Newton was right—equations cannot rationally describe how parameters are related. Rational equations require a paradigm shift.

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Abstract
Until 1822, engineers and scientists such as Newton agreed that equations cannot describe how parameters are related because parameter dimensions cannot be multiplied or divided. (That is why Newton’s laws were initially proportions instead of equations.) In 1822, Fourier claimed (without proof) that dimensions can be multiplied and divided, and parameters such as \( h \) and \( E \) can be created by assigning dimensions to numbers. Fourier’s unproven claims are the only reason that, since 1822, it has been widely agreed that equations can describe how parameters are related. However, critical analysis in the text proves that dimensions cannot be multiplied and divided. Also, it has for many years been generally agreed that dimensions must not be assigned to numbers. Therefore equations cannot describe how parameters are related, and equations must not include parameters such as \( h \) and \( E \). The paradigm shift described in the text requires that parameter symbols in equations represent only numerical values, and all parameters created by assigning dimensions to numbers be abandoned. After the paradigm shift, all proportions and equations are inherently dimensionally homogeneous because they contain only numbers. All laws are analogs of the rational equation \( y = f(x) \) which states that the numerical value of parameter \( y \) is a function of the numerical value of parameter \( x \), and the function may be proportional, linear, or nonlinear. If an equation is quantitative, the dimension units that underlie parameter symbols (that are not in dimensionless groups) must be specified in an accompanying nomenclature.

Key words: Dimensional homogeneity, dimensions, irrational equations, irrational laws, Newton’s laws, parameters, proportions, rational equations, rational laws.

1. Introduction
Until 1822, it was generally agreed that equations cannot describe how parameters are related because parameter dimensions cannot be multiplied or divided. In 1822, Fourier [1] made the revolutionary and unproven claims that dimensions can be multiplied and divided, and dimensions can be assigned to numbers. Therefore equations can describe how parameters are related, and laws can be made dimensionally homogeneous by assigning dimensions to numbers.

Critical analysis in the text proves that dimensions cannot be multiplied and divided, and therefore equations cannot describe how parameters are related.

Sometime before 1951, it was widely agreed that dimensions must not be assigned to numbers. Therefore modern engineering laws are irrational for two reasons: because parameter
dimensions in the laws cannot be multiplied or divided, and because coefficients in the laws (such as $h$, $E$, and $R$) were created by assigning dimensions to numbers.

The paradigm shift described in the text results in rational equations that describe how the numerical values of parameters are related, and rational laws that are analogs of $y = f(x)$. It also results in the abandonment of all parameters (such as $h$, $E$, and $R$) that were created by assigning dimensions to numbers.

2. **Dimensional homogeneity until 1822.**

Until 1822, scientists and engineers such as Newton and his colleagues agreed that:

- Parameter symbols in proportions and equations represent numerical value and dimension.
- Proportions need not be dimensionally homogeneous.
- Equations must be dimensionally homogeneous.
- Dimensions cannot be multiplied or divided.
- Because parameter dimensions cannot be multiplied or divided, equations that describe how parameters are related are irrational.

Because proportions need not be dimensionally homogeneous, and because proportions that relate two parameters do not require that parameter dimensions be multiplied or divided, proportions are generally used instead of equations. That is why Hooke’s [2] law is Proportion (1) instead of an equation, Newton’s [3] law of cooling\(^1\) is Proportion (2) instead of Eq. (3), and Newton’s [4] second law of motion is Proportion (4) instead of Eq. (5)\(^2\).

\[ \sigma \propto \varepsilon \quad (1) \]
\[ (dT_{body}/dt) \propto (T_{air} - T_{body}) \quad (2) \]
\[ q = h\Delta T \quad (3) \]
\[ a \propto f \quad (4) \]
\[ f = ma \quad (5) \]

\(^1\) American heat transfer texts generally refer to Eq. (3) as “Newton’s law of cooling”. However, Eq. (3) cannot be Newton’s law of cooling because cooling is a transient phenomenon whereas Eq. (3) is a steady-state equation, and because Eq. (3) was irrational in Newton’s time.

\(^2\) When Proportion (4) was transformed to Eq. (5), the transformed version should have been $a = f/m$ in order to correctly indicate that force is the independent variable, in accordance with Newton’s Proportion (4).
3. Fourier’s heat transfer experiment, and the proportion and equation that resulted.

Fourier performed a heat transfer experiment in which a warm, solid body is cooled by the steady-state forced convection of ambient air. Fourier concluded that Proportion (6) and Eq. (7) correlated the data.

\[ q \propto \Delta T \]  
\[ q = c\Delta T \] 

Newton and his colleagues would have been satisfied by Proportion (6), but it did not satisfy Fourier because he wanted an equation, and it had to be dimensionally homogeneous. Equation (7) did not satisfy Fourier because it is not dimensionally homogeneous.

4. How Fourier transformed dimensionally inhomogeneous Eq. (7) to dimensionally homogeneous Eq. (8).

Fourier recognized that Eq. (7) could be transformed to a dimensionally homogeneous equation only if it were rational to assign dimensions to number \( c \) in Eq. (7), and rational to multiply and divide parameter dimensions. Fourier [1] describes his revolutionary and unproven view of dimensional homogeneity in the following:

\[ \ldots \text{every undetermined magnitude or constant has one dimension proper to itself, and the terms of one and the same equation could not be compared, if they had not the same exponent of dimension.} \ldots \text{this consideration is derived from primary notions on quantities; for which reason, in geometry and mechanics, it is the equivalent of the fundamental lemmas which the Greeks have left us without proof.} \]

The above claims that it is rational to assign dimensions to numbers, and rational to multiply or divide parameter dimensions. Note that Fourier points out that there is no proof that his revolutionary view of dimensional homogeneity is rational. In accordance with his unproven view of dimensional homogeneity, Fourier assigned the symbol \( h \) and the dimension of \( q/\Delta T \) to number \( c \) in Eq. (7), and multiplied \( h \) and \( \Delta T \), resulting in dimensionally homogeneous Eq. (8).

\[ q = h\Delta T \] 

Even though Fourier’s unproven view of dimensional homogeneity has never been proven, it is a fundamental tenet in modern engineering science. It is the only reason that, since 1822, it has been widely agreed that Newton and his colleagues were wrong—equations can describe how parameters are related because parameter dimensions can be multiplied or divided.

5. Proof that equations cannot describe how parameters are related because parameter dimensions cannot be multiplied or divided.

“Multiply four times seven” means “add seven four times”. Therefore “multiply meters times kilograms” must mean “add kilograms meters times”. Because “add kilograms meters times” has no meaning, dimensions cannot be multiplied.
“Divide forty by five” means “how many fives are in forty”. Therefore “divide meters by seconds” must mean “how many seconds are in meters”. Because “how many seconds are in meters” has no meaning, dimensions cannot be divided.

The above proves that parameter dimensions cannot be multiplied or divided. Therefore Newton and his colleagues were right—equations cannot describe how parameters are related.

6. Why dimensions must not be assigned to numbers.

Langhaar [5] explains why dimensions must not be assigned to numbers:

*Dimensions must not be assigned to numbers, for then any equation could be regarded as dimensionally homogeneous.*

7. What equations can describe about how parameters are related.

Equations can describe only how the numerical values of parameters are related. If an equation is quantitative, the dimension units that underlie parameter symbols (that are not in dimensionless groups) must be specified in an accompanying nomenclature.

8. What charts can describe about how parameters are related.

Charts can describe only how the numerical values of parameters are related. If a chart is quantitative, the dimension units on each axis must be specified on the chart, or in an accompanying nomenclature.

9. The proposed paradigm shift.

The proposed paradigm shift requires the following:

- Parameter symbols in proportions and equations must represent only numerical values.

- Parameters (such as $h$, $E$, and $R$) that were created by assigning dimensions to numbers must be abandoned.

- Engineering laws must be analogs of Eq. (9) which states that the numerical value of parameter $y$ is a function of the numerical value of parameter $x$, and the function may be proportional, linear, or nonlinear.

$$y = f(x)$$ (9)

- If an equation is quantitative, the dimension units that underlie parameter symbols (that are not in dimensionless groups) must be specified in an accompanying nomenclature.
10. The engineering science that results from the proposed paradigm shift.

The engineering science that results from the proposed paradigm shift is described by the following:

- Parameter symbols in proportions and equations represent *only* numerical values.
- *All* proportions and equations are *inherently* dimensionally homogeneous.
- There are *no* parameters that were *created* by assigning dimensions to numbers.
- If an equation is quantitative, the dimension units that underlie parameter symbols (that are *not* in dimensionless groups) are specified in an *accompanying* nomenclature.
- *All* laws are analogs of Eq. (9) which states that the *numerical value* of parameter $y$ is a function of the *numerical value* of parameter $x$, and the function may be proportional, linear, or nonlinear.

$$y = f(x)$$ \hspace{1cm} (9)

11. How conventional engineering equations are transformed to equations based on the proposed paradigm shift.

Conventional heat transfer equations are readily transformed to heat transfer equations based on the proposed paradigm shift by substituting $q/\Delta T$ for $h$ and/or $k/t$, then *separating* $q$ and $\Delta T$. For example, when $q$ and $\Delta T$ are separated, Eq. (10) is transformed to Eq. (11). Note that Eqs. (10) and (11) are *identical*—they state *exactly* the same thing.

$$U = (1/h_1 + t_{wall}/k_{wall} + 1/h_2)^{-1}$$ \hspace{1cm} (10)

$$\Delta T_{total} = \Delta T_1(q) + \Delta T_{wall}(q) + \Delta T_2(q)$$ \hspace{1cm} (11)

And similarly for other branches of engineering.

12. The laws of conventional engineering vs the laws based on the paradigm shift.

The laws of conventional engineering are analogs of Eq. (12) in which parameters such as $h$ (the symbol for $q/\Delta T$) and $E$ (the symbol for $\sigma/\varepsilon$) are analogs of $(y/x)$.

$$y = (y/x)x$$ \hspace{1cm} (12)

Equation (9) is *often* used in pure mathematics because it describes the relationship between $y$ and $x$ whether the relationship is proportional, linear, or nonlinear. Equation (12) is *never* used in pure mathematics because it describes the relationship between $y$ and $x$ *only* if $y$ is *proportional* to $x$. If $y$ is a *nonlinear* function of $x$, Eq. (12) does *not* describe the relationship between $y$ and $x$. It merely states that $y = y$. 
When Fourier created Eq. (8), he emphasized that it applies only if $q$ is proportional to $\Delta T$. However, sometime near the beginning the twentieth century, the heat transfer community decided to apply Eq. (8) even if $q$ is a nonlinear function of $\Delta T$. When that change took place, Eq. (8) should have been replaced by Eq. (13) in order to correctly indicate that the relationship between $q$ and $\Delta T$ may be proportional, linear, or nonlinear, and $h$ may be either a constant or a variable.

$$q = h[\Delta T]\Delta T$$  

(13)

Note that Eq. (13) correctly indicates that, if $q$ is a nonlinear function of $\Delta T$, $h$ is a third and extraneous variable that greatly complicates problem solutions. Also note that Eq. (9) never includes an extraneous third variable because it always includes only two variables.

13. Conclusions

Engineering science should be founded on the proposed paradigm shift because it results in equations and laws that are rational and much simpler, whereas the equations and laws of conventional engineering science are irrational and much more difficult because they include extraneous variables when applied to nonlinear phenomena. The proposed paradigm shift is described in Section 9. The engineering science that results from the proposed paradigm shift is described in Section 10.

Nomenclature

$a$ acceleration
$c$ arbitrary constant
$E$ modulus
$F$ force
$h$ $q/\Delta T$
$m$ mass
$q$ heat flux
$T$ temperature
$t$ time
$x$ arbitrary
$y$ arbitrary
$\varepsilon$ strain
$\sigma$ stress

References


