Newton was *right*—equations *cannot* rationally describe how parameters are related. Rational equations *require* a paradigm shift.

Eugene F. Adiutori
Ventuno Press
efadiutori@aol.com
1-239-537-4107
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Abstract
Until 1822, engineers and scientists such as Newton agreed that equations *cannot* describe how parameters are related because parameter *dimensions cannot* be multiplied or divided. (That is why Newton's laws were initially *proportions* instead of *equations*.) In 1822, Fourier claimed (*without proof*) that dimensions *can* be multiplied and divided, and parameters such as *h* and *E* *can* be *created* by assigning dimensions to numbers. Fourier's *unproven* claims are the *only* reason that, since 1822, it has been widely agreed that equations *can* describe how parameters are related. However, critical analysis in the text proves that dimensions *cannot* be multiplied and divided. Also, it has for many years been generally agreed that dimensions must *not* be assigned to numbers. Therefore equations *cannot* describe how parameters are related, and equations must *not* include parameters such as *h* and *E*. The paradigm shift described in the text *requires* that parameter symbols in equations represent *only* numerical values, and *all* parameters created by assigning dimensions to numbers be *abandoned*. After the paradigm shift, *all* proportions and equations are *inherently* dimensionally homogeneous because they contain *only* numbers. *All* laws are analogs of the *rational* equation $y = f\{x\}$ which states that the *numerical value* of parameter *y* is a function of the *numerical value* of parameter *x*, and the function may be proportional, linear, or nonlinear. If an equation is *quantitative*, the dimension units that underlie parameter symbols (that are *not* in dimensionless groups) *must* be specified in an *accompanying* nomenclature.

Key words: Dimensional homogeneity, dimensions, irrational equations, irrational laws, Newton's laws, parameters, proportions, rational equations, rational laws.

1. Introduction

Until 1822, it was generally agreed that equations *cannot* describe how parameters are related because parameter *dimensions cannot* be multiplied or divided. In 1822, Fourier [1] made the revolutionary and *unproven* claims that dimensions *can* be multiplied and divided, and dimensions *can* be assigned to numbers. Therefore equations *can* describe how parameters are related, and laws *can* be made dimensionally homogeneous by *assigning* dimensions to *numbers*.

Critical analysis in the text proves that dimensions *cannot* be multiplied and divided, and therefore equations *cannot* describe how parameters are related.

Sometime before 1951, it was widely agreed that dimensions must *not* be assigned to numbers. Therefore modern engineering laws are *irrational* for *two* reasons: because parameter dimensions in the laws *cannot* be multiplied or divided, *and* because coefficients in the laws (such as *h, E,* and *R*) were created by *assigning* dimensions to *numbers*.

The paradigm shift described in the text results in *rational* equations that describe how the *numerical values* of parameters are related, and *rational* laws that are analogs of $y = f\{x\}$. It also results in the abandonment of *all* parameters (such as *h, E,* and *R*) that were *created* by *assigning* dimensions to *numbers*.

2. Dimensional homogeneity until 1822.

Until 1822, scientists and engineers such as Newton and his colleagues agreed that:

- Parameter symbols in proportions and equations represent numerical value *and* dimension.
- Proportions need *not* be dimensionally homogeneous.
- Equations *must* be dimensionally homogeneous.
- Dimensions *cannot* be multiplied or divided.
- Because parameter dimensions *cannot* be multiplied or divided, equations that describe how parameters are related are *irrational*.

Because proportions need *not* be dimensionally homogeneous, and because proportions that relate two parameters do *not* require that parameter dimensions be multiplied or divided, proportions are generally used instead of equations. That is why Hooke's [2] law is Proportion (1) instead of an equation, Newton's [3] law of cooling[1] is Proportion (2) instead of Eq. (3), and Newton's [4] second law of motion is Proportion (4) instead of Eq. (5)[2].

$$\sigma \,\alpha\, \varepsilon \tag{1}$$

$$(dT_{body}/dt) \;\alpha\; (T_{air} - T_{body}) \tag{2}$$

$$q = h\Delta T \tag{3}$$

$$a \,\alpha\, f \tag{4}$$

$$f = ma \tag{5}$$

[1] American heat transfer texts generally refer to Eq. (3) as "Newton's law of cooling". However, Eq. (3) *cannot* be Newton's law of cooling because cooling is a transient phenomenon whereas Eq. (3) is a steady-state equation, and because Eq. (3) was *irrational* in Newton's time.

[2] When Proportion (4) was transformed to Eq. (5), the transformed version should have been $a = f/m$ in order to correctly indicate that force is the independent variable, in accordance with Newton's Proportion (4).

3. Fourier's heat transfer experiment, and the proportion and equation that resulted.

Fourier performed a heat transfer experiment in which a warm, solid body is cooled by the steady-state forced convection of ambient air. Fourier concluded that Proportion (6) and Eq. (7) correlated the data.

$$q \propto \Delta T \tag{6}$$

$$q = c\Delta T \tag{7}$$

Newton and his colleagues would have been satisfied by Proportion (6), but it did *not* satisfy Fourier because he wanted an *equation*, and it *had* to be dimensionally homogeneous. Equation (7) did *not* satisfy Fourier because it is *not* dimensionally homogeneous.

4. How Fourier transformed dimensionally *in*homogeneous Eq. (7) to dimensionally homogeneous Eq. (8).

Fourier recognized that Eq. (7) could be transformed to a dimensionally homogeneous equation *only* if it were rational to *assign* dimensions to *number c* in Eq. (7), *and* rational to *multiply and divide* parameter dimensions. Fourier [1] describes his revolutionary and *unproven* view of dimensional homogeneity in the following:

*. . . every undetermined magnitude or constant has one dimension proper to itself, and the terms of one and the same equation could not be compared, if they had not the same exponent of dimension. . . this consideration is derived from primary notions on quantities; for which reason, in geometry and mechanics, it is the equivalent of the fundamental lemmas which the Greeks have left us **without proof**.*

The above claims that it is *rational* to assign dimensions to numbers, and *rational* to multiply or divide parameter dimensions. Note that Fourier points out that there is ***no proof*** that his revolutionary view of dimensional homogeneity is rational. In accordance with his *unproven* view of dimensional homogeneity, Fourier *assigned* the symbol h and the dimension of $q/\Delta T$ to *number c* in Eq. (7), and multiplied h and ΔT, resulting in dimensionally homogeneous Eq. (8).

$$q = h\Delta T \tag{8}$$

Even though Fourier's *unproven* view of dimensional homogeneity has *never* been proven, it is a fundamental tenet in modern engineering science. It is the *only* reason that, since 1822, it has been widely agreed that Newton and his colleagues were *wrong*—equations *can* describe how parameters are related because parameter dimensions *can* be multiplied or divided.

5. Proof that equations *cannot* describe how parameters are related because parameter *dimensions cannot* be multiplied or divided.

"Multiply four times seven" means "add seven four times". Therefore "multiply meters times kilograms" *must* mean "add kilograms meters times". Because "add kilograms meters times" has *no meaning*, dimensions *cannot* be multiplied.

"Divide forty by five" means "how many fives are in forty". Therefore "divide meters by seconds" *must* mean "how many seconds are in meters". Because "how many seconds are in meters" has *no meaning*, dimensions *cannot* be divided.

The above proves that parameter dimensions *cannot* be multiplied or divided. Therefore Newton and his colleagues were *right*—equations *cannot* describe how parameters are related.

6. Why dimensions *must not* be assigned to numbers.

Langhaar [5] explains why dimensions must *not* be assigned to numbers:

Dimensions ***must not*** *be assigned to numbers, for then any equation could be regarded as dimensionally homogeneous.*

7. What equations *can* describe about how parameters are related.

Equations *can* describe *only* how the *numerical values* of parameters are related. If an equation is *quantitative*, the dimension units that underlie parameter symbols (that are not in dimensionless groups) *must* be specified in an accompanying nomenclature.

8. What charts *can* describe about how parameters are related.

Charts can describe *only* how the *numerical values* of parameters are related. If a chart is *quantitative*, the dimension units on each axis *must* be specified on the chart, or in an accompanying nomenclature.

9. The proposed paradigm shift.

The proposed paradigm shift requires the following:

- Parameter symbols in proportions and equations *must* represent *only* numerical values.

- Parameters (such as *h, E,* and *R*) that were *created* by *assigning* dimensions to *numbers must* be *abandoned*.

- Engineering laws *must* be analogs of Eq. (9) which states that the *numerical value* of parameter y is a function of the *numerical value* of parameter x, and the function may be proportional, linear, or nonlinear.

$$y = f\{x\} \tag{9}$$

- If an equation is quantitative, the dimension units that underlie parameter symbols (that are *not* in dimensionless groups) *must* be specified in an *accompanying* nomenclature.

10. The engineering science that results from the proposed paradigm shift.

The engineering science that results from the proposed paradigm shift is described by the following:

- Parameter symbols in proportions and equations represent *only* numerical values.
- *All* proportions and equations are *inherently* dimensionally homogeneous.
- There are *no* parameters that were *created* by *assigning* dimensions to numbers.
- If an equation is quantitative, the dimension units that underlie parameter symbols (that are *not* in dimensionless groups) are specified in an *accompanying* nomenclature.
- *All* laws are analogs of Eq. (9) which states that the *numerical value* of parameter *y* is a function of the *numerical value* of parameter *x*, and the function may be proportional, linear, or nonlinear.

$$y = f\{x\} \tag{9}$$

11. How conventional engineering equations are transformed to equations based on the proposed paradigm shift.

Conventional heat transfer equations are readily transformed to heat transfer equations based on the proposed paradigm shift by substituting $q/\Delta T$ for h and/or k/t, then *separating* q and ΔT. For example, when q and ΔT are separated, Eq. (10) is transformed to Eq. (11). Note that Eqs. (10) and (11) are *identical*—they state *exactly* the same thing.

$$U = (1/h_1 + t_{wall}/k_{wall} + 1/h_2)^{-1} \tag{10}$$

$$\Delta T_{total} = \Delta T_1\{q\} + \Delta T_{wall}\{q\} + \Delta T_2\{q\} \tag{11}$$

And similarly for other branches of engineering.

12. The laws of conventional engineering vs the laws based on the paradigm shift.

The laws of conventional engineering are analogs of Eq. (12) in which parameters such as h (the symbol for $q/\Delta T$) and E (the symbol for σ/ε) are analogs of (y/x).

$$y = (y/x)x \tag{12}$$

Equation (9) is *often* used in pure mathematics because it describes the relationship between y and x whether the relationship is proportional, linear, or nonlinear. Equation (12) is *never* used in pure mathematics because it describes the relationship between y and x *only* if y is *proportional* to x. If y is a *nonlinear* function of x, Eq. (12) does *not* describe the relationship between y and x. It merely states that $y = y$.

When Fourier created Eq. (8), he *emphasized* that it applies *only* if q is proportional to ΔT. However, sometime near the beginning the twentieth century, the heat transfer community decided to apply Eq. (8) even if q is a *nonlinear* function of ΔT. When that change took place, Eq. (8) *should* have been replaced by Eq. (13) in order to correctly indicate that the relationship between q and ΔT may be proportional, linear, or nonlinear, and h may be either a constant or a *variable*.

$$q = h\{\Delta T\}\Delta T \tag{13}$$

Note that Eq. (13) *correctly* indicates that, if q is a *nonlinear* function of ΔT, h is a *third* and *extraneous variable* that greatly complicates problem solutions. Also note that Eq. (9) *never* includes an extraneous *third* variable because it *always* includes only *two* variables.

13. Conclusions

Engineering science should be founded on the proposed paradigm shift because it results in equations and laws that are rational and *much* simpler, whereas the equations and laws of conventional engineering science are irrational and *much* more difficult because they include *extraneous variables* when applied to nonlinear phenomena. The proposed paradigm shift is described in Section 9. The engineering science that results from the proposed paradigm shift is described in Section 10.

Nomenclature

a acceleration
c arbitrary constant
E modulus
F force
h $q/\Delta T$
m mass
q heat flux
T temperature
t time
x arbitrary
y arbitrary
ε strain
σ stress

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