How to find the radius of a circular object? Radius Meter

Dinesh Katugampala[a], Udita N. Katugampola[b]

[a] *Institute of Technology, University of Moratuwa, Sri Lanka.*
[b] *Department of Mathematical Sciences, University of Delaware, Newark, DE 19716, U.S.A.*

Abstract

There is no instrument that can measure the inner or outer radius of a circular object taking only one measurement, given only a portion of it. We discuss a new approach for such an instrument, now known as Radius Meter. This device can be used to find radii of circular objects ranging from 5mm - 10m with a relatively small error.

Keywords: Measuring inner radius, Measuring outer radius

1. Introduction

The curiosity and urge of understanding the nature goes back to several millennia, if not the beginning of the civilization. The Greeks tries to develop the ideas of straight lines, arcs, polygons, circles, conic sections and several other two- and three- dimensional figures and objects [1].

Understanding of mathematical objects mainly requires the knowledge of their characteristics: the length, width, radius, angles, area, volume and etc.

(a) Earthenware clay pot

(b) Colosseum in Italy

Figure 1: Historical artifacts

There are measuring instruments that can be used to find most of the above features. For example, the Vernier Calliper, Micrometer, and Spherometer (see Figure 2) can be used to measure some inner and outer diameters of a small circular object with diameter ranging from $1\mu m - 15cm$. One of the problems we come across in engineering applications is how to find the radius of a circular object of various sizes, for example, from a test tube to a large vertical pillar to the ramp of a road when only a piece of the object interested is given. For example, what is the inner radius of the opening of an old clay pot, the base radius of one of the pillars of Parthenon in Greece, or the base radius of the Colosseum in Italy (see Figure 1)? But we did not

Email addresses: `dskatugampala@gmail.com` (Dinesh Katugampala), `uditanalin@yahoo.com` (Udita N. Katugampola)

have a devise that finds the radius of a circular object, especially large, just by taking few measurements. The purpose of this paper is to discuss such an approach that would ease some of the laboratory works, especially the work of explorers and engineers.

(a) Vernier Calliper (b) Micrometer (c) Spherometer

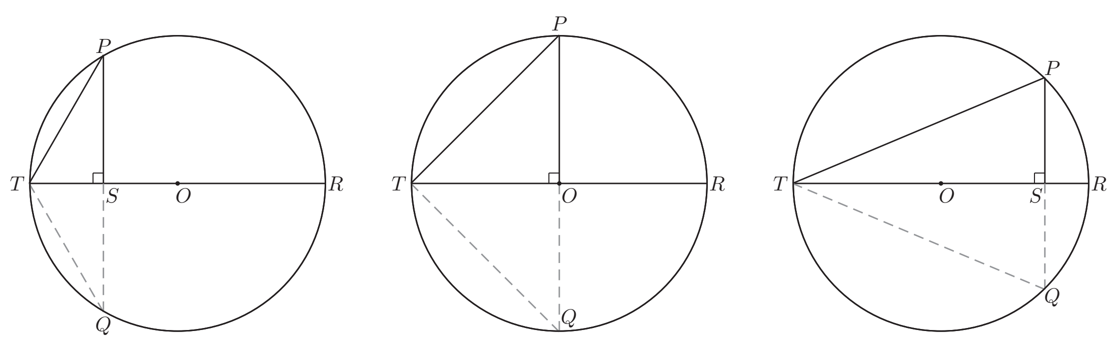
Figure 2: Diameter measuring devices

Question 1.1. Assume that PQ is a chord (minor or major) of a circle, say of length $2x$. Can we find the radius of the circle, say r, by taking only one measurement? For example, what is the inner radius of the outlet of clay pot in Figure 1? Put it in another way, can we derive an expression for r, using only a practically measurable measurement, such as the sagitta (h), while other measurements are predefined?

We need some definitions to begin with.

Definition 1.2 (Right-Symmetric Triangles)**.** Consider a circle with center O. Let T be any point on the circle. Suppose the diameter that passes through T meets the circle again at R. Let P be any point on the circle different from T and R. Drop a perpendicular from P on the diameter TR so that it meets the circle again at the point Q. The line PQ meets the line TR at S. Then it is easy to see that the two triangles, PTS and QTS are right triangles. We say such a triangle is a `right-symmetric triangle`.

There are three cases to consider as shown in Figure 3 below. It can be seen that the derivation of the formula of the radius r is depend on the configuration, but not the the formula itself.



(a) (b) (c)

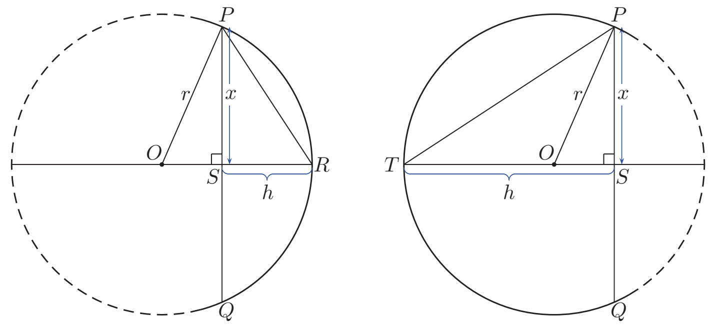
Figure 3: Three configurations of the right-symmetric triangles

1.1. Case a: When PRQ is a minor arc

In Figure 4(a), let the radius of the circle be r. Then $PO = RO = r$. Also let $PS = x$ and $RS = h$.

Now consider the right triangle PSR. Then, due to the Pythagorean identity, we have $OS^2 = r^2 - x^2$. We also have $r = RO = RS + SO = h + OS = h + \sqrt{r^2 - x^2}$. Then, $\sqrt{r^2 - x^2} = r - h$. Thus, $r^2 - x^2 = (r-h)^2 = r^2 - 2rh + h^2$. That is,

$$r = \frac{h^2 + x^2}{2h} \tag{1}$$



(a) When PQ chord is a minor arc (b) When PQ chord is a major arc

Figure 4: Two positions of the arc.

Considering the right triangle PSR and using the Pythagorean identity again, we have $x^2 + h^2 = PR^2$. Substituting this in Eq. (1), we arrive at

$$r = \frac{PR^2}{2h} = \frac{(\text{Length of PR hypotenuse})^2}{2 \cdot \text{Length of SR}} \tag{2}$$

1.2. Case c: When PTQ is a major arc

Considering the PST right triangle in Figure 4(b), we have

$$SO = \sqrt{r^2 - x^2} \tag{3}$$

We also have $h = SO + OT = SO + r$. Substituting for SO by Eq. (3), we get $h = \sqrt{r^2 - x^2} + r$. Thus, we have $r^2 - x^2 = (r - h)^2 = r^2 - 2rh + h^2$. That is,

$$r = \frac{h^2 + x^2}{2h} \tag{4}$$

Considering the right triangle PST and using the Pythagorean identity again, we have $x^2 + h^2 = PT^2$. Substituting this in Eq. (4), we arrive at

$$r = \frac{PT^2}{2h} = \frac{(\text{Length of PT hypotenuse})^2}{2 \cdot \text{Length of ST}}. \tag{5}$$

It can be seen that a similar result is true for the case (b) as well, and is left for the readers to verify. Due to the similarity of triangles RSQ and PSR in the first case or the similarity of triangles PST and QST in the second case, we may obtain similar results for all of the above cases in Figure 3.

We shall restate the above observation as a theorem below.

Theorem 1.3 (Square Dual Theorem of Arc Radius)**.** *The radius of curvature can be obtained by the square of the length of the hypotenuse (l) of the corresponding right-symmetric triangle divided by twice the length of the sagitta (h), that is $\frac{l^2}{2h}$.*

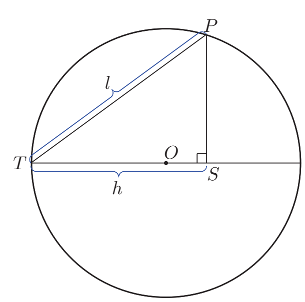


Figure 5: The square dual theorem, $r = \frac{l^2}{2h}$

1.3. Radius Meter

For practical purposes, we may adopt the equation (1) (or (4)), written in another form as,

$$r = \frac{h}{2} + \frac{x^2}{2h} \tag{6}$$

to invent an instrument that can be used to measure the radius of a circular object by measuring the base length and the height of a right-symmetric triangle. The instrument developed using such an argument is now known as the `Radius Meter` [2]. The instrument has multifunctions features such as reading outer

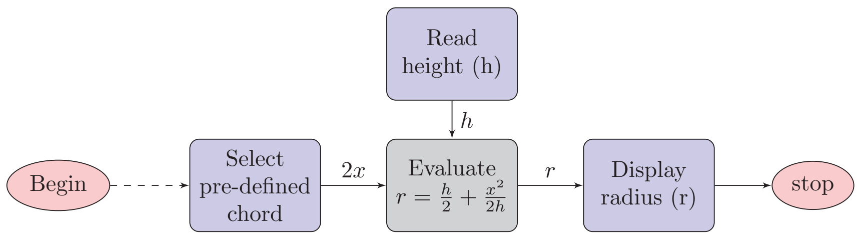
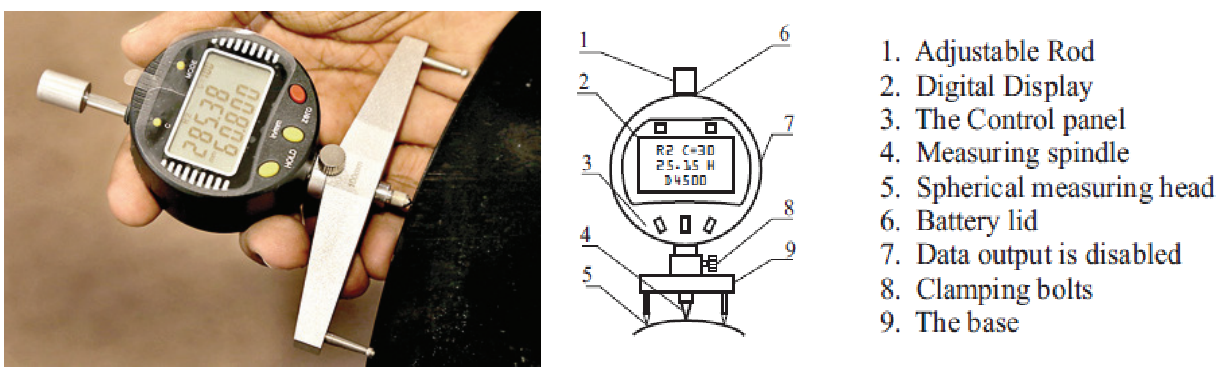


Figure 6: Schematic diagram

radius, inner radius and crown radius, which will be used in several engineering applications, commercial or domestic applications, and also academic purposes. The key features of the instrument is in Figure 7(b) below. The measurements in the first column of Table 1 were taken by using a roller bearings made out of



(a) Instrument (b) Diagram

Figure 7: Radius Meter

Exact value (mm)	Radius meter reading (mm)
10	10.01
40	39.98
200	200.03
800	799.96
1250	1249.76

Table 1: Table of Values

stainless steel. The radii values marked in the bearing were taken as the exact values. Assuming $\pm 0.05mm$ reading error in $2x$ and h, the errors in measured radii (r) can be calculated at $\delta r = \pm 0.05 r \sqrt{\frac{2}{h^2} + \left(\frac{2}{x}\right)^2}$ mm, if all the measures were taken in millimeters (mm). The instrument in question has received international recognition with four awards in recent years, that include Silver medal in world innovations exhibition “Inventions Geneva” in Switzerland in 2012, Sri Lanka presidential awards (Gold medal) in 2011, “The Ray Award 2015” and the most innovative product of INCO (Institute of Incorporated Engineers in Sri Lanka) in 2012.

References

[1] C.B. Boyer, *A History of Mathematics*, 2nd Edition, Wiley, 1991.
[2] Dinesh Katugampala, Radius Meter, Sri Lanka Patent number 15150 (2008)