -\hat{w},_x(1+\hat{\mathrm{v}},_y) + \hat{\mathrm{v}},_x\,\hat{w},_y \\ \psi &= -\hat{w},_y(1+\hat{u},_x) + \hat{u},_y\,\hat{w},_x \\ \chi &= \hat{u},_x + \hat{\mathrm{v}},_y + \hat{u},_x\,\hat{\mathrm{v}},_y - \hat{u},_y\,\hat{\mathrm{v}},_x \end{aligned} \tag{4}$$

and

$$\begin{aligned}\varepsilon_{xx} &= \hat{\varepsilon}_{xx} + z\kappa_{xx} + z^2\eta_{xx} \\ \varepsilon_{yy} &= \hat{\varepsilon}_{yy} + z\kappa_{yy} + z^2\eta_{yy} \\ \varepsilon_{xy} &= \hat{\varepsilon}_{xy} + z\kappa_{xy} + z^2\eta_{xy}\end{aligned} \tag{5}$$

The mid-plane strains in (5) are

$$\begin{aligned}\hat{\varepsilon}_{xx} &= \hat{u},_x + \tfrac{1}{2}\left[\hat{u},_x^2 + \hat{\mathrm{v}},_x^2 + \hat{w},_x^2\right] \\ \hat{\varepsilon}_{yy} &= \hat{\mathrm{v}},_y + \tfrac{1}{2}\left[\hat{u},_y^2 + \hat{\mathrm{v}},_y^2 + \hat{w},_y^2\right] \\ 2\hat{\varepsilon}_{xy} &= \hat{u},_y + \hat{\mathrm{v}},_x + \hat{u},_y\, \hat{u},_x + \hat{\mathrm{v}},_x\, \hat{\mathrm{v}},_y + \hat{w},_x\, \hat{w},_y\end{aligned} \tag{6}$$

and other quantities in (5):

$$\begin{aligned}\kappa_{xx} &= \theta,_x + \hat{u},_x\, \theta,_x + \hat{\mathrm{v}},_x\, \psi,_x + \hat{w},_x\, \chi,_x \\ \kappa_{yy} &= \psi,_y + \hat{u},_y\, \theta,_y + \hat{\mathrm{v}},_y\, \psi,_y + \hat{w},_y\, \chi,_y \\ 2\kappa_{xy} &= \theta,_y + \psi,_x + \hat{u},_x\, \theta,_y + \hat{u},_y\, \theta,_x + \hat{\mathrm{v}},_x\, \psi,_y + \hat{\mathrm{v}},_y\, \psi,_x + \hat{w},_x\, \chi,_y + \hat{w},_y\, \chi,_x\end{aligned} \tag{7}$$

$$\begin{aligned}\eta_{xx} &= \tfrac{1}{2}\left[\theta,_x^2 + \psi,_x^2 + \chi,_x^2\right] \\ \eta_{yy} &= \tfrac{1}{2}\left[\theta,_y^2 + \psi,_y^2 + \chi,_y^2\right] \\ 2\eta_{xy} &= \theta,_x\, \theta,_y + \psi,_x\, \psi,_y + \chi,_x\, \chi,_y\end{aligned} \tag{8}$$

In (Novozhilov, 1961, 2011) it is indicated that for small elongations and small shears the last terms in every relation of (5) are small enough and may be neglected. Then (5) is transformed into

$$\begin{aligned}\varepsilon_{xx} &= \hat{\varepsilon}_{xx} + z\kappa_{xx} \\ \varepsilon_{yy} &= \hat{\varepsilon}_{yy} + z\kappa_{yy} \\ \varepsilon_{xy} &= \hat{\varepsilon}_{xy} + z\kappa_{xy}\end{aligned} \tag{9}$$

Substituting (4) in (7) and keeping in mind the small in-plane rotations, after cumbersome transformations we obtain (see Selyugin 2021)

$$|\chi| << 1 \tag{10}$$

and

$$\begin{aligned}\kappa_{xx} &= -\theta(x,y)\hat{u},_{xx} - \psi(x,y)\hat{\mathrm{v}},_{xx} - \hat{w},_{xx} \\ \kappa_{yy} &= -\theta(x,y)\hat{u},_{yy} - \psi(x,y)\hat{\mathrm{v}},_{yy} - \hat{w},_{yy} \\ \kappa_{xy} &= -\theta(x,y)\hat{u},_{xy} - \psi(x,y)\hat{\mathrm{v}},_{xy} - \hat{w},_{xy}\end{aligned} \tag{11}$$

Below we will call (11) the generalized curvatures.

The internal forces and moments are introduced:

$$\begin{aligned}N_x &= \int_{-h/2}^{h/2} \sigma_{xx} dz, \qquad M_x = \int_{-h/2}^{h/2} z\sigma_{xx} dz, \\ N_y &= \int_{-h/2}^{h/2} \sigma_{yy} dz, \qquad M_y = \int_{-h/2}^{h/2} z\sigma_{yy} dz, \\ N_{xy} &= \int_{-h/2}^{h/2} \sigma_{xy} dz, \qquad M_{xy} = \int_{-h/2}^{h/2} z\sigma_{xy} dz\end{aligned} \tag{12}$$

The following notations are used in the paper:

$$\vec{N}=\begin{pmatrix} N_x \\ N_y \\ N_{xy} \end{pmatrix} \quad ; \quad \vec{M}=\begin{pmatrix} M_x \\ M_y \\ M_{xy} \end{pmatrix} \tag{13}$$

$$\vec{N}=A\vec{\varepsilon} \quad ; \quad \vec{M}=D\vec{\kappa} \tag{14}$$

where the vectors $\vec{\kappa}$ and $\vec{\varepsilon}$ are:

$$\vec{\kappa}=\begin{pmatrix} \kappa_{xx} \\ \kappa_{yy} \\ 2\kappa_{xy} \end{pmatrix} \tag{15}$$

$$\vec{\varepsilon}=\begin{pmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ 2\varepsilon_{xy} \end{pmatrix} \tag{16}$$

We also consider the tensor of generalized curvatures $\boldsymbol{\kappa}$ composed of $\kappa_{xx}, \kappa_{xy}, \kappa_{yy}$:

$$\kappa=\begin{bmatrix} \kappa_{xx} & \kappa_{xy} \\ \kappa_{xy} & \kappa_{yy} \end{bmatrix} \tag{17}$$

2.2. LAY-UP OPTIMIZATION PROBLEM

The structural potential energy is (Selyugin, 2021):

$$U=\int_{\Gamma}\left(\frac{1}{2}\hat{\vec{\varepsilon}}^{T}A\hat{\vec{\varepsilon}}+\frac{1}{2}\vec{\kappa}^{T}D\vec{\kappa}\right)d\Gamma-W \tag{18}$$

where the integral is the total strain energy *Π*

$$\Pi=\int_{\Gamma}\left[\frac{1}{2}\left(\hat{\vec{\varepsilon}}^{T}A\hat{\vec{\varepsilon}}\right)+\frac{1}{2}\left(\vec{\kappa}^{T}D\vec{\kappa}\right)\right]d\Gamma \tag{19}$$

W is the potential of external "dead" forces and the superscript *T* means a transposition.

The kinematic variational principle (the structural potential energy stationary principle derived in the paper of Selyugin, 2021), describes the plate postbuckling nonlinear behavior also.

The variational principle for the composite plates under the above assumptions is formulated in the following way. The quantities $(\hat{u},\hat{v},\hat{w})$ are the corresponding plate mid-surface displacements in the Cartesian coordinates (*x, y, z*), respectively; $\hat{w}$ is the out-of-plane deflection. The corresponding components of the Green strain tensor in the plate mid-plane are given in (6). The quantities of the $\hat{\vec{\kappa}}$ vector are in (15).

The matrices *A* and *D* in (18), (19) are the (3*3) matrices describing the 2D-mid-plane behavior and the bending-twisting of the plate (Gibson, 1994).

The kinematic variational principle (Selyugin, 2021) states that in equilibrium the structure satisfies

$$\delta U=\delta\Pi-\delta W=0 \tag{20}$$

for all kinematically admissible displacements $(\hat{u},\hat{v},\hat{w})$.

The principle (20) leads to the equilibrium equations for the mid-surface displacements $(\hat{u},\hat{v},\hat{w})$.

In the present paper we consider the structural potential energy U from (18) as a measure of the structural stiffness. The boundary conditions are assumed to be the clamping ones for all three displacements $(\hat{u}, \hat{v}, \hat{w})$ (see Selyugin, 2019, 2021).

The goal of the lay-up optimization is to maximize the postbuckling stiffness value. The ply orientation angles α_i are varied in the point-wise way. We assume, that the lamina orientation angles α_i are the smooth functions of the mid-plane location (x, y), $i=1,\dots,N$.

2.3. OPTIMALITY CONDITIONS

As it was shown in (Selyugin, 2019), the derivatives of the stiffness matrices A, D w.r.t. the lamina orientation angles α_n are, $ij=11, 12, 22; n=1,\dots,N$:

$$\begin{aligned} \frac{dA_{ij}}{d\alpha_n} &= (z_n - z_{n-1})\frac{d(\overline{Q}_{ij})_n}{d\alpha_n} \\ \frac{dD_{ij}}{d\alpha_n} &= \frac{1}{3}(z_n^3 - z_{n-1}^3)\frac{d(\overline{Q}_{ij})_n}{d\alpha_n} \end{aligned} \tag{21}$$

where $\overline{Q}_{ij}$ in brackets with the subscript n is the ply stiffness matrix for the n-th ply (Gibson, 1994), z_n is the top coordinate of the n-th ply.

Following the usual variational procedure (Selyugin, 2019) and taking into account the variational principle (20), we obtain the first order necessary conditions of local optimality in the case. The maximization of the structural stiffness means that the first variation of U with respect to the orientation angles is equal to zero.

The variation of the structural potential energy is:

$$\delta U = \delta U\Big|_{\substack{A-non-\text{varied} \\ D-non-\text{varied}}} + \delta U\Big|_{(\hat{u},\hat{v},\hat{w})-non-\text{varied}} = 0 \tag{22}$$

The first item in (22) is equal to zero due to (20). In the second item, the matrices A, D are functions of the lamina orientation angles only. From the second item, we obtain a relation for the orientation angles of i-th layer at the current point, $i=1,\dots,N$:

$$\delta U\Big|_{(\hat{u},\hat{v},\hat{w})-non-\text{varied}} = \sum_{i=1}^{N}\int dS\left(\frac{1}{2}\hat{\vec{\varepsilon}}^{\,T}\frac{dA}{d\alpha_i}\hat{\vec{\varepsilon}} + \frac{1}{2}\vec{\kappa}^{\,T}\frac{dD}{d\alpha_i}\vec{\kappa}\right)\delta\alpha_i = 0 \tag{23}$$

and, keeping in mind the independence of the α_i components from each other, $i=1,\dots,N$, and the basic Lemma of calculus of variations, we obtain the first order necessary conditions of local optimality for the post-buckled plate stiffness maximization:

$$\frac{1}{2}\hat{\vec{\varepsilon}}^{\,T}\frac{dA}{d\alpha_i}\hat{\vec{\varepsilon}} + \frac{1}{2}\vec{\kappa}^{\,T}\frac{dD}{d\alpha_i}\vec{\kappa} = 0\ ,\ i=1,\dots,N \tag{24}$$

Further, we calculate the first item in (24) (the item corresponds to the mid-plane 2D-strains) in the axes of the principal 2D mid-plane strains $\varepsilon_1, \varepsilon_2$, where $\varepsilon_1 \geq \varepsilon_2$. The second item in (24) (the item corresponds to the plate bending and twisting) we calculate in the axes of the principal plate generalized curvatures k_1, k_2, where $k_1 \geq k_2$. Making necessary transformations in (24) and remembering that the absolute value of the Jacobian of a transition from the principal mid-plane strain coordinates to the principal generalized curvature coordinates is equal to 1.0 (the transition is a simple rotation), we obtain for $i=1,\dots,N$:

$$\frac{1}{2}\left\{(\varepsilon_1,\varepsilon_2)\begin{bmatrix}\frac{dA_{11}^{pr.str.}}{d\alpha_i} & \frac{dA_{12}^{pr.str.}}{d\alpha_i}\\ \frac{dA_{12}^{pr.str.}}{d\alpha_i} & \frac{dA_{22}^{pr.str.}}{d\alpha_i}\end{bmatrix}\begin{pmatrix}\varepsilon_1\\ \varepsilon_2\end{pmatrix}\right\}+\frac{1}{2}\left\{(k_1,k_2)\begin{bmatrix}\frac{dD_{11}^{pr.cur.}}{d\alpha_i} & \frac{dD_{12}^{pr.cur.}}{d\alpha_i}\\ \frac{dD_{12}^{pr.cur.}}{d\alpha_i} & \frac{dD_{22}^{pr.cur.}}{d\alpha_i}\end{bmatrix}\begin{pmatrix}k_1\\ k_2\end{pmatrix}\right\}=0 \qquad (25)$$

where *pr.str.*, *pr.cur.* correspond to the principal mid-plane strain axes and to the principal generalized curvature axes.

Analyzing (25), we consider at the beginning the first item there.

Substituting in (25) the relations (21) and ones used earlier in (Selyugin, 2019), we obtain for every *i*-th layer, *i*=1,…,*N*:

$$\left.\begin{cases}\varepsilon_1^2\left[-2U_2\sin 2(\alpha_i-\varphi)-4U_3\sin 4(\alpha_i-\varphi)\right]+\\ +\varepsilon_2^2\left[2U_2\sin 2(\alpha_i-\varphi)-4U_3\sin 4(\alpha_i-\varphi)\right]+2\varepsilon_1\varepsilon_2 4U_3\sin 4(\alpha_i-\varphi)\end{cases}\right\}\frac{1}{2}\left(z_i-z_{i-1}\right) \qquad (26)$$

where φ is the angle between the global *x*-axis and the ε_1 axis; U_2, U_3 are some material constants of the ply (see Gibson, 1994, §2.6).

Analogous the second item gives for every *i*-th layer, *i*=1,…,*N*:

$$\left.\begin{cases}k_1^2\left[-2U_2\sin 2(\alpha_i-\xi)-4U_3\sin 4(\alpha_i-\xi)\right]+\\ +k_2^2\left[2U_2\sin 2(\alpha_i-\xi)-4U_3\sin 4(\alpha_i-\xi)\right]+2k_1k_2 4U_3\sin 4(\alpha_i-\xi)\end{cases}\right\}\frac{1}{6}\left(z_i^3-z_{i-1}^3\right) \qquad (27)$$

where ξ is the angle between the global *x*-axis and the k_1 axis.

The sum of (26) and (27), equated to zero, is the first order necessary optimality condition for every layer, *i*=1,...,*N*

$$\begin{aligned}&\left.\begin{cases}\varepsilon_1^2\left[-2U_2\sin 2(\alpha_i-\varphi)-4U_3\sin 4(\alpha_i-\varphi)\right]+\\ +\varepsilon_2^2\left[2U_2\sin 2(\theta_i-\varphi)-4U_3\sin 4(\alpha_i-\varphi)\right]+2\varepsilon_1\varepsilon_2 4U_3\sin 4(\alpha_i-\varphi)\end{cases}\right\}\frac{1}{2}\left(z_i-z_{i-1}\right)+\\ &+\left.\begin{cases}k_1^2\left[-2U_2\sin 2(\alpha_i-\xi)-4U_3\sin 4(\alpha_i-\xi)\right]+\\ +k_2^2\left[2U_2\sin 2(\alpha_i-\xi)-4U_3\sin 4(\alpha_i-\xi)\right]+2k_1k_2 4U_3\sin 4(\alpha_i-\xi)\end{cases}\right\}\frac{1}{6}\left(z_i^3-z_{i-1}^3\right)=0\end{aligned} \qquad (28)$$

As we see, the conditions for every layer are highly nonlinear ones. Transforming (28), we obtain:

$$\begin{aligned}&2\sin 2(\alpha_i-\varphi)\left.\begin{cases}\varepsilon_1^2\left[-U_2-4U_3\cos 2(\alpha_i-\varphi)\right]+\\ +\varepsilon_2^2\left[U_2-4U_3\cos 2(\alpha_i-\varphi)\right]+8\varepsilon_1\varepsilon_2 U_3\cos 2(\alpha_i-\varphi)\end{cases}\right\}\frac{1}{2}\left(z_i-z_{i-1}\right)+\\ &+2\sin 2(\alpha_i-\xi)\left.\begin{cases}k_1^2\left[-U_2-4U_3\cos 2(\alpha_i-\xi)\right]+\\ +k_2^2\left[U_2-4U_3\cos 2(\alpha_i-\xi)\right]+8k_1k_2U_3\cos 2(\alpha_i-\xi)\end{cases}\right\}\frac{1}{6}\left(z_i^3-z_{i-1}^3\right)=0\end{aligned} \qquad (29)$$

and, finally, dividing by $-\left(z_i-z_{i-1}\right)4U_3$:

$$\begin{aligned}&\sin 2(\alpha_i-\varphi)\left[\frac{U_2}{4U_3}\left(\varepsilon_1^2-\varepsilon_2^2\right)+\left(\varepsilon_1-\varepsilon_2\right)^2\cos 2(\alpha_i-\varphi)\right]+\\ &+\sin 2(\alpha_i-\xi)\left[\frac{U_2}{4U_3}\left(k_1^2-k_2^2\right)+\left(k_1-k_2\right)^2\cos 2(\alpha_i-\xi)\right]\frac{1}{3}\left(z_i^2+z_iz_{i-1}+z_{i-1}^2\right)=0\end{aligned} \qquad (30)$$

As we see, the dependence from material comes through the parameter $\frac{U_2}{4U_3}$. It is interesting to note that all multipliers of the trigonometric functions in (30) are non-negative. The explicit dependence on the layer *z* location comes through the last multiplier of the second item. The coupling between the layers occurs due to the principal 2D-strain lines, to the principal generalized curvature lines, to the values of the principal 2D-strains and to the principal generalized curvatures. Further we discuss the ply locations with dominating one or both items of the optimality conditions.

Analyzing (30), one may say also that the innermost layers and the outermost layers play different roles there. In particular, it follows from the first item in (30) that the innermost layers are most sensitive to the principal 2D-strains and their principal directions. As to the outermost layers, they are most sensitive (due to z^2 multiplier of the second item) to the principal plate generalized curvatures and their principal directions.

If we make a sum of (29) for all k and use the definition of A and D of $\overline{Q}_{ij}$ (Gibson, 1994, §2.6), we obtain analogous to (Selyugin, 2019) the following united condition

$$\left[A_{16}^{pr.str.}\varepsilon_1^2 - A_{26}^{pr.str.}\varepsilon_2^2 - \left(A_{16}^{pr.str.} - A_{26}^{pr.str.}\right)\varepsilon_1\varepsilon_2\right] + \left[D_{16}^{pr.cur.}k_1^2 - D_{26}^{pr.cur.}k_2^2 - \left(D_{16}^{pr.cur.} - D_{26}^{pr.cur.}\right)k_1k_2\right] = 0 \tag{31}$$

or, after using the definitions of the stiffness matrices A and D, the relation with the clear physical meaning

$$N_{xy}^{pr.str.}(\varepsilon_1 - \varepsilon_2) + M_{xy}^{pr.cur.}(k_1 - k_2) = 0 \tag{32}$$

The relation (32) is a linear combination of the optimality conditions (29) or (30) and, hence, also is the optimality condition uniting the ones for all layer. The first item in (32), if considered separately as equal to zero, means that the nonlinear mid-plane strain tensor is co-axial with the force tensor (or, for $\varepsilon_1 \neq \varepsilon_2$, the shear flow in the principal strain axes is equal to zero). The second item in (32), if considered separately as equal to zero, means that the tensor of the generalized curvatures is co-axial with the moment tensor (or, for $k_1 \neq k_2$, the twisting moment in the principal generalized curvature axes is equal to zero).

It should be noted that the result, in the case of an orthotropic plate with a point-wise orthotropic orientation, follows from (32) with the number of layers N=1.

A generalization of (30) to the plate-wise ply angle variation case is written as follows, $i=1,\ldots,N$:

$$\int_\Gamma d\Gamma \begin{bmatrix} \sin 2(\alpha_i - \varphi)\left[\dfrac{U_2}{4U_3}\left(\varepsilon_1^2 - \varepsilon_2^2\right) + \left(\varepsilon_1 - \varepsilon_2\right)^2 \cos 2(\alpha_i - \varphi)\right] + \\ + \sin 2(\alpha_i - \xi)\left[\dfrac{U_2}{4U_3}\left(k_1^2 - k_2^2\right) + \left(k_1 - k_2\right)^2 \cos 2(\alpha_i - \xi)\right]\dfrac{1}{3}\left(z_i^2 + z_i z_{i-1} + z_{i-1}^2\right) \end{bmatrix} = 0 \tag{33}$$

A generalization of (32) for the case is

$$\int_\Gamma d\Gamma\left[N_{xy}^{pr.str.}(\varepsilon_1 - \varepsilon_2) + M_{xy}^{pr.cur.}(k_1 - k_2)\right] = 0 \tag{34}$$

3. A SOLUTION

Analysis of the optimality conditions (30) allows identifying a solution of them. Namely, when

$$\alpha_i = (\varphi \text{ or } \xi),\ i=1,\ldots,N \tag{35}$$

and

$$\varphi = \xi \tag{36}$$

or

$$\varphi = \xi + \pi/2 \tag{37}$$

then (30) is satisfied. In this case the sines in front of the square brackets in (30) are equal to zero.

The optimal lay-up identified by (35)-(37) is a locally orthotropic one. The orthotropy axes of the solution coincide the principal 2D-strain axes and the principal generalized curvature axes.

4. CONCLUSIONS

- The first order necessary local optimality conditions for the considered postbuckling stiffness maximization problem with large deflections are highly nonlinear ones;
- The optimality conditions contain two terms. One of them corresponds to the plate bending-twisting and is mainly governed by the outermost layers; another one corresponds to the mid-plane strains and is mainly governed by the innermost layers; the layers in the middle are governed by both terms;
- The principal mid-plane strain lines and the principal generalized curvature lines play an important role in the conditions;
- A locally orthotropic lay-up is identified as an optimal solution;
- The united optimality condition, being a linear combination of the conditions for the plies, contains two terms. The first term corresponds to the bending-twisting and equals zero when the tensor of the generalized curvatures is co-axial with the moment tensor. The second term corresponds to the mid-plane strains and equals zero when the mid-plane strain tensor is co-axial with the force tensor;
- The united optimality condition has a particular solution when the both pairs of the tensors (strain - forces and generalized curvature - moments) are co-axial.

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