Why conventional engineering laws are *irrational*, and a *paradigm shift* that results in *rational* laws.

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Abstract
Until 1822, engineers and scientists agreed that equations *cannot* describe how parameters are related because parameter dimensions *cannot* be multiplied or divided. In 1822, Fourier claimed (*without proof)* that equations *can* rationally describe how parameters are related because parameter dimensions *can* be multiplied or divided, and dimensions *can* be assigned to numbers. Fourier's *unproven* claims are the *only* reason that, since 1822, equations have been used to describe how engineering parameters are related. However, for more than 70 years, it has been widely agreed that dimensions must *not* be assigned to numbers. Because parameters such as *h* and *E* were *created* by assigning dimensions to *numbers,* they are *irrational*, and equations in which they appear should be *abandoned*. The proposed paradigm shift *requires* that parameter symbols represent *only* numerical values, and results in engineering laws that are analogs of $y = f\{x\}$. The new laws state that the *numerical value* of parameter *y* is a function of the *numerical value* of parameter *x*, and the function may be proportional, linear, or nonlinear. Because parameter symbols represent *only* numerical value, *all* proportions and equations are *dimensionally homogeneous* because they are *inherently* dimensionless. If an equation is *quantitative*, the dimension units that underlie parameter symbols *must* be specified in an *accompanying nomenclature*. The proposed paradigm shift results in a *rational* engineering science that is *much* easier to learn and apply because *irrational* parameters such as *h* and *E* are *abandoned*. They are *not* replaced because they are *not* necessary.

Key words: Dimensional homogeneity, dimensions, irrational equations, irrational laws, Newton's laws, paradigm shift, parameters, proportions, rational equations, rational laws.

1. Introduction

Until 1822, it was generally agreed that equations *cannot* describe how parameters are related because parameter dimensions *cannot* be multiplied or divided. In 1822, Fourier [1] claimed that equations *can* rationally describe how parameters are related because:

- Parameter dimensions *can* be multiplied or divided.

- Dimensions *can* rationally be assigned to *numbers.*

These *revolutionary* claims made it possible *for the first time in history* to create dimensionally homogeneous equations that quantitatively describe how parameters are related.

However, since sometime before 1951, it has been widely agreed that dimensions must *not* be assigned to numbers. Therefore *all* conventional engineering laws and equations that include parameters created by *assigning dimensions to numbers* (such as h and E) are *irrational*, and should be *replaced* by the *rational* laws and equations that result from the paradigm shift.

The proposed paradigm shift *requires* that parameter symbols in equations represent *only* numerical values, and results in laws that are analogs of Eq. (1) in which symbols represent *only* the *numerical values* of parameters.

$$y = f\{x\} \tag{1}$$

These laws make it *much simpler* to learn and apply engineering science because *all* parameters (such as h and E) created by *assigning dimensions to numbers* are *abandoned*. They are *not* replaced because they are *not* necessary.

2. Dimensional homogeneity until 1822.

Until 1822, scientists and engineers such as Galileo and Newton agreed that:

- Parameter symbols in proportions and equations represent numerical value *and* dimension.

- Parameters *cannot* be multiplied or divided because parameter dimensions *cannot* be multiplied or divided.[1]

- Because equations generally *require* that parameters be multiplied or divided, and because parameter dimensions *cannot* be multiplied or divided, equations *cannot* describe how parameters are related.

- Proportions need *not* be dimensionally homogeneous.

- Equations *must* be dimensionally homogeneous.

Because proportions need *not* be dimensionally homogeneous, and because proportions that relate two parameters do *not* require that parameters be multiplied or divided, proportions are generally used instead of equations. That is why Hooke's [2] law is Proportion (1) instead of an equation, Newton's [3] law of cooling[2] is Proportion (2) instead of Eq. (3), and Newton's [4] second law of motion is Proportion (4) instead of Eq. (5).

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[1] However, a dimension *can* be divided by the *same* dimension. For example, meters *can* be divided by meters, and seconds *can* be divided by seconds, but meters *cannot* be divided by seconds. In the qualitative equations that result, all terms are dimensionless ratios. This methodology was used by Galileo.

[2] American heat transfer texts generally refer to Eq. (3) as "Newton's law of cooling", and claim that Newton created h. However, Eq. (3) *cannot* be Newton's law of cooling because cooling is a *transient* phenomenon, and Eq. (3) is a *steady-state* equation. Also because Eq. (3) *requires* that h and ΔT be multiplied, whereas in Newton's time, it was *irrational* to multiply parameters. Newton could *not* have created h because he could *not* rationally have multiplied h times another parameter.

$$\sigma \,\alpha\, \varepsilon \tag{1}$$

$$(dT_{body}/dt) \,\alpha\, (T_{air} - T_{body}) \tag{2}$$

$$q = h\Delta T \tag{3}$$

$$a \,\alpha\, f \tag{4}$$

$$f = ma \tag{5}$$

3. Fourier's heat transfer experiment, and the proportion and equation that resulted.

Fourier performed a heat transfer experiment in which a warm, solid body is cooled by the steady-state forced convection of ambient air. Fourier concluded that Proportion (6) and Eq. (7) correlate the data.

$$q \,\alpha\, \Delta T \tag{6}$$

$$q = c\Delta T \tag{7}$$

Newton and his colleagues would have been satisfied by Proportion (6), but it did *not* satisfy Fourier because he wanted an *equation*, and it *had* to be dimensionally homogeneous. Equation (7) did *not* satisfy Fourier because it is *not* dimensionally homogeneous.

4. Fourier's *revolutionary* and *unproven* view of dimensional homogeneity that enabled him to transform *in*homogeneous Eq. (7) to *homogeneous* Eq. (8).

Fourier recognized that Eq. (7) could be transformed to a dimensionally homogeneous equation *only* if it were rational to *assign* dimensions to *number c* in Eq. (7), *and* rational to *multiply and divide* parameter dimensions. Fourier [1] describes his *revolutionary* and *unproven* view of dimensional homogeneity in the following:

. . . every undetermined magnitude or constant has one dimension proper to itself, and the terms of one and the same equation could not be compared, if they had not the same exponent of dimension. . . this consideration is derived from primary notions on quantities; for which reason, in geometry and mechanics, it is the equivalent of the fundamental lemmas which the Greeks have left us ***without proof****.*

It is important to note that, in Fourier's nearly 500 page treatise, *The Analytical Theory of Heat* [1], he made *no effort* to prove that his view of dimensional homogeneity is rational. He did *not* include the Greek lemmas, he did *not* cite a reference where the Greek lemmas could be found, and he did *not* include his own proof.

Fourier's *revolutionary* view of dimensional homogeneity includes the following *unproven* claims:

- Parameter *dimensions can* be multiplied or divided.

- Dimensions *can* be assigned to *numbers*.

In accordance with his *revolutionary* and *unproven* view of dimensional homogeneity, Fourier *assigned* the symbol h and the dimension of $q/\Delta T$ to *number* c in Eq. (7), then *multiplied* h and ΔT, resulting in dimensionally homogeneous Eq. (8).

$$q = h\Delta T \tag{8}$$

5. The definition of *h*.

American heat transfer texts generally do not define h. Nomenclatures in heat transfer texts generally state only that h is "heat transfer coefficient". However, rearranging Eq. (8) results in Eq. (9).

$$h = q/\Delta T \tag{9}$$

Equations (8) and (9) state that h and $q/\Delta T$ are *identical* and *interchangeable*. They also state that h is a symbol for the *dimensional group* $q/\Delta T$.

Combining Eqs. (8) and (9) results in Eq. (10). Note that Eqs. (8) and (10) are *identical* because h and $q/\Delta T$ are *identical* and *interchangeable*.

$$q = (q/\Delta T)\Delta T \tag{10}$$

The nomenclature in *every* conventional heat transfer text should state "h is a symbol for the dimensional group $q/\Delta T$—*i.e.* h and $q/\Delta T$ are *identical* and *interchangeable*".

6. What Eq. (8) meant in most of the nineteenth century.

In most of the nineteenth century, Eq. (8) was a *proportional* equation, and h was a proportionality *constant*. Fourier warned that Eq. (8) applies *only* if a solid, warm body is cooled by the *steady-state forced convection* of *ambient air*. He *emphasized* that Eq. (8) does *not* apply if a solid, warm body is cooled by the *natural convection* of ambient air because the coolant flow rate would *vary*, and consequently the relationship between q and ΔT would *not* be proportional.

7. What Eq. (8) has meant since sometime near the end of the nineteenth century.

Sometime near the end of the nineteenth century, the heat transfer community decided to ignore Fourier's warning that Eq. (8) applies *only* if the heat transfer behavior is proportional. It decided to apply Eq. (8) even if the relationship between q and ΔT is *nonlinear*.

When Eq. (8) is applied to *nonlinear* heat transfer phenomena, it is *not* an equation because a *proportional* equation *cannot* describe *nonlinear* behavior. Even though Eq. (8) is a proportional equation, it *must now* be interpreted to mean that the relationship between q and ΔT *may* be proportional, linear, or nonlinear, and h *may* be a *constant* or a *variable*.

8. The equation that *should* have replaced Eq. (8) when it began to be applied to *nonlinear* phenomena.

When the decision was made to apply Eq. (8) to nonlinear phenomena, Eq. (8) *should* have been *abandoned* because it obviously *cannot* describe *nonlinear* behavior. Equation (8) should have been replaced by Eq. (11) because it correctly states that the relationship between q and ΔT may be *proportional, linear, or nonlinear,* and h may be a *constant* or a *variable*.

$$q = h\{\Delta T\}\Delta T \tag{11}$$

Note that Eqs. (11) and (11a) are *identical.* They *both* state that q is a function of ΔT, and the function may be proportional, linear, or nonlinear.

$$q = f\{\Delta T\} \tag{11a}$$

However, Eq. (11a) could *not* rationally have replaced Eq. (8) because, based on conventional parameter symbolism, Eq. (11a) is *not* dimensionally homogeneous.

9. Why conventional engineering laws are mathematically *undesirable*.

Substituting $q/\Delta T$ for h in Eq. (11) results in Eq. (12).

$$q = (q/\Delta T)\{\Delta T\}\Delta T \tag{12}$$

Equation (12) is a *rigorously correct* expression of the modern law of convective heat transfer. Note that Eq. (12) is an analog of Eq. (13), and $q/\Delta T$ (i.e. h) is an analog of $(y/x)\{x\}$.

$$y = (y/x)\{x\}x \tag{13}$$

In mathematics, if y is a *nonlinear* function of x, Eq. (13) is *never* used because $(y/x)\{x\}$ is a *third* variable, and it *greatly* complicates problem solutions. Equation (14) is *always* used because it *always* has only *two* variables, and therefore it *always* allows nonlinear problems to be solved in the simplest possible way—ie with y and x *separated* rather than *combined* in an analog of $(y/x)\{x\}$.

$$y = f\{x\} \tag{14}$$

Laws such as Eqs. (8), (10), (11), and (12) are *mathematically undesirable* because, if q is a nonlinear function of ΔT, they include *three* variables (q, $q/\Delta T$, and ΔT) to describe the relationship between *two* variables (q and ΔT). And similarly for *all proportional* engineering laws that are applied to *nonlinear* phenomena.

10. Proof that Fourier was *wrong*. Dimensions *cannot* rationally be assigned to numbers.

Langhaar [5] explains why dimensions *cannot* rationally be assigned to numbers:

*Dimensions **must not** be assigned to numbers, for then any equation could be regarded as dimensionally homogeneous.*

11. Proof that Fourier was *wrong*. Equations *cannot* describe how parameters are related because parameter dimensions *cannot* rationally be multiplied or divided.

Conventional engineering laws and equations are based in part on Fourier's *unproven* claim that parameters *can* be multiplied or divided because parameter *dimensions can* be multiplied or divided. Fourier was *wrong*. As demonstrated by the following, parameter dimensions *cannot* rationally be multiplied or divided.

"Multiply four times seven" means "add seven four times". Therefore "multiply meters times kilograms" *must* mean "add kilograms meters times". Because "add kilograms meters times" has *no meaning*, dimensions *cannot* be multiplied.

"Divide forty by five" means "how many fives are in forty". Therefore "divide meters by seconds" *must* mean "how many seconds are in meters". Because "how many seconds are in meters" has *no meaning*, dimensions *cannot* be divided. If meters could be divided by seconds, it would be possible to determine how many seconds are in meters.

12. Irrational views in conventional engineering.

The following are irrational views in conventional engineering.

- Hooke's law, Proportion (15), *and* Young's law, Eq. (16), are *identical*. They *both* state that stress equals a constant times strain. Therefore it is *irrational* to *require* Young's law to be dimensionally homogeneous, and *not* require Hooke's law to be dimensionally homogeneous.

$$\sigma \, \alpha \, \varepsilon \tag{15}$$

$$\sigma = E_{elastic}\varepsilon \tag{16}$$

- A chart of q vs ΔT is a picture of Eq. (17).

$$q = f\{\Delta T\} \tag{17}$$

It is *irrational* to *reject* Eq. (17) because it is *not* dimensionally homogeneous, and to *accept* a *chart* of Eq. (17) that is *not* dimensionally homogeneous.

- Charts are *dimensionless* because they describe how the *numerical value* of parameter group y is related to the *numerical value* of parameter group x. If a chart is to be *quantitative*, the dimension units that underlie parameter groups x and y *must* be specified on the chart, or in an accompanying nomenclature.

Because charts are pictures of equations, and because charts are *dimensionless*, it is *irrational* to *not* have equations in which parameter symbols are *dimensionless*.

The above *irrational* views have *no place* in the engineering science that results from the paradigm shift.

13. What equations *can* rationally describe about how parameters are related.

Equations *can* rationally describe how the *numerical values* of parameters are related because equations are *inherently* dimensionally homogeneous if parameter symbols represent *only* numerical value. If a dimensionless equation is *quantitative*, the dimension units that underlie parameter symbols (that are *not* in dimensionless groups) *must* be specified in an accompanying nomenclature.

14. The proposed paradigm shift.

The proposed paradigm shift requires that parameter symbols in proportions and equations represent *only* numerical values.

15. The engineering science that results from the proposed paradigm shift.

The engineering science that results from the proposed paradigm shift is described by:

- *All* proportions and equations are *dimensionless* because *all* parameter symbols represent *only* numerical value.

- *All* proportions and equations are *dimensionally homogeneous* because they are dimensionless.

- If an equation is *quantitative*, the dimension units that underlie parameter symbols are specified in an accompanying nomenclature (except for symbols in dimensionless groups).

- *All* engineering laws are replaced by analogs of Eq. (18) which states that the *numerical value* of parameter *y* is a function of the *numerical value* of parameter *x*, and the function may be proportional, linear, or nonlinear.

$$y = f\{x\} \tag{18}$$

- There are *no* parameters that were created by assigning dimensions to numbers.

16. How to transform conventional texts to texts based on the proposed paradigm shift.

To transform conventional engineering texts to texts based on the proposed paradigm shift:

- Replace laws with analogs of $y = f\{x\}$.

- In equations that include analogs of *(y/x)*, replace analogs with *y/x*, then separate *x* and *y*.

For example, to transform Eq. (19) to a paradigm shift equation, replace *h* and k_{wall}/t_{wall} with $q/\Delta T$, then separate *q* and ΔT, resulting in Eq. (20).

$$U = (1/h_1 + t_{wall}/k_{wall} + 1/h_2)^{-1} \tag{19}$$

$$\Delta T_{total} = \Delta T_1\{q\} + \Delta T_{wall}\{q\} + \Delta T_2\{q\} \tag{20}$$

It is important to note that Eqs. (19) and (20) are *identical*. They mean *exactly the same thing*. Equation (19) is written in the *opaque* language of conventional engineering. Equation (20) is written in the *transparent* language of the proposed paradigm shift. (Convection heat transfer correlations are generally in the form $\Delta T\{q\}$ because that is the form required by Eq. (20.))

Textbooks for other branches of engineering are transformed to paradigm shift texts in the same way heat transfer texts are transformed—i.e. by replacing conventional laws with laws that are analogs of Eq. (14), and transforming equations by *separating* *x* and *y*.

17. How data are correlated in conventional engineering, and in engineering based on the proposed paradigm shift.

Experimenters *cannot* obtain *h* data or *E* data because there is *no such thing* as *h or E*. They are *symbols* for the *dimensional groups* $q/\Delta T$ and σ/ε.

In conventional engineering, experimenters obtain *q* data and ΔT data, and use it to determine $q/\Delta T$ values and $(q/\Delta T)\{\Delta T\}$ correlations—ie to determine *h* values and $h\{\Delta T\}$ correlations.

In engineering based on the proposed paradigm shift, experimenters obtain *q* data and ΔT data, and use it to determine $\Delta T\{q\}$ correlations. And similarly for other engineering branches.

18. Correlation transformations and experiments.

It is important to note that the proposed paradigm shift does *not* require that experiments that resulted in *current* correlations be *repeated*. It requires merely that current correlations be transformed *analytically* as described in Section 15, or that the data that resulted in current correlations be used to determine correlations that are analogs of $y = f\{x\}$.

20. Conclusions

Conventional engineering science works well when applied to problems that concern *proportional* behavior because it is founded on laws that are *proportional* equations, and the coefficients in the laws (such as *h* and *E)* are *proportionality constants*. It does *not* work well when applied to problems that concern *nonlinear* behavior because the coefficients in the laws (such as *h* and *E*) are *extraneous variables,* and they *greatly* complicate problem solutions.

Engineering science should be founded on the proposed paradigm shift because it results in laws that work well with *all* forms of behavior—proportional, linear, and nonlinear.

Nomenclature

a acceleration
c arbitrary constant
E modulus
F force
h $q/\Delta T$
k $q/(dT/dx)$
m mass
q heat flux
T temperature
t time or wall thickness
x arbitrary variable
y arbitrary variable
ε strain
σ stress

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