DERIVATION OF DYNAMIC PROPERTIES OF STEEL ASYMETRIC PERFORATED ULTRA SHALLOW FLOOR BEAMS (USFB[TM]) VIA FINITE ELEMENT MODAL ANALYSIS AND EXPERIMENTAL VERIFICATION

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1. ABSTRACT

Σε αυτή την εργασία διενεργείται πειραματική και υπολογιστική μελέτη των χαρακτηριστικών ιδιοταλάντωσης των USFB. Συγκεκριμένα, ένα αμφιέρειστο δοκίμιο USFB υποβάλλεται σε πειραματική ιδιομορφική ανάλυση για την εύρεση των σημαντικών ιδιοσυχνοτήτων και ιδιομορφών σχετιζόμενα με την καμπτική τάλαντωση στο κατακόρυφο επίπεδο της δοκού. Επιπλέον, ένα ομοίωμα πεπερασμένων στοιχείων (ΠΣ) του δοκιμίου δημιουργείται στο πρόγραμμα ANSYS με τη χρήση ΠΣ κελύφους. Η ακρίβεια με την οποία αυτό το ομοίωμα ΠΣ προσεγγίζει τη δυναμική συμπεριφορά του δοκιμίου εξασφαλίζεται όπως προκύπτει από τις πειραματικές μετρήσεις με αντίστοιχα αποτελέσματα ιδιομορφικής ανάλυσης ΠΣ. Στη συνέχεια, παραμετρική ιδιομορφική ανάλυση 4 USFB διατομών για διάφορα μήκη δοκού πραγματοποιείται στο ANSYS. Τέλος, παρατίθενται ισοδύναμα γεωμετρικά χαρακτηριστικά των υπό ανάλυση USFBs τα οποία υπολογίζονται από την ικανοποίηση συγκεκριμένων κριτηρίων ισοδυναμίας μεταξύ των USFBs και απλών πρισματικών δοκών Euler-Bernoulli σταθερής διατομής. Τα ισοδύναμα αυτά χαρακτηριστικά μπορούν εύκολα να χρησιμοποιηθούν για να διευκολύνουν σημαντικά τη μοντελοποίηση των USFBs σε προγράμματα ΠΣ του εμπορίου για τη μελέτη μεταλλικών κατασκευών υποκείμενα σε δυναμικές καταπονήσεις. Η παρούσα εργασία αποτελεί το πρώτο βήμα για τη μελέτη της ταλάντωσης σύμμικτων δαπέδων με χρήση USFBs που αποτελεί κρίσιμο ζήτημα στη διαστασιολόγηση τους έναντι οριακών καταστάσεων λειτουργικότητας.

2. INTRODUCTION

Requirements for maximum space utilization, efficiency during construction, and cost-effectiveness demand the use of long-spanned, shallow, light steel beams in steel and composite structures. In achieving optimum integration of structure and services within the

same horizontal zone, perforated beams having large closely spaced web openings along their length is a commonly used structural element in long span light roof as well as in composite floor systems. Typically, such structural systems are sensitive to dynamic loads induced during their service life by personnel activities (e.g. walking, jumping, e.t.c.) as well as by broadband forces from turbulence in piping and ducting supported within floor slabs. In this respect, their response to such excitations needs to be considered at design.

In recent years, a new breed of composite floors have been introduced by ASDWestok Ltd. utilizing the Ultra Shallow Floor Beam (USFB). The USFB steel section is fabricated by welding two highly asymmetric cellular tees together along the webs resulting in a large bottom flange. Either precast concrete floor units or profiled steel decking rest on the bottom flange of the USFB creating a very shallow floor beam construction system, thus minimizing the overall structural depth (*Fig. 1*). As the floors are being cast, the in-situ concrete passes through the web openings, which may or may not include a tie-bar or duct. This concrete plug and tie-bar forms a unique mechanism for transferring longitudinal shear force along the beam. A special end diaphragm is used for deep decking floor applications so that the concrete fully surrounds the steel section, apart from the bottom plate. 'Arching' action is occurred through the concrete partial encasement, which is resisted by the end plate connections. The web openings provide a passage for reinforcing tie-bars and service ducts within the depth of the beam. In-situ concrete fills the web openings as the floor slabs are cast. This concrete passing through the web openings, together with the reinforcing bars and ducts, contribute to the longitudinal shear resistance when the beam is subjected to axial bending. The free web openings under the profiled steel decking are utilized for the integration of services. [1,2,3,4,5]
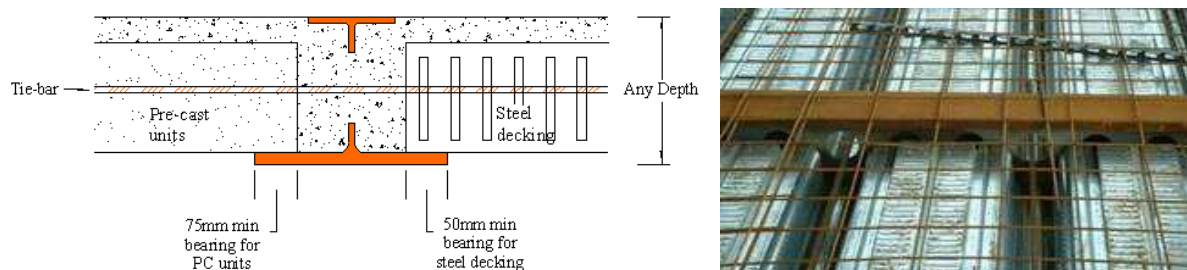

Fig. 1: Cross-section configuration of composite floor construction using asymmetric USFBs (left panel) and deep decking floor application (right panel) (ASDWestok Ltd.)

Herein, a first step towards the study of the vibration behavior of the aforementioned composite floor structures is undertaken by probing into the dynamic properties of commonly used bare steel USFBs. For this purpose, experimental impact hammer modal testing is conducted in the Heavy Structures Lab at City University London to characterize the first few flexural modes of vibration of a simply supported USFB steel specimen. Next, a finite element (FE) model of this specimen is developed in ANSYS and its effectiveness to capture the vibration behavior of the considered USFB is verified by comparing the experimental data with results from FE modal analysis. Then, parametric modal analyses based on the above FE model is undertaken in ANSYS to obtain the natural frequencies of several USFBs commonly used in building engineering practice. Finally, equivalent geometric properties of these USFBs corresponding to equivalent Euler-Bernoulli beams of constant cross-section are derived by relying on specific criteria of equivalency involving inertial and dynamic properties.

3.   EXPERIMENTAL MODAL TESTING

A simply supported bare steel Ultra Shallow Floor Beam (USFB) spanning 6m is subject to impact experimental modal testing to extract the modal characteristics of the transverse flexural modes of vibration of the beam specimen. The geometry of the beam specimen and the experimental setup are shown in *Fig. 2*. Three uni-axial high sensitivity shear accelerometers, henceforth acc1, acc2, and acc3, are placed on the middle of the upper flange in the mid-span and in the quarters of the 6m (positions #2,#4 and #6 as shown in *Fig. 2*). An impact hammer (model 5803A by Dytran Instruments) equipped with an embed force sensor is used to excite the transverse flexural modes of vibration of the beam specimen by hitting it downwards along the gravitational axis at the seven positions indicated in *Fig. 2*. Five "qualified" hits are recorded at each position. An appropriately hard hammer head was used to ensure that an adequately broadband input signal is achieved to excite modes at least up to 300Hz. A USB-9234 device by National Instruments connected to a regular PC unit is used for simultaneous data acquisition of the four channels (three output/acceleration and one input/force) and storage in ASCII format using specialized software onto the hard disk of the PC. In this manner, 30s long digital recorded signals sampled at a rate of 1024Hz are obtained from all four channels for the 35 hammer hits considered in total. This duration was found to be sufficiently long for the response transient signals to die out eliminating the need to apply an exponentially decaying time window. The stored data are then processed off-line to construct a 3-by-7 matrix $\mathbf{H} = \begin{bmatrix} H_{ij} \end{bmatrix}$ containing frequency response functions (FRFs) corresponding to location *i* due to an impact at location *j* using a custom-made script in MATLAB. The latter follows the common analysis steps used by standard FFT analysers [6,7,8].
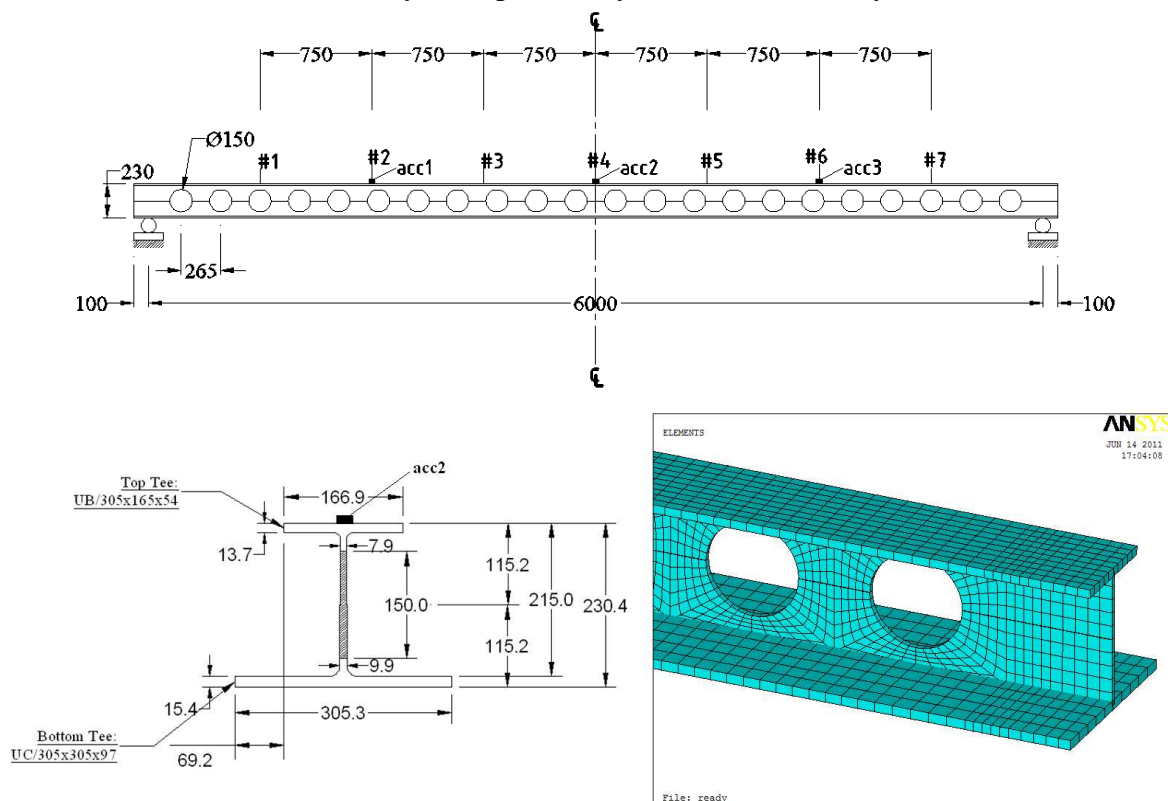
Fig. 2: Longitudinal view, cross-sectional properties and finite element mesh of the USFB considered for experimental modal testing (units in mm)

Specifically, the raw signals are processed by a 5th order band-pass Butterworth filter with cut-off frequencies at 5Hz and at 400Hz. The filtered signals, although deterministic and non-stationary (transient) in nature, are treated as random (i.e. as samples of certain underlying stochastic processes); a common consideration in the field of impact experimental modal testing [6]. This allows for computing FRF estimates in the domain of frequencies ω by the following widely used expressions:

$$H_1(\omega) = \frac{G_{xf}(\omega)}{G_{xx}(\omega)}, \dotfill (1)$$

and

$$H_2(\omega) = \frac{G_{xx}(\omega)}{G_{xf}(\omega)}, \dotfill (2)$$

where G_{xx} and G_{ff} are the (real) auto- power spectral density (PSD) functions of the acceleration and the input hammer force, respectively, and G_{xf} is their (complex) cross-PSD.
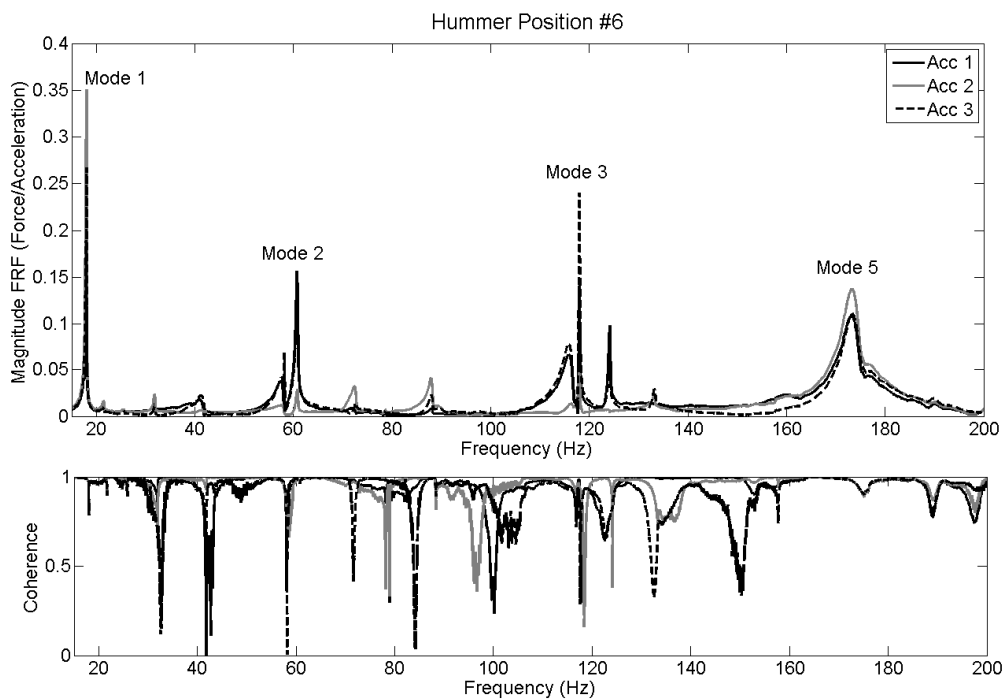

Fig. 3 : Average frequency response functions and coherence corresponding to position #6

Fig. 3 plots the magnitude of H$_2$ estimators linearly averaged over an ensemble of five functions corresponding to the five hammer hits at position #6 obtained from the three accelerograms (the 6th column of the **H** matrix constructed from the experimental data). A plot of the coherence function defined as the ratio H$_1$/H$_2$ is also shown to provide an indication of the reliability of the plotted FRF curves (the closer the coherence lies to unity, the more reliable the FRF functions). The 1st, 3rd and 5th natural frequencies are clearly identified in the aforementioned FRFs. The 2nd natural frequency is picked up only by the acc1 and acc3 channels as the acc2 lies at a nodal point of the second mode (a point where the corresponding mode shape attains a zero). The 4th natural frequency cannot be determined by any of the accelerograms as they all lie at nodal points of the 4th mode shape. However, it can be retrieved by Finite Element modal analysis as discussed in the

next section. Similar observations are made to the averaged sets of FRFs corresponding to the rest of the columns of the **H** matrix.

Next, the FRF data around the four natural frequencies of interest indicated in *Fig.* 3 are isolated and a standard SDOF circle-fitting method on the complex (Argand) plane implemented in MATLAB is employed to extract estimates of natural frequencies, damping, and modal constants (coordinates of the mode shapes) corresponding to the first three and the 5th flexural modes of vibration. The average values obtained from the total 21 FRFs of these natural frequencies and damping are collected in *Table 1*. Furthermore, *Fig. 4* plots the first two mode shapes normalized to their peak absolute value.

Mode	Damping (%) Experimental				Natural Frequency (Hz) (in-plane flexural modes) Experimental				FE (ANSYS)	FE (ANSYS) Solid beam
	acc 1	acc 2	acc 3	average	acc 1	acc 2	acc 3	average		
1st	0.98	0.98	0.98	0.98	17.954	17.948	17.948	17.950	18.184	18.449
2nd	0.94	0.80	0.98	0.91	60.705	61.135	60.699	60.846	63.565	69.628
3rd	0.23	0.24	0.23	0.23	118.03	117.93	118.02	117.99	120.74	141.87
4th	-	-	-	-	-	-	-	-	167.86	179.37
5th	1.58	1.58	1.49	1.55	173.91	173.9	173.86	173.89	184.17	231.94

Table 1. Experimentally derived damping and natural frequencies vis-à-vis results from finite element based modal analysis.
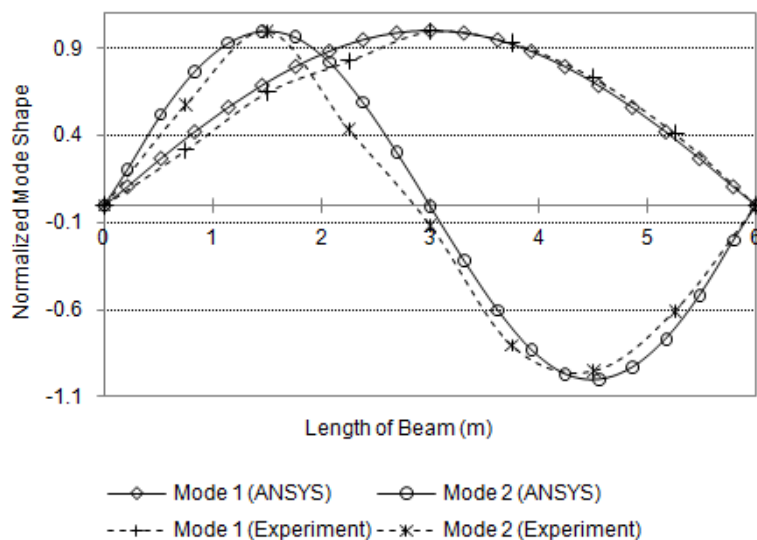

Fig. 4 : Mode shapes normalized to their peak absolute value

4.   FINITE ELEMENT MODELLING AND ANALYSES

Following standard practices in modal analysis [6,9,10,11], a Finite Element (FE) model of the USFB used in the experimental modal test is developed in ANSYS. Eight-node shell elements (SHELL281) and mapped meshing is implemented to design the bare steel beams. A snapshot of the model is included in *Fig. 2*. The mass density is computed using the total measured weight of the beam specimen (404 kg) and the dimensional data of the cross sectional properties shown in *Fig. 2*. The accuracy of this FE model compares well

against the experimental data (*Table 1* and *Fig. 4*). Moreover, natural frequencies for the same asymmetric beam but without web openings (solid) are also included in *Table 1*. Evidently, it is found that the existence of the web openings influence significantly the higher modes of vibration.

4.1 Parametric Finite Element analysis

Having established a FE model for the beam specimen which compares well with the experimental modal testing, a parametric FE modal analysis is undertaken considering four commonly used in practical applications cross-sections for USFBs of four different lengths. In total, 16 models are constructed in ANSYS. Key geometric properties of the considered models are reported in the first four columns of *Table 2*. The first five natural frequencies and typical results from modal analysis are also reported in *Table 2*.

(FE Model) Top / Bottom Tee-Section (mm)	Volume (m^3)	Number of web openings	Length (m)	Natural Frequencies (Hz) (in-plane flexural modes)					A_{eq} (cm^2)	I_{eq} (cm^4)
				1st	2nd	3rd	4th	5th		
(CAT1) UB 254x102x22/ UC 254x254x73	0.0231	15	4	36.468	119.68	214.01	255.57	306.65	57.750	3107.71
	0.0342	22	6	16.999	61.413	120.45	171.75	187.2	57.000	3374.05
	0.0451	30	8	9.7153	36.096	74.92	118.99	132.39	56.375	3444.96
	0.0561	37	10	6.2704	24.071	50.89	83.643	105.51	56.100	3486.41
(CAT2) UB 254x102x22/ UC 254x254x73	0.0351	15	4	38.412	120.00	206.27	247.16	287.98	87.750	5238.97
	0.0518	22	6	18.184	63.565	120.74	167.86	184.17	86.333	5847.73
	0.0684	30	8	10.454	38.444	76.962	118.71	131.68	85.500	6049.45
	0.0851	37	10	6.7679	25.611	53.111	85.369	104.81	85.100	6161.16
(CAT3) UB 254x102x22/ UC 254x254x73	0.0454	11	4	51.732	142.1	222.98	243.66	291.52	113.500	12290.77
	0.0668	17	6	25.518	80.458	139.74	168.34	197.18	111.333	14850.75
	0.0881	23	8	15.005	51.126	94.762	127.42	140.9	110.125	16052.50
	0.1100	28	10	9.8448	35.318	68.807	101.17	108.49	110.000	16851.22
(CAT4) UB 254x102x22/ UC 254x254x73	0.0659	9	4	63.399	160.00	217.17	253.58	327.04	164.750	26795.08
	0.0966	14	6	32.559	92.416	150.81	164.37	218.63	161.000	34962.13
	0.1270	19	8	19.597	61.956	107.92	123.43	157.48	158.750	39470.96
	0.1580	24	10	13.005	43.64	79.941	99.678	119.67	158.000	42237.93

Table 2. Natural frequencies from FE modal analysis and equivalent properties corresponding to Euler-Bernoulli beams of constant cross-section for various USFBs

4.2 Equivalent Geometric Properties of USFB beams

Relying on the previously discussed FE parametric study, a further step of practical importance is taken to obtain equivalent geometric properties for the considered USFBs corresponding to a prismatic Euler-Bernoulli beam of constant cross-section along its longitudinal axis [12]. These properties can be readily used in conjunction with commercial FE codes to model the bare steel USFBs. For this purpose, an equivalent cross-sectional area A_{eq} and an equivalent second moment of area I_{eq} are defined based on the two following criteria: (i) The total mass of the equivalent prismatic beam is equal to the

mass of the USFB assuming that both beams are of the same length, and (ii) The natural frequency of the first in-plane flexural mode shape of the USFB and its prismatic surrogate are equal.

In satisfying criterion (i), the A_{eq} is determined as the ratio of the volume over the length of the USFB. The volumes of the herein considered USFBs are reported in *Table 2* along with the thus obtained A_{eq}. Furthermore, assuming simply supported beams, criterion (ii) is met by determining the I_{eq} from the expression:

$$I_{eq} = \frac{4 f_1^2 \rho A_{eq} L^4}{E \pi^2} . \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots(3)$$

The latter equation is derived from the well known expression of continuum dynamics for the first natural frequency (f_1) of the free transverse vibration of a simply supported Euler-Bernoulli beam with mass density ρ, modulus of elasticity E, cross-sectional area A_{eq}, and length L. *Table 2* includes values of I_{eq} for the various USFBs considered obtained using eq. (3) assuming ρ=7800 kg/m^3 and E=200GPa corresponding to nominal steel material properties.

It is noted in passing that other criteria of equivalency can be utilized in deriving equivalent geometric properties which will generally yield different values for A_{eq} and I_{eq} [12]. In this work, the aforementioned criteria have been selected as they relate to the dynamic behavior of the USFB beams under investigation in a straightforward manner.

5. CONCLUDING REMARKS

The vibration behavior of asymmetrical ultra shallow floor beams (USFBs) has been investigated. Experimental data obtained from impact hammer modal test on a specific simply supported USFB specimen have been considered to extract certain of the first five natural frequencies and first two mode shapes corresponding to the flexural in-plane free vibration motion using purpose-made MATLAB scripts. These compare well with results from finite element (FE) modal analysis undertaken in ANSYS on a detailed FE model developed to capture the dynamic behavior of the considered specimen. Furthermore, the first five natural frequencies of four different USFBs commonly used in practice for various lengths have been obtained via modal analyses in ANSYS. These analyses have been applied on detailed FE models produced by relying on the initial model pertaining to the experimentally tested beam. Finally, equivalent geometric properties (cross-sectional area and second moment area) for all the considered USFBs in the parametric study corresponding to prismatic Euler-Bernoulli beams with constant cross-sections have been computed. These properties are derived by enforcing equality of the total mass and the natural frequency of the first flexural mode of vibration between the USFBs and their prismatic surrogates. Clearly, these equivalent properties can be used to incorporate USFBs in FE models of structures developed in FE software commonly used by the practicing engineers for structural design against dynamic loads, such as those induced by earthquakes. Future experimental and computational work will be directed towards the characterization of the dynamic properties of composite floors incorporating USFBs similar to those shown in *Fig. 1* [13,14].

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