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# GMPE-Consistent Hard-Rock Site Adjustment Factors for Western North America

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2 Linda Al Atik,<sup>1</sup> M.EERI, Nicholas J. Gregor,<sup>2</sup> M.EERI, Norman A.  
3 Abrahamson,<sup>3</sup> Albert R. Kottke,<sup>4</sup> M.EERI,

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## Abstract

Empirical ground-motion prediction equations [GMPEs] such as the Next Generation Attenuation-West2 [NGA-West2] GMPEs are limited in the number of recordings on hard-rock stations used to develop the models. Therefore, the site response scaling in the GMPEs cannot be reliably extrapolated to hard-rock conditions. The state of practice for the development of hard-rock adjustment factors involves the use of analytical methods that typically assign small values to the high-frequency small strain damping [ $\kappa_0$ ] for hard-rock sites resulting in large scaling factors at short periods. Alternatively, the hard-rock scaling factors developed in [Ktenidou and Abrahamson \(2016\)](#) [KA16] based on empirical ground-motion data are used. These empirical factors, developed for a broad rock site category, show that the average hard-rock scaling factors observed in ground-motion data are small in amplitude.

To address the shortcomings in the current state of practice, we present a methodology to develop hard-rock linear site adjustment factors to adjust the NGA-West2 GMPEs from  $V_{S30}$  of 760 m/sec to target hard-rock site conditions with  $V_{S30}$  ranging from 1000 to 2200 m/sec. These factors are analytically derived using the IRVT approach of [Al Atik et al. \(2014\)](#) but with inputs constrained using the empirical KA16 factors and normalized to the scaling of the NGA-West2 GMPEs for  $V_{S30}$  of 1000 m/sec. The proposed factors merge the results of the NGA-West2 site response scaling for  $V_{S30} \leq 1000$  m/sec with the KA16 hard-rock category factors to produce a site factor model that is a continuous function of  $V_{S30}$ . The epistemic uncertainty of these factors is evaluated.

## Keywords

Site response, site adjustment, site amplification,  $\kappa_0$ , hard rock, GMPE consistent, NGA-West2

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<sup>1</sup>Linda Alatik Consulting, San Francisco, CA

<sup>2</sup>Consultant, Oakland, CA

<sup>3</sup>University of California, Berkeley, CA

<sup>4</sup>Pacific Gas & Electric, San Francisco, CA

## Corresponding author:

Linda Al Atik, San Francisco, CA

Email: linda.alatik@gmail.com

## 6 Introduction

7 Modern ground-motion prediction equations [GMPEs] such as the Next Generation  
8 Attenuation-West2 [NGA-West2] GMPEs characterize site response as a continuous  
9 function of the time-averaged shear-wave velocity over the top 30 m of the site profile  
10 [ $V_{S30}$ ]. Other parameters, such as the depth to a shear-wave velocity of 1.0 or 2.5 km/s  
11 horizon [ $Z_{1.0}$  and  $Z_{2.5}$ ], are used to characterize the long-period site amplification due  
12 to basin effects. Nonlinear site response is modeled in the GMPEs as a function of  
13  $V_{S30}$  and the median spectral acceleration or peak ground acceleration on rock. The  
14 histogram of the number of recording stations in the different  $V_{S30}$  bins used in the  
15 development of the [Abrahamson et al. \(2014\)](#) GMPE [ASK14] is shown in Figure  
16 [1](#). This figure shows that the number of recording stations with  $V_{S30} > 1000$  m/sec  
17 is limited. The  $V_{S30}$  dependence of the site factors is modeled in the GMPEs as a  
18 linear function of  $\ln(V_{S30})$ . With the sampling of  $V_{S30}$  in the NGA-West2 dataset,  
19 the coefficient for  $\ln(V_{S30})$  is constrained by empirical ground-motion data for  $V_{S30}$   
20 values between 200 and 800 m/s. For hard-rock sites with  $V_{S30} > 1000$  m/sec, the site  
21 factor is based on an extrapolation of this slope to high  $V_{S30}$  values with little empirical  
22 constraints. To reflect the limited hard-rock data, some GMPEs limit the reduction of  
23 the site factor at high  $V_{S30}$  values (e.g. 1500 m/s).

24 Empirical hard-rock adjustment factors were developed in [Ktenidou and](#)  
25 [Abrahamson \(2016\)](#) [KA16] to adjust GMPEs from  $V_{S30}$  of 760 m/sec to average  
26 hard-rock conditions based on the Next Generation Attenuation-East Project [NGA-  
27 East] and the BCHydro British Columbia ground-motion datasets. These factors were  
28 developed using the average of total ground-motion residuals with  $V_{S30}$  greater than  
29 1000 m/sec relative to median predictions from an interim NGA-East GMPE and the  
30 [Chiu and Youngs \(2014\)](#) [CY14] GMPE with  $V_{S30}$  of 760 m/sec for the NGA-East  
31 and the BCHydro datasets, respectively. These scaling factors account for differences in  
32 the  $V_S$  profiles and the high-frequency small-strain damping [ $\kappa_0$ ] between the reference  
33 site condition in the GMPEs with  $V_{S30}$  of 760 m/sec and an average hard-rock site  
34 in the NGA-East and the BCHydro datasets. Figure [2](#) presents the two hard-rock  
35 scaling models proposed by KA16: model 2 is based on the residual analysis of the  
36 BCHydro dataset and model 1 is based on an average of the scaling obtained for  
37 the combined BCHydro and the NGA-East datasets. KA16 indicate that the average  
38  $V_{S30}$  for the BCHydro and the NGA-East data used to develop the models is 2380 and  
39 1975 m/sec, respectively. The KA16 models shown in Figure [2](#) were developed as an  
40 interim measure to provide an alternative to the typically large scale factors computed

41 using analytical methods for hard-rock sites. These models show that observed ground-  
42 motion data on hard-rock sites, on average, do not show the large scale factors at short  
43 periods typically found in analytical studies that assign small  $\kappa_0$  values, on the order  
44 of 0.006 sec, for hard-rock sites.

45 The KA16 hard-rock adjustment factors suffer from shortcomings stemming from  
46 the poor characterization of site conditions at the recordings stations in the NGA-  
47 East and BCHydro datasets and the low sample rates at some seismic stations that  
48 limit the useable frequency band. Most of the hard-rock stations in the NGA-East  
49 dataset do not have measured  $V_{S30}$  values, and the NGA-East ground motions suffer  
50 from frequency-bandwidth limitations affecting data quality at the low and the high  
51 frequencies. Similarly, all rock stations in the BCHydro dataset are classified based on  
52 surface geology (Ktenidou and Abrahamson 2016). Errors in the  $V_{S30}$  estimates for the  
53 hard-rock sites could lead to misclassification of the sites and affect the resulting scale  
54 factors and average  $V_{S30}$  values. Another limitation of the KA16 factors is that they  
55 are not a continuous function of  $V_{S30}$  and instead apply to a broad hard-rock site class  
56 with  $V_{S30} > 1000$  m/sec. This is a result of the limited recordings on hard-rock sites in  
57 the empirical datasets.

58 The current state of practice for the development of site adjustment factors for hard-  
59 rock site conditions involves three different approaches: (1) use of  $V_S$ - $\kappa_0$  correction  
60 factors developed using analytical methods with assigned target  $\kappa_0$  values and  $V_S$   
61 profiles; (2) use of empirical hard-rock factors such as the KA16 factors which are  
62 developed for a broad category of hard-rock sites; and (3) use of the  $V_{S30}$  scaling in  
63 the NGA-West2 GMPEs extrapolated to hard-rock  $V_{S30}$  despite the limited empirical  
64 data on hard-rock site conditions available to constrain the  $V_{30}$  scaling and the lack  
65 of explicit  $\kappa_0$  scaling in the GMPEs. To address the shortcomings in the current state  
66 of practice, we present a methodology to develop hard-rock site factors to adjust the  
67 NGA-West2 GMPEs from  $V_{S30}$  of 760 m/sec to target hard-rock site conditions with  
68  $V_{S30}$  ranging from 1000 to 2200 m/sec. These factors are analytically derived using  
69 the IRVT approach of Al Atik et al. (2014) but constrained using the empirical KA16  
70 factors and also normalized to the NGA-West2 site factors for  $V_{S30}$  of 1000 m/sec.  
71 These empirical constraints allow for the calibration of the hard-rock properties in the  
72 analytical study so that they are consistent with the observed ground-motion scaling for  
73 these sites. The proposed factors merge the results of the NGA-West2  $V_{S30}$  scaling for  
74  $V_{S30} \leq 1000$  m/sec with the KA16 hard-rock category factors to produce a model for  
75 the site factors that is a continuous function of  $V_{S30}$  and consistent with the empirical  
76 hard-rock factors.

77 Our approach starts with an evaluation of the average hard-rock site conditions  
78 representative of the KA16 scaling factors, by inverting for the average  $V_{S30}$ ,  $V_S$   
79 profile, and  $\kappa_0$  implicit in the KA16 models. Next, the target site conditions ( $V_S(z)$   
80 profile and  $\kappa_0$ ) are defined for a range of hard-rock site conditions with  $V_{S30}$  of 1000  
81 to 2200 m/sec based on a literature review of hard-rock  $V_S$  profiles and  $\kappa_0$  estimates  
82 in Western North America [WNA]. Target site parameters are adjusted using empirical  
83 constraints, and hard-rock site adjustment factors are derived and presented for a suite  
84 of thirteen target  $V_{S30}$  values between 1000 and 2200 m/sec.

85 The site factors presented in this paper extend the NGA-West2 linear site response  
86 scaling to hard-rock site conditions with target hard-rock sites defined based on WNA  
87 average rock site properties. Therefore, our derived hard-rock site adjustment factors  
88 are applicable for target sites in WNA. We use the KA16 models derived from empirical  
89 NGA-East and BChydro data as constraints because of the scarcity of empirical  
90 ground-motion data on hard rock sites in WNA. Moreover, KA16 showed that average  
91 hard-rock scaling outside of WNA is not drastically different than what would be  
92 expected for WNA sites. We note that the hard-rock factors presented in this paper are  
93 intended for use at sites with limited site characterization, such as sites with measured  
94 or estimated  $V_{S30}$  values, but without measured  $V_S(z)$  profiles. For hard-rock sites  
95 with measured  $V_S(z)$  profiles, site-specific hard-rock adjustment factors should be  
96 computed based on the site profile with its appropriate epistemic uncertainty.

## 97 **Vs Profile and $\kappa_0$ Inversion for the KA16 Models**

98 The use of the KA16 models to constrain the analytical hard-rock site adjustment  
99 factors requires an evaluation of the implied average site conditions representative of  
100 these scaling factors. Because a large number of the stations used in the KA16 analysis  
101 did not have measured  $V_{S30}$  and  $\kappa_0$  values, we use the  $\kappa_0$  and  $V_S$  profile inversion  
102 methodology of Al Atik and Abrahamson (2021) to invert for representative  $V_{S30}$ ,  
103  $V_S(z)$  profile, and  $\kappa_0$  for the average site conditions implied by the KA16 hard-rock  
104 scaling factors.

105 The inversion is performed using the CY14 GMPE because KA16 model 2 is based  
106 on the residual analysis of the BChydro dataset with respect to CY14 and because  
107 the spectral shape of the CY14 GMPE generally falls in the center of the range of  
108 spectral shapes from the NGA-West2 GMPEs. The first step involves converting the  
109 KA16 hard-rock scaling factors from pseudo-spectral acceleration [PSA] domain to  
110 Fourier amplitude spectra [FAS] domain. As such, CY14 median response spectra for  
111 strike-slip scenarios with magnitude 5, 6, and 7, distance of 5, 10, and 20 km and

112  $V_{S30}$  of 760 m/sec are computed. These scenarios are selected to capture the short-  
113 distance  $\kappa_0$  scaling from a range of hazard-significant magnitudes. The CY14 median  
114 response spectra are corrected to hard-rock conditions by multiplying them with the  
115 KA16 model 1 and model 2 factors. Next, the IRVT approach of [Al Atik et al. \(2014\)](#)  
116 is used to convert the GMPE's response spectra for  $V_{S30} = 760$  m/sec and the spectra  
117 corrected to hard-rock conditions into corresponding FAS. Duration estimates for the  
118 different scenarios are calculated using estimates of source and path durations with  
119 generic Western US [WUS] parameters based on [Campbell \(2003\)](#). The peak factor of  
120 [Vanmarke \(1975\)](#) is used in the IRVT method. The FAS for the scenarios with  $V_{S30} =$   
121 760 m/sec and those corrected to hard-rock conditions are presented in Figure 3. These  
122 IRVT-based FAS show a change in their spectral shape at frequency of about 50 Hz.  
123 This is likely due to saturation effects in the IRVT process discussed in [Al Atik et al.](#)  
124 [\(2014\)](#). For the hard-rock FAS, the sharp change observed around 50 Hz is caused by  
125 the sharp changes in the KA16 factors in the same frequency range, particularly for  
126 KA16 model 2.

127 For each earthquake scenario, the ratio of FAS for the hard-rock site condition  
128 relative to FAS for  $V_{S30} = 760$  m/sec is computed. Figure 4 presents these ratios for  
129 each of the nine scenarios considered for model 1 and model 2 along with the average of  
130 the ratios over the nine scenarios. These ratios approximate the FAS linear site factors  
131 for hard rock relative to the reference site condition with  $V_{S30}$  of 760 m/sec. These  
132 relative site factors represent the differences in the  $V_S$  profile and  $\kappa_0$  scaling between  
133 the average hard-rock site implied by the KA16 models and the average site condition  
134 for CY14 for  $V_{S30}$  of 760 m/sec. The FAS ratios are stable over all nine scenarios for  
135 frequencies up to about 20-30 Hz as shown in Figure 4. For frequencies greater than  
136 20 Hz, the FAS ratios start diverging due to potential saturation effects in the IRVT-  
137 derived FAS discussed in [Al Atik et al. \(2014\)](#). The average relative site factors are  
138 smoothed as shown in Figure 5. We note that these average relative FAS site factors are  
139 considered reliable for frequencies between 0.6 than 30 Hz. The upper limit is imposed  
140 to avoid potential saturation effects in the IRVT-based FAS and the lower limit is based  
141 on the KA16 factors being constrained by data for frequencies up to 0.6 Hz.

142 The next step involves converting the hard-rock site factors relative to 760 m/sec  
143 to total site factors (relative to the  $V_S$  and density at the source depth). As such, we  
144 use the CY14-compatible  $V_S$  profile and  $\kappa_0$  of [Al Atik and Abrahamson \(2021\)](#) as  
145 representative of the reference site condition with  $V_{S30}$  of 760 m/sec. The reference  
146  $V_S$  profile and corresponding quarter-wavelength [QWL] linear site amplification for  
147 CY14 for  $V_{S30}$  of 760 m/sec are shown in Figure 10. The QWL site amplification is  
148 computed according to [Boore \(2003\)](#) with a zero angle of incidence and source  $V_S$  and

149 density set at 3.5 km/sec and 2.75 g/cm<sup>3</sup>, respectively. The  $\kappa_0$  for CY14 for  $V_{S30}$  of 760  
150 m/sec is 0.039 sec. The total FAS site factors for the average hard-rock site condition  
151 representative of the KA16 models 1 and 2 are obtained by multiplying the relative  
152 hard-rock site factors with the site factors of the CY14 reference site condition and are  
153 shown in Figure 6.

### 154 *Inversion of KA16 Model 1*

155 The total linear site factors represent the combined effects of the linear site  
156 amplification of the  $V_S$  profile and the attenuation due to damping, parameterized by  
157  $\kappa_0$ . To reduce the trade-off between the  $V_S$  profile and  $\kappa_0$  at high frequencies, we  
158 assume that the depth dependence of the  $V_S$  profile follows a power law (e.g.,  $a \cdot z^b$ ).  
159 With this assumption, we have use an analytical solution for the combined effects of  
160 the site amplification of the  $V_S$  profile in the top 30 m and the  $\kappa_0$  attenuation given the  
161  $V_{S30}$  value. The methodology is described in Al Atik and Abrahamson (2021).

162 Using the total linear site factors for KA16 model 1 shown in Figure 6, the inversion  
163 is performed to estimate the average  $\kappa_0$  and  $V_S$  profile representative of the average  
164 hard-rock site condition in the model. A zero angle of incidence and a source  $V_S$   
165 and density of 3.5 km/sec and 2.75 g/cm<sup>3</sup>, respectively, are used in the inversion.  
166 The density- $V_S$  relationship used in Al Atik and Abrahamson (2021) is used in this  
167 inversion. Because  $V_{S30}$  is unknown for the KA16 models, the inversion is performed  
168 to estimate  $V_{S30}$  as well as for different assumed  $V_{S30}$  values. Using the frequency  
169 range of 10 to 20 Hz (10 Hz roughly corresponds to the frequency associated with  
170 QWL amplification for the top 30 m of the profile and 20 Hz was chosen to avoid the  
171 unreliable higher frequencies in the IRVT-based FAS),  $\kappa_0$ ,  $V_{S30}$  and the  $V_S$  profile  
172 in the top 30 m are estimated analytically by fitting the site response function in  
173 the 10-20 Hz frequency range assuming that the top 30 m of the  $V_S$  profile follows  
174 a power law function. The estimated  $\kappa_0$  and  $V_{S30}$  are 0.032 sec and 1300 m/sec,  
175 respectively. Figure 7(a) shows the high-frequency fit compared to the total site factors  
176 for frequencies > 10 Hz. For frequencies < 10 Hz, the fit uses the initial site factors as  
177 shown in the pink curve.

178 The site factors modified for frequencies greater than 10 Hz to follow the high-  
179 frequency fit (pink curve in Figure 7(a)) are divided by the  $\kappa_0$  operator to obtain the  
180 linear site amplification function due only to the  $V_S$  profile which is subsequently  
181 smoothed as shown in Figure 7(b). The inverse QWL approach outlined in Al Atik  
182 and Abrahamson (2021) is then applied to invert for the  $V_S$  profile working from high  
183 to low frequencies of the site amplification and solving for the shallow to deep layers

184 of the profile. The inverted  $V_S$  profile, which is subsequently smoothed, is shown by  
185 the pink curve in Figure 7(c). Because linear site amplifications are considered reliable  
186 for frequencies  $> 0.6$  Hz, the  $V_S$  profile could only be inverted to a depth of 1.06 km.  
187 A comparison of the initial relative site factors of KA16 model 1 to those obtained  
188 using the inversion results is shown in Figure 7(d). This plot shows that the inverted  $V_S$   
189 profile and  $\kappa_0$  representative of the hard-rock condition for KA16 model 1 used along  
190 with the reference  $V_S$  profile and  $\kappa_0$  for CY14 at  $V_{S30}$  of 760 m/sec can approximate  
191 reasonably well the initial relative site factors of KA16 model 1 for frequencies up to  
192 30 Hz.

193 Next, the inversion of KA16 model 1 described in this section is repeated using  
194 different assumed  $V_{S30}$  values instead of inverting for  $V_{S30}$  as shown above. This  
195 sensitivity analysis allows for a more robust estimation of  $\kappa_0$  from the high-frequency  
196 site factors as well as an evaluation of the range of  $V_{S30}$  and  $\kappa_0$  values that can fit  
197 the hard-rock site factors of KA16 model 1 relative to the reference site condition  
198 with  $V_{S30}$  of 760 m/sec. Assumed  $V_{S30}$  values of 1500, 1700, and 1975 m/sec are  
199 used in this sensitivity analysis. The value of 1975 m/sec is used because it represents  
200 the average  $V_{S30}$  of the NGA-East hard-rock data used in KA16. Figure 8 shows a  
201 comparison of the initial relative site factors of KA16 model 1 to those obtained using  
202 the inversion for the derived and assumed  $V_{S30}$  values. The inversion results for the  
203 different assumed  $V_{S30}$  values indicate that, as the assumed  $V_{S30}$  increases, the derived  
204  $\kappa_0$  value decreases and the slope of the inverted  $V_S$  profile in the top 30 m becomes less  
205 steep approaching a single constant layer. The sum-of-squared errors (SSE) between  
206 the inversion-based relative site factors and the initial site factors in the frequency range  
207 of 0.6 to 30 Hz are calculated and listed in the plots of Figure 8. An evaluation of the  
208 SSE values for the different inversion cases as well as the corresponding shapes of the  
209 inverted  $V_S$  profiles indicates that the assumed  $V_{S30}$  of 1975 m/sec does not represent  
210 the average hard-rock site conditions of KA16 model 1. The average  $V_{S30}$  of 1975  
211 m/sec obtained using the NGA-East hard-rock data in KA16 is likely biased high due  
212 to the large number of stations with estimated or assigned  $V_{S30}$  values. As a result, we  
213 conclude that, within the context of the QWL approach used in these inversions and the  
214 related assumptions made, a  $V_{S30}$  of 1300 m/sec (with a range of 1300 to 1500 m/sec)  
215 and  $\kappa_0$  of 0.032 sec (with a range of 0.03 to 0.032 sec) are representative of the average  
216 site conditions of KA16 hard-rock model 1.



### 217 *Inversion of KA16 Model 2*

218 An inversion approach similar to that described in the previous section is applied to  
219 estimate the average hard-rock site characteristics representative of KA16 model 2. The  
220 first inversion case is performed to estimate  $V_{S30}$  along with  $\kappa_0$  for the total site factors  
221 of KA16 model 2 for the high-frequency range of 12 to 25 Hz shown in Figure 6. The  
222 frequency range of 12 to 25 Hz is chosen to capture the smaller- $\kappa_0$  scaling expected for  
223 this model while staying below the high-frequency limit of 30 Hz. The inversion for  
224 KA16 model 2 results in an average  $V_{S30}$  estimate of 1600 m/sec and  $\kappa_0$  of 0.025 sec.  
225 We note that, for KA16 model 2, the inverted  $V_{S30}$  value is sensitive to the frequency  
226 range used to fit the site factors with the analytical function that assumes that the top  
227 30 m of the  $V_S$  profile can be approximated with a power-law function. Moreover, the  
228 inversion of KA16 model 2 generally required more smoothing than that of model 1  
229 due to the shape of the KA16 model 2 hard-rock factors with bigger jumps in the site  
230 factors in the high-frequency range and less smooth transitions.

231 Next, KA16 model 2 is inverted using different assumed  $V_{S30}$  values of 1500, 1700,  
232 1850, 2000, and 2380 m/sec. The  $V_{S30}$  of 2380 m/sec is reported in KA16 as the  
233 average  $V_{S30}$  of the BCHydro data used to derive model 2 scaling factors. Inverted  $\kappa_0$   
234 values and calculated SSE values for the different inversion cases are listed in Table 1.  
235 Similar to the trends observed for model 1, the inversion results for KA16 model 2  
236 indicate that the inverted  $\kappa_0$  value decreases with increasing  $V_{S30}$  and that KA16 model  
237 2 cannot be well represented with hard-rock conditions with large average  $V_{S30}$  values,  
238 particularly greater than 2000 m/sec. Based on a qualitative evaluation of the inversion  
239 results as well as the SSE values for the different cases, we conclude that the inversion  
240 results for  $V_{S30}$  of 1700 m/sec (range of 1600 to 1850 m/sec) and  $\kappa_0$  of 0.024 sec  
241 (range of 0.022 to 0.025 sec) best represent the average hard-rock site conditions of  
242 KA16 model 2. The best-case inversion results for KA16 model 2 in terms of  $\kappa_0$  fit,  
243 site amplification, and inverted  $V_S$  profile are shown in Figure 9.

### 244 *Discussion of $V_S$ Profile and $\kappa_0$ Inversions of KA16 Models*

245 The inverted  $V_S$  profiles and  $\kappa_0$  values presented in this section are representative of  
246 the average hard-rock site conditions of KA16 models 1 and 2 within the context of  
247 the QWL method used in the inversion and the assumptions employed to solve for  
248 the multiple unknowns in this process. These assumptions are related to the assigned  
249 half-space  $V_S$  and density values, density- $V_S$  relationship, vertical angle of incidence,  
250 smooth  $V_S$  profiles, and the representation of the top 30 m of the  $V_S$  profile with a  
251 power-law function. While these assumptions are reasonable, they do introduce a level

252 of uncertainty in the resulting inverted  $V_S$  profiles and  $\kappa_0$  values. Moreover, due the  
253 frequency limitations of the KA16 hard-rock factors and their jagged appearance, the  
254 inverted  $V_S$  profiles are limited in their depth range.

255 **Boore (2013)** compared site amplifications calculated using the QWL method to  
256 those obtained from theoretical simulations of wave propagation in layered media  
257 accounting for the constructive and destructive interference of all reverberations in the  
258 layers (full resonant [FR] method). For velocity models made up of gradients, **Boore**  
259 **(2013)** found that the QWL method systematically underestimates the theoretical FR  
260 site amplification over a wide frequency range. This underestimation can be on the  
261 order of 20%. Based on that, the QWL-based inversion can potentially underestimate  
262 the derived  $V_S$  profiles compared to those expected from the FR method for the same  
263 site amplification. The use of the QWL method in the inversion is, however, consistent  
264 with the approach used to develop analytical site adjustment factors presented in the  
265 next section. Therefore, we consider the inverted profiles and  $\kappa_0$  values presented in  
266 this section as appropriate values for use with the QWL method to represent the average  
267 hard-rock site conditions of the KA16 factors. We use these inverted profiles and  $\kappa_0$   
268 values to constrain the inputs to the analytical calculations for the hard-rock factors.

## 269 **Development of GMPE-Consistent Analytical Hard-Rock Site** 270 **Adjustment Factors**

271 The inversion of the KA16 empirical hard-rock factors indicates that these factors can  
272 be used to scale response spectra from a reference  $V_{S30}$  of 760 m/sec to target  $V_{S30}$   
273 of about 1300 (model 1) or 1700 m/sec (model 2). To develop rock site adjustment  
274 factors that are a continuous function of  $V_{S30}$  between 1000 and 2200 m/sec, we use  
275 the analytical IRVT method of **Al Atik et al. (2014)** with empirical constrains based on  
276 the KA16 scaling factors for  $V_{S30}$  of 1300 and 1700 m/sec and the NGA-West2 scaling  
277 factors for  $V_{S30}$  of 1000 m/sec. Because the spectral shape for CY14 generally lies  
278 in the center of the range of spectral shapes of the NGA-West2 GMPEs, we develop  
279 the rock scaling factors using the CY14 GMPE and assume the resulting factors are  
280 applicable to the other NGA-West2 GMPEs.

281 The development of analytical site adjustment factors requires the definition of host  
282 and target site conditions in terms of  $V_S$  profiles and  $\kappa_0$  values. For the host site  
283 condition, the  $V_S$  profile and  $\kappa_0$  value of 0.039 sec inverted for CY14 for  $V_{S30}$  of  
284 760 m/sec in **Al Atik and Abrahamson (2021)** are used. Thirteen target site conditions  
285 are defined having  $V_{S30}$  ranging between 1000 and 2200 m/sec. The  $V_S$  profiles for  
286 the target sites are obtained using **Boore (2016)** based on a  $V_{S30}$ -based interpolation

287 between generic WUS and Eastern US profiles with  $V_{S30}$  of 618 and 2780 m/sec,  
288 respectively. Figure 10 presents host CY14  $V_S$  profile along with the target  $V_S$  profiles  
289 and their corresponding QWL site amplifications. Figure 10 shows that there is a  
290 significant difference between the host  $V_S$  profile for CY14 for  $V_{S30}$  of 760 m/sec  
291 and the target profile for  $V_{S30}$  of 1000 m/sec. This difference is due to CY14 having a  
292 relatively high  $V_S$  scaling from 1000 to 760 m/sec resulting in higher site amplification  
293 and softer  $V_S$  profile for  $V_{S30}$  of 760 m/sec compared to the target profile  $V_{S30}$  of 1000  
294 m/sec. These effects are discussed in Al Atik and Abrahamson (2021).

### 295 *Target $\kappa_0$*

296 The estimation of site-specific  $\kappa_0$  is a complex process that often involves a large  
297 degree of uncertainty and trade-offs. The origins of  $\kappa_0$  and the relationship between the  
298 observed high-frequency attenuation in FAS ( $\kappa_0$  scaling) and the low-strain damping at  
299 a site are subject of ongoing debate . The current paradigm assumes that  $\kappa_0$ , estimated  
300 with the source, path, and site effects removed, is due only to damping at the site  
301 (EPRI). As a result, a low  $\kappa_0$  implies low damping that must lead to an increase in  
302 the high-frequency ground motion. Hard-rock to soft-rock site factors of 2-3 at the  
303 frequency range of 20-40 Hz are common (Biro and Renault 2012). When the current  
304 paradigm was established in the 1990s, there were only a four hard-rock recordings  
305 with low  $\kappa_0$  values and they were consistent with the large factors of 2-3 amplification  
306 for hard-rock sites relative to soft-rock sites. The current data sets for hard-rock sites  
307 are much larger with over 100 recordings, and they do not show the large site factors at  
308 high frequencies that are predicted for hard-rock sites with  $\kappa_0$  in the 0.006-sec range.  
309 This indicates that estimated  $\kappa_0$  values are not just the result of damping; they also  
310 reflect the errors in the assumed source, path, and site effects on the slope of the FAS  
311 used to estimate  $\kappa_0$ . The negative values of  $\kappa_0$  estimated for some sites also indicate  
312 that there is more than just damping controlling the  $\kappa_0$  values (Ktenidou et al. 2021).

313 To avoid the common tendencies for underestimating  $\kappa_0$ , We use target  $\kappa_0$  values  
314 that are consistent with the observed ground-motion scaling at high-frequencies for  
315 rock site conditions. By using the amplitude of the ground motion and not just the  
316 high-frequency slope of the FAS, the  $\kappa_0$  values can be interpreted as effects of damping  
317 and used in the traditional  $\kappa_0$  scaling methodology. We note that our resulting target  $\kappa_0$   
318 values are not site-specific; they are average values that can be expected for hard-rock  
319 sites with different  $V_{S30}$  values. We also account for the uncertainty in the average  $\kappa_0$   
320 value for a rock site condition as described below.

321 For this study, target  $\kappa_0$  values are estimated based on a review of [Silva and Darragh](#)  
 322 (1995) with additional empirical constrains. [Silva and Darragh \(1995\)](#) analyzed 49 rock  
 323 sites in WNA and 22 rock sites in Eastern North America [ENA]. Table 5-3 of [Silva](#)  
 324 [and Darragh \(1995\)](#) lists the median and range of  $\kappa_0$  values for average site conditions  
 325 in WNA and ENA. It indicates that average  $\kappa_0$  values for WNA rock site conditions  
 326 are not small and are larger than those for ENA. [Silva and Darragh \(1995\)](#) interpreted  
 327 the  $\kappa_0$  to be the result of damping in the top 1-2 km below the site and proposed  
 328 two Q models ( $Q = \gamma \cdot V_S$ ) with  $\gamma = 0.007$  and  $0.029$  sec/m for soft-rock and hard-  
 329 rock sites, respectively. Their soft-rock and hard-rock sites are representative WNA  
 330 and ENA generic  $V_S$  profiles, respectively, and are shown in Figure 10(a).

331 For each of the 13 target  $V_S$  profiles in this study, we estimate  $\kappa_0$  by summing up  
 332 the damping in the profile layers over the top 1 and 2 km of the profile as shown in  
 333 Equations 1 and 2. Two profile depths are used to capture the uncertainty in the total  
 334 depth of the profile contributing to damping. Two alternative Q models are used: a  
 335 linear Q model with gamma = 0.007 sec/m representative of WNA soft-rock condition  
 336 and a bilinear Q model with gamma of 0.007 sec/m for the profile layers with  $V_S \leq$   
 337 2700 m/sec and 0.029 sec/m for larger  $V_S$ . This results in a total of four  $\kappa_0$  estimates  
 338 for each target  $V_S$  profile. The alternative target  $\kappa_0$  estimates as a function of  $V_{S30}$   
 339 are shown in Figure 11 (a) and are compared to empirical  $\kappa_0$  estimates inferred from  
 340 ground-motion data. Empirical  $\kappa_0$  estimates shown in Figure 11 are based on  $\kappa_0$   
 341 estimates for the 4 NGA-West2 GMPEs in [Al Atik and Abrahamson \(2021\)](#) for  $V_{S30}$   
 342 of 760 and 1000 m/sec and on  $\kappa_0$  and  $V_{S30}$  inverted for the KA16 models. The upper  
 343 estimates of target  $\kappa_0$  values for this study shown in Figure 11 are the result of using  
 344  $\gamma = 0.007$  sec/m and a profile depth of 2 km contributing to damping while the lower  
 345 estimates are the result of the bilinear Q model with a profile depth of 1 km contributing  
 346 to  $\kappa_0$ .

$$\kappa_0 = \sum_i \frac{H_i}{V_{S,i} Q_i} \quad (1)$$

$$Q = \gamma * V_S \quad (2)$$

348 Figure 11(a) indicates that the target  $\kappa_0$  values have a similar trend with  $V_{S30}$  as the  
 349 empirical  $\kappa_0$  estimates, but with the average target  $\kappa_0$  values falling below the average  
 350 empirical  $\kappa_0$  estimates, indicating an underestimation of the target  $\kappa_0$  values compared  
 351 to the empirical data. Because this study uses CY14 to develop analytical hard-rock  
 352 site adjustment factors, we constrain the average target  $\kappa_0$  for  $V_{S30} = 1000$  m/sec to  
 353 match that of CY14 (0.0345 sec). As a result, the target  $\kappa_0$  values are scaled up by a

354 constant factor and the adjusted target  $\kappa_0$  values are shown in Figure 11(b). We note  
355 that the trend of the empirical  $\kappa_0$  values as a function of  $V_{S30}$  is still different from  
356 that of the scaled target  $\kappa_0$  values for this study. Our ultimate goal is not to match the  
357 exact empirical  $\kappa_0$  values but to have a good match between the analytical and the  
358 empirical rock site adjustment factors. We aim to match the hard-rock scaling observed  
359 in empirical data reflecting the combined effects of  $\kappa_0$  and  $V_S$  profile scaling. We  
360 also note that the upper estimates of the scaled target  $\kappa_0$  are within the range of  $\kappa_0$   
361 values for WNA rock from Silva and Darragh (1995) and are considered reasonable.  
362 Table 2 lists the four  $\kappa_0$  values for the different target  $V_S$  profiles along with their  
363 average and standard deviation.

### 364 *Hard-Rock Site Adjustment Factors*

365 For each of the target  $V_{S30}$  values ranging from 1000 to 2200 m/sec, four sets of  
366 adjustment factors are developed using the IRVT approach of Al Atik et al. (2014)  
367 corresponding to the four target  $\kappa_0$  values listed in Table 2. Strike-slip earthquake  
368 scenarios with magnitude 5, 6, and 7, distance of 5, 10, and 20 km, and  $V_{S30}$  of 760  
369 m/sec are used in the IRVT approach. CY14 median response spectra are computed for  
370 the nine scenarios considered for the linear site response. These response spectra are  
371 converted into compatible FAS using the IRVT approach as described in the previous  
372 sections. Then, each FAS is scaled to adjust for the differences in the linear site  
373 amplification and  $\kappa_0$  scaling between the host and target  $V_S$  profiles and  $\kappa_0$  values.  
374 The  $V_S$ - $\kappa$  scaled FAS are then converted into a  $V_S$ - $\kappa$  scaled response spectra using  
375 random vibration theory. The  $V_S$ - $\kappa$  scaling factors are calculated as the ratio of the  
376 scaled response spectra to the initial GMPE response spectra and averaged over the  
377 nine scenarios considered.

378 For each target  $V_S$  profile, four sets of  $V_S$ - $\kappa$  scaling factors are computed  
379 corresponding to the four target  $\kappa_0$  values. Average  $V_S$ - $\kappa$  scaling factors are derived  
380 assuming equal weights for the four target  $\kappa_0$  values. Figure 12 shows the  $V_S$ - $\kappa$  scaling  
381 factors for the individual target  $\kappa_0$  values as well as the average scaling factors for  
382  $V_{S30}$  of 1700 m/sec compared to the empirical hard-rock factors of KA16. Figure 12  
383 indicates a good agreement between the average analytical factors for  $V_{S30}$  of 1700  
384 m/sec and the KA16 model 2 factors which have a representative  $V_{S30}$  of about 1700  
385 m/sec. Figure 13 compares the set of average analytical hard-rock adjustment factors  
386 for the range of  $V_{S30}$  of 1000 to 2200 m/sec to the CY14 empirical site factors for  $V_{S30}$   
387 of 1000 m/sec and the KA16 hard-rock factors. While some mismatch can be observed  
388 in Figure 13 between the analytical factors for  $V_{S30}$  of 1300 m/sec and the KA16

389 model 1 factors, there is good agreement between the analytical hard-rock factors for  
390  $V_{S30}$  of 1000 m/sec and the corresponding CY14 site factors for frequencies less than  
391 20 Hz and between the analytical factors for  $V_{S30}$  of 1700 m/sec and the KA16 model 2  
392 factors for 15-30 Hz. We conclude that, on average, the analytical hard-rock factors are  
393 reasonable based on their comparison with empirical scaling for rock site conditions  
394 (CY14 for  $V_{S30} = 1000$  m/sec and KA16 factors).

## 395 Implementation

396 The hard-rock site adjustment factors derived in this study are used to extrapolate the  
397 average NGA-West2 empirical site factors to hard-rock conditions in a relative sense  
398 to ensure a smooth transition in the scaling factors to hard-rock sites. As such, the  
399 ratios of hard-rock analytical factors relative to those for  $V_{S30}$  of 1000 m/sec are used  
400 to model the scaling of the hard-rock site factors. These ratios are then applied to the  
401 empirical site factors for  $V_{S30}$  of 1000 m/sec relative to reference  $V_{S30}=760$  m/sec. This  
402 normalization of the analytical site factors allows the site factors from the analytical  
403 modeling to be centered on the GMPEs which provides a smooth scaling from soft-rock  
404 to hard-rock site conditions. The empirical linear site factors for  $V_{S30}$  of 1000 m/sec  
405 are obtained by averaging the ratio of median response spectra for  $V_{S30}$  of 1000 m/sec  
406 relative to 760 m/sec for 4 NGA-West2 GMPEs (Abrahamson et al. 2014; Boore et al.  
407 2014; Campbell and Bozorgnia 2014; Chiou and Youngs 2014). The resulting rock-site  
408 adjustment factors are shown in Figure 14 and included as an electronic appendix to this  
409 paper. Figure 14 also shows the average empirical linear site factors of the NGA-West2  
410 GMPEs for  $V_{S30}$  of 680 to 1000 m/sec relative to the reference 760 m/sec. The GMPEs  
411 nonlinear site response is not included in the calculation of the average empirical site  
412 factors. Figure 14 indicates a smooth extrapolation of the empirical average GMPE site  
413 factors to hard-rock conditions based on the analytical factors described in this paper.

414 Figure 15 shows the linear  $V_S$  scaling of the NGA-West2 GMPEs relative to  $V_{S30}$   
415 of 760 m/sec and extrapolated to hard-rock conditions. Also plotted in Figure 15 are  
416 the average of the scaling from the 4 NGA-West2 GMPEs and the hard-rock scaling  
417 proposed in this study. Comparisons of the linear  $V_S$  scaling are shown for frequencies  
418 of 0.2, 1, 5, and 25 Hz. These comparisons indicate that, for  $V_{S30}$  values  $> 1000$   
419 m/sec, linear  $V_S$  scaling varies among the NGA-West2 GMPEs reflecting the different  
420 hard-rock extrapolation constraints imposed in the models. The extrapolated hard-rock  
421 scaling in the NGA-West2 GMPEs is unconstrained with empirical data for hard-rock  
422 conditions and is, therefore, unreliable for application to hard-rock sites. In contrast to  
423 the hard-rock factors proposed in this study, the NGA-West2 scaling does not follow

424 expected trends with  $\kappa$  for hard-rock sites at the high frequency of 25 Hz as shown in  
425 Figure 15 (a). Therefore, the NGA-West2 linear  $V_S$  scaling should not be extrapolated  
426 to hard-rock sites and the factors presented in this paper should be used instead.

427 The average hard-rock adjustment factors from this study, presented in Figure 14 and  
428 included as an electronic appendix to this paper, can be applied to correct the average  
429 median ground motion predicted by the NGA-West2 GMPEs with  $V_{S30}$  of 760 m/sec  
430 to a hard-rock site with  $V_{S30}$  between 1000 and 2200 m/sec. Nonlinear site response  
431 should be disabled when calculating the NGA-West2 ground-motion predictions for  
432  $V_{S30}$  of 760 m/sec before applying the hard-rock adjustment factors. For target  $V_{S30}$   
433 values not explicitly listed in the electronic appendix, hard-rock factors can be obtained  
434 using a log-log interpolation of the provided factors for the neighboring  $V_{S30}$  values.  
435 For hard-rock sites with qualitative assessment of site conditions, hard-rock adjustment  
436 factors for a range of target  $V_{S30}$  values can be enveloped to estimate the median hard-  
437 rock adjustment factors.

### 438 *Site-to-Site Uncertainty*

439 The adjustment of median ground-motion predictions for hard-rock sites is presented  
440 in this paper. To evaluate the uncertainty in the hard-rock adjustment factors, we  
441 examine the site-to-site variability [ $\phi_{S2S}$ ] in the NGA-West2 GMPEs for soil versus  
442 rock sites. Site terms are obtained using a mixed-effects regression on the within-event  
443 residuals of the NGA-West2 GMPEs with the station term as the random effect and  
444 using earthquakes with magnitude  $\geq 5$  and stations with a minimum of 3 recordings  
445 as described in Al Atik (2015). Ground-motion data with magnitude  $< 3$  are not  
446 used in this analysis to reduce the dependence of linear site factors on earthquake  
447 magnitude. This effect was examined in Stafford et al. (2017) and was found to be  
448 most pronounced at short periods and for small magnitude scenarios. Soil sites in the  
449 NGA-West2 database are classified with  $V_{S30} < 680$  m/sec while rock sites have  $V_{S30}$   
450  $\geq 680$  m/sec. Site terms for each NGA-West2 GMPE are divided in these two site  
451 categories and the resulting  $\phi_{S2S}$  are computed.

452 The  $\phi_{S2S}$  for soil and rock sites obtained using the residuals of ASK14, Boore et al.  
453 (2014) [BSSA14] and CY14 for magnitude  $\geq 5$  were examined and the comparison  
454 using CY14 residuals is shown in Figure 16. We note  $\phi_{S2S}$  for Campbell and Bozorgnia  
455 (2014) [CB14] is not shown due to the limited CB14 dataset as a result of restricting  
456 the residuals to magnitudes  $> 5$  and stations with a minimum of 3 recordings. This  
457 impacted the stability of the  $\phi_{S2S}$  estimates for CB14. The large error bars in Figure 16  
458 reflect the smaller subset of stations with  $V_{S30} \geq 680$  m/sec compared to the number

459 of softer sites in the NGA-West2 dataset. For example, using the CY14 residuals,  
460 the average  $V_{S30}$  is about 390 m/sec for soil sites and 830 m/sec for rock sites. The  
461 comparison of  $\phi_{S2S}$  for soil and rock sites indicates that the NGA-West2  $\phi_{S2S}$  values  
462 are generally comparable for the two site groups at high frequencies as shown in  
463 Figure 16. At periods greater than 1 sec,  $\phi_{S2S}$  values for rock sites are lower than  
464 those for soil sites. We note that the subsets of data for rock sites are very limited  
465 in number of stations for periods  $> 4$  sec. We conclude that, for hazard significant  
466 scenarios with magnitudes  $\geq 5$ ,  $\phi_{S2S}$  obtained from the NGA-West2 residuals for all  
467  $V_{S30}$  can be used to estimate  $\phi_{S2S}$  for hard-rock sites with modifications associated  
468 with the expected spectral shapes of site variability for hard-rock.

469 The average  $\phi_{S2S}$  obtained using residuals of ASK14, BSSA14 and CY14 for  
470 magnitude  $\geq 5$  and for all  $V_{S30}$  values is shown in Figure 17 (a). We note that the  
471 peak in  $\phi_{S2S}$  at frequency 5-10 Hz is likely related to the variability of the resonance  
472 frequency of shallow layers for soil and soft-rock sites. For hard-rock sites, this peak is  
473 expected to be shifted to higher frequencies reflecting the variability in kappa scaling  
474 for hard-rock conditions. We examine this effect using  $\phi_{S2S}$  obtained from a residual  
475 analysis of Japanese surface and borehole data. A discussion of the residual analysis  
476 of the Japanese dataset is presented in Goulet et al. (2018). Figure 17 (b) presents a  
477 comparison of  $\phi_{S2S}$  for the surface and borehole Japanese data with magnitude  $\geq 5$ .  
478 Borehole  $\phi_{S2S}$  values obtained using stations with  $V_S \geq 1000$  m/sec are also shown.  
479 Figure 17 (b) shows a shift in the peak of  $\phi_{S2S}$  to higher frequencies for the rock  
480 borehole data compared to the surface data. As a result, we correct the average  $\phi_{S2S}$   
481 for NGA-West2 to follow the high-frequency scaling of the Japanese borehole  $\phi_{S2S}$   
482 for frequencies greater than 2.5 Hz. For frequencies less than 2.5 Hz, the  $\phi_{S2S}$  shape  
483 is based on the NGA-West2 data. The resulting proposed  $\phi_{S2S}$  model for use for hard-  
484 rock sites is shown in Figure 17 (a) and listed in Table 3. This proposed model can  
485 be used to characterize the epistemic uncertainty of the average rock-site adjustment  
486 factors presented in this study if additional site-specific information is not available to  
487 constrain the epistemic uncertainty of the site factors.

488 For hard-rock adjustments of the NGA-West2 GMPEs using the ergodic aleatory  
489 variability model, the standard deviation models in the NGA-West2 GMPEs, calculated  
490 for  $V_{S30}$  of 760 m/sec without including effects of nonlinear site response, could be  
491 used for hard-rock sites. The use of the ergodic NGA-West2 sigma models is likely  
492 conservative for some frequency ranges and might not capture the expected peaks in  
493 the variability for hard-rock sites. Alternatively, ergodic sigma for hard-rock sites can  
494 be constructed using  $\phi_{S2S}$  proposed in this study along with NGA-West2 Tau models  
495 and published single-station sigma models for WUS ((Al Atik 2015)). We note that the



496  $\phi_{S2S}$  model proposed in this study for hard-rock site adjustment factors is a simplified  
497 model based on adjusting the NGA-West2  $\phi_{S2S}$ . A more detailed study of the ground-  
498 motion variability and its components for hard-rock sites is warranted.

## 499 Conclusions and Discussion

500 Hard-rock adjustment factors are derived to adjust the NGA-West2 GMPEs from  
501 their average host site conditions with  $V_{S30}$  of 760 m/sec to target sites with  $V_{S30}$   
502 ranging from 1000 to 2200 m/sec. These analytical factors are obtained using the IRVT  
503 approach [Al Atik et al. \(2014\)](#) and are consistent with empirical scaling observed in  
504 ground-motion data. These factors can be applied to adjust median NGA-West2 ground  
505 motions at  $V_{S30}$  of 760 m/sec to hard-rock conditions and can be assumed to have the  
506 same overall site-to-site uncertainty inherent in the NGA-West2 GMPEs.

507 The site adjustment factors developed in this study are computed using generic  $V_S$   
508 profiles and  $\kappa_0$  values that would be representative of average site response in WUS  
509 for rock site conditions. The KA16 Scaling factors obtained using ENA and BChydro  
510 data are used as empirical constraints for this study because of the scarcity of empirical  
511 data on hard-rock sites in WUS and because KA16 showed that average hard-rock  
512 scaling in ENA is comparable to what would be expected for WUS sites. The proposed  
513 hard-rock factors are intended for use at sites with measured or estimated  $V_{S30}$  or  
514 sites with qualitative assessment of site condition. For hard-rock sites with site-specific  
515 measurements of  $V_S$  profiles extending below the shallow 20 to 30 m of the profile,  
516 the hard-rock adjustment factors presented here are not recommended to be used. For  
517 such sites, site-specific adjustments need to be developed following a characterization  
518 of the target site-specific conditions in terms of best estimates and uncertainty of  $V_S$   
519 profiles and  $\kappa_0$ . Also, the use of  $\phi_{S2S}$  for site-specific adjustments is conservative and  
520 can potentially be reduced based on the uncertainty in the site-specific characterization.

## 521 Data and Resources

522 The `pyrvt` program used to perform the inverse random vibration theory (IRVT) and  
523 random vibration theory (RVT) calculations ([Kottke 2020](#)). An Excel file containing the  
524 hard rock adjustment factors for the NGA-West2 GMPEs is included as a supplemental  
525 material.

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**Table 1.** Results of the inversion for KA16 model 2 for the different cases analyzed.

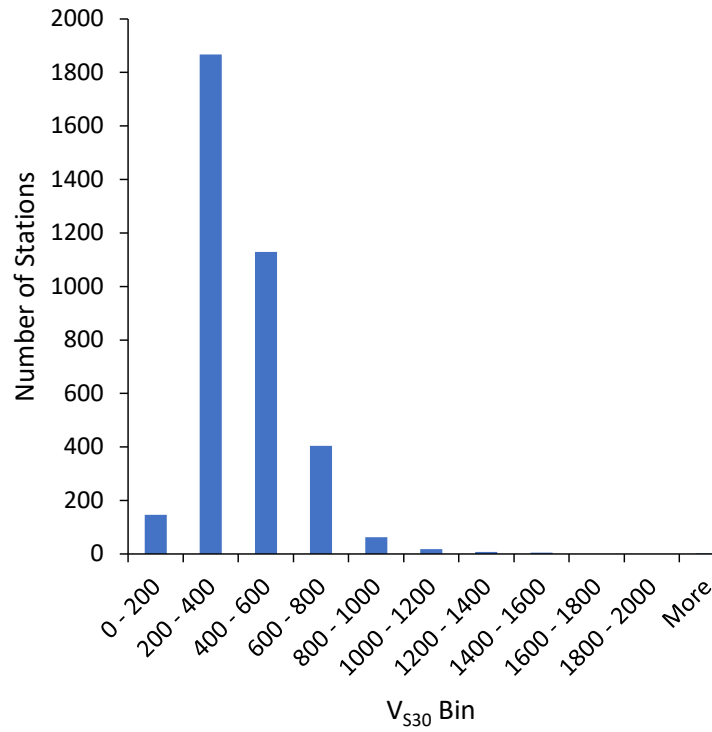
Case	$V_{S30}$ (m/sec)	Inverted $\kappa_0$ (sec)	SSE (0.6 to 30Hz)
Inverted $V_{S30}$	1602	0.025	0.325
Assumed $V_{S30}$	1500	0.026	0.371
Assumed $V_{S30}$	1700	0.024	0.350
Assumed $V_{S30}$	1850	0.022	0.414
Assumed $V_{S30}$	2000	0.021	0.477
Assumed $V_{S30}$	2380	0.019	0.769

**Table 2.** Target  $\kappa_0$  values used in the development of the analytical rock site adjustment factors.

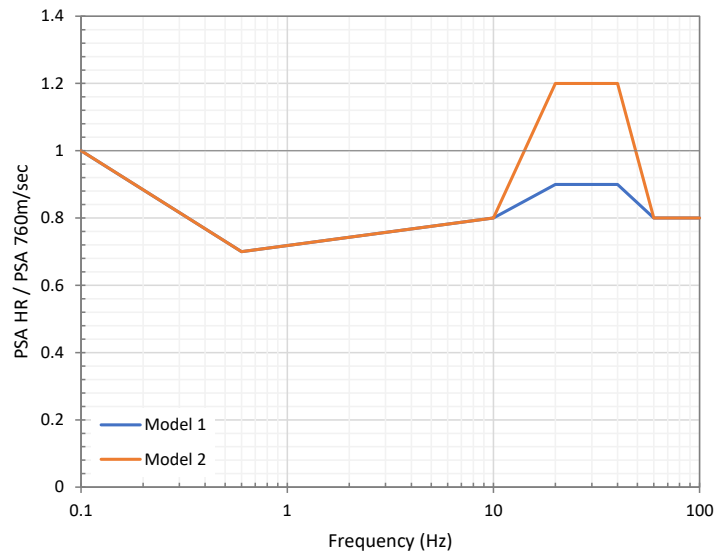
$V_{S30}$ (m/sec)	$\kappa_0$ -1 (sec)	$\kappa_0$ -2 (sec)	$\kappa_0$ -3 (sec)	$\kappa_0$ -4 (sec)	Average $\kappa_0$ (sec)	Standard Deviation (LN units)
1100	0.0296	0.0462	0.0235	0.0276	0.0307	0.289
1200	0.0275	0.0436	0.0206	0.0245	0.0279	0.322
1300	0.0258	0.0416	0.0182	0.0221	0.0256	0.353
1400	0.0245	0.0399	0.0162	0.0200	0.0237	0.387
1500	0.0233	0.0385	0.0144	0.0182	0.0220	0.420
1600	0.0223	0.0373	0.0129	0.0165	0.0205	0.458
1700	0.0215	0.0363	0.0116	0.0153	0.0193	0.490
1800	0.0208	0.0354	0.0106	0.0142	0.0182	0.521
1900	0.0202	0.0346	0.0095	0.0131	0.0172	0.560
2000	0.0196	0.0339	0.0088	0.0124	0.0164	0.584
2100	0.0192	0.0333	0.0082	0.0117	0.0157	0.611
2200	0.0187	0.0327	0.0075	0.0110	0.0150	0.640

**Table 3.** Proposed site-to-site uncertainty model ( $\phi_{S2S}$ ) model.

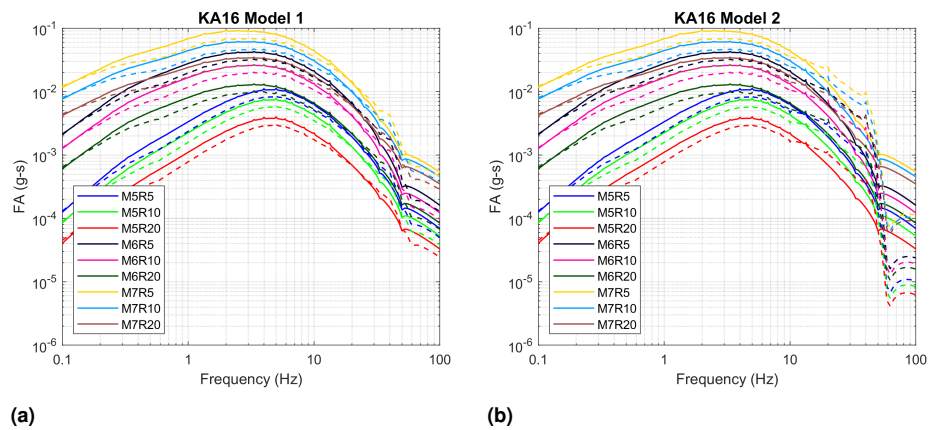
Frequency (Hz)	Period (sec)	$\phi_{S2S}$ (LN units)
100.00	0.010	0.3110
50.00	0.020	0.3110
33.33	0.030	0.3275
20.00	0.050	0.3901
13.33	0.075	0.3894
10.00	0.100	0.3627
6.67	0.150	0.3308
5.00	0.200	0.3182
4.00	0.250	0.3182
3.33	0.300	0.3182
2.50	0.400	0.3182
2.00	0.500	0.3312
1.33	0.750	0.3446
1.00	1.000	0.3739
0.67	1.500	0.4001
0.50	2.000	0.4185
0.33	3.000	0.4232
0.25	4.000	0.4065
0.20	5.000	0.3965
0.13	7.500	0.3480
0.10	10.000	0.2877



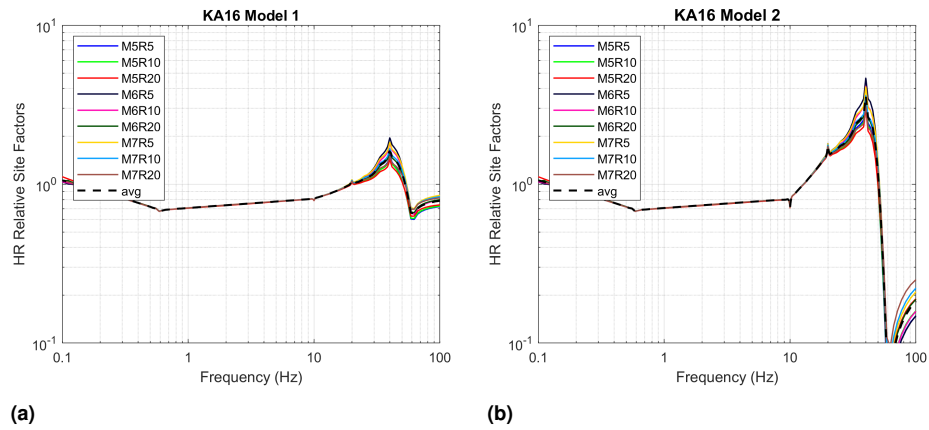
**Figure 1.** Histogram of the number of stations in different  $V_{S30}$  bins in the ASK14 dataset.



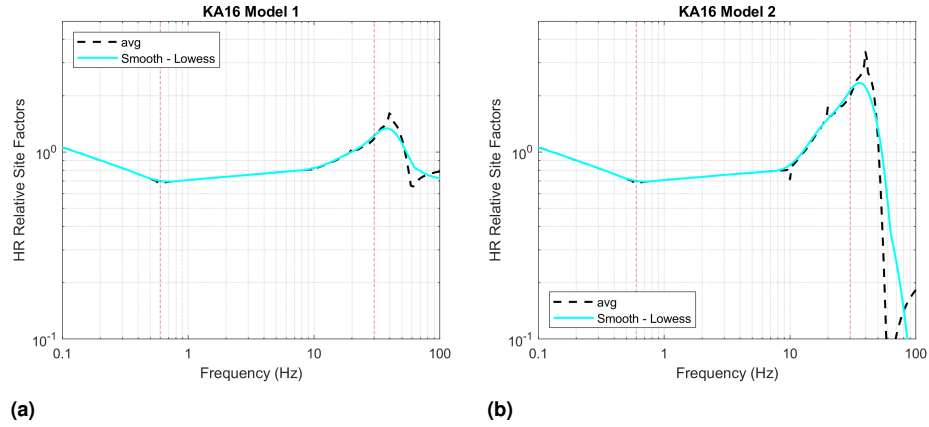
**Figure 2.** KA16 hard-rock scaling factors relative to  $V_{S30}$  of 760 m/sec.



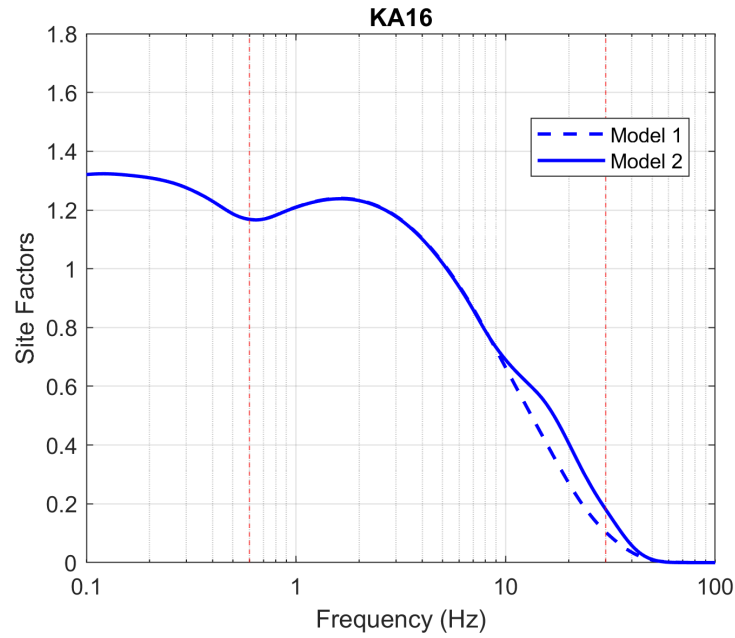
**Figure 3.** CY14 IRVT-based Fourier amplitude spectra for  $V_{S30} = 760$  m/sec (solid lines) and for spectra corrected to hard-rock conditions (dashed lines) using KA16 model 1 (a) and model 2 (b).



**Figure 4.** Hard-rock site factors in FAS domain relative to  $V_{S30} = 760$  m/sec for a suite of scenarios (solid lines) and average relative site factors over all scenarios (dashed lines) for KA16 model 1 (a) and model 2 (b).

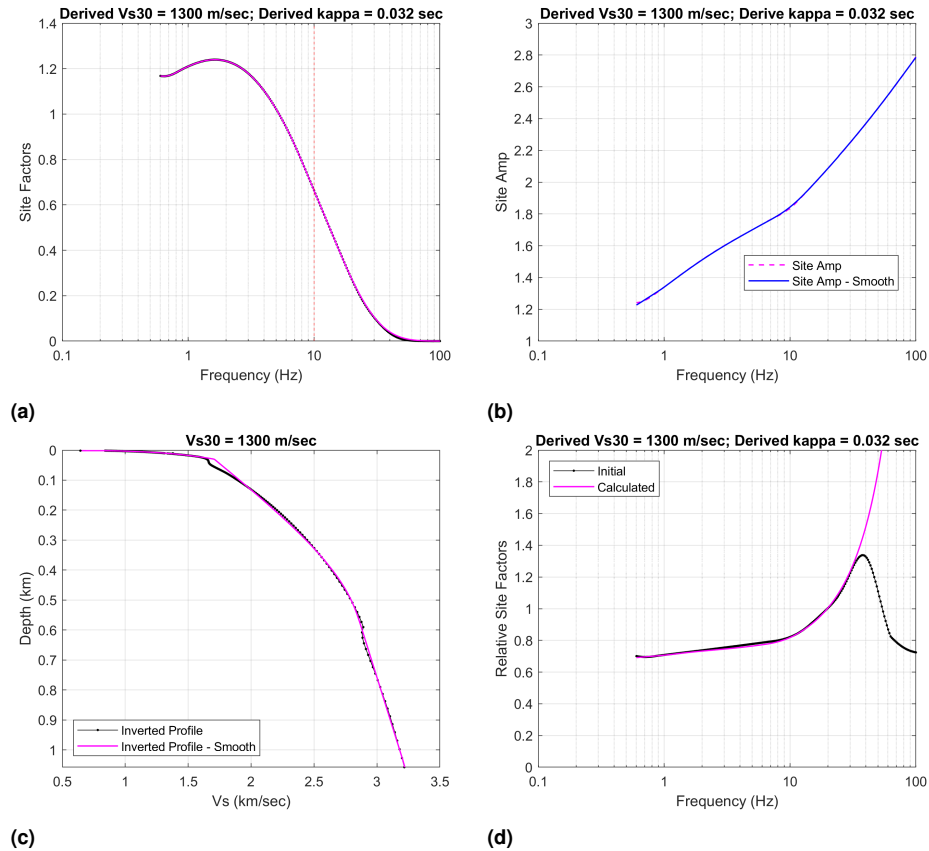


**Figure 5.** Average hard-rock site factors relative to 760 m/sec in FAS domain (dashed lines) and smoothed factors (solid lines) for KA16 model 1 (a) and model 2 (b). Dashed red vertical lines indicate the frequency range used in the analysis (0.6 to 30 Hz).

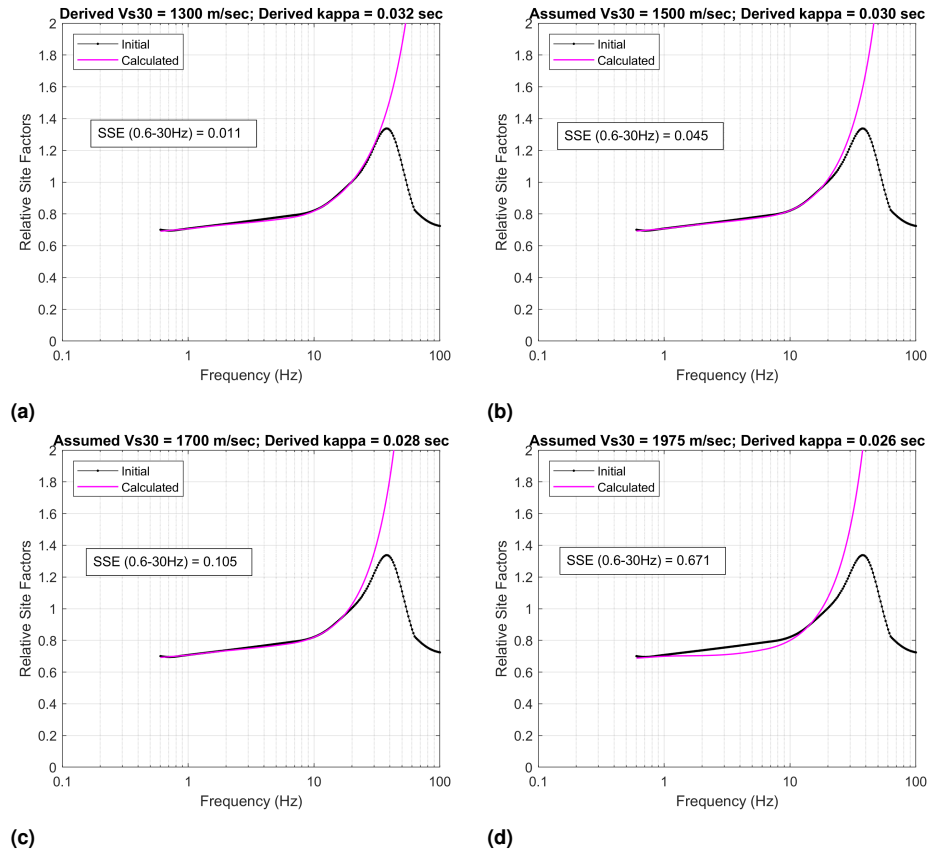


**Figure 6.** Total FAS site factors for the average hard-rock site conditions representative of the KA16 models. Dashed red vertical lines indicate the reliable frequency range (0.6 to 30 Hz).

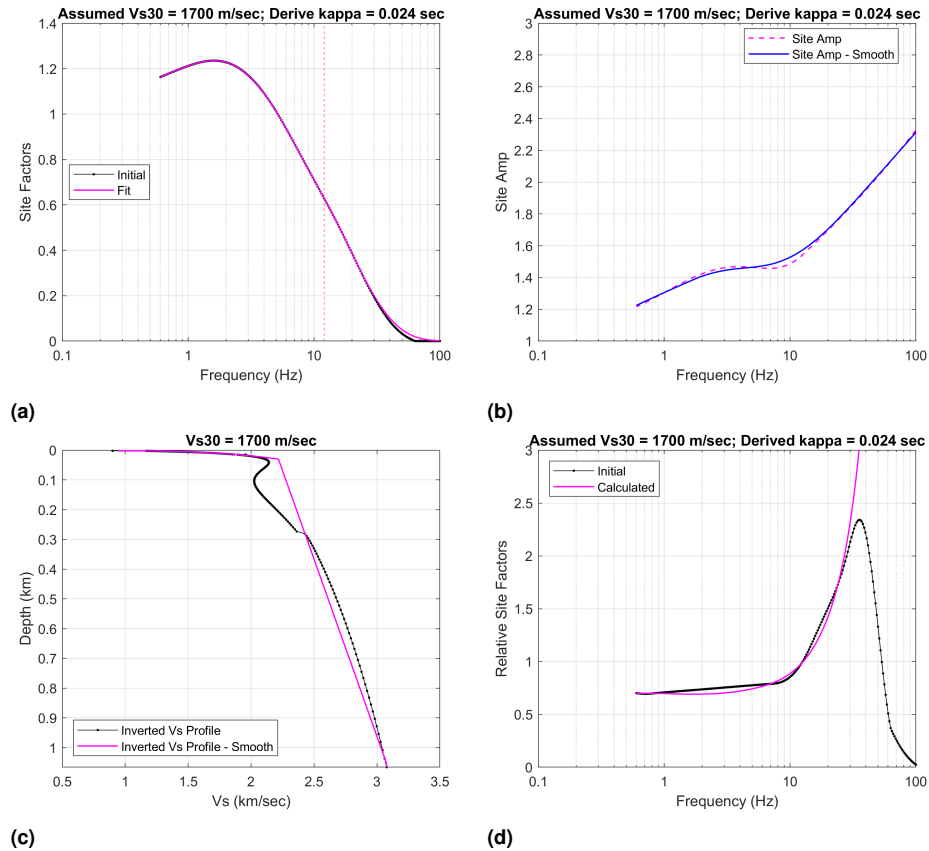




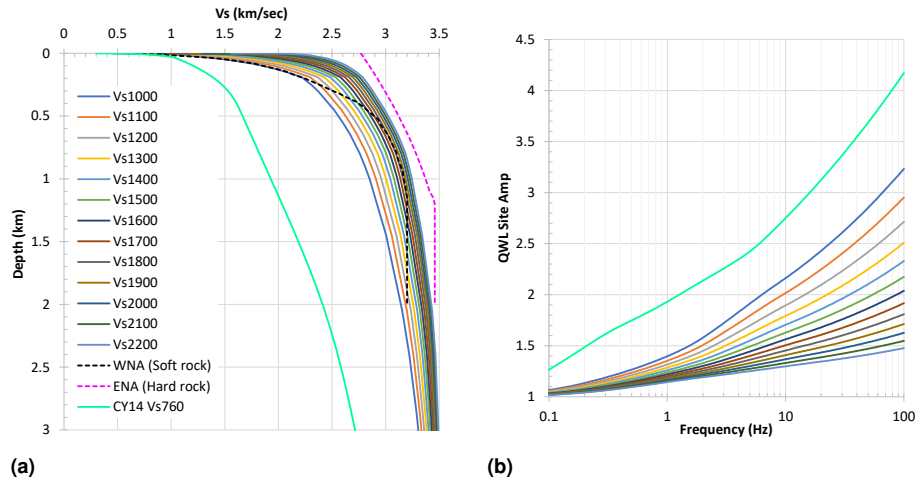
**Figure 7.** Inversion results for KA16 model 1. (a) Hard-rock site factors and high-frequency fit to estimate  $\kappa_0$  and  $V_{S30}$ . (b) Site amplification function obtained by dividing the fitted site factors by the  $\kappa_0$  operator. (c) Inverted  $V_S$  profile and smoothed. (d) comparison of the hard-rock site factors relative to  $V_{S30}$  of 760 m/sec obtained from the inversion (calculated) to the initial relative site factors.



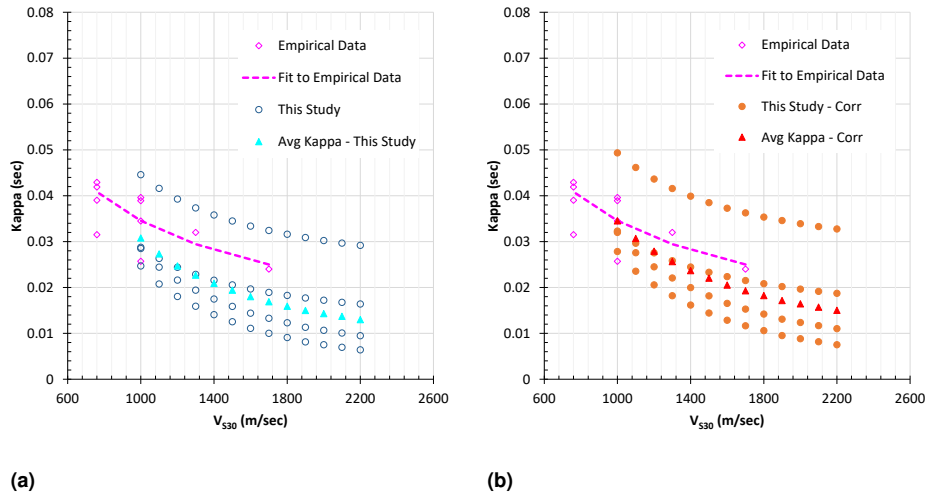
**Figure 8.** Comparison of the KA16 model 1 hard-rock site factors relative to  $V_{S30}$  of 760 m/sec to the relative site factors obtained from the inversions for the cases of (a) derived  $V_{S30}$  and assumed  $V_{S30}$  values of (b) 1500, (c) 1700, and (d) 1975 m/sec. Derived  $\kappa_0$  values and calculated SSE are included in the plots.



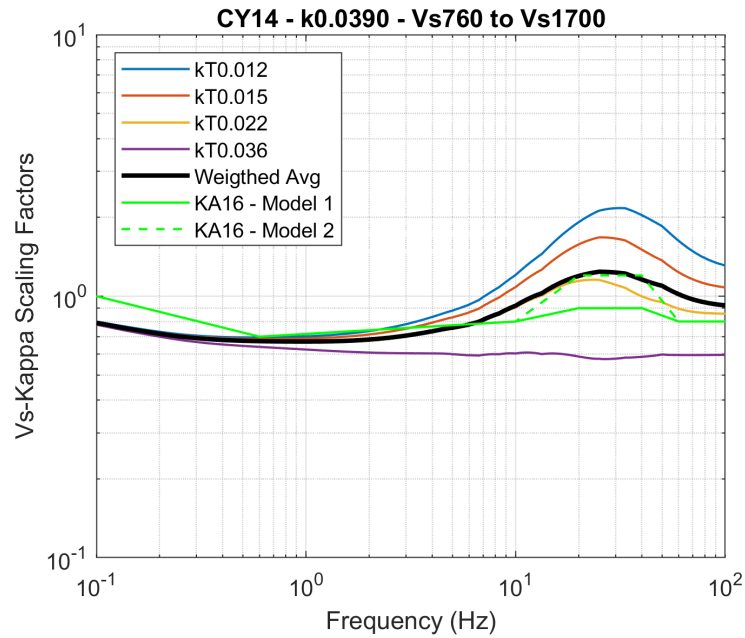
**Figure 9.** Inversion results for KA16 model 2. (a) Hard-rock site factors and high-frequency fit to estimate  $\kappa_0$  for an assumed  $V_{S30}$  of 1700 m/sec. (b) Site amplification function obtained by dividing the fitted site factors by the  $\kappa_0$  operator. (c) Inverted  $V_S$  profile and smoothed. (d) comparison of the hard-rock site factors relative to  $V_{S30}$  of 760 m/sec obtained from the inversion (calculated) to the initial relative site factors.



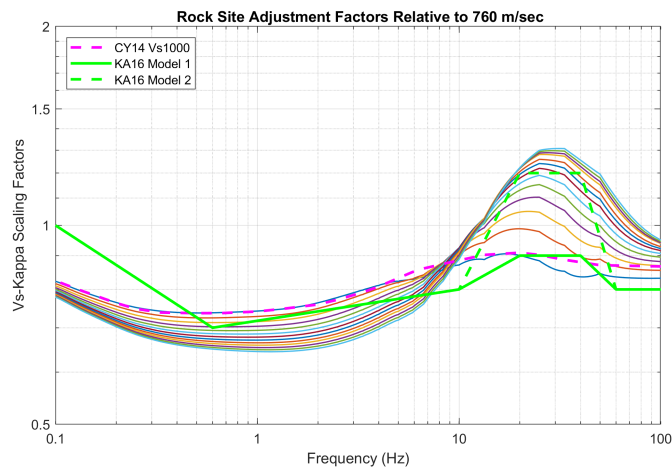
**Figure 10.** (a) Host (CY14 Vs760) and target  $V_S$  profiles compared to the WNA and ENA  $V_S$  profiles of Silva and Darragh (1995) (b) Corresponding QWL linear site amplification.



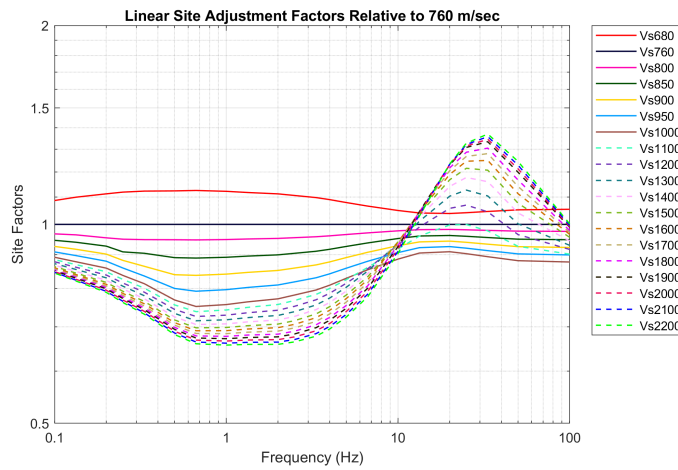
**Figure 11.** (a) Comparison of target  $\kappa_0$  values as a function of  $V_{S30}$  to  $\kappa_0$  inferred from empirical ground motion data (b) Scaled target  $\kappa_0$  values such that their average matches CY14  $\kappa_0$  at  $V_{S30}$  of 1000 m/sec.



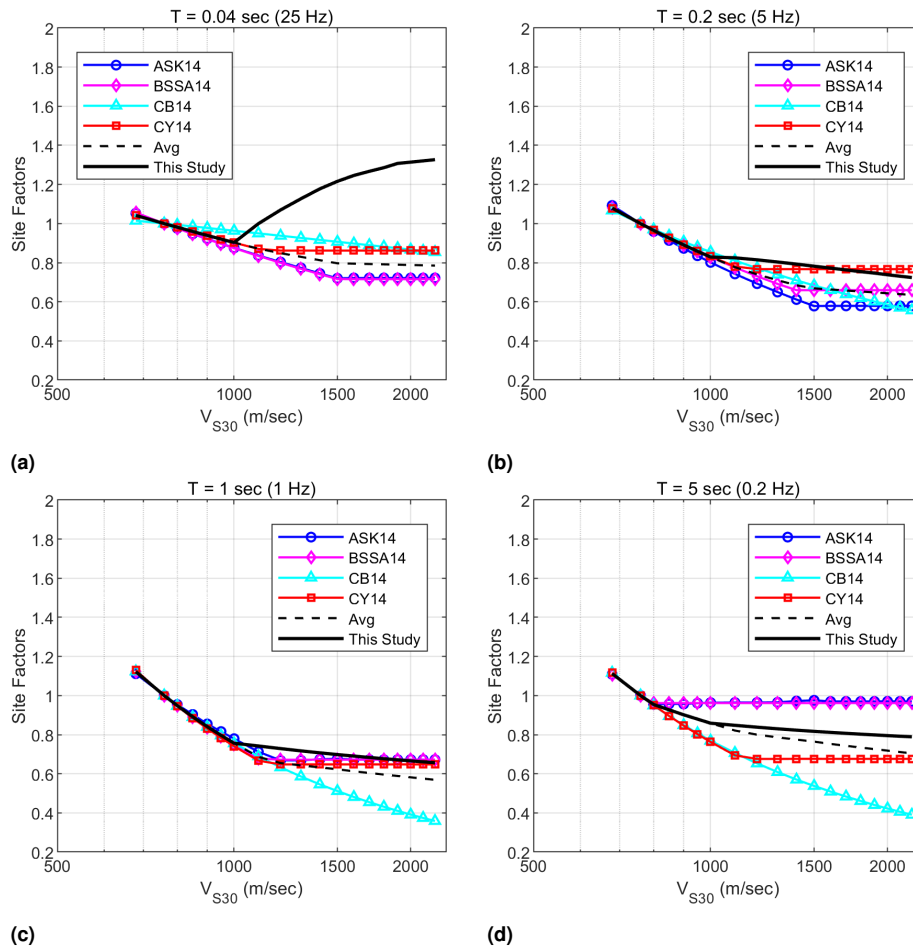
**Figure 12.** Comparison of the analytical adjustment factors for target  $V_{S30} = 1700$  m/sec to the KA16 rock site adjustment factors.



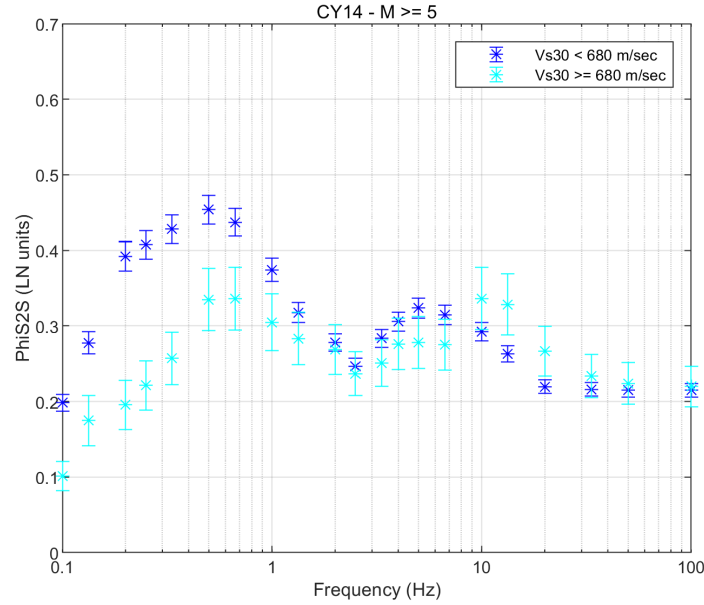
**Figure 13.** Analytical hard-rock site adjustment factors for target  $V_{S30}$  of 1000 to 2200 m/sec compared to the CY14 site factors for  $V_{S30} = 1000$  m/sec and the KA16 hard-rock rock site adjustment factors.



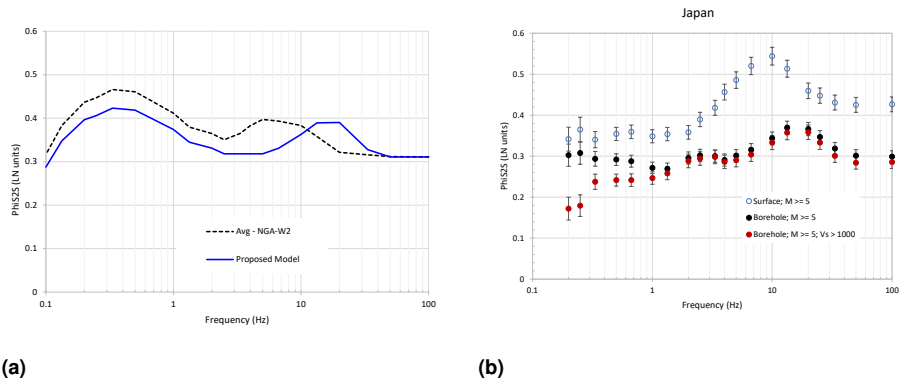
**Figure 14.** Proposed linear site adjustment factors for  $V_{S30} = 680$  to 2200 m/sec relative to 760 m/sec. Solid and dashed lines show empirical and analytical factors, respectively.



**Figure 15.** Linear  $V_S$  scaling factors relative to  $V_{S30}$  of 760 m/sec for the NGA-West2 GMPEs extrapolated to hard-rock conditions compared to hard-rock scaling factors from this study for frequencies of 25 Hz (a), 5 Hz (b), 1 Hz (c), and 0.2 Hz (d).



**Figure 16.** Site-to-site uncertainty ( $\phi_{S2S}$ ) of CY14 for soil sites with  $V_{S30} < 680$  m/sec and rock sites with  $V_{S30} \geq 680$  m/sec using data with magnitude  $\geq 5$ . Error bars show one standard error around the  $\phi_{S2S}$  estimates..



(a)

(b)

**Figure 17.** (a) Average  $\phi_{S2S}$  based on the NGA-West2 residuals and proposed  $\phi_{S2S}$  model for hard-rock sites adjusted at high frequencies (b)  $\phi_{S2S}$  for the Japanese surface and borehole data.