Note on the generation of long gravity waves by breaking and shoaling of short-wave groups in gently-sloping beaches: the long-wave similarity parameter

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Abstract

It is proposed a long-wave similarity parameter based on the surf-beat similarity and Ursell parameters. By including the Ursell number, the long-wave similarity allows distinguishing between breaking and shoaling generation of long-waves for conditions rendering similar values of surf-beat similarity. The proposed parameter is tested with three cases of wave conditions published in the literature, with promising results.

The Surf-beat Similarity Parameter

Previous theoretical and experimental works (Baldock 2006; Baldock and Huntley 2002; Schäffer 1993; Symonds et al. 1982; Zou 2011) have identified the release of long-gravity waves by the shoaling and breaking of short-wave groups in sloping beaches. In a relatively simple case, the surf beat similarity parameter, $\xi_{surfbeat}$ (Baldock 2012), was proposed to account for the shoaling-breaking dichotomy in the nearshore generation of long-gravity-waves. This parameter was thought as the product of the normalized bed slope, $\beta_{norm} = \frac{h_x}{\omega_{long}}\sqrt{\frac{g}{h}}$, and the square root of the short-wave steepness $\sqrt{\frac{H}{L}}$ as a representative of short-wave shoaling conditions. In that parameterization, β_{norm} represents the slope regime for long waves, where h_x is the bed slope, ω_{long} is the angular frequency of the long wave, g is the acceleration due to the gravity, and h is a characteristic water depth (van Dongeren et al. 2007). The normalized bed slope has been previously used to distinguish between mild and steep regimes in the long wave transformation (Battjes et al. 2004), while the surf beat similarity parameter has been utilized to discern the role of shoaling and breaking in long-wave generation (Contardo and Symonds 2013).

Long waves have been found to be “released” either due to variations in water depth within short-wave groups along with their propagation as shallow water waves, or by the time-varying breakpoint mechanism (Baldock 2006 and references therein). It is further argued that the release by short-wave shoaling occurs when wave groups and its bound long waves propagate as shallow water waves. That idea supports the inclusion of the short-wave steepness in the parameterization in pioneer works (Baldock 2012). However, the reason of this parameter inclusion appears to reside in its presence in the well-known surf similarity parameter (Battjes 1974). The author suggests replacing $\sqrt{\frac{H}{L}}$ in $\xi_{surfbeat}$ with the Ursell number, therefore allowing the similarity parameter to represent short-wave shoaling conditions.

The Ursell Number and the Long Wave Similarity Parameter

Pioneer work (Ursell 1953) proposed the ratio of a representation of nonlinearity, a/h, to a surrogate of frequency dispersion, $(kh)^2$, as representative of the regime in which waves propagate. This number may also be thought as the ratio of the free-surface elevation amplitudes of the secondary-order $\left(\eta_2 = \frac{\epsilon^2 ch}{8\gamma\kappa^3}\cos 2\theta\right)$ to the amplitude of the leading-order $(\eta_1 = \epsilon h \cos\theta)$ Stokes solutions to the Korteweg-deVries equation for progressive, sinusoidal waves, where c is the phase celerity for constant water depth h, $\epsilon = a/h$ is considered small (so $a \ll h$), a is the wave amplitude, $\gamma = ch^2/6$, κ is the wavenumber, $\theta = \kappa x - \omega t$, and ω is the short-wave radian frequency (Eq. after Fig. 6 in Doering and Bowen 1986; see also Whitham 1999 section 13.13). In such case, the Ursell number is given by

$$Ur = \frac{|\eta_2|_{max}}{|\eta_1|_{max}} = \frac{3}{4}\frac{ak}{(\kappa h)^3}. \tag{1}$$

After accounting for the wave amplitude in terms of the significant wave height (H_S) as $a = \sqrt{2}H_S/4$ (Holthuijsen 2007 Eq. 4.2.26) and $\kappa = 2\pi/L$, this parameter reads

$$Ur = \alpha\frac{H_S}{L}\left(\frac{L}{h}\right)^3, \tag{2}$$

with $\alpha = \frac{3\sqrt{2}}{64\pi^2} \approx 0.007$. Note that the frequency-dispersion term, represented in Eq. (2) by $\left(\frac{L}{h}\right)^3$, is relatively more important than the wave steepness for $H_S \approx h$, which is the case of sea-swell in the nearshore. Although a more recent parameterization that resolves intermediate water waves has been proposed (Beji 1995), the author preferred the original formulation since typically

$0 < Ur < O(10^1)$ and $0 < \xi_{surfbeat} < O(1)$, and both lower and upper bounds relate to the same regimes: deep water (frequency dispersive) and shallow water (amplitude dispersive) waves, respectively.

The author then proposes the long-wave similarity parameter, $\xi_{longwave}$, as the product of the normalized bed slope and the Ursell parameter as

$$\xi_{longwave} = \alpha \frac{h_x}{\omega_{low}} \sqrt{\frac{g}{h}} \frac{H_S}{L} \left(\frac{L}{h}\right)^3. \tag{3}$$

For comparison purposes, the long-wave similarity parameter was calculated for published data related to $\xi_{surfbeat}$. Extreme values of $\xi_{longwave}$ generally follow the shoaling-breaking differentiation. By including Ur, the long-wave similarity parameter further allowed differencing among several datasets with a relatively broad range of short-wave conditions and similar values of surf beat similarity.

Three cases are reported in Table 1. The first case represents two field conditions with markedly different normalized slopes and wave steepness. The surf-beat similarity parameter cannot distinguish between conditions and renders a value ~0.015 for both. After including Ur, it appears evident that 10 s waves propagated in more amplitude-dispersive conditions with a tendency towards the breaking generation mechanism. Long-wave similarity for that case was 0.188, compared to 0.027 for the other case.

Second case also represents two field conditions with relatively similar normalized slopes and slightly different wave steepness that give $\xi_{surfbeat} \approx 0.032$. Ursell number for one condition was, however, twice as large as the other (1.48 versus 0.66), therefore representing more amplitude-dispersive conditions. Values of $\xi_{longwave}$ were 0.859 and 0.192. Lastly, third case represents both a field and a laboratory experiment with surf-beat similarities of ~0.14. Normalized slopes and wave steepness were practically equal (therefore with similar values of $\xi_{surfbeat}$), as well as the Ursell numbers. In this case, $\xi_{longwave}$ values were similar, but with opposite tendencies when compared to reported $\xi_{surfbeat}$ values, as were values of Ur.

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Tables

Table 1. The inclusion of $\xi_{longwave}$, given by Eq. (3), allows to distinguish reported cases with similar values of $\xi_{surfbeat}$. Values of Ur were calculated according to Eq. (2) (data published in Baldock 2012).

Case	f_{long} (mHz)	T_{short} (s)	H (m)	β_{norm}	H/L	kh_b	$\xi_{surfbeat}$	Ur	$\xi_{longwave}$
1	14.0	10.00	1.00	0.20	0.006	0.22	0.015	0.94	0.188
	20.0	7.00	3.00	0.08	0.390	0.58	0.016	0.33	0.027
2	5.0	15.00	1.00	0.58	0.003	0.15	0.032	1.48	0.859
	10.0	7.00	1.00	0.29	0.013	0.32	0.033	0.66	0.192
3	100.0	1.67	0.10	0.91	0.023	0.44	0.138	0.45	0.409
	8.3	10.00	5.00	0.80	0.032	0.52	0.143	0.34	0.303