Thermal Buckling Analysis of Laminated Composite Plates using Isogeometric Analysis

Presented in honor of Prof. Romesh C. Batra's 70th Birthday

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Outline

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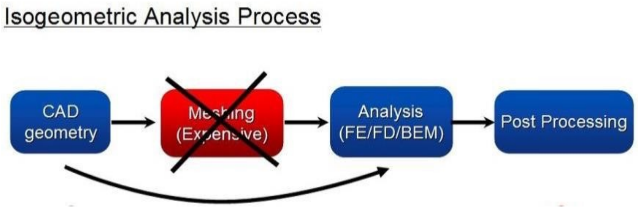
- Bezier Extraction : Motivation
- Bezier Extraction : Theory
- Plate Problem
- Numerical Investigation
- Analysis Procedure
- Results

Isogeometric Analysis



- Unifies the computer aided design (CAD) and the computer aided engineering (CAE)[1].
- Significant decrease in computational cost as the meshes are generated within the CAD.
- Higher accuracy results because of the smoothness and the higher order continuity between elements

1. Cottrell, J. Austin, Thomas JR Hughes, and Yuri Bazilevs. Isogeometric analysis toward integration of CAD and FEA. John Wiley Sons, 2009.

Isogeometric Analysis

B-spline basis functions are expressed by a set of non-decreasing values called the knot vector

$$\Xi = \{\xi_1, \xi_2, \ldots, \xi_{n+p+1}\} \quad (1)$$

B-spline basis functions are defined by the following recursive form

$$N_{i,0}(\xi) = \begin{cases} 1 \text{ if } \xi_i \leq \xi < \xi_{i+1}, & \textit{for } p = 0 \\ 0 \quad \text{otherwise}, \end{cases} \quad (2)$$

$$N_{i,p}(\xi) = \frac{\xi - \xi_i}{\xi_{i+p} - \xi_i} N_{i,p-1}(\xi) + \frac{\xi_{i+p+1} - \xi}{\xi_{i+p+1} - \xi_{i+1}} N_{i+1,p-1}(\xi) \text{ for } p \geq 1 \quad (3)$$

where ξ_1 is the ith knot, n is the number of basis functions and p is the polynomial order.

Bezier Extraction : A Primer

- IGA is implemented with the presence of C^0 continuous Bézier elements
- The Bezier extraction operator decomposes the Non Uniform Rational B-Splines(NURBS) into C^0 -continuous Bezier elements using linear combinations of Bernstein polynomials
- Bezier elements bear a close resemblance to the Lagrange elements used in finite element methods
- Allows easy incorporation into existing finite element codes without adding many changes as IGA

Bezier Extraction : Theory

Bézier decomposition [2] is accomplished by the insertion of new knots into the B-spline curve until k = p. Wherek is the multiplicity of the knot

$$\bar{P}_i = \begin{cases} P_1 & i = 1 \\ a_i P_i + (1 - a_i) P_{i-1} & 1 < i < n+1 \\ P_n & i = n+1 \end{cases}, \tag{4}$$

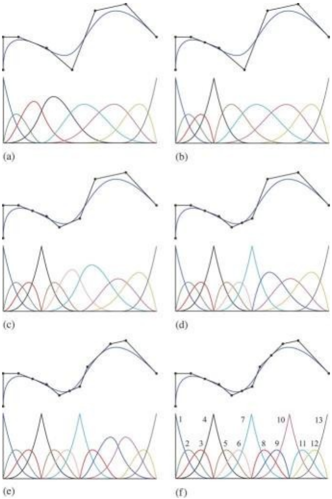
$$a_i = \begin{cases} 1 & 1 \leq i \leq k-p \\ \frac{\bar{\xi} - \xi_i}{\xi_{i+1} - \xi_i} & k-p+1 \leq i \leq k \\ 0 & i \geq k+1 \end{cases}, \tag{5}$$

The new control points are expressed in matrix form as,

$$\mathbf{P}^b = \mathbf{C}^T \mathbf{P} \tag{6}$$

2. Borden, Michael J., Michael A. Scott, John A. Evans, and Thomas JR Hughes. "Isogeometric finite element data structures based on Bézier extraction of NURBS." International Journal for Numerical Methods in Engineering 87, no. 1-5 (2011) : 15-47.

Knot Insertion : Visual Description



Bezier Extraction : Theory

The 2D NURBS basis functions which are written as

$$R_{i,j}(\xi,\eta) = \frac{N_{i,p}(\xi)M_{j,q}(\eta)w_{i,j}}{\sum_{i=1}^{n}\sum_{j=1}^{m} N_{i,p}(\xi)M_{j,q}(\eta)w_{i,j}}. \tag{7}$$

are now written in terms of Bernstein polynomials as

$$\mathbf{R}(\xi,\eta) = \frac{1}{W(\xi,\eta)}\mathbf{WN}(\xi,\eta) = \frac{1}{W(\xi,\eta)}\mathbf{WCB}(\xi,\eta), \tag{8}$$

The new control points are expressed as,

$$\mathbf{P}^{b} = (\mathbf{W}^{b})^{-1}\mathbf{C}^{T}\mathbf{WP}, \tag{9}$$

Where $\mathbf{W}^{b}$ and $\mathbf{W}$ are derived from the weights of the control points

Geometry : Construction and Mesh

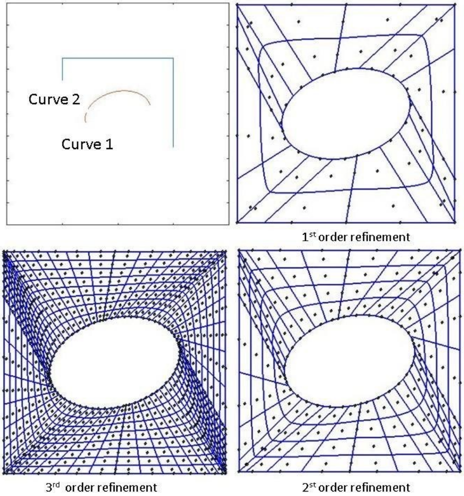
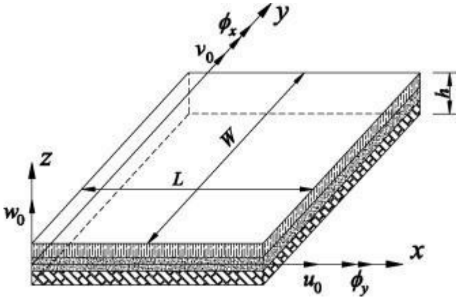


Plate Problem

Three-dimensional displacement field (u, v, w) of a four-layered unit squareareexpressedin terms of five unknown variables as :

$$u(x,y,z) = u_0(x,y) + z\varphi_x(x,y)$$
$$v(x,y,z) = v_0(x,y) + z\varphi_y(x,y)$$
$$w(x,y,z) = w_0(x,y)$$

where , $\frac{-h}{2} \leq z \leq \frac{h}{2}$



Numerical Investigation

- Material Property

$$E_L/E_T = 15,\ G_{LT}/E_T = 0.5,\ G_{TT}/E_T = 0.3356,$$
$$\nu_{LT} = 0.3,\ \nu_{TT} = 0.49,\ \alpha_L/\alpha_0 = 0.015,\ \alpha_T/\alpha_0 = 1 \qquad (10)$$

- Boundary Condition

$$\textit{Clamped}: \quad \begin{aligned} u_0 = v_0 = w_0 = \varphi_y = \varphi_x = 0 \text{ on } x = 0, L \\ u_0 = v_0 = w_0 = \varphi_x = \varphi_y = 0 \text{ on } y = 0, W \end{aligned} \qquad (11)$$

- Loads : A unit temperature rise of 1 unit is applied on all the control points.

Analysis Procedure

- For the symmetric laminated plate subjected to a uniform temperature rise, only membrane forces are generated
- ABD matrix is calculated using the material properties
- A linear static analysis is then solved to determine the thermal in-plane resultants
- The stress resultants are then used to compute the initial stress stiffness matrix
- The stiffness matrix and the geometric stiffness matrix are used to solve the eigenvalue problem

Results

The critical buckling temperature is normalized as $\overline{T_{cr}} = \alpha_0 T_{cr}$

L/h	FSDT[3]	HSDT[3]	Present
100	0.3348	0.3352	0.3349
10	0.1655	0.1601	0.1656

Table – critical buckling temperature of a symmetric four-layer [0/90/90/0] laminated plate.10x10 mesh and $p = 2$

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Thermal buckling mode shapes for various L/h ratios are presented.

3. Kant, T., and C. S. Babu. "Thermal buckling analysis of skew fibre-reinforced composite and sandwich plates using shear deformable finite element models." Composite Structures 49, no. 77 (2000) : 85.

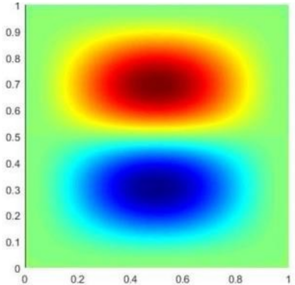

Figure – L/h= 100 (Present)

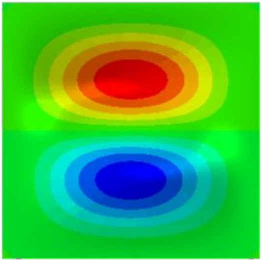

Figure – L/h= 100 (ABAQUS)

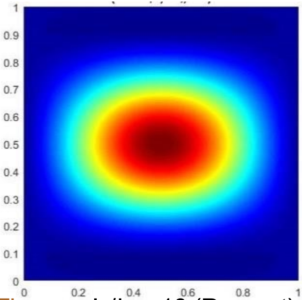

Figure – L/h= 10 (Present)

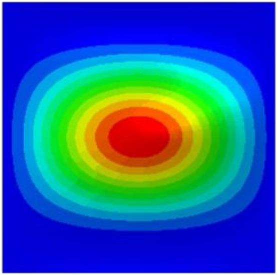

Figure – L/h= 10 (ABAQUS)

Plate with a Central Circular Cutout

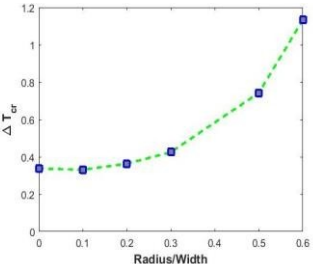

Figure – [0º/90º/90º/0º]

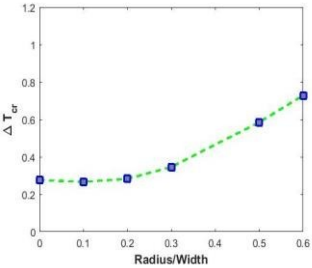

Figure – [45º/-45º/-45º/45º]

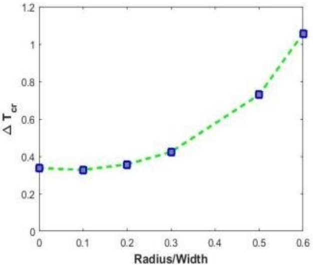

Figure – [90º/0º/0º/90º]

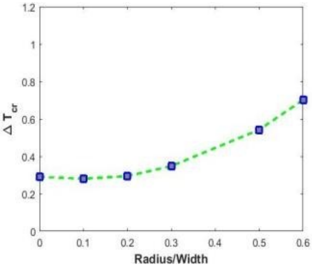

Figure – [60º/-60º/-60º/60º]

Numerical Investigation : Hybrid Composites

- Material Property - I

$$E_L/E_T = 15,\ G_{LT}/E_T = 0.5,\ G_{TT}/E_T = 0.3356,$$
$$\nu_{LT} = 0.3,\ \nu_{TT} = 0.49,\ \alpha_L/\alpha_0 = 0.015,\ \alpha_T/\alpha_0 = 1 \qquad (12)$$

- Material Property - II (Isotropic)

$$E_L/E_T = 1,\ G_{LT}/E_T = 0.3846,\ G_{TT}/E_T = 0.3846,$$
$$\nu_{LT} = 0.3,\ \nu_{TT} = 0.3,\ \alpha_L/\alpha_0 = 1,\ \alpha_T/\alpha_0 = 1 \qquad (13)$$

- Boundary Condition

Clamped :

$$u_0 = v_0 = w_0 = \varphi_y = \varphi_x = 0 \text{ on } x = 0, L$$
$$u_0 = v_0 = w_0 = \varphi_x = \varphi_y = 0 \quad \text{on } y = 0, W \qquad (14)$$

- 0/90/90/0 Laminate where the 1st and the 3rd layer have Material Property -I and the 2nd and 4th layer have Material Property -II

Results : Hybrid Composites

The critical buckling temperature is normalized as

$$\bar{T}_{cr} = 1000 * \alpha_0 * T_{cr}$$

Mode	IGA	abaqus
1	0.241	0.245
2	0.310	0.308
3	0.390	0.387
4	0.423	0.425
5	0.485	0.481

Table – critical buckling temperature of a symmetric four-layer [0/90/90/0] laminated plate. Radius/Width=0.15

Acknowledgement

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References

1. Devarajan, B. and Kapania, R.K., 2020. Thermal buckling of curvilinearly stiffened laminated composite plates with cutouts using isogeometric analysis. *Composite Structures*, *238*, p.111881.
2. Miglani, J., Devarajan, B. and Kapania, R.K., 2018. Thermal buckling analysis of periodically supported composite beams using Isogeometric analysis. In *2018 AIAA/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference* (p. 1224).
3. Li, Q., Devarajan, B., Zhang, X., Burgos, R., Boroyevich, D. and Raj, P., 2016. *Conceptual design and weight optimization of aircraft power systems with high-peak pulsed power loads* (No. 2016-01-1986). SAE Technical Paper.
4. Devarajan, B. and Kapania, R.K., 2022. Analyzing thermal buckling in curvilinearly stiffened composite plates with arbitrary shaped cutouts using isogeometric level set method. *Aerospace Science and Technology*, p.107350.
5. Devarajan, B., 2019. *Thermomechanical and vibration analysis of stiffened unitized structures and threaded fasteners* (Doctoral dissertation, Virginia Tech).
6. Miglani, J., Devarajan, B. and Kapania, R.K., 2021. Isogeometric Thermal Buckling and Sensitivity Analysis of Periodically Supported Laminated Composite Beams. *AIAA Journal*, pp.1-10.
7. Devarajan, B., 2021. Vibration Analysis of Timoshenko Beams using Isogeometric Analysis. *arXiv preprint arXiv:2104.12860*.
8. Devarajan, B., 2021. Free Vibration analysis of Curvilinearly Stiffened Composite plates with an arbitrarily shaped cutout using Isogeometric Analysis. *arXiv preprint arXiv:2104.12856*.
9. Devarajan, B., Locatelli, D., Kapania, R.K. and Meritt, R.J., 2016. Thermo-Mechanical Analysis and Design of Threaded Fasteners. In *57th AIAA/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference* (p. 0579).