DECODING SATELLITE IMAGES OF EARTHQUAKES, PART-IV: UNDERSTANDING CONVOLUTION

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This document is part IV of a series of documents providing a lowdown on image processing in the context of understanding satellite images of earthquakes.

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FIG. 1: Annualized Earthquake Losses (AEL: estimated long-term value of earthquake losses to the general building stock) by County. Sources: (1) <https://www.usgs.gov/news/usgs-collaborates-fema-national-earthquake-loss-estimate> and (2) <https://www.fema.gov/hazus>. The analysis yielded an estimate of the national AEL of \$6.1 billion per year. Loss estimates based on the best science and engineering available (during 2016-2017). The study only addresses direct economic losses to buildings, and does not cover damage and losses to critical facilities, transportation and utility lifelines or indirect economic losses.

The earthquake cycle, from slow deformation associated with interseismic behavior to rapid deformation associated with earthquake rupture, spans spatial scales ranging from fractions of a meter associated with the size of contact asperities on faults and individual grains to hundreds of kilometers associated with plate boundaries (Kanamori and Brodsky (2004)). Similarly, temporal scales range from fractions of a second associated with slip at a point during earthquake rupture to hundreds of years of strain accumulation between earthquakes. In many cases, earthquakes are triggered after pore pressure perturbations activate critically stressed seismogenic faults (Kanamori and Brodsky (2004)), not just due to natural causes like earth tides (Scholz et al. (2019)), rainfall (Hainzl et al. (2006)), snow-fall (Montgomery-Brown et al. (2019)), typhoons (Liu et al. (2009)), but also due to human activity (Foulger et al. (2018)). Ground motion and failure due to *earthquakes lead to huge losses to the tune of billions of dollars annually in the United States* (see Fig. 1). Understanding the causality between the events leading to fault slip, the location of

the slipping fault and the earthquake recording is important for seismic design and monitoring of underground structures (Hashash et al. (2001)), bridges (Priestley et al. (1996)) and reinforced concrete buildings (Moehle (2015)) as well as climate mitigation projects like carbon sequestration (Zoback and Gorelick (2012)) and energy technologies like enhanced geothermal systems (Ellsworth et al. (2019)) or oilfield wastewater disposal (Keranen and Weingarten (2018)). A holistic framework to study earthquakes would incorporate seismic wave propagation and subsurface pressure perturbations, and have a dialogue with the satellite image processing-based inverse modeling framework. The first step is working quasi-static simulations (Dana (2018, 2019a,b); Dana et al. (2018, 2020a); Dana and Jha (2021); Dana and Lythakula (2021); Dana et al. (2020b); Dana and Wheeler (2018a,b,c, 2019, 2020a,b); Dana et al. (2021)) in which wave propagation is ignored and the solution is a time series of solutions to static problems with potentially time-varying physical properties and boundary conditions. The next step is to gradually leverage this subject matter expertise to render fully dynamic simulations. The next step is to arrive at deployable satellite image techniques as an inverse modeling framework to work with the aforementioned forward modeling framework. One of the critical aspects in image processing is "convolution".

0.1 Discrete Fourier transform

The basic idea behind the Fourier transform method is that an image can be thought of as a 2D function, f , that can be expressed as a weighted sum of sines and cosines (Fourier basic functions) along two dimensions. We can transition from a set of grayscale pixel values in the image (spatial/time domain) to a set of Fourier coefficients (frequency domain) using the Discrete Fourier Transform (DFT), and it is discrete since the spatial and the transform variables to be used can only take a set of discrete consecutive integer values (typically the locations of a 2D array representing the image). In a similar way, the frequency domain 2D array of Fourier coefficients can be converted back into the spatial domain using the Inverse Discrete Fourier Transform (IDFT), which is also known as reconstruction of the image using the Fourier coefficients. The low frequencies in the frequency domain correspond to the average gross level of information in an image, whereas the higher frequencies correspond to edges, noise, and more detailed information. Typically, images by nature are smooth and that is why most images can be represented using a handful of DFT coefficients and all of the remaining higher coefficients tend to be almost negligible/zeros. This is very useful in image compression, particularly for Fourier-sparse images, where only a handful of Fourier coefficients are required to reconstruct an image, hence only those frequencies can be stored and others can be discarded, leading to high compression (for example, in the JPEG image compression algorithm a similar transformation, Discrete Cosine Transform (DCT) is used. The Fast Fourier Transform (FFT) is a divide and conquer algorithm to recursively compute the DFT much quicker (with $O(N \log_2 N)$ time complexity) than the much slower $O(N^2)$ naive computation for an $n \times n$ image.

1. CONVOLUTION

FIG. 2: A 1D convolution with a kernel sized 3 and stride 1

Convolution is an operation that operates on two images, one being an input image and the other one being a mask (also called the kernel) as a filter on the input image, producing an output image. Convolution filtering is used to modify the spatial frequency characteristics of an image. It works by determining the value of a central pixel

Convolution theorem

$$f(x, y) * h(x, y) \Leftrightarrow F(u, v)H(u, v)$$

Space convolution = frequency multiplication

FIG. 3: Convolution theorem says that convolution in an image domain is equivalent to a simple multiplication in the frequency domain. Photo source: Dey (2018)

FIG. 4: We have the original image, F , and a kernel (a mask or a degradation/enhancement function) as input. First, both input items need to be converted into the frequency domain with DFT, and then the convolution needs to be applied, which by convolution theorem is just an (element-wise) multiplication. This outputs the convolved image in the frequency domain, on which we need to apply IDFT to obtain the reconstructed image (with some degradation or enhancement on the original image). Photo source: Dey (2018)

by adding the weighted values of all of its neighbors together to compute the new value of the pixel in the output image. The pixel values in the output image are computed by traversing the kernel window through the input image. Convolution applies a general-purpose filter effect on the input image. This is done in order to achieve various effects with appropriate kernels on an image, such as smoothing, sharpening, and embossing, and in operations such as edge detection.

1.1 Filtering

Filtering refers to transforming pixel intensity values to reveal certain image characteristics such as:

- ✓ Enhancement: This image characteristic improves contrast
- ✓ Smoothing: This image characteristic removes noise
- ✓ Template matching: This image characteristic detects known patterns

The filtered image is described by a discrete convolution and the filter is described by a $n \times n$ discrete convolution mask. "High-Pass Filter (HPF)" allows only high frequencies from the frequency domain representation of the image (obtained with DFT) and blocks all low frequencies beyond a cut-off value. The image is reconstructed with inverse

Convolution in the Frequency Domain

FIG. 5: The framework in a nutshell. Photo source: Dey (2018)

FIG. 6: Sine signal with additive noise after processing with a low-pass filter and a high-pass filter. Photo source: Maier et al. (2018)

DFT, and since the high-frequency components correspond to edges, details, noise, and so on, HPFs tend to extract or enhance them. "Low-pass filter (LPF)" allows only the low frequencies from the frequency domain representation of the image (obtained with DFT), and blocks all high frequencies beyond a cut-off value. The image is reconstructed with inverse DFT, and since the high-frequency components correspond to edges, details, noise, and so on, LPF tends to remove these.

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