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**Bounding Surface Elasto-Viscoplasticity: A General Constitutive
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Bounding Surface Elasto-Viscoplasticity: A General Constitutive Framework for Rate-Dependent Geomaterials

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ABSTRACT

A general framework is proposed to incorporate rate and time effects into bounding surface (BS) plasticity models. For this purpose, the elasto-viscoplasticity (EVP) overstress theory is combined with bounding surface modeling techniques. The resulting constitutive framework simply requires the definition of an overstress function through which BS models can be augmented without additional constitutive hypotheses. The new formulation differs from existing rate-dependent bounding surface frameworks in that the strain rate is additively decomposed into elastic and viscoplastic parts, much like classical viscoplasticity. Accordingly, the proposed bounding surface elasto-viscoplasticity (BS-EVP) framework is characterized by two attractive features: (1) the rate-independent limit is naturally recovered at low strain rates; (2) the inelastic strain rate depends exclusively on the current state. To illustrate the advantages of the new framework, a particular BS-EVP constitutive law is formulated by enhancing the Modified Cam-clay model through the proposed theory. From a qualitative standpoint, this simple model shows that the new framework

is able to replicate a wide range of time/rate effects occurring at stress levels located strictly inside the bounding surface. From a quantitative standpoint, the calibration of the model for over-consolidated Hong Kong marine clays shows that, despite the use of only six constitutive parameters, the resulting model is able to realistically replicate the undrained shear behavior of clay samples with OCR ranging from 1 to 8, and subjected to axial strain rates spanning from 0.15%/hr to 15%/hr. These promising features demonstrate that the proposed BS-EVP framework represents an ideal platform to model geomaterials characterized by complex past stress history and cyclic stress fluctuations applied at rapidly varying rates.

Keywords: rate-dependent, bounding surface, elasto-viscoplasticity, geomaterials

INTRODUCTION

Classical plasticity is rooted in the definition of the yield surface, i.e., a threshold in stress space delineating the region where purely elastic strains are expected. Despite the intuitive appeal of such a simple conceptual scheme, a sharp distinction between elastic and plastic regimes is known to be inaccurate in that permanent strains taking place at stress levels located strictly inside the yield surface have been extensively documented for a wide variety of solids (Karsan and Jirsa 1969; Dafalias and Popov 1976; Banerjee and Stipho 1978; Banerjee and Stipho 1979; Scavuzzo et al. 1983; McVay and Taesiri 1985). Noticeable examples include the deformation responses of overconsolidated soils, the deterioration of yielding stress of metal subjected to unloading/reloading, and the deformation responses of cyclically loaded materials. Such “early” inelastic behavior has motivated the development of various constitutive frameworks (e.g., multi-surface kinematic hardening plasticity (Prévost 1977; Mróz et al. 1978; Mróz et al. 1981), sub-loading surface plasticity (Hashiguchi 1989; Hashiguchi and Chen 1998) and bounding surface plasticity (Dafalias and Popov 1975; Dafalias and Popov 1976; Dafalias 1986)). Bounding surface (BS) plasticity maps the current stress state to an image point on the so-called bounding surface, through which a plastic modulus is computed as a function of the plastic modulus at the image stress and the distance between the current and image stress points. Radial mapping BS plasticity (Dafalias 1981) represents a special class of bounding surface models, in which a projection center radially projects

the current stress to its corresponding image stress. By virtue of the use of a single function (i.e., the bounding surface) and the consequent mathematical simplicity, radial mapping BS plasticity has been used in a variety of constitutive models for history-dependent materials (e.g., Dafalias and Herrmann 1986; Bardet 1986; Pagnoni et al. 1992), and it will therefore be the focus of the following discussion.

Despite the wide development of rate-independent BS models, their rate-dependent counterparts have received little attention. One of the few examples in this direction is the pioneering work of Dafalias (1982), where a bounding surface elastoplasticity-viscoplasticity (BS-EP/VP) formulation was proposed, through which BS models originally conceived for inviscid materials were enhanced to replicate rate/time effects. Subsequently, this formulation was employed to represent the rate-dependent behavior of cohesive soils (Kaliakin and Dafalias 1990; Alshamrani and Sture 1998; Jiang et al. 2017). A fundamental assumption underlying the BS-EP/VP formulation is that the strain rate is additively decomposed into elastic and inelastic parts, with the latter being a summation of plastic (instantaneous) and viscoplastic (time-dependent) strain rates. In other words, plastic strain contributions are computed through rate-independent BS theory, whereas the viscoplastic strain rate is related to an overstress (Perzyna 1963), a distance measured between the current stress state and an elastic nucleus defined in stress space. An important advantage of defining rate-dependent BS models by enhancing existing inviscid baseline models is that the baseline deformation behavior can be recovered under special conditions. Accordingly, experimental observations which meet these conditions can be used to separate the calibration of material constants associated with the baseline models from those relevant to time/rate effects, thus significantly simplifying the calibration. For BS-EP/VP models, as a result of the adopted hypothesis on strain rate splitting, an inviscid baseline behavior is recovered when the loading rate tends to infinity. Accordingly, experiments allowing the above-mentioned parameter separation have to be performed under high enough loading rates, thereby posing considerable experimental challenges. Furthermore, laboratory tests conducted at high loading rates tend to exhibit heterogeneous stress-strain fields due to inertial effects and/or insufficient time for pore fluid drainage and pore pressure equalization (Kolsky 1949; Kimura and

78 [Saitoh 1983](#)). As a result, global measurements may not be reliably interpreted as elemental stress-
79 strain relations, thus posing major obstacles for model calibration. Further complications in the
80 use of BS-EP/VP models derive from the fact that it leads to potentially mesh-dependent numerical
81 solutions. When the current stress is located on the bounding surface and the loading rate is high
82 enough, the convergence to the rate-independent BS models allows the derivation of a tangent
83 constitutive tensor that relates strain rate and stress rate ([Dafalias 1982](#)). When this tangent moduli
84 satisfies certain bifurcation criteria, the quasi-static governing equations for incremental equilibrium
85 lose ellipticity, while under dynamic loading conditions the wave propagation velocity becomes
86 imaginary ([Hill 1962](#); [Rudnicki and Rice 1975](#); [Rice 1977](#); [Needleman 1988](#)). Consequently, even
87 if rate-dependence has been considered, mesh dependence associated with the numerical solutions
88 of localization problems cannot be fully ruled out by the BS-EP/VP framework, especially within
89 the dynamic regimes, where regularization is likely to become mandatory.

90 An alternative assumption for strain rate splitting is that the total strain rate is decomposed into
91 elastic and viscoplastic parts, which fundamentally states that the development of any irrecoverable
92 deformations requires a characteristic elapsed time. Such assumption is at the core of elasto-
93 viscoplasticity (EVP) overstress theories ([Perzyna 1963](#)) and it has two major differences compared
94 to BS-EP/VP models. First, EVP models converge to their rate-independent baseline models
95 when the loading rate tends to zero. Consequently, the parameters of inviscid models can be
96 independently calibrated from laboratory tests conveniently conducted at low loading rates, for
97 which global measurements can be more confidently interpreted as elemental constitutive responses.
98 Second, the plastic strain rate computed from the EVP overstress framework does not depend
99 on incremental quantities (e.g., strain/stress rates or rates of internal variables). This trait not
100 only significantly simplifies the model implementation, but also eliminates nonuniqueness and
101 regularizes pathological mesh dependence associated with numerical solutions of localization
102 under static and dynamic loading conditions ([Needleman 1988](#); [Needleman 1989](#); [Loret and](#)
103 [Prevost 1990](#)). Despite the foregoing advantages of the EVP overstress framework, its application
104 has been limited to standard elasto-plastic models (e.g., [Zienkiewicz and Corneau 1974](#); [Eisenberg](#)

and Yen 1981; Adachi and Oka 1982; Desai and Zhang 1987; Yin and Graham 1999; di Prisco and Imposimato 1996).

This work proposes a general framework to incorporate time/rate effects into bounding surface models based on the EVP overstress theory, which hereafter is referred to as bounding surface elastoviscoplasticity (BS-EVP) framework. The proposed framework enables existing rate-dependent BS models to be enhanced without introducing additional constitutive hypotheses, except the definition of an overstress function. In the remainder of this paper, a systematic description of the key attributes of the BS-EVP constitutive models is provided. The simulation capacities of the new framework are detailed with reference to a particular Cam-clay model based on the critical state theory, eventually discussing its performance in replicating the rate-dependent behavior of overconsolidated Hong Kong marine clays.

GENERAL FORMULATION OF RATE-INDEPENDENT BOUNDING SURFACE PLASTICITY

This section presents the general aspects of rate-independent bounding surface plasticity, thus providing the basis for its extension to the rate-dependent regime. Similar to classical elastoplasticity, bounding surface plasticity assumes that the strain rate can be additively decomposed into elastic and plastic parts:

$$\dot{\epsilon}_{ij} = \dot{\epsilon}_{ij}^e + \dot{\epsilon}_{ij}^p \quad (1)$$

where superscripts e and p stand for elastic and plastic, respectively, and the superposed dot indicates a time derivative. The computation associated with each strain component is discussed in the following two sections.

Elastic Response

The elastic strain rate is related to the stress rate by

$$\dot{\epsilon}_{ij}^e = C_{ijkl} \dot{\sigma}_{kl}; \quad \dot{\sigma}_{ij} = E_{ijkl} \dot{\epsilon}_{ij}^e \quad (2)$$

where the fourth order tensors C_{ijkl} and E_{ijkl} are the elastic compliance and stiffness moduli, respectively.

Plastic Response

Similar to classical plasticity, the plastic strain rate is computed according to a plastic flow rule, which can be written as follows:

$$\dot{\epsilon}_{ij}^p = \langle \Lambda \rangle R_{ij} \quad (3)$$

where the symbol $\langle \rangle$ indicates Macaully brackets such that $\langle x \rangle = x$ if $x \geq 0$ and $\langle x \rangle = 0$ if $x < 0$, R_{ij} denotes the direction of the plastic flow, and Λ is a plastic multiplier. To evaluate the plastic multiplier for an incremental loading path, a bounding surface is defined in stress space:

$$F(\bar{\sigma}_{ij}, q_n) = 0 \quad (4)$$

where q_n represents the plastic internal variables (PIVs) (Dafalias and Popov 1976) and the image stress $\bar{\sigma}_{ij}$ is the radial projection of the current stress σ_{ij} onto $F = 0$ (Fig. 1(a)). This radial mapping rule can be analytically expressed as

$$\bar{\sigma}_{ij} = b(\sigma_{ij} - \alpha_{ij}) + \alpha_{ij} \quad (5)$$

where α_{ij} is the projection center with respect to which the current stress state is mapped to the bounding surface. The loading direction at the current state is assumed to coincide with the gradient of the bounding surface at the image stress (L_{ij} in Fig. 1(a)):

$$L_{ij} = \frac{\partial F}{\partial \bar{\sigma}_{ij}} \quad (6)$$

The combination of this assumption with the radial mapping rule implies a loading surface ($f = 0$ in Fig. 1(a)), which passes through the current stress and is homothetic to the bounding surface with the projection center as the center of homothecy. The variable b in Eq. (5) can be further interpreted as the similarity ratio between the bounding surface and the loading surface. It varies from 1 to ∞ , with these two end conditions being attained when the current stress coincides with either the image stress ($b = 1$) or the projection center ($b = \infty$).

The plastic multiplier Λ is evaluated by

$$\Lambda = \frac{1}{K_p} L_{ij} \dot{\sigma}_{ij} \quad (7)$$

where K_p , the plastic modulus at the current stress, is a function of the plastic modulus at the image stress, $\bar{K}_p$, and the similarity ratio b . A convenient example of such an equation linking K_p to $\bar{K}_p$ and b was suggested by Dafalias (1986):

$$K_p = \bar{K}_p + \hat{H} \left\langle \frac{b}{b-1} - s \right\rangle^{-1} \quad (8)$$

where $\hat{H}$ is a positive shape hardening scalar function that may depend on stress state and PIVs, while the material constant $s \leq 1$ defines the size of the “elastic nucleus” (i.e., the region where bracketed terms become negative and thus Eq. (8) yields $K_p = \infty$). This domain of purely elastic response in stress space is again homothetic to the bounding surface with reference to the projection center, and when $s = 1$, the elastic nucleus degenerates to the projection point, thus modeling materials with vanishing elastic range. This “elastic nucleus” was used by Dafalias (1982) to define the overstress for calculating the viscoplastic strain rate in the BS-EP/VP framework discussed previously. The variable $\bar{K}_p$ in Eq. (8) is evaluated by enforcing the consistency condition at the bounding surface:

$$\bar{K}_p = -\frac{\partial F}{\partial q_n} \bar{q}_n \quad (9)$$

where $\bar{q}_n$ denotes the direction of the rate of PIVs and is specified by certain evolution rules, which can be written as follows:

$$\dot{q}_n = \langle \Lambda \rangle \bar{q}_n \quad (10)$$

The evolution of α_{ij} can be expressed in a similar form, thus treating the projection center as a particular PIV:

$$\dot{\alpha}_{ij} = \langle \Lambda \rangle \bar{\alpha}_{ij} \quad (11)$$

GENERAL FORMULATION OF BOUNDING SURFACE ELASTO-VISCOPLASTICITY

A general formulation of bounding surface elasto-viscoplasticity (BS-EVP) will be presented here, which is applicable to incorporate time/rate effects into inviscid bounding surface models. As stressed before, a fundamental assumption of the proposed BS-EVP framework is that the strain rate can be additively decomposed into elastic and viscoplastic parts, as follows:

$$\dot{\epsilon}_{ij} = \dot{\epsilon}_{ij}^e + \dot{\epsilon}_{ij}^{vp} \quad (12)$$

where the superscript vp stands for viscoplastic. Note that the elastic response is still governed by Eq. (2). In accordance with Perzyna's overstress theory, the viscoplastic strain rate and the rate of change of the internal variables can be related to a viscous nucleus function, as follows:

$$\begin{aligned} \dot{\epsilon}_{ij}^{vp} &= \langle \Phi \rangle R_{ij} \\ \dot{q}_n &= \langle \Phi \rangle \bar{q}_n \\ \dot{\alpha}_{ij} &= \langle \Phi \rangle \bar{\alpha}_{ij} \end{aligned} \quad (13)$$

where R_{ij} , $\bar{q}_n$ and $\bar{\alpha}_{ij}$ retain the same definitions previously provided with reference to the rate-independent framework. The viscous nucleus Φ is a function of the overstress, y , that satisfies the following requirements:

$$\begin{cases} \langle \Phi(y) \rangle = 0 & \text{if } y \leq 0 \\ \langle \Phi(y) \rangle > 0 & \text{if } y > 0 \end{cases} \quad (14)$$

This work employs a unique viscous nucleus expression to control the evolution of both viscoplastic strains and internal variables. In principle, however, different expressions of Φ could be hypothesized for each of these variables.

Static Loading Surface and Overstress

This work introduces a *static loading surface* ($f_s = 0$ in Fig. 1(b)) and uses the departure of the current stress σ_{ij} from $f_s = 0$ to define the overstress, y , such that

$$\begin{cases} y \leq 0 & \text{if } f_s(\sigma_{ij}) \leq 0 \\ y > 0 & \text{if } f_s(\sigma_{ij}) > 0 \end{cases} \quad (15)$$

Note that the static loading surface resembles the loading surface in the rate-independent framework, however it distinctly differs from the loading surface in two key aspects: (1) the static loading surface independently evolves as a function of viscoplastic deformations; (2) the current stress σ_{ij} is allowed to lie outside the static loading surface. The changes in size and location of the static loading surface are related to the evolution of the internal variables. Therefore, the combination of Eq. (13)-(15) implies that the static loading surface will tend to approach the current stress, when the viscous nucleus is not null, accompanied by delayed plasticity until when the current stress lies on the static loading surface. As a result, at low loading rates (i.e., when sufficient time is allowed for viscoplastic strains to develop), the current stress will tend to remain on the static loading surface. Consequently, to ensure convergence to the underlying bounding surface rate-independent behavior at low loading rates, the static loading surface ($f_s = 0$) and the loading surface ($f = 0$) in the underlying rate-independent models have to share the same analytical expression and evolution rules with plastic strains. This condition is here referred to as the *identity condition*. The analytical expression of f_s is obtained by inserting the radial mapping of Eq. (5) into $F = 0$ (i.e., Eq. (4)):

$$f_s = F([b_s(\sigma_{ij,s} - \alpha_{ij}) + \alpha_{ij}], q_n) = 0 \quad (16)$$

where the variable b_s replaces b in Eq. (5), denoting the homothetic ratio between the bounding surface and the static loading surface. In contrast to the variable b in rate-independent BS models, b_s has to be treated as an *independent* PIV, because the current stress is not required to lie on the static loading surface. Similar to other PIVs, the evolution of b_s can be related to the viscous

nucleus, as follows:

$$\dot{b}_s = \langle \Phi \rangle \bar{b}_s \quad (17)$$

where the expression of $\bar{b}_s$ has to be defined in agreement with the underlying bounding surface formulation (see next section). The variable $\sigma_{ij,s}$ in Eq. (16) denotes stress states laying on the static loading surface. A particular instance of $\sigma_{ij,s}$ (see Fig. 1(b)), which is here referred to as the static stress, is the radial projection of the current stress onto $f_s = 0$ by using α_{ij} as the projection center. As will be discussed in next section, this stress variable will replace the appearance of the current stress in the expression of $\bar{b}_s$.

Similar to the static loading surface, a dynamic loading surface (i.e., $f_d = 0$ in Fig. 1(b)) exists, which always passes through the current stress σ_{ij} and is homothetic to the bounding surface, with α_{ij} being the homothetic center. The dynamic loading surface can be written as follows:

$$f_d = F([b_d(\sigma_{ij} - \alpha_{ij}) + \alpha_{ij}], q_n) = 0 \quad (18)$$

where b_d denotes the similarity ratio between the bounding surface and the dynamic surface, and its value can be computed for any given stress state from Eqs. (4) and (5) simply by replacing b in Eq. (5) with b_d .

As the static stress and the current stress lie along the same radial projection (Fig. 1(b)), inserting b_s and the static stress $\sigma_{ij,s}$ or b_d and the current stress σ_{ij} into Eq. (5) yields the same image stress. Accordingly, $\sigma_{s,ij}$ can be related to the current stress σ_{ij} by knowing the values of b_s and b_d :

$$\sigma_{s,ij} = \frac{b_d}{b_s}(\sigma_{ij} - \alpha_{ij}) + \alpha_{ij} \quad (19)$$

where b_d/b_s can be further interpreted as the similarity ratio of the static loading surface over the dynamic loading surface, through which a normalized overstress can be defined as:

$$y = \frac{b_s}{b_d} - 1 \quad (20)$$

This overstress function satisfies the requirement in Eq. (15), in that when the current stress lies outside the static loading surface (i.e., the dynamic loading surface encloses the static loading surface), the ratio $b_s/b_d > 1$ and consequently $y > 0$.

Evolution Rule of the Internal Variables

To fulfill the identity condition, the evolution of the static loading surface with plastic strains has to be identical to that of the loading surface in the rate-independent models, and thus $\bar{b}_s$, $\bar{q}_n$ and $\bar{\alpha}_{ij}$ should be defined consistently with the hardening rules employed in inviscid baseline models (i.e., Eqs. (10) and (11)).

Since the value of b in rate-independent models is directly computed from the current stress σ_{ij} , an explicit definition of its evolution is not required. This rule, which also governs the evolution of b_s in the BS-EVP framework, can be derived by enforcing the consistency condition of the loading surface (i.e., Eq. (16) with b_s and $\sigma_{s,ij}$ replaced by b and σ_{ij}):

$$\dot{f} = \frac{\partial f}{\partial \sigma_{ij}} \dot{\sigma}_{ij} + \frac{\partial f}{\partial q_n} \dot{q}_n + \frac{\partial f}{\partial b} \dot{b} + \frac{\partial f}{\partial \alpha_{ij}} \dot{\alpha}_{ij} = 0 \quad (21)$$

Using the chain rule and the radial mapping of Eq. (5), the partial derivatives in Eq. (21) can be expressed as

$$\begin{aligned} \frac{\partial f}{\partial \sigma_{ij}} &= \frac{\partial F}{\partial \bar{\sigma}_{ij}} \frac{\partial \bar{\sigma}_{ij}}{\partial \sigma_{ij}} = b \frac{\partial F}{\partial \bar{\sigma}_{ij}} \\ \frac{\partial f}{\partial q_n} &= \frac{\partial F}{\partial q_n} \\ \frac{\partial f}{\partial b} &= \frac{\partial F}{\partial \bar{\sigma}_{ij}} \frac{\partial \bar{\sigma}_{ij}}{\partial b} = (\sigma_{ij} - \alpha_{ij}) \frac{\partial F}{\partial \bar{\sigma}_{ij}} \\ \frac{\partial f}{\partial \alpha_{ij}} &= \frac{\partial F}{\partial \bar{\sigma}_{ij}} \frac{\partial \bar{\sigma}_{ij}}{\partial \alpha_{ij}} = (1 - b) \frac{\partial F}{\partial \bar{\sigma}_{ij}} \end{aligned} \quad (22)$$

By substituting Eq. (22) into Eq. (21), and taking Eq. (7) into account, the following relation is obtained:

$$\left[\frac{\partial F}{\partial q_n} \dot{q}_n + (\sigma_{ij} - \alpha_{ij}) \frac{\partial F}{\partial \bar{\sigma}_{ij}} \dot{b} + (1 - b) \frac{\partial F}{\partial \bar{\sigma}_{ij}} \dot{\alpha}_{ij} \right] = -\langle \Lambda \rangle b K_p \quad (23)$$

Considering the rate equations of q_n and α_{ij} given in Eq. (10) and (11), respectively, and the definition of $\bar{K}_p$ given in Eq. (9), the evolution function $\bar{b}$ controlling the rate of the similarity ratio

b (i.e., $\dot{b} = \langle \Lambda \rangle \bar{b}$) can be expressed as:

$$\bar{b} = \left[-bK_p + \bar{K}_p - (1-b) \frac{\partial F}{\partial \bar{\sigma}_{ij}} \bar{\alpha}_{ij} \right] \left/ \left[(\sigma_{ij} - \alpha_{ij}) \frac{\partial F}{\partial \bar{\sigma}_{ij}} \right] \right. \quad (24)$$

Accordingly, the evolution rule of b_s is obtained by replacing b in Eq. (24) with b_s . Moreover, the current stress σ_{ij} appearing in the expression needs to be replaced by another stress variable, because $f_s = 0$ evolves independently of the current stress. This stress needs to be located on $f_s = 0$ and coincide with the current stress σ_{ij} when the loading rate is sufficiently low (i.e., rate-independent BS models are recovered). The stress that satisfies the above condition is the static stress $\sigma_{ij,s}$ introduced in previous section. Lastly, the image stress appearing in the expression of $\bar{K}_p$ is readily obtained by inserting b_s and $\sigma_{ij,s}$ into Eq. (5).

When the static loading surface expands to be coincident with the bounding surface under monotonic loading (i.e., $b_s = 1$ and consequently $K_p = \bar{K}_p$ by recalling Eq. (8)), Eq. (24) yields $\dot{b}_s = 0$, thus implying that the static loading surface will be identical to the bounding surface throughout the entire loading path. In other words, in the above mentioned circumstance the overstress is simply measured with respect to the bounding surface, and the formulation converges to a classical viscoplastic model where the yield surface is used to compute the overstress appearing in the viscoplastic flow rules.

A key feature of the proposed framework that should be emphasized is that the rate equations of the internal variables are fully defined from information derived from the rate-independent baseline models, thus allowing a straightforward incorporation of rate effects into existing bounding surface models without introducing additional hypotheses.

Stress Reversal and Relocation of Static Loading Surface

Combination of the viscoplastic strain rate equation of Eq. (13) and the viscous nucleus of Eq. (14), leads to vanishing viscoplastic strain rate once the stress state is on or inside the static loading surface $f_s = 0$. Consequently, any stress reversal that brings σ_{ij} back into the interior of $f_s = 0$ will result in purely elastic response until σ_{ij} moves outside $f_s = 0$. The latter

feature, however, differs from rate-independent bounding surface plasticity, which can resume the development of irrecoverable deformations after stress reversals as soon as the loading index $L_{ij}\dot{\sigma}_{ij}^{trial}$ becomes positive, where L_{ij} is the loading direction defined in Eq. (6) and $\dot{\sigma}_{ij}^{trial}$ is the trial stress rate, evaluated by ignoring plastic deformation rate (i.e., $\dot{\sigma}_{ij}^{trial} = E_{ijkl}\dot{\epsilon}_{kl}$). This intrinsic difference results from the absence in the proposed BS-EVP framework of the loading/unloading criterion typical of rate-independent plasticity. The lack of such criterion prevents BS-EVP models from converging to their inviscid counterparts upon stress paths other than monotonic loading. This inconvenience can be removed by devising a relocation of the static loading surface. Specifically, it is here proposed to apply a relocation of the static loading surface $f_s = 0$ whenever the following two conditions are met simultaneously: (i) the current stress σ_{ij} is within $f_s = 0$; (ii) the above-mentioned loading index attains positive values (i.e., $L_{ij}\dot{\sigma}_{ij}^{trial} > 0$). As depicted in Fig. 2 (b), after relocation, the static loading surface will pass through the current stress while its homothecy to the bounding surface with respect to the projection center remains preserved. After the surface relocation, the trial stress rate points outwards with respect to $f_s = 0$ and plastic deformation consequently starts to develop.

A theoretical interpretation of the above surface relocation can be made in light of the concept of “plastic equilibrium state”. Eisenberg and Yen (1981) defines the stress-strain relation obtained under sufficiently slow loading rates as a series of equilibrium states, such that each increment of stress is applied after the total plastic strain due to the previous stress increment has time to develop fully. Within the context of the BS-EVP framework, materials achieve a plastic equilibrium state once the current stress returns back to the static loading surface (i.e., all delayed plasticity has been fully developed). Stress reversal from this equilibrium state then will temporarily shut off plasticity, resulting in only elastic deformations. A new plastic equilibrium state (i.e., a static loading surface passing through the current stress) is however reconstructed once plastic loading is reactivated, which is here signaled by a positive loading index.

SPECIALIZATION OF BS-EVP FRAMEWORK TO MODIFIED CAM-CLAY

This section presents a simple bounding surface model based on critical state theories for

clays and discusses its enhancement to incorporate rate/time effects using the proposed BS-EVP framework. For simplicity, the model will be discussed with reference to triaxial stress conditions, in which the mean effective stress $p = (\sigma_a + 2\sigma_r)/3$ and deviatoric stress $q = \sigma_a - \sigma_r$ are stress measures, and the volumetric strain $\epsilon_v = \epsilon_a + 2\epsilon_r$ and deviatoric strain $\epsilon_d = 2(\epsilon_a - \epsilon_r)/3$ are the work-conjugate strain measures. Subscripts a and r denote axial and radial components, while v and d denote volumetric and deviatoric terms, respectively. All the stress quantities are regarded as effective stresses, and compression is assumed positive for both stress and strain measures.

Rate-Independent Model

The yield surface of the Modified Cam-clay (MCC) model (Roscoe and Burland 1968) is adopted as bounding surface. The latter can be represented in triaxial stress space as an ellipse (Fig. 3) characterized by the following expression:

$$F = \bar{q}^2 - M^2 \bar{p}(p_0 - \bar{p}) \quad (25)$$

where M is the stress ratio at critical state, while p_0 is an internal variable governing the size of the bounding surface. The variables $\bar{p}$ and $\bar{q}$ in Eq. (25) denote the image stress. By assuming a projection center coincident with the origin of stress space, the image stress is related to the actual stress by

$$\bar{p} = bp; \quad \bar{q} = bq \quad (26)$$

The plastic modulus at the current stress state K_p is related to the plastic modulus at the image stress $\bar{K}_p$ by

$$K_p = \bar{K}_p + hp_0^3(1 + e)(b - 1) \quad (27)$$

where h is a material constant. Equation (27) implies that in Eq. (8), $s = 1$ (i.e., vanishing elastic region). The variable $\bar{K}_p$ can be expressed as

$$\bar{K}_p = -\frac{\partial F}{\partial p_0} \bar{p}_0 = M^2 \bar{p} \bar{p}_0 \quad (28)$$

where $\bar{p}_0$ is specified in the isotropic hardening rule of p_0 typically used in critical state plasticity models (Roscoe and Burland 1968; Schofield and Wroth 1968):

$$\bar{p}_0 = \frac{1+e}{\lambda-\kappa} p_0 R_p \quad (29)$$

In Eq. (29), e denotes the void ratio, while λ and κ are material constants representing the slopes of the normal compression line and swelling/recompression line in $e - \ln(p)$ space, respectively. The variable R_p is the volumetric component of the plastic flow direction, which is given by the following flow rule:

$$\epsilon_v^p = \langle \Lambda \rangle R_p; \quad \epsilon_d^p = \langle \Lambda \rangle R_q \quad (30)$$

where

$$R_p = L_p = \frac{\partial F}{\partial \bar{p}} = \bar{p}(M^2 - \bar{\eta}^2); \quad R_q = L_q = \frac{\partial F}{\partial \bar{q}} = 2\bar{p}\bar{\eta} \quad (31)$$

in which $\bar{\eta}$ is the image stress ratio (i.e., $\bar{\eta} = \bar{q}/\bar{p}$). Equation (31) implies that an associative flow rule is employed, i.e., the loading direction coincides with the plastic flow direction.

Finally, the elastic response is governed by a hypoelastic law often used in critical state models for clay (Roscoe and Burland 1968; Schofield and Wroth 1968):

$$\dot{p} = K \dot{\epsilon}_v^e; \quad \dot{q} = 3G \dot{\epsilon}_d^e \quad (32)$$

where the elastic bulk moduli K and shear moduli G are given by

$$K = \frac{1+e}{\kappa} p; \quad G = \frac{3(1-2\nu)}{2(1+\nu)} K \quad (33)$$

In Eq. (33), ν is the Poisson's ratio.

Rate-Dependent Enhancement of the Bounding Surface MCC Model

The rate equation of the viscoplastic strain is defined by:

$$\epsilon_v^{vp} = \langle \Phi \rangle R_p; \quad \epsilon_d^{vp} = \langle \Phi \rangle R_q \quad (34)$$

where R_p and R_q have been specified in Eq. (31). Here a simple linear viscous nucleus is used:

$$\Phi = \nu y \quad (35)$$

where ν is a material parameter related to viscosity. To compute y from Eq. (20), b_d is computed by substituting Eq. (26) into Eq. (25), while the value of b_s is readily obtained from its evolution rule:

$$\dot{b}_s = \langle \Phi \rangle \bar{b}_s \quad (36)$$

The expression of $\bar{b}_s$ is obtained by specifying Eq. (24):

$$\bar{b}_s = \frac{-b_s K_p + \bar{K}_p}{M^2 p_s p_0} \quad (37)$$

In Eq. (37), the static stress p_s and q_s are related to p and q according to Eq. (19):

$$p_s = \frac{b_d}{b_s} p; \quad q_s = \frac{b_d}{b_s} q \quad (38)$$

The plastic modulus $\bar{K}_p$ and K_p in Eq. (37) are given by Eqs. (28) and (27), respectively. The image stress $\bar{p}$ and $\bar{q}$ appearing in the expression of $\bar{K}_p$ are obtained by inserting b_s , p_s and q_s into Eq. (26).

Finally, the rate equation of the internal variable p_0 is given by

$$\dot{p}_0 = \langle \Phi \rangle \bar{p}_0 \quad (39)$$

These few relations are sufficient to generate a rate-dependent bounding surface model based on the BS-EVP framework. It should be emphasized that such an enhancement is straightforward and merely requires the definition of a viscous nucleus.

KEY FEATURES OF THE BS-EVP MODIFIED CAM-CLAY MODEL

This section describes some of the main features of the rate-dependent BS Cam-clay model, thus illustrating two major characteristics of the BS-EVP formulation: (i) its convergence to the rate-independent plasticity model under sufficiently slow loading rate and (ii) its capacity to capture time- and rate-dependent plastic responses of material with stress state located inside the bounding surface. For this purpose, the responses of saturated clays under drained and undrained conditions are simulated. Moreover, clays at normally consolidated (NC) and overconsolidated (OC) states are both considered, with the former referring to stress states located on the bounding surface, and the latter referring to stress states located within the bounding surface.

Figure 4 compares the effective stress paths (ESP) and the stress-strain curves for an undrained triaxial compression test computed by the rate-dependent BS model and its rate-independent counterpart. For stress states initially located both on and inside the bounding surface (dashed line in Fig. 4), the simulations conducted with the rate-dependent model converge to those simulated by the inviscid model as the loading rate diminishes. The same feature can also be observed from the simulation of an undrained cyclic triaxial test shown in Fig. 5, in which the majority of the ESP is enclosed by the bounding surface. When employing a small enough loading frequency, the EVP bounding surface model identically reproduces the gradual decrease of effective mean stress and the accumulation of axial strains that are computed from the inviscid model. By contrast, higher loading rates cause a non-negligible departure of the computed cyclic response from its rate-independent limit (i.e., smaller deformations and pore pressure are accumulated). Such rate-dependent responses are consistent with reported observations for clays (Li et al. 2011; Ni et al. 2014).

Figure 6 presents simulations of undrained triaxial compression tests on OC clays under three different strain rates. The simulations show that the undrained strength increases with increasing

values of strain rate, whereas the shape of the stress-strain curves tends to be rate-independent. Similar characteristics are also observed in experiments on clays (e.g., Vaid et al. 1979; Sheahan et al. 1996; Graham et al. 1983; Lefebvre and LeBoeuf 1987; Zhu and Yin 2000). Fig. 6 (b) shows that at the early stage of shearing, the computed ESP has already deviated from the purely elastic response (i.e., a vertical ESP resulting from the volumetric and deviatoric uncoupling in the elastic model), thus indicating that rate-dependent plasticity is initiated long before the stress state reaches the bounding surface. Fig. 6 also includes a simulation of shearing with step-wise changes of strain rate from 0.1%/hr, to 5%/hr, to 10%/hr and finally back to 0.1%/hr. Note that the simulation reproduces the widely observed “isotach” behavior of cohesive soils (Graham et al. 1983)

Figure 7 shows the simulation of isotropic compression tests with three different strain rates on OC clays. A yielding point corresponding to the abrupt elasto-plastic transition cannot be observed in the simulation, because plasticity develops right from the early stages of loading. Nevertheless, if yielding is interpreted graphically as the point of maximum curvature in the stress-strain curves, consistently with experimental observations (e.g., Vaid et al. 1979; Graham et al. 1983; Aboshi et al. 1970; Leroueil et al. 1985; Adachi et al. 1995) the EVP model predicts increasing values of yielding stress at higher loading rates. Fig. 7 also includes a simulation of isotropic compression test interrupted by intermediate stages of creep and stress relaxation. During creep, compressive strain keeps developing under constant mean stress, thus replicating the so-called phenomenon of “secondary compression” or “delayed compression” observed for clays (Bjerrum 1967; Graham et al. 1983), rocks (Hamilton and Shafer 1991) and other cement-based materials (Bernard et al. 2003). Similarly, when constant volumetric strains are enforced, a decrease of the mean stress is observed. During both creep and stress relaxation, the material stress-strain state eventually converges back to the limiting stress-strain curve given by the inviscid plasticity model, indicating that a plastic equilibrium state is achieved once delayed plasticity has fully developed.

In contrast to classical plasticity models augmented by the overstress theory (e.g., see Adachi and Oka 1982; Sekiguchi 1984), in which creep can be triggered only after stress reaching the yield surface, the BS-EVP model allows simulating creep responses even at stress states located inside

421 bounding surface. This unique feature enables the use of the model to study the influence of the
422 overconsolidation ratio (OCR) on secondary compression. Fig. 8 illustrates such simulation, where
423 different initial values of OCR are created by adjusting the value of mean stress with respect to a
424 fixed bounding surface (see inset in Fig. 8). To activate secondary compression, for each case a
425 mean stress increment of 10 kPa is applied in a nearly instantaneous manner (i.e., within a time step
426 of 1 second). The simulated results show that the magnitude of secondary compression, as well
427 as the rate of this delayed deformation (i.e., the so-called coefficient of secondary compression,
428 C_α), both increase when the current stress approaches the maximum past pressure (i.e., as the OCR
429 decreases). Such simulated responses are consistent with experimental evidence available for a
430 number of fine-grained soils (e.g., [Leroueil et al. 1985](#); [Mesri et al. 1997](#)).

431 EVALUATION OF THE BS-EVP MODIFIED CAM-CLAY MODEL

432 This section presents a quantitative assessment of the model, in which the simulations are
433 compared against experimental data available for overconsolidated Hong Kong marine (HKM)
434 clays. Fig. 9 to 11 displays the measurements of undrained triaxial compression responses of
435 HKM clays (discrete symbols) for three axial strain rates (0.15%/hr, 1.5%/hr and 15%/hr) and three
436 values of overconsolidation ratio (OCR=1, OCR=4 and OCR=8) ([Zhu and Yin 2000](#)). These figures
437 also include the outcome of model simulations based on the BS-EVP Modified Cam-clay model
438 calibrated with model constants in Table 1. In the experiments, specimens were isotropically loaded
439 and unloaded to reproduce stress history corresponding to various OCR values. The elasticity and
440 bounding surface plasticity parameters are calibrated based on material responses under axial strain
441 rate of 0.15%/hr (detailed calibration methods of these parameters can be found in similar critical-
442 state bounding surface models (e.g., [Dafalias and Herrmann 1986](#); [Kaliakin and Dafalias 1989](#))),
443 whereas the viscosity parameter v is obtained by fitting NC clays (OCR=1) responses under a strain
444 rate of 15% /hr.

445 For NC clays (Fig. 9), the model satisfactorily replicates the increase of undrained strength
446 as well as the simultaneous decrease of excess pore pressure generated by increasing strain rates.
447 Despite its simple form, the employed overstress function (Eq. (35)) successfully captures the

rate-insensitive behavior when the strain rate increases from 0.15 %/hr to 1.5%/hr. On the other hand, the model simulation slightly overestimates the stiffness when stresses approach failure. This mismatch could be attributed to the simplicity of the hardening rule (Eq. (29)) employed in this work.

Figure 10 and 11 compare model simulations and experimental data for OC clays. These simulations also capture the undrained strength increase resulting from higher strain rates. Moreover, the computed pore pressure responses qualitatively match the trends observed in the experiments (i.e., excess pore pressure first increases then decreases, and the increase in loading rate suppresses the generation of positive pore pressure). Nevertheless, quantitative mismatches between simulations and test data can also be observed. For example, the computed excess pore pressure initially overestimates the measurements, whereas it eventually underestimates them at larger strains. Despite these mismatches, it must be emphasized that the simple BS-EVP model characterized only by six parameters reasonably captures the first-order features of the response of HKM clays with OCR ranging from 1 up to 8 and tested at strain rates spanning three orders of magnitude. If desired, the aforementioned mismatches can be removed or mitigated by adopting a projection center which is not fixed at stress space origin and/or by employing a more sophisticated bounding surface geometry (e.g., [Dafalias and Herrmann \(1986\)](#)).

CONCLUSIONS

A bounding surface elasto-viscoplasticity (BS-EVP) framework has been presented in its most general form, thus providing a platform to incorporate time and rate effects into otherwise inviscid bounding surface plasticity models. An attractive characteristic of the proposed framework is that BS-EVP models will converge to the corresponding rate-independent plasticity models as the loading rate decreases, thus facilitating the calibration of the material constants that control plastic effects and rate-sensitivity. Moreover, the new formulation leads to a plastic strain rate which solely depends on the current state of material, and which therefore can be conveniently used as a regularization tool for numerical analyses. What distinguishes this formulation from classical plasticity models augmented by Perzyna's EVP theory is its capability to represent rate-dependent

plastic responses corresponding to stress paths falling within the bounding surface. A simple isotropic BS-EVP bounding surface model has been formulated to illustrate the key features of the proposed framework. A qualitative assessment of the simple model has demonstrated its capacity to represent a wide range of rate-dependent behaviors of clays, including strain rate effects, isotach behavior, creep and stress relaxation. A quantitative comparison with experimental data available for overconsolidated Hong Kong marine clays has also shown that this simple BS-EVP model can captures satisfactorily the responses of clay specimens with OCR ranging from 1 up to 8 and subjected to undrained compression tests conducted at strain rates spanning from 0.15%/hr to 15%/hr.

The BS-EVP framework proposed in this paper can be considered as a highly versatile platform readily adaptable to different soil models. For example, future development of the framework may include its application to anisotropic soils, for the purpose of capturing anisotropic rate- and time-dependent behavior resulting from an oriented micro-structure (e.g., contact fabric, particle aggregates). Furthermore, the framework can be readily used to incorporate rate-sensitivity into rate-independent constitutive models designed to replicate the mechanical responses of cyclically loaded materials, thus enabling the analyses of a number of rate processes for which accurate simulation platforms are not yet available (e.g., dependence on the loading frequency, rate-dependent strength deterioration).

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TABLE 1. Material constants for Hong Kong marine clays

κ	ν	M	λ	h	ν
–	–	–	–	–	$1/s$
0.044	0.2	1.2	0.3	6	7E-7

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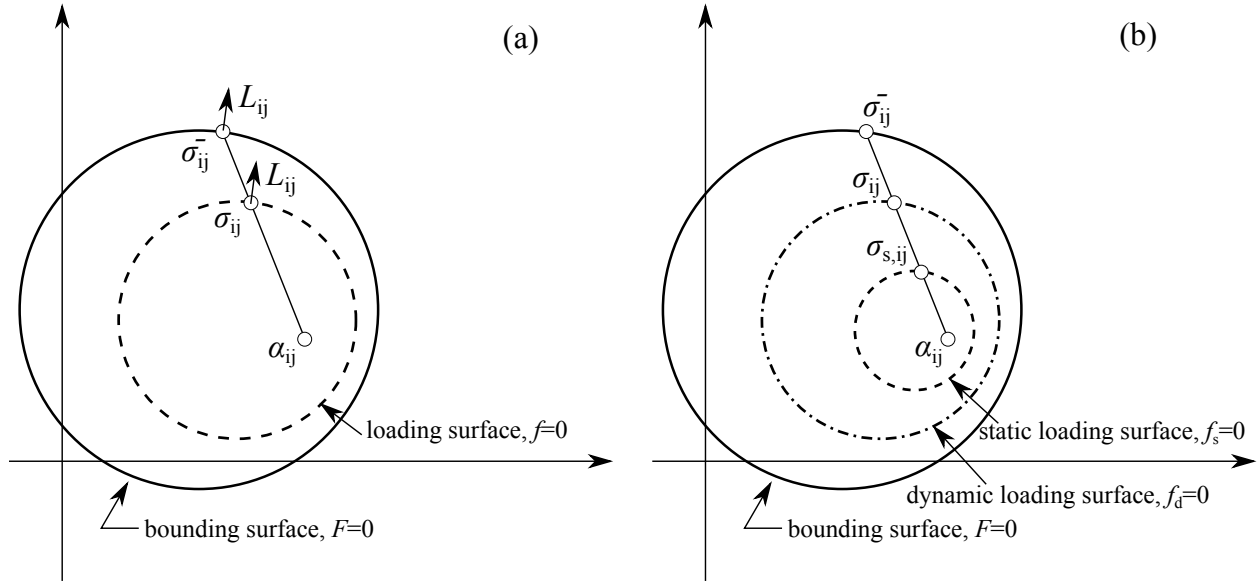


Fig. 1. Schematic illustration of (a) bounding surface, loading surface and radial mapping rule in rate-independent bounding surface plasticity and (b) static and dynamic loading surfaces in bounding surface elasto-viscoplasticity

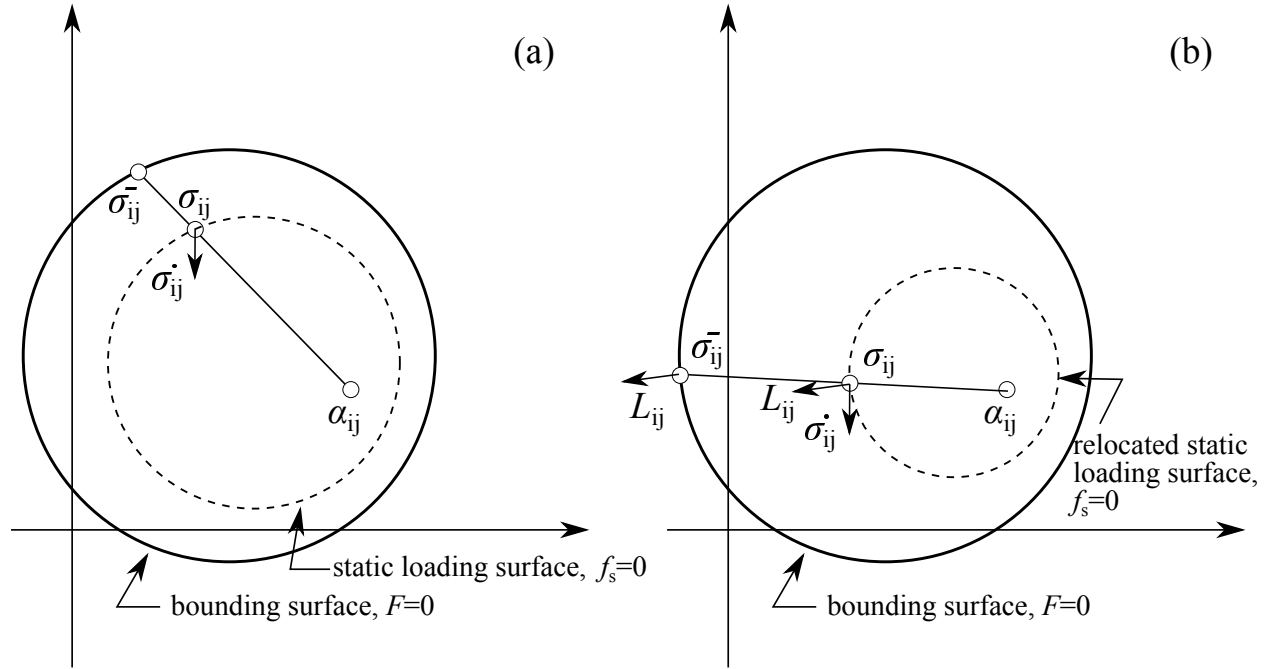


Fig. 2. Schematic illustration of (a) initiation of stress reversal and (b) relocation of the static loading surface upon attaining a positive loading index

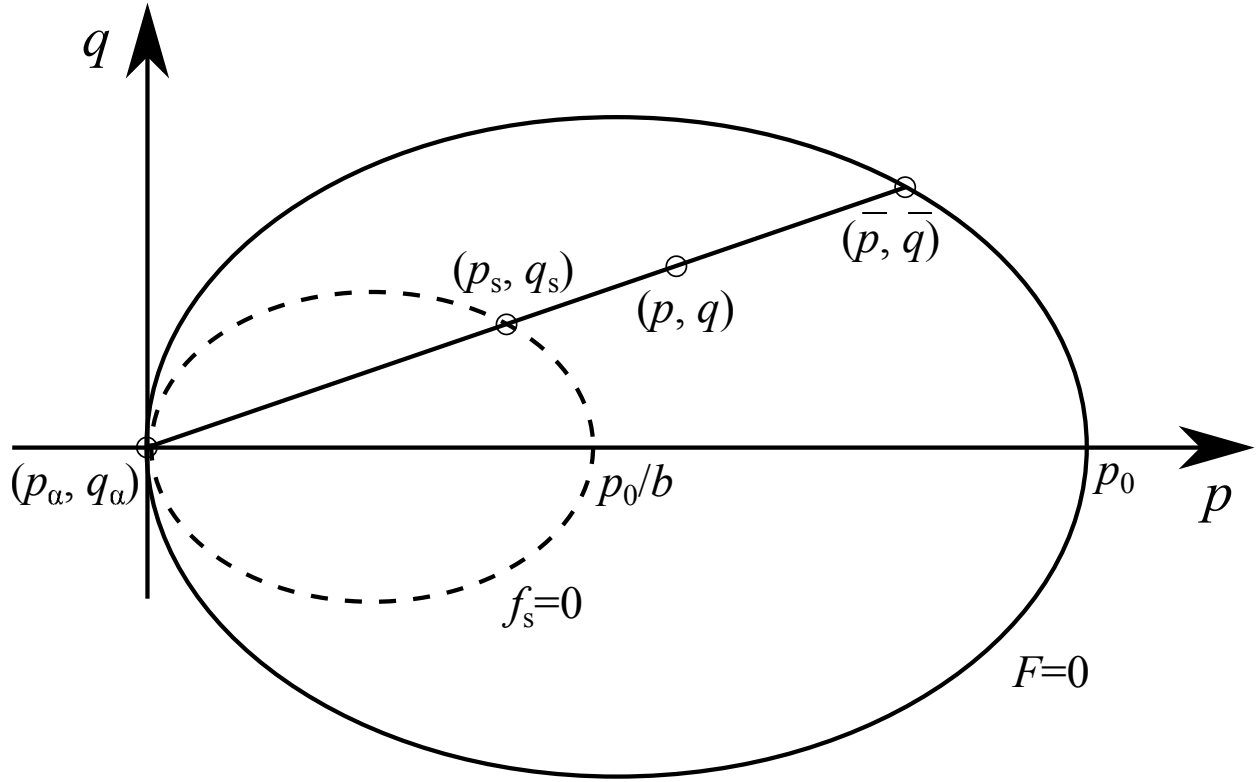


Fig. 3. Schematic illustration of an isotropic bounding surface, static loading surface and associated radial mapping rule based on a projection center fixed at the origin of stress space

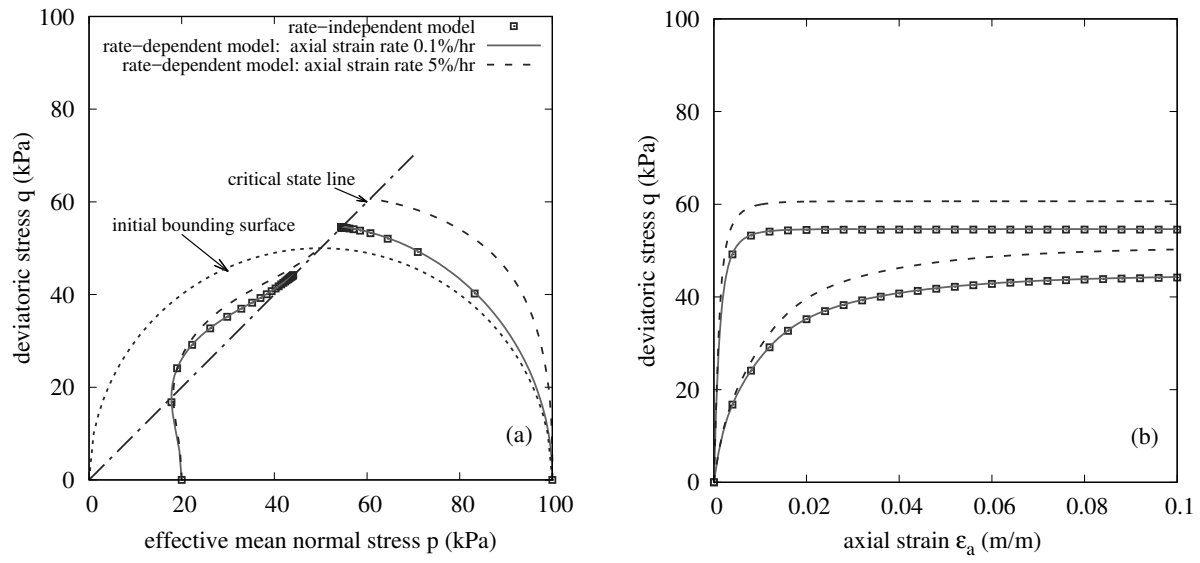


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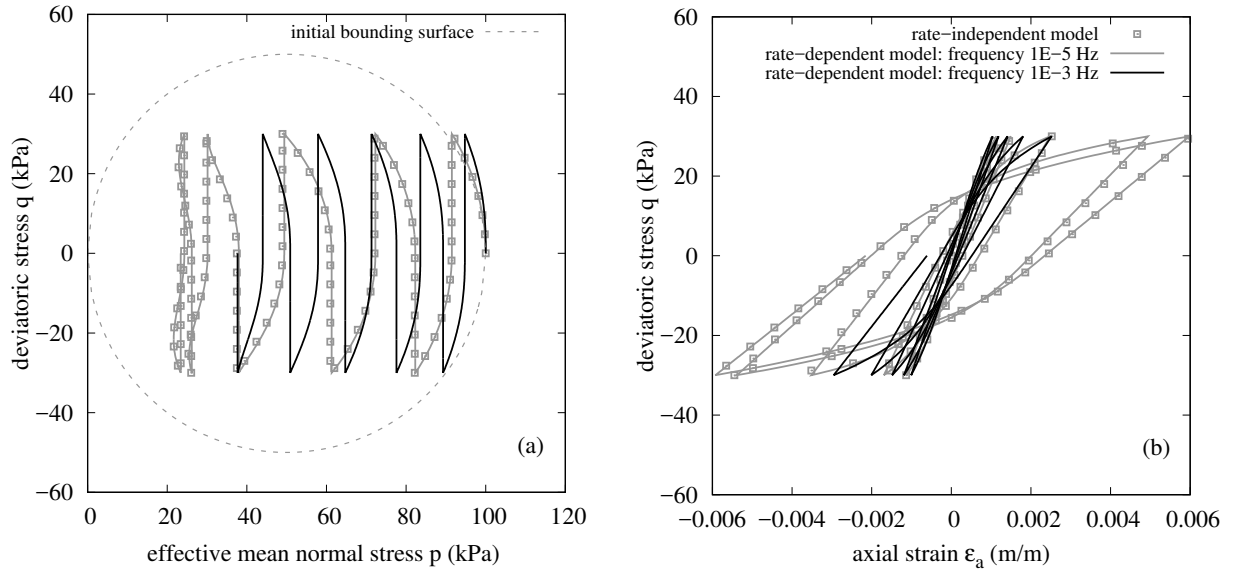


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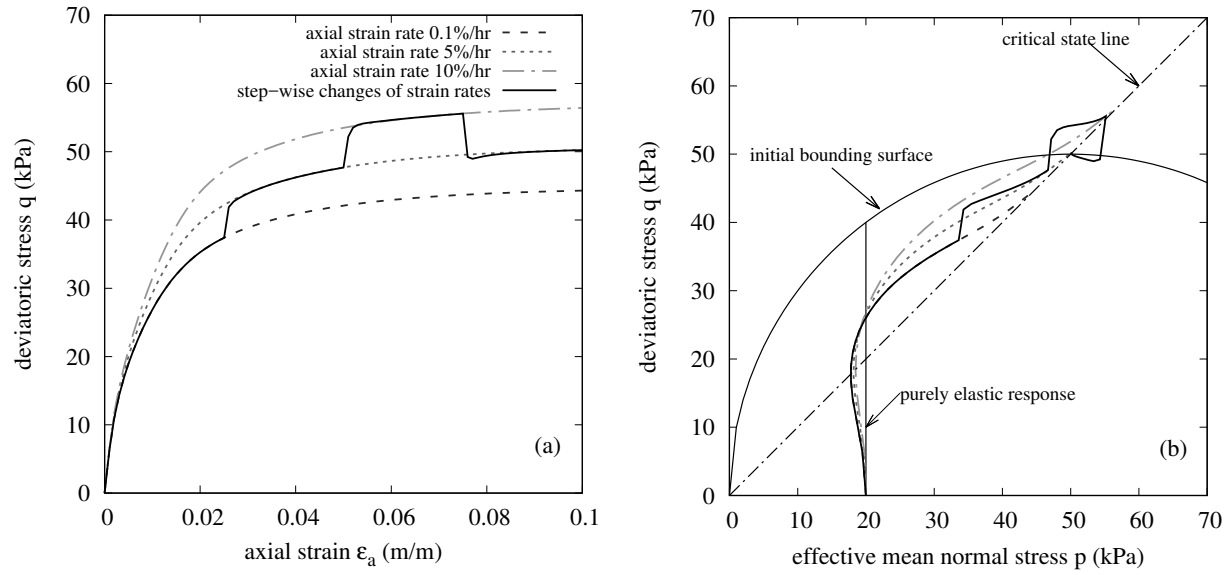


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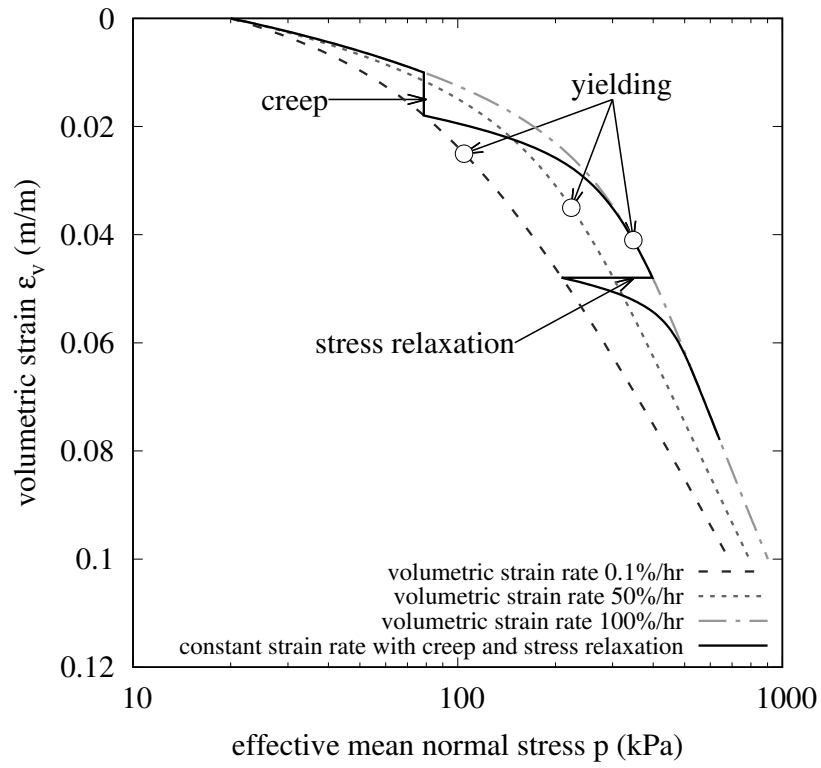


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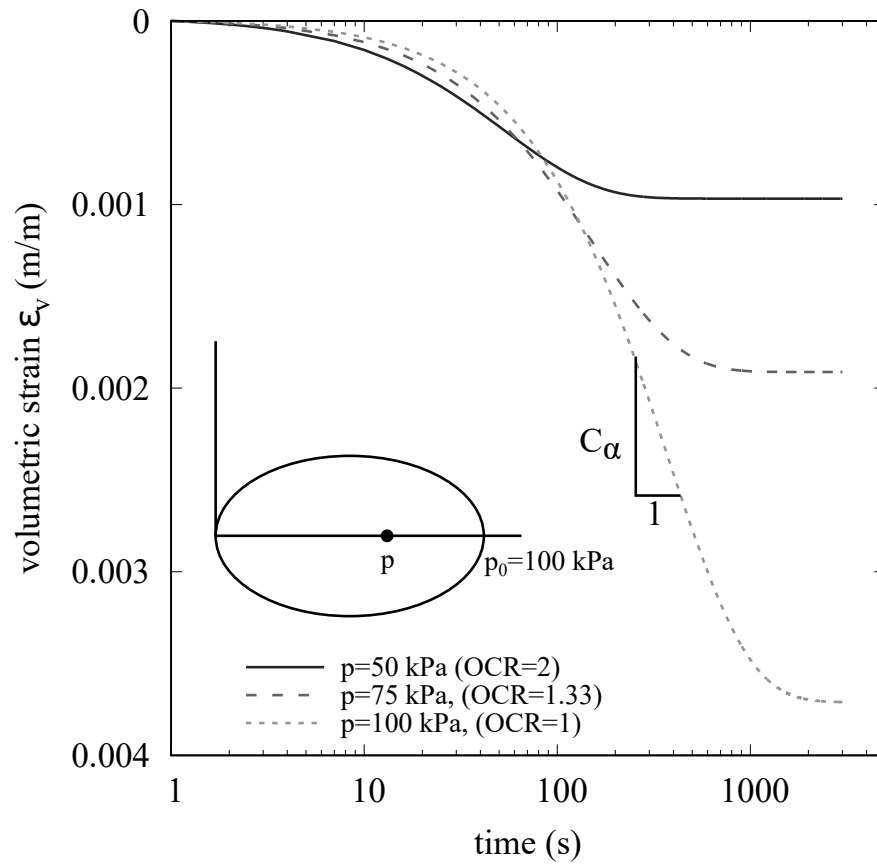


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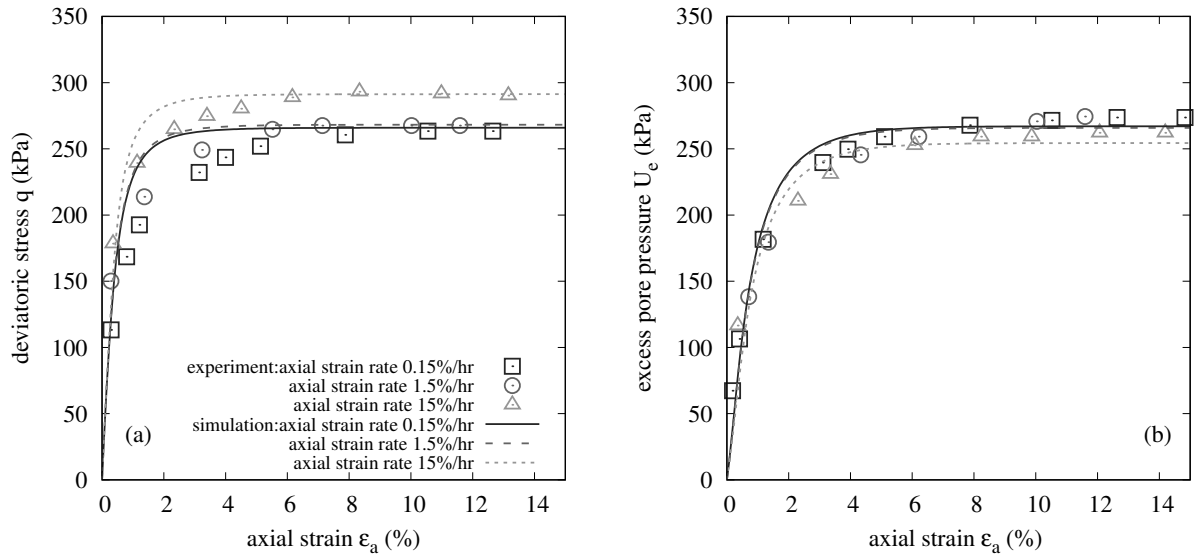


Fig. 9. Comparison between experimental observations and model simulations for undrained triaxial compression tests on HKM clay (OCR=1) with different strain rates: (a) stress-strain response and (b) excess pore pressure-strain response

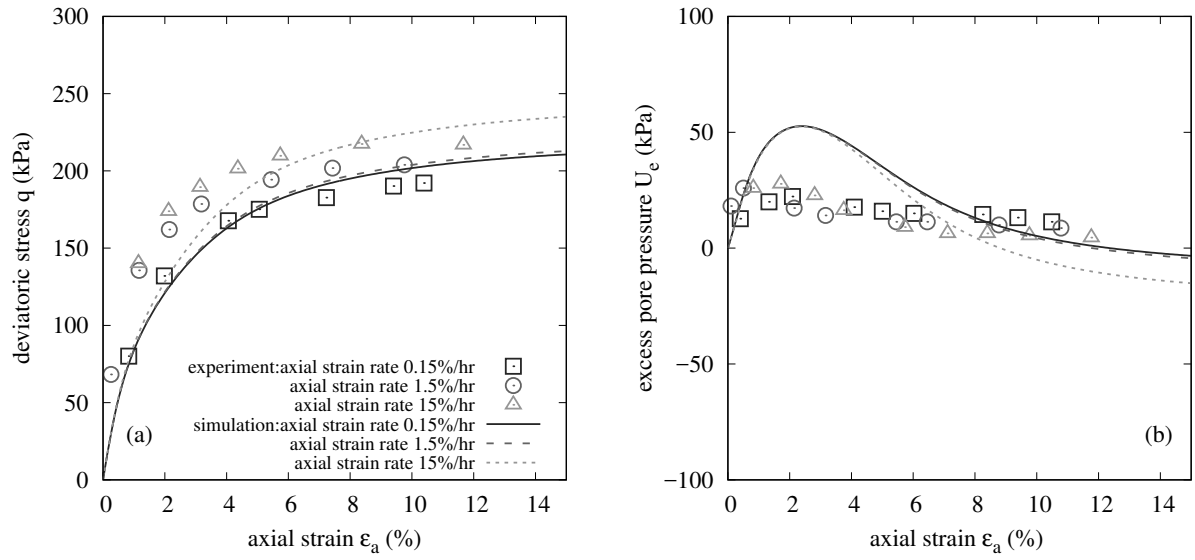


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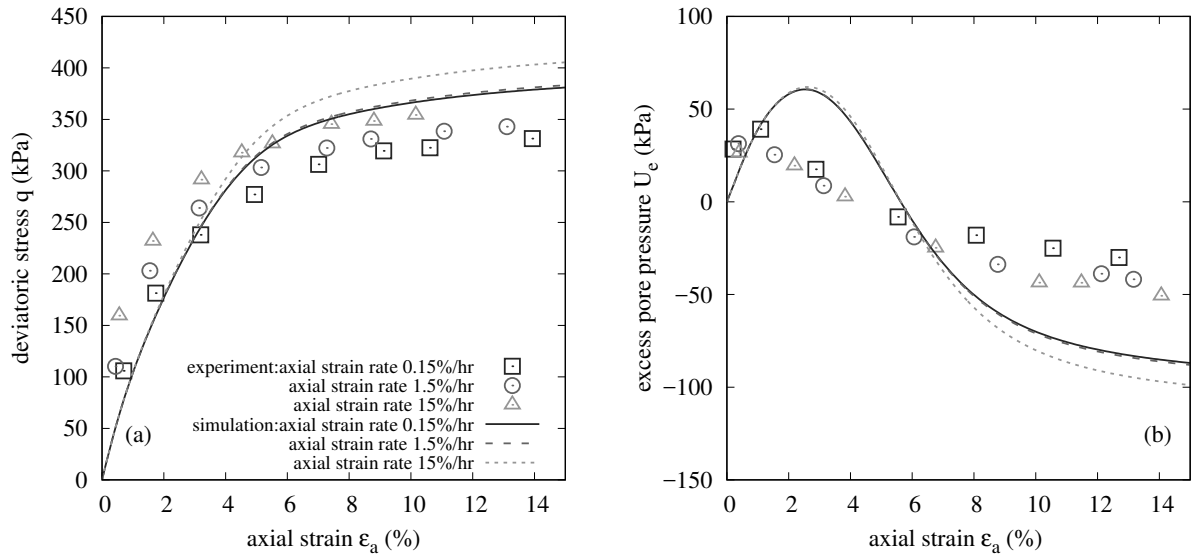


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