A Brief Survey on Recent Applications of Linear Robust Control Theory In Commercial Automotive Applications

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***Abstract*—This technical note offers a brief survey on some recently disseminated research works regarding the applications of the linear robust control theory to the ground automotive control system design. We pay particular attention to papers in which the linear matrix inequality is employed as the toolset to solve the linear robust controller design problems. Papers reviewed in this note are roughly classified into two categories - longitudinal motion control and lateral motion control. At the end of the review, some authors' opinions are stated as discussions regarding potential future research topics in such applications.

***Index Terms*—Automotive control, linear matrix inequality, linear robust control, motion control**

I. Introduction

AUTOMOTIVE industry is a fundamental social component that acts as an important thrusting momentum to levitate human civilization and public welfare [1]-[3].

In modern times, the control systems design for the automobile has been an increasingly trending topic that has engrossed continuous research efforts since the last century [4]-[7]. For engineers and industrial practitioners, the linear robust control theory is unquestionably the eldest, most classical, and most widely adopted/favoured control methodology which has been universally implemented in various engineering and technological fields. In spite of its theoretical maturity, the wide recognition/utilization of the linear robust control has not surged until the 21st century. One core toolset which greatly helps the LC get wide attention is the linear matrix inequality (LMI). Essentially, the LMI enables the practitioners to handle the synthesis of linear robust control systems as convex optimization problems, thus introducing a uniform strategy of undertaking a wide spectrum of engineering challenges [8],[9]. What's more, the LMI problems are computationally tractable (via the interior point method) and many commercial software packages are readily developed to work out such problems efficiently and inexpensively [10].

The main purpose of this technical communique is to provide the readers with some recently published papers (published beyond 2020) regarding the LMI-based automotive control systems. The selected research works are roughly categorized into two categories - longitudinal motion control and lateral control. At the end of the note, some remarks are presented to discuss prospective open research points.

II. Review of Recent Papers

The main course of this technical note is offered in this section, i.e., several scholarly papers on the topic of commercial automotive controllers which are grouped into categories of longitudinal motion control, lateral control, and other vehicular electronics control. In what follows, we will briefly review those papers in order.

A. LMI-based longitudinal controllers

Thanks to the innovation and advancement in both hardware/software capabilities and control methodology, the vehicular longitudinal motion control systems have been greatly evolved. In the current market, more and more commercial automobiles are standardly equipped with adaptive cruise control (ACC) systems or more futuristic cooperative adaptive cruise control (CACC) systems [11].

In [12], an innovative ACC system, which simultaneously considered disturbance, measurement noise, and measurement delay, was designed via the multi-objective $\mathcal{H}_2/\mathcal{H}_\infty$ linear robust control method. The derivation of the linear robust control law was based on the LMI. Concerning the CACC design, ref. [13] employed the $\mathcal{H}_\infty$ theory to analyze the so-called string stability in face of time delays within the control loop. Specifically, a set of LMIs from the Lyapunov-Krasovskii functional analysis was derived as the constraint for the final semidefinite programming (SDP) optimization problem. Lately, ref. [14] has delved into the ACC design problem via the LMI-based linear robust control strategy ($\mathcal{H}_\infty$ to be more specific) for the electrical vehicles with domain-based electronic and electrical (E/E) architectures.

It should be noted that the $\mathcal{H}_\infty$ method is the most frequently utilized method for designing ACC and CACC. For outlook, LMI-based linear control methods based on other performance criteria should be investigated.

B. LMI-based lateral controllers

The lateral control for automotive applications is considered more challenging than its longitudinal counterpart ever since the plant is open-loop unstable (due to the positional state and possibly oversteer-type lateral stability). Hence, ensuring the robust stability of the lateral motion control loop is of great importance. linear control methods are generally considered

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superb candidates as they can offer guaranteed robust performance against undesired effects such as model uncertainties, disturbances/noises, and delays. In [15], Cheng *et al.* investigated the vehicular lateral control problem with the $\mathcal{H}_\infty$ method. The developed controller was carefully designed such that its robustness against model parametric uncertainties was ensured. A novel unknown input observer based linear robust controller was proposed in [16] to stabilize the vehicle lateral dynamics. The state variables were strictly confined within pre-established safe envelopes. In [17], an LMI-based path-following controller that was resilient against intermittent denial-of-service attacks was originated. In [18], an observer based $\mathcal{H}_\infty$ path-following controller was proposed with explicit considerations on steering actuator dynamics. [19] tackled the robust lateral motion control problem with explicit consideration of the kinematic nonlinearity. Such a controller was based on both $\mathcal{H}_\infty$ and Popov stability criterion. In [20], a fuzzy static output feedback control for the path following objective was proposed by unifying the $\mathcal{H}_2$ theorem and the Takagi-Sugeno fuzzy form. The control problem is formed in LMI wherein the final optimal control gain can be practically solved by a single line parameter search. Soon after, a Takagi-Sugeno $\mathcal{H}_\infty$ fuzzy vehicle lateral motion controller was proposed with explicit considerations delays [21]. In [22], an LMI-based output feedback vehicle lateral controller was designed. Such a controller was built to be resilient against both transmission delay and data dropout. [23] studied the lateral motion control problem of an automated parking system with the linear-parameter-varying $\mathcal{H}_2$ method.

III. Future View

Those research works surveyed in this brief have mostly focused on the robustness, stability, and fault-tolerant perspectives of automotive control systems. The future study may concentrate on other perspectives such as: First, the human factors, especially human driver's behaviours (biological and psychological), should be analysed further and considered more explicitly into the automotive control system design. Furthermore, the synergistic collaborations between the energy management systems and automobile controllers should be explored further.

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