FDTD Scheme for Interfaces Formed by Space-Time Modulations

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Abstract

Emerging metamaterials formed by space-time modulation involve interfaces that conjunctly move in space and time. This creates a fundamental discretization issue in numerical methods such as the Finite Difference Time Domain (FDTD) method. This paper highlights this issue and resolves it using a numerical frame-hopping scheme. This scheme is validated by comparison with exact analytical results from previous works.

1 Introduction

Space-time metamaterials are generalizations of pure-space and pure-time metamaterials Caloz and Deck-Léger (2019a). They represent one of the latest and most promising advances in the area of metamaterials Caloz and Deck-Léger (2019b). Such metamaterials are formed by the (electronic, acousto-optic, electro-optic) modulation of a host medium in the form of a traveling-wave perturbation of one of its constitutive parameters (e.g., index of refraction), with the modulation typically consisting of a periodically repeated space-time "slab pair" (e.g., slabs of refractive indices n_1 and n_2) unit cell that is eventually made sub-wavelength and subperiod for metamaterial homogeneity. Given this background periodic structure, the *space-time interface* that separates the two media of the slab-pair unit cell forms the building brick of a space-time metamaterial, and it represents therefore the primary related problem to address.

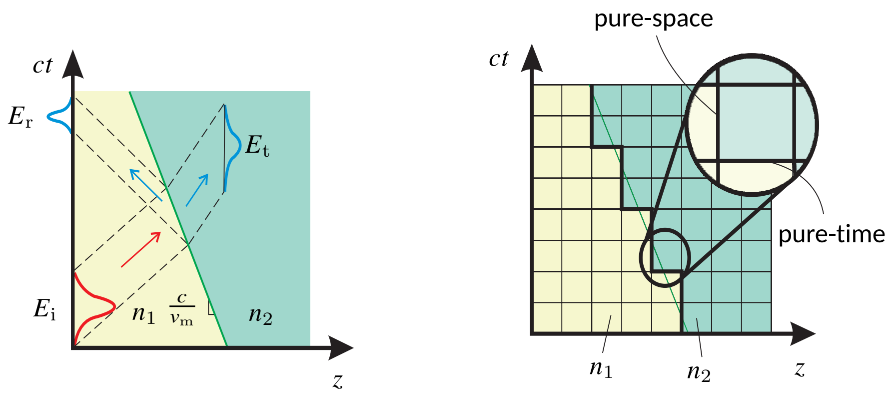
As their pure-space counterparts, space-time metamaterials require numerical schemes for the resolution of non-canonical problems. The Finite Difference Time Domain (FDTD) technique Yee (1966); Taflove et al. (2005) appears to be a natural choice because of its a priori straightforward capability to handle arbitrary spatial *and* temporal variations. However, a fundamental *discretization* problem occurs when the space and time variations appear conjointly. In the case of *moving matter*, this problem is circumvented by transformation into an equivalent *stationary* (pure space) bianisotropic problem Zhao and Chaimool (2018); Teixeira (2008), but the problem of a *moving perturbation interface* has not yet been addressed so far.

This paper solves this problem. Section II highlights the aforementioned problem, Sec. III resolves it and Sec. IV provides discussions and prospects.

2 Spacetime Discretization Issue

Figure 1 depicts the overall problem under consideration. Figure 1(a) shows the physical problem to be solved, namely the determination of the fields scattered at a space-time interface between two different media. Such a simple problem admits compact analytical solutions Caloz and Deck-Léger (2019a,b); Deck-Léger et al. (2019), but a numerical resolution becomes necessary for more complex interfaces (e.g., nonuniform-velocity interfaces or interfaces between bianisotropic media) that locally involve the same structure.

A naive approach would be to attempt solving this space-time problem by discretizing its space-time $(z-t)$ discontinuity in the same way as a space-space $(z-x)$ discontinuity Taflove et al. (2005), as illustrated in Fig. 1(b). What happens then with the (conventional) FDTD algorithm Yee (1966) is the following. The usual stationary boundary conditions (continuity of the tangential $\mathbf{E}$ and $\mathbf{H}$ fields) are applied at each (time) iteration in the resolution of the discretized Maxwell curl equations. In this process, the motion of the interface is properly accounted for, so that we might expect right Doppler shifting, but the applied boundary conditions are the *wrong* ones[1] and the scattering coefficients can therefore not be expected to be right[2].



(a) (b)

Figure 1: FDTD space-time discretization issue for a space-time interface (here moving in the $-z$-direction) between two different media (here of refractive indices n_1 and n_2). (a) Physical interface. (b) Naive staircase approximation in FDTD.

Figure 2 illustrates the failure of the naive space-time FDTD discretization in Fig. 1(b) by comparing its result with the results from the analytical scattering coefficients Caloz and Deck-Léger (2019a,b); Deck-Léger et al. (2019).

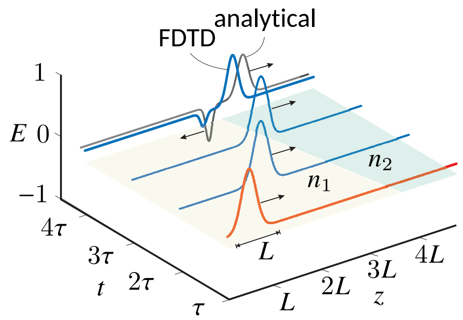


Figure 2: Illustration of the failure of the naive FDTD as compared with the analytical results given in Deck-Léger et al. (2019) of a single interface between two media of $n_1 = 3$ and $n_2 = 5$ moving at $v_m = -0.3c$.

3 Modified Frame Hopping Resolution

We shall present here a resolution of the FDTD problem described in Sec. II using frame hopping Kong (1990). The resolution procedure is composed of the following steps:

1. Transpose the incident source from the lab frame, K, where only the modulation moves (without any net motion of matter), to the comoving frame, K', using the Lorentz transformations, e.g., $\mathbf{E}'_{\mathrm{src}} = \gamma\,(\mathbf{E}_{\mathrm{src}} - v_{\mathrm{m}}\mathbf{B}_{\mathrm{src}})$ for the $\mathbf{E}$ field;

2. FDTD-solve the problem in the K' frame, where the interface is purely spatial (or purely temporal) but the media are bianisotropic Kong (1990) due to the motion of matter (molecules and atoms forming the modulation host media) in the opposite direction; this involves the Maxwell iteration equations
$$\mathbf{B}'^{\mathrm{new}} = \mathbf{B}'^{\mathrm{old}} + \Delta t'\nabla\times\mathbf{E}' \quad \text{and} \quad \mathbf{D}'^{\mathrm{new}} = \mathbf{D}'^{\mathrm{old}} - \Delta t'\nabla\times\mathbf{H}', \tag{1}$$
with the constitutive relations

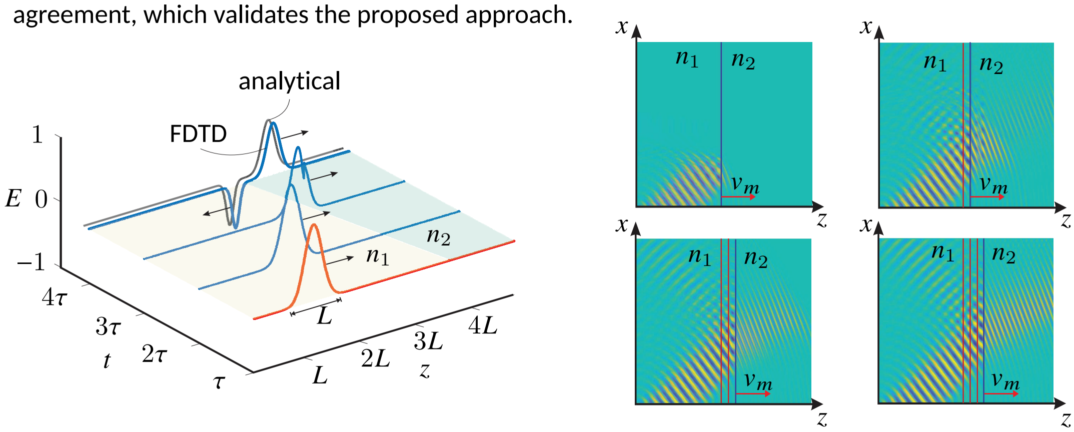
[1]The correct boundary conditions for a space-time moving discontinuity are established in the theory of relativity Kong (1990); Deck-Léger et al. (2019) and will be used in Sec. III.

[2]Note that this is a *fundamental* issue, and not just a numerical (convergence) issue, which could be fixed with finer meshing.

$$\mathbf{H}' = \overline{\overline{\mu}}^{-1}\mathbf{B}' - \overline{\overline{\mu}}^{-1}\overline{\overline{\zeta}}\mathbf{E}' \quad \text{and} \quad \mathbf{E}' = \overline{\overline{\epsilon}}^{-1}\mathbf{D}' - \overline{\overline{\epsilon}}^{-1}\overline{\overline{\xi}}\mathbf{H}'. \tag{2}$$

3. Record the computed fields at each (time) step.
4. Numerically transpose these fields to the K frame using inverse Lorentz transforms.

Figure 3 shows the results obtained with this approach for a 1+1D interface problem. Figure 3(a) compares the FDTD field with the closed-form results. The two curves, in contrast to the case of Fig. 2, are in perfect agreement, which validates the proposed approach.



(a) (b)

Figure 3: Results for the proposed FDTD resolution scheme. (a) 1+1D scattering of a Gaussian pulse from an interface moving at $v_\mathrm{m} = -0.3c$ between media of refractive indices $n_1 = 3$ and $n_2 = 5$. (b) 1+2D scattering of a monochromatic-plane beam from an interface moving at $v_\mathrm{m} = 0.2c$ between media of of refractive indices $n_1 = 1.5$ and $n_2 = 2.5$, with the four panels corresponding to different time snapshots.

This method can be straightforwardly extended to higher dimensions, which involves spatial obliqueness. Figure 3(b) shows results for a 2+1D interface problem.

4 Discussion and Prospects

We have presented a frame-hopping – FDTD method to circumvent the fundamental issue of the space-time staircase approximations that occurs in the conventional FDTD method in the presence of conjoint spatial and temporal medium variations.

This method can be extended to include more complex motions, such as accelerated (e.g., Rindler) motion and non-rectilinear (e.g., parabolic, elliptic) motion, and to actual space-time metamaterials.

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