Axiomatic principle of multi field coupling dynamics analysis

Peng Yue*
University of Electronic Science and technology of China, Chengdu, China and China Aerodynamics Research and Development Center, Mianyang, China

Jinghui Zhang
University of Electronic Science and technology of China, Chengdu, China

Sibei Wei
National Aerospace University "Kharkiv Aviation Institute", Kharkiv, Ukraine

Multi field coupling dynamics analysis has always been a problem of great concern. As the basic elements of machinery, the properties of materials such as mechanics, thermodynamics, and electromagnetism play an essential role in mechanical design. On the one hand, in recent years, the material property system in a single discipline has tended to be perfect, but machinery usually works under the action of multi-field coupling, and there are often multiple design indicators that are difficult to reconcile in terms of weight and scope of action. Therefore, it is quite difficult to use the single-field performance index of the material to achieve the design goal of multi-field coupled machinery at one time. On the other hand, due to the development of science and technology, the natural balance has been greatly changed. It is foreseeable that the important direction of the next industrial revolution should be to develop in harmony with nature on the basis of meeting the needs of human survival and production. While developing new materials, we must also pay attention to the comprehensive utilization rate of materials, and cannot blindly pursue the ultimate performance in a certain aspect. Therefore, with the help of the proposed axiomatic principle of dynamic behavior of materials, a new description method of multi-field coupling index is proposed. In the future, one of these indicators can be used to develop multi-field coupled mechanical design methods for mechanical design, and the second is to use these indicators to develop evaluation criteria that can describe the environmental conditions of materials, which has certain practical significance.

I. INTRDOCTION

At present, machinery manufacturing is one of the most important production activities for human beings. German mechanic Reuleaux F [1] believed that machinery is a collection of components connected in various ways. The movement of a certain component drives other components to achieve mechanical functions, and the relative motion relationship depends on the connection state between the component units. A set of excellent mechanical design scheme can greatly improve the operaten efficiency, reliability and service life of machinery [2], and can also reduce the cost of manufacturing, maintenance and energy consumption as much as possible [3]. Therefore, mechanical design is an indispensable and important part in the further development of science and technology in the future.

Usually, the mechanical design is to select reasonable modes and routes for energy transmission by comparison, and combine the functional input and output mechanism to achieve the purpose of realizing the mechanical function. Therefore, the essence of mechanical design is to choose an appropriate energy transfer method to achieve high-efficiency realization of mechanical functions. In related research, Petrescu F I et al. [4] proposed a method to determine gear efficiency, gear force, speed, and power. Esmail E L et al. [5] proposed a two-degree-of-freedom transmission efficiency calculation formula, proving the dependence between kinematics and efficiency. Bharti S et al. [6] studied the wear performance of gears under non-lubricated and wet-lubricated conditions. Federico G et al. [7] proposed a new mathematical model to estimate the mechanical efficiency of harmonic drives.

In general, the efficiency of transmission equipment is the main evaluation index for the strength of mechanical design functions at present [8], and the improvement of mechanical efficiency means the enhancement of mechanical functions [9]. The mechanical efficiency is the percentage of the output and input work of the machine when the machine is running stably [10]. It is defined from the perspective of energy and is a key parameter in the design process of general energy power machinery [11] and transmission machinery [12], not a key parameter of general dynamic balance machinery [13]. Since efficiency cannot express the degree of action of physical quantities, when designing new machinery, it is still not possible to rely solely on mechanical efficiency to design machinery, and other important physical parameters are also considered. For example, the design of aero-engine needs to comprehensively consider the boost rate of the compressor [14]; the design of the gear transmission mechanism should comprehensively consider physical quantities such as speed [15]. Therefore, it is a very important scientific

* Emial:pengyu.yue@outlook.com

problem to clarify the design index of mechanical function.And with the help of the research on the essence of function, it is one of the problems to be solved in this research to clarify the design index of mechanical function.

According to the current design method, the mechanical structure design is carried out after the mechanical function design is completed [16]. In the mechanical structure design, a streamlined and suitable mechanical structure can effectively reduce the mechanical quality [17] and improve the mechanical function stability [18] and efficiency. The complicated and improper mechanical structure will improve the mechanical quality [19], and may endanger the normal operaten of the machinery. When facing different usage scenarios, mechanical structural design problems can be divided into lightweight design [20]; traditional structural design based on different loads [21]; structural stability design[22] and reliable/durable design [23] problems.

The yield limit is the stress that resists a small amount of plastic deformation [24]. The strength limit is the maximum stress value that a member can withstand while maintaining the mechanical strength of the member [25]. In order to reduce the probability of plastic deformation or fracture failure in mechanical design, the more direct and simple approach is to increase the amount of material. The reduction of material utilization rate will lead to a series of economic problems such as the increase of cost [26], and even affect the development of precision [27] and batch [28] of machinery.

In order to further reduce the design weight of machinery, lightweight design has gradually become a key link in the mechanical design process [29]. In the mechanical design of transportation, aerospace and other fields, in order to keep the mechanical function unaffected, lightweight design is a recognized and necessary design process [30]. However, due to the changes in the geometric parameters of lightweight design, the current lightweight design results may lead to local instability [31], local stress concentraten [32] and other phenomena of the structure, which in turn causes reliability problems such as structural stability or durability.

The stability of the structure refers to the ability of the structure to maintain the original equilibrium state under the action of load [33]. The durability of the structure refers to the long time that the structure can be used without failure [34], which is the longevity of the mechanical structure. Cracks or fractures may occur in mechanical parts under the action of alternating stress for a long time [35], thereby reducing the durability of the mechanical structure. The reliability of the structure refers to the ability of the structure to complete the function under the specified conditions and within the specified time [36].

In the mechanical structure design, the irreconcilable contradictions will occur between the structural indicators at the structural design level. Therefore, unifying the design indicators of the mechanical structure and avoiding the interference between the indicators are the problems that need to be solved in this research.

Fundamentally, the mechanical design quality determines the operatenal quality of a machine [37]. Schulte F et al. [38] propose a more detailed system coping strategy that includes response functions, describing system functions and their interconnections through signals, materials, and energy flow. Yoqubjon Y [39] proposed a structural optimization design method based on structural optimization and parameter optimization. Yan G et al. [40] proposed a design method of bio-inspired vibraten isolation machinery from the perspective of mechanics. The development of these methods mainly relies on traditional mechanical design methods and supplements or optimizes them with industry technology.

The traditional mechanical design method is mainly reflected in the serial design idea, that is, the functional design and structural design of the machine are carried out step by step [41]. With the continuous development of mechanical design research, scholars have gradually realized that there are numerous repetitive structures among the functional structures of machinery [42]. On the one hand, this problem will lead to the extension of the design cycle and the increase of the design cost. On the other hand, the simple combination of structural elements is easy to interact with each other.

Therefore, some scholars have proposed the design idea of functional structure integraten. For example, Gu D et al. [43] proposed an integrated design method of functional structure by means of additive manufacturing in *Science*. Liu H et al. [44] elaborated on the principle, structure, output performance and advantages of various hybrid energy harvesting systems. And Zaman H R R et al. [45] proposed an improved particle swarm optimization method for mechanical optimization design. Thomas J P [46] proposed a method for deriving an index. In addition, multidisciplinary fusion methods such as collaborative design [47] are also current research hotspots, but they are not essentially separated from the design idea of tandem. Researches on electromechanical coupling [48] and fluid-structure coupling [49] are widely carried out, but the accuracy of the algorithm still causes certain uncertainty in mechanical design [50].

At present, the difficulty of functional structure integrated mechanical design is mainly concentrated in the inability to establish a direct connection between the mechanical structure and the realization of functions [51][52]. Therefore, it is necessary to effectively construct a multi-field coupling design system between the structure and the function in the mechanical design.

II. AXIOMATIC PRINCIPLES OF DYNAMIC BEHAVIOR OF MATERIALS

In order to further improve the quality of mechanical design, it is necessary to find a unified mechanical structure and function design index, which requires clarifying the basic principles of mechanical design, so as to obtain new mechanical design indicators.

A. Function and Structure

Definition 1. *The bounded closed space composed of all points in the machine is called* ***the ontology space*** *of the machine; the bounded open space composed of all the points that the machine may act on is called* ***the environment space*** *of the machine. It can be seen from this that the ontology space of a machine is a subspace of its environment space.*

Definition 2. *All machines are called* ***design machines****, the complete set of machines is called* ***machine space****, machines in n-dimensional space are called n-dimensional* ***space machines****, and n-dimensional* ***space machines*** *must be n-dimensional design machines.*

Definition 3. *The set of infinite points in the mechanical environment space is $\Omega \subseteq \mathbb{R}^n$. Since the function of the machine is the effect of the mechanical part $P \subseteq \Omega$ on the environmental space Ω , and the two spaces are homeomorphic, that is, the function is the mapping between the functional part of the machine and the environmental space. Then, the* ***function*** *of the machine can be defined as*

$$\varphi : p \mapsto \omega, p \in P, \omega \in \Omega.$$

If the set composed of all functions is called function set Φ , then $\varphi \in \Phi$.

As shown in Fig.1, at this time, point A in a certain area of the static boom forms a mapping relationship with point B in the corresponding area of the environment space of the boom. Assuming that the mapping relationship is displacement, then the second derivative of this mapping relationship to time is acceleraten. According to D'Alembert's principle, this acceleraten is produced by inertial forces. Therefore, this mapping can reveal the force of the boom on the object, even if the object lifts this function.

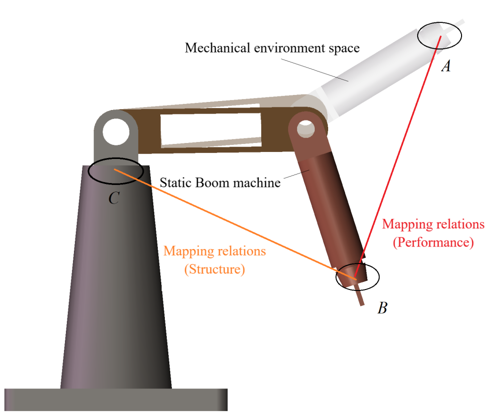


FIG. 1: Boom machinery and its environment space

Definition 4. *The normed vector space composed of infinite points in the ontology space of the design machine is $Q \subseteq \mathbb{R}^n$. The structural part $S \subseteq Q$ of the machine is to support and connect the whole machine Q and complete the function of Q . Since there must be load flow conduction in supports and connections, there is a mapping relationship between the structural part of the machine and the entire machine. Then, the* ***structure*** *of the machine can be defined as*

$$\Psi_a : s \mapsto q, s \in S, q \in Q,$$

among them, a is called structural design index, then the set composed of all structural targets is called structural set Ψ , then $\psi_a \in \Psi$.

Considering that the part of the mechanically generated function is also realized by the mechanical structural part, the intersection of the part of the mechanically generated function and the structural part is called the **functional structure** part R , that is,

$$R = P \cap S.$$

B. Density and Tensor-Change Rate(T-C Rate)

In physics, density is a common concept. In mechanics, and it can be obtained from this that density can describe the effect of scalar physical quantities, such as the greater the density, the greater the effect of mass.

Definition 5. *Let ρ be the scalar field of any scalar field 4 that changes with time and space in bounded n-dimensional space $Q \subseteq \mathbb{R}^n$ and time $T \subset [0, T_t]$. When the scalar field refers to the physical field, it satisfies*

$$\rho = \lim_{\Delta\tau \to 0} \frac{\Delta\varphi}{\Delta\tau} = \frac{d\varphi}{d\tau}, \tag{1}$$

then ρ is called the ***density*** *of the physical quantity φ.*

When the amount of the physical quantity flowing out of the object is greater or less than the amount flowing into the object, it means that the source or sink of the physical quantity is formed inside the object, and the source and sink are the effect of the physical quantity on the object (Fig.2).

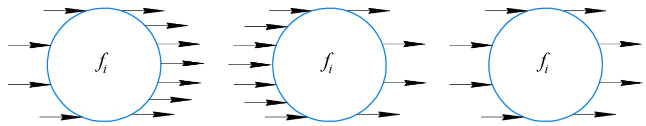
Theorem 1. *The divergence of any vector can be described as the density of the flux of the vector function.*

Proof: Set $\varphi_i \in C^m(\mathbb{R}^n)$,then the flux of the vector function can be expressed as $A = \oint_\sigma \varphi_i \, \mathrm{d}\sigma_i$,and according to Gauss's theorem, $\oint_\sigma \varphi_i \, \mathrm{d}\sigma_i = \int_\tau \varphi_{i,i} \, \mathrm{d}\tau$,namely

$$\varphi_{i,i} = \lim_{\Delta\tau \to 0} \frac{\oint_\sigma \varphi_i \, \mathrm{d}\sigma_i}{\Delta\tau}$$

.

Then the divergence of any vector can be described as the density of the flux of the vector function. □



(a) Flux is positive (b) Flux is negative (c) Flux is zero

FIG. 2: The role of flux and vector physical quantities.

Definition 6. *Let ρ be a scalar function of n-dimensional bounded space $\Omega \subset \mathbb{R}^n$ and time $T \subset [0, T_t]$ about any vector $\varphi_i \in C^m(\mathbb{R}^n \times [0, T_t])$ that changes with time and space, when vector φ_i is a physical quantity, it satisfies*

$$\rho = \varphi_{i,i}, \tag{2}$$

then ρ is called the density of the physical quantity φ_i.

As shown in Fig.3, the points with the greatest local effect of the vector field are mainly concentrated in the six points A, B, C, D, E and F. Considering that convergence and dispersion are the sink and source points of the vector field, the magnitude of the divergence of the vector field indicates the degree of action of the vector field, which is the density of the vector physical field.

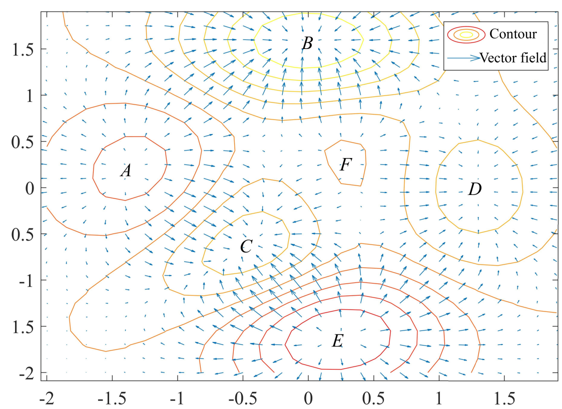


FIG. 3: The role of vector fields

Considering that tensors can be represented by vectors as

$$\varphi_{ij} = (\varphi_i)_j \,, \tag{3}$$

that is, each component of the vector is also a vector, so it can be considered that the density of the tensor physical quantity is the double divergence.

Definition 7. *Let ρ be a function of n-dimensional bounded space $\Omega \subset \mathbb{R}^n$ and time $T \subset [0, T_t]$ about any vector $\varphi_i \in C^m(\mathbb{R}^n \times [0, T_t])$ that changes with time and space, when vector $\varphi_{ij} \in C^m(\mathbb{R}^{n\times n} \times [0, T_t])$ that changes with time and space, when tensor φ_{ij} is a physical quantity, there are*

$$\rho = \varphi_{ij,ij}, \tag{4}$$

then ρ is called the density of the physical quantity φ_{ij}.

Mean while, the volume is also a very important reference factor.

Theorem 2. *The volume fraction of density can represent the effect of physical quantity formation.*

Proof. Under the framework of Euler mechanics, the change of physical quantity is realized by the distribution of the field. The function or structure of the machine, as an effect on the environment or itself, must change the field distribution under the Euler framework. Therefore, the effect formed by any physical quantity can be converted into another scalar to measure, that is, the volume fraction of density. □

Definition 8. *The volume fraction of the density of a physical quantity in the environmental space is called the mass of the physical quantity, that is, for any physical quantity $\varphi_{ij} \in C^m(\mathbb{R}^{n\times n} \times [0, T_t])$,use the density of the physical quantity to construct a mapping relationship $\boldsymbol{\vartheta}$: $\varphi_{ij} \mapsto \gamma$,$\gamma$ is called the mass of the physical quantity, which can be expressed as*

$$\gamma = \int_\tau \rho \mathrm{d}\tau, \tag{5}$$

where ρ refers to the density of the physical quantity, and refers to the volume of the space where the physical quantity acts.

Theorem 3. *When the mass of the physical quantity is a certain value, the mass of the physical quantity can be reduced to the form of the line integral of the infinite field, that is, let ρ be any scalar in the bounded space $\Omega \subset \mathbb{R}$,$\tau$ is the volume of the space, then*

$$\int_\tau \rho \mathrm{d}\tau = \int_{-\infty}^{+\infty} \rho \mathrm{d}x. \tag{6}$$

Proof. Suppose a cylinder with base area $S = 1$ and height of infinite length, since the two open sets are homeomorphic, the space is homeomorphic to the environment space, then

$$\int_\tau \rho \mathrm{d}\tau = \int_x \rho S \ \mathrm{d}x = \int_{-\infty}^{+\infty} \rho \mathrm{d}x.$$

□

Definition 9. *Treating the mechanical design process as a stochastic process, the probability for a single function and a single structure is*

$$P(x) = \int_{-\infty}^{x} \sigma_p(\xi)\mathrm{d}\xi; P(x) = \int_{-\infty}^{x} \sigma_s(\xi)\mathrm{d}\xi. \tag{7}$$

where σ_p and σ_s are the density of physical quantity to realize the function and structure.

As shown in Fig.4, in the random process of mechanical design, there are two probability distributions of functional design and structural design. Its rate reveals the strength of mechanical function and structure.

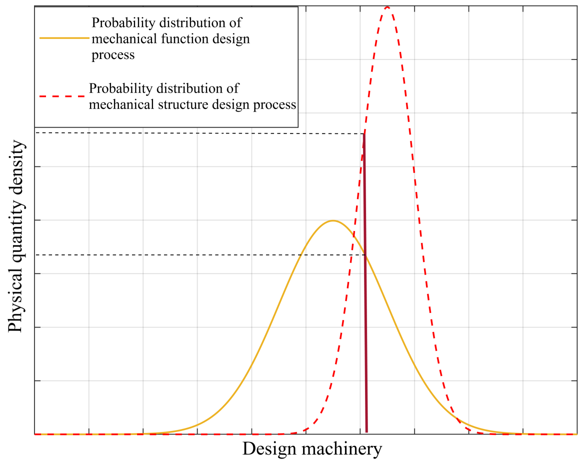


FIG. 4: Schematic diagram of the probability distribution relationship between physical quantity density and design machinery

Definition 10. *For a quasi-single functional machine, the rate of the density of functional physical quantities to the density of structural physical quantities is called the tensor-change rate of mechanical function to structure, that is*

$$z_{p,s} = \frac{\sigma_p}{\sigma_s}. \tag{8}$$

The density of the structural physical quantity is higher than the density of the functional physical quantity is called the mechanical structure-to-function tensor-change rate, namely

$$z_{s,p} = \frac{\sigma_s}{\sigma_p}, \tag{9}$$

where σ_p and σ_s are the density of physical quantity to realize the function and structure.

The tensor-change rate can not only intuitively describe the functional or structural strength of a quasi-single functional machine, but also directly participate in the expression of the basic dynamic equations. That is to say, the two laws can be used to design machinery.

Definition 11. *The function tensor-change rate z_p or the structure tensor-change rate z_s can be defined as*

$$z_p = \frac{\sigma_p}{\rho}; z_s = \frac{\sigma_s}{\rho}, \tag{10}$$

where σ is the physical quantity density to achieve the function or structure; ρ is the mass density.

C. Analysis of Tensor-Change Rate

Suppose displacement $x_k \in \Omega \subseteq \mathbb{R}^n$,$(x_k)_j$,$(\dot{x}_k)_j$ and $(\ddot{x}_k)_j$ represent the displacement, velocity and acceleration of the j_{th} element. So the component is regarded as a multi-degree-of-freedom body composed of multiple elements, and the basic dynamic equation is expressed as:

$$\begin{aligned}\left\{(F_k^p)_i\right\} - \left\{(F_k^s)_i\right\} = [M_{ij}^\alpha]\left\{(\ddot{x}_k)_j\right\} + [C_{ij}]\left\{(\dot{x}_k)_j\right\} \\ + [K_{ij}]\left\{(x_k)_j\right\},\end{aligned} \tag{11}$$

where $[C_{ij}]$ is the damping tensor of the i_{th} unit to the j_{th} unit, reflecting the damping effect. $[K_{ij}]$ is the stiffness tensor of the i_{th} element to the j_{th} element, reflecting the deformation effect of the part. $[M_{ij}^\alpha]$ is the square matrix composed of the component element mass. $(F_k^p)_i$ and $(F_k^s)_i$ are the functional or structural forces acting on the i_{th} element.

The functional force here refers to the external force applied to the input part or the external force of the output part, and the structural force is the force caused by stress and strain. Suppose the environmental space is τ and its boundary is A, considering that the structural force is the form of stress acting in the environmental space. Therefore, the flux of the structural force can be expressed as

$$\oint_A F_k^s dA_k = \oint_A \oint_A \sigma_{k\alpha} dA_\alpha dA_k = \gamma^s, \tag{12}$$

where F_k^s is the structural force; $\sigma_{k\alpha}$ is the stress state that produces the force; σ^s is the density of the force; and γ^s is the mass of the force. Similarly, if σ^p is the density of the functional force and γ^p is the mass of the functional force, the flux of the functional force F_i^p can be expressed as follows

$$\oint_A F_i^p dA_i = \int_\tau F_{i,i}^p = \int_\tau \sigma^p d\tau = \gamma^p. \tag{13}$$

Since mass has no flux effect, assuming that σ^a,σ^ν and σ^x are used to represent the density of acceleration, velocity, and displacement, respectively, the flux on both sides can be obtained:

$$\begin{aligned}\int_\tau \left\{\sigma_i^p\right\} d\tau - \int_\tau \left\{\sigma_i^s\right\} d\tau = \int_\tau \left[M_{ij}^\alpha\right]\left\{\sigma_j^a\right\} d\tau \\ + \int_\tau [C_{ij}]\left\{\sigma_j^\nu\right\} d\tau + \int_\tau [K_{ij}]\left\{\sigma_j^x\right\} d\tau.\end{aligned} \tag{14}$$

Eq.(14) is equal to the following equation.

$$\left\{\sigma_i^p\right\} - \left\{\sigma_i^s\right\} = \left[M_{ij}^\alpha\right]\left\{\sigma_j^a\right\} + [C_{ij}]\left\{\sigma_j^\nu\right\} + [K_{ij}]\left\{\sigma_j^x\right\}. \tag{15}$$

Multiply both sides of the equal sign above by the inverse matrix $\rho_{ij}^{-\alpha}$ of the density, and use z_i^p and z_i^s to represent the functional tensor rate and structural tensor rate of the i_{th} unit. Correspondingly, z_j^a , z_j^v and z_j^x are used to denote the acceleraten, velocity and displacement tensor of the j_{th} element. Then

$$\left\{z_i^p\right\} - \left\{z_i^s\right\} = \left[M_{ij}^\alpha\right]\left\{z_j^a\right\} + [C_{ij}]\left\{z_j^v\right\} + [K_{ij}]\left\{z_j^x\right\}. \tag{16}$$

A diagonalized square matrix $[Z_{ij}]$ is constructed:

$$[Z_{ij}] = diag(z_1^{-s}, z_2^{-s}, \cdots, z_n^{-s}). \tag{17}$$

Multiply both sides of the tensor-change rate equation equation by this square matrix, and use the function of the i_{th} element of $z_i^{p,s}$ to the structure's tensor rate, and use $z_j^{a,s}$,$z_j^{v,s}$ and $z_j^{x,s}$ to denote the acceleraten, velocity, and displacement of the j_{th} element to the structure. The rate of change of the tensor, then we can get

$$\left\{z_i^{p,s}\right\} = \left[M_{ij}^{\alpha}\right]\left\{z_j^{a,s}\right\} + \left[C_{ij}\right]\left\{z_j^{v,s}\right\} + \left[K_{ij}\right]\left\{z_j^{x,s}\right\} + \left\{\lambda_i\right\}, \tag{18}$$

where $\{\lambda_i\}$ is a column vector with all 1 elements. This is the first kind of dynamic tensor-change rate equation for mechanical design.

Derivation of dynamic tensor-change rate equation of the second kind for mechanical design. Assuming that if the functional force can be described as

$$F_k^p = \Gamma^p \cdot \int_\tau \sigma^p d\tau \cdot p_k, \tag{19}$$

where Γ^p is the density of the physical quantity environment, σ^p is the density of the physical quantity, and p_k is the direction unit vector of the force.

Consider that the dot product of the physical quantity vector field θ_k and the unit vector field l_k is equal to the derivative of the physical quantity mass to the area equal to the product of the physical quantity density σ and the effective thickness δ. Here, a closed surface A is constructed with the unit vector field as the normal vector, and the volume inside the closed surface A is τ , namely:

$$\sigma\delta = \sigma\frac{\mathrm{d}\tau}{\mathrm{d}A} = \frac{\mathrm{d}\int_\tau \theta_{k,k} d\tau}{\mathrm{d}A} = \frac{\mathrm{d}\oint_A \theta_k l_k dA}{\mathrm{d}A} = \theta_k l_k. \tag{20}$$

Therefore, the structural force can be expressed as

$$F_k^s l_k = \oint_A \sigma_{k\alpha} l_k dA_\alpha = \int_\tau \sigma_{k\alpha,\alpha k} \delta d\tau = \sigma^s \int_\tau \delta d\tau = \Gamma^s \sigma^s \tau, \tag{21}$$

where Γ^s is the geometric density of the thickness (which can be non-uniform), which can be expressed as

$$\Gamma^s = \frac{\int_\tau \delta d\tau}{\tau}. \tag{22}$$

At this time, after decomposing the basic equation of dynamics according to the principle of vector decomposition, it can be transformed into

$$\left\{\Gamma_i^p z_i^p\right\} - \left\{\Gamma_i^s z_i^s\right\} = \left[\delta_{ij}\right]\left\{\ddot{x}_j\right\} + \left[C_{ij}^m\right]\left\{\dot{x}_j\right\} + \left[K_{ij}^m\right]\left\{x_j\right\}. \tag{23}$$

This is the second type of kinetics tensor-change rate equation for component-level mechanical design.

When facing the whole machine, the machine is divided into functional parts consisting of units numbered 1,...,n_p, structural parts and additional parts consisting of units numbered n_p+1,...,$n_p + n_s$, and it is composed of units numbered $n_p + n_s$+1,...,$n_p + n_s + n_0$. The functional components can be further divided into functional input components. Number 1,...,n_{p1}; function transmission part, number n_{p1}+1,...,$n_{p1} + n_{p2}$; and function output part, number $n_{p1} + n_{p2}$+1,...,n_p. For the convenience of mathematical derivation, it is assumed here that the structural components only act on the functional transmission components.

Considering n_{p1} functional input components, the first input functional unit, as a functional input component, receives the combined action of functional force $(F_k^p)_1$ and structural force $(F_k^s)_1$ to generate inertial force $m_1(\ddot{x}_k)_1$, damping force $C_{1j}\left(\dot{x}_k\right)_j$ and elastic force $K_{1j}\left(x_k\right)_j$. Among them, the damping force and elastic force satisfy Einstein's summation rule, then the basic dynamic equation satisfied by the first input functional unit can be expressed as

$$\left(F_k^p\right)_1 - \left(F_k^s\right)_1 = m_1(\ddot{x}_k)_1 + C_{1j}\left(\dot{x}_k\right)_j + K_{1j}\left(x_k\right)_j. \tag{24}$$

By analogy, the basic dynamic equations of all functional input components from 2 to n_{p1} can be obtained

$$\left(F_k^p\right)_i - \left(F_k^s\right)_i = m_i(\ddot{x}_k)_i + C_{ij}\left(\dot{x}_k\right)_j + K_{ij}\left(x_k\right)_j. \tag{25}$$

By superimposing the above equations, we can get

$$\sum_{i=1}^{n_{p1}}\left(\left(F_k^p\right)_i - \left(F_k^s\right)_i\right) = \sum_{i=1}^{n_{p1}}\left(m_i(\ddot{x}_k)_i + C_{ij}\left(\dot{x}_k\right)_j + K_{ij}\left(x_k\right)_j\right). \tag{26}$$

In the same way, considering n_{p2} function transmission components, the functional transmission unit, as a function transmission component, is affected by the structural reaction force $\sum\limits_{i=1}^{n_{p1}}\left(F_k^s\right)_i$ of the functional input component, the reaction force $-\sum\limits_{i=n_p+1}^{n_p+n_s}\left(F_k^s\right)_i$ of the structural component and the structural reaction force $-\sum\limits_{i=n_{p1}+n_{p2}+1}^{n_p}\left(F_k^s\right)_i$ of the functional output component, resulting in an inertial force $\sum\limits_{i=n_p+1}^{n_{p1}+n_{p2}}\left(m_i(\ddot{x}_k)_i\right)$,damping force $\sum\limits_{i=n_p+1}^{n_{p1}+n_{p2}}\left(C_{ij}\left(\dot{x}_k\right)_j\right)$ and elastic force $\sum\limits_{i=n_p+1}^{n_{p1}+n_{p2}}\left(K_{ij}\left(x_k\right)_j\right)$. Then the basic dynamic equation of the functional transmission component can be expressed as

$$\begin{aligned}&\sum_{i=1}^{n_{p1}}\left(F_k^s\right)_i - \sum_{i=n_p+1}^{n_p+n_s}\left(F_k^s\right)_i - \sum_{i=n_{p1}+n_{p2}+1}^{n_{p1}+n_{p2}+n_{p3}}\left(F_k^s\right)_i = \\ &\sum_{i=n_{p1}+1}^{n_{p1}+n_{p2}}\left(m_i(\ddot{x}_k)_i + C_{ij}\left(\dot{x}_k\right)_j + K_{ij}\left(x_k\right)_j\right).\end{aligned} \tag{27}$$

For the n_{p3} functional output components, the functional output unit as the functional output component is affected by the structural reaction force $\sum\limits_{i=n_{p1}+n_{p2}+1}^{n_{p1}+n_{p2}+n_{p3}}\left(F_k^s\right)_i$ of the functional transmission component, and the output functional force, inertia force, damping force are

$-\sum_{i=n_{p1}+n_{p2}+1}^{n_{p1}+n_{p2}+n_{p3}} (F_k^p)_i$, $\sum_{i=n_{p1}+n_{p2}+1}^{n_{p1}+n_{p2}+n_{p3}} (m_i(\ddot{x}_k)_i)$, $\sum_{i=n_{p1}+n_{p2}+1}^{n_{p1}+n_{p2}+n_{p3}} \left(C_{ij}\left(\dot{x}_k\right)_j\right)$,and elastic force $\sum_{i=n_{p1}+n_{p2}+1}^{n_{p1}+n_{p2}+n_{p3}} \left(K_{ij}\left(x_k\right)_j\right)$ respectively. The index j of damping force and elastic force satisfies Einstein's summation rule, then the basic dynamic equation of the functional output component can be expressed as

$$\begin{aligned}&\sum_{i=n_{p1}+n_{p2}+1}^{n_{p1}+n_{p2}+n_{p3}} \left((F_k^s)_i - (F_k^p)_i\right) = \\ &\sum_{i=n_{p1}+n_{p2}+1}^{n_{p1}+n_{p2}+n_{p3}} \left(m_i(\ddot{x}_k)_i + C_{ij}\left(\dot{x}_k\right)_j + K_{ij}\left(x_k\right)_j\right).\end{aligned} \tag{28}$$

In summary, the basic dynamic equations satisfied by the n_p functional components are:

$$\begin{aligned}&\sum_{i=1}^{n_{p1}} (F_k^p)_i - \sum_{i=n_p+1}^{n_p+n_s} (F_k^s)_i - \sum_{i=n_{p1}+n_{p2}+1}^{n_p} (F_k^p)_i = \\ &\sum_{i=1}^{n_p} \left(m_i(\ddot{x}_k)_i + C_{ij}\left(\dot{x}_k\right)_j + K_{ij}\left(x_k\right)_j\right).\end{aligned} \tag{29}$$

$n_p = n_{p1}+n_{p2}+n_{p3}$,considering the self-contained system of n_s structural parts, it can be considered that the structural parts satisfy the basic equations of dynamics

$$\sum_{i=n_p+1}^{n_p+n_s} (F_k^s)_i = \sum_{i=n_p+1}^{n_p+n_s} \left(m_i(\ddot{x}_k)_i + C_{ij}\left(\dot{x}_k\right)_j + K_{ij}\left(x_k\right)_j\right). \tag{30}$$

In the same way,n_0 additional parts form a self-contained system, so it can be considered that the additional parts satisfy the basic equations of dynamics

$$\sum_{i=n_p+n_s+1}^{n} \left(m_i(\ddot{x}_k)_i + C_{ij}\left(\dot{x}_k\right)_j + K_{ij}\left(x_k\right)_j\right) = 0. \tag{31}$$

For the whole machine, the machine composed of n units satisfies the basic equation of dynamics, and the functional force and structural force of the whole machine can be expressed as

$$P_k = \sum_{i=1}^{n_{p1}} (F_k^p)_i - \sum_{i=n_{p1}+n_{p2}+1}^{n_p} (F_k^p)_i, \tag{32}$$

$$S_k = \sum_{i=1}^{n_{p1}} (F_k^s)_i - \sum_{i=n_p+1}^{n_p+n_s} (F_k^s)_i - \sum_{i=n_{p1}+n_{p2}+1}^{n} (F_k^s)_i. \tag{33}$$

It can be obtained that for the whole machine, the basic equation of dynamics is

$$P_k = \sum_{i=1}^{n} \left(m_i(\ddot{x}_k)_i + C_{ij}\left(\dot{x}_k\right)_j + K_{ij}\left(x_k\right)_j\right). \tag{34}$$

For important transfer functional components, that is, the basic equation of functional transfer kinetics is

$$S_k = \sum_{i=n_{p1}+1}^{n_{p1}+n_{p2}} \left(m_i(\ddot{x}_k)_i + C_{ij}\left(\dot{x}_k\right)_j + K_{ij}\left(x_k\right)_j\right). \tag{35}$$

Considering that for a quasi-single function machine, according to the first kind of dynamic tensor-change rate equation of mechanical design, if the whole machine has the same dynamic function, so

$$\ddot{x}_k \approx (\ddot{x}_k)_1 \approx (\ddot{x}_k)_2 \approx ... \approx (\ddot{x}_k)_i \approx ... \approx (\ddot{x}_k)_n. \tag{36}$$

When ignoring damping and elastic effects, the basic dynamic equation can be written as

$$P_k = M\ddot{x}_k. \tag{37}$$

This is the common second law of dynamics. When ignoring the dynamic effect and the elastic effect, the basic dynamic equation is:

$$P_k = \sum_{i=1}^{n} \left(C_{ij}\left(\dot{x}_k\right)_j\right). \tag{38}$$

This situation is commonly used in the field of deceleraten. When ignoring the damping effect and the dynamic effect, the basic dynamic equation is

$$P_k = \sum_{i=1}^{n} \left(K_{ij}\left(x_k\right)_j\right). \tag{39}$$

This situation is commonly used in the field of plate machinery.

Since the first case has a wide range of applications, the basic equations of kinetics are discussed in detail here.

$$P_k = M\ddot{x}_k. \tag{40}$$

As mentioned earlier, the functional force can be expressed in the basic tensor-change rate equation of the second type of dynamic in mechanical design as

$$P_k = \Gamma^p \cdot \int_\tau \sigma^p d\tau \cdot p_k. \tag{41}$$

When the functional density distribution is uniform, the functional force can be expressed as

$$P_k = \Gamma^p \sigma^p \tau p_k. \tag{42}$$

Substituting it into the kinetic equation, we get

$$\Gamma^p z^p p_k = \ddot{x}_k. \tag{43}$$

This is the second type of dynamic tensor-change rate equation for complete machine design, which reveals the relationship between functional force and function.

[1] F. Reuleaux. *The kinematics of machinery: outlines of a theory of machines*. Courier Corporation, 2013.

[2] K. Franz, R. Hörnschemeyer, A. Ewert, M. Fromhold-Eisebith, M. G. Böckmann, R. Schmitt, K. Petzoldt, C. Schneider, J. E. Heller, J. Feldhusen, et al. Life cycle engineering in preliminary aircraft design. In *Leveraging Technology for a Sustainable World*, pages 473–478. Springer, 2012.

[3] J. Xu and I. Kroo. Aircraft design with active load alleviation and natural laminar flow. *Journal of Aircraft*, 51(5):1532–1545, 2014. https://doi.org/10.2514/1.C032402.

[4] F. I. Petrescu and R. V. Petrescu. Gear design. *Engevista*, 16(4):313–328, 2014.

[5] E. L. Esmail, E. Pennestrì, and M. Cirelli. Power-flow and mechanical efficiency computation in two-degrees-of-freedom planetary gear units: New compact formulas. *Applied Sciences*, 11(13):5991, 2021. https://doi.org/10.3390/app11135991.

[6] S. Bharti and S. Senthilvelan. Transmission efficiency and surface damage of polymer–polymer gear pair under wet lubrication. In *Machines, Mechanism and Robotics*, pages 847–855. Springer, 2022.

[7] F. Gravagno, V. H. Mucino, and E. Pennestrì. The mechanical efficiency of harmonic drives: A simplified model. *Journal of Mechanical Design*, 143(6), 2021. https://doi.org/10.1115/1.4048412.

[8] K. Sengupta, D. K. Singh, A. K. Mondal, D. Bose, and B. Ghosh. Analysis of mechanical property of electrically assisted friction stir welding to enhance the efficiency of joints. *Materials Today: Proceedings*, 38:2263–2270, 2021. https://doi.org/10.1016/j.matpr.2020.06.321.

[9] L. A. Peyré-Tartaruga, A. H. Dewolf, P. E. di Prampero, G. Fábrica, D. Malatesta, A. E. Minetti, A. Monte, G. Pavei, V. Silva-Pereyra, P. A. Willems, et al. Mechanical work as a (key) determinant of energy cost in human locomotion: recent findings and future directions. *Experimental Physiology*, 106(9):1897–1908, 2021. https://doi.org/10.1113/EP089313.

[10] M. A. ElBahloul, E. S. Aziz, and C. Chassapis. Mechanical efficiency prediction methodology of the hypocycloid gear mechanism for internal combustion engine application. *International Journal on Interactive Design and Manufacturing (IJIDeM)*, 13(1):221–233, 2019. https://doi.org/10.1007/s12008-018-0508-2.

[11] R. Jaron, A. Moreau, S. Guérin, L. Enghardt, T. Lengyel-Kampmann, T. Otten, and E. Nicke. Multidisciplinary design optimization of a low-noise and efficient next-generation aero-engine fan. *Journal of Turbomachinery*, 144(1), 2022. https://doi.org/10.1115/1.4051935.

[12] W. Cai, F. Liu, and S. Hu. An analytical investigation on energy efficiency of high-speed dry-cutting cnc hobbing machines. *International Journal of Sustainable Engineering*, 11(6):412–419, 2018. https://doi.org/10.1080/19397038.2017.1349205.

[13] T. Sathish, S. D. Kumar, and S. Karthick. Modelling and analysis of different connecting rod material through finite element route. *Materials Today: Proceedings*, 21:971–975, 2020. https://doi.org/10.1016/j.matpr.2019.09.139.

[14] Z. Luo. Application and development of electronic computers in aero engine design and manufacture. *ASP Transactions on Computers*, 1(1):6–11, 2021. https://doi.org/10.52810/TC.2021.100025.

[15] J. Liu, Z. Tong, Y. Gao-hong, X. Zhao, and H. Zhou. Design of a non circular gear mechanism with twice unequal amplitude transmission ratio. *Journal of Mechanisms and Robotics*, pages 1–13, 2022. https://doi.org/10.1115/1.4053472.

[16] J. Zhu and T. Gao. *Topology optimization in engineering structure design*. Elsevier, 2016.

[17] F. Wu, Z. Wang, D. Song, and H. Lian. Lightweight design of control arm combining load path analysis and biological characteristics. *Reports in Mechanical Engineering*, 3(1):71–82, 2022. https://doi.org/10.31181/rme2001210122w.

[18] M. Cid Montoya, F. Nieto, S. Hernández, A. Fontán, J. Jurado, and A. Kareem. Aero-structural optimization of streamlined twin-box deck bridges with short gap considering flutter. *Journal of Bridge Engineering*, 26(6):04021028, 2021. https://doi.org/10.1061/(ASCE)BE.1943-5592.0001705.

[19] L. Duan, X. Liu, W. Xu, D. Xu, L. Shi, and H. Jiang. Thickness-based subdomian hybrid cellular automata algorithm for lightweight design of biw under side collision. *Applied Mathematical Modelling*, 102:170–193, 2022. https://doi.org/10.1016/j.apm.2021.09.038.

[20] F. Wang, G. Yang, J. Ge, and F. Xu. Lightweight design of a certain mortar base plate based on sensitivity analysis. *Journal of the Brazilian Society of Mechanical Sciences and Engineering*, 43(3):1–13, 2021. https://doi.org/10.1007/s40430-021-02864-2.

[21] Q. Pang, M. Xu, Z. Mi, J. Cui, and J. Guo. Microstructure evolution of ultra-high-strength twinning-induced plasticity (twip) steel under dynamic loading. *Journal of Iron and Steel Research International*, 28(1):66–75, 2021. https://doi.org/10.1007/s42243-020-00457-2.

[22] Y. Pan. Role of hydrogen on the structural stability, mechanical and thermodynamic properties of the cubic tm3si silicides. *International Journal of Hydrogen Energy*, 46(55):28338–28345, 2021. https://doi.org/10.1016/j.ijhydene.2021.06.038.

[23] A. Togaev and A. Shermukhamedov. Mathematical model for calculating the reliability of the characteristics of the designed trailer frame. In *E3S Web of Conferences*, volume 264, page 04054. EDP Sciences, 2021. https://doi.org/10.1051/e3sconf/202126404054.

[24] A. V. Demchyshyn, L. D. Kulak, and V. A. Yavor. Structure and mechanical properties of thick metal condensates strengthened with various dispersed particles. i. structure and yield strength of dispersion-strengthened thick metal condensates. *Powder Metallurgy and Metal Ceramics*, 60(1):112–127, 2021. https://doi.org/10.1007/s11106-021-00221-5.

[25] L. Feng andD. Huang, X. Chen, H. Shi, and S. Wang. Residual ultimate strength investigation of offshore pipeline with pitting corrosion. *Applied Ocean Research*, 117:102869, 2021. https://doi.org/10.1016/j.apor.2021.102869.

[26] S. K. Sahu and S. K. Natarajan. Design and development of a low-cost solar parabolic dish concentrator system with manual dual-axis tracking. *Inter-*

national Journal of Energy Research, 45(4):6446–6456, 2021. https://doi.org/10.1002/er.6164.

[27] Y. Xie, Y. Li, C. F. Cheung, Z. Zhu, and X. Chen. Design and analysis of a novel compact xyz parallel precision positioning stage. *Microsystem Technologies*, 27(5):1925–1932, 2021. https://doi.org/10.1007/s00542-020-04968-6.

[28] T. Xia, K. Zhang, B. Sun, X. Fang, and L. Xi. Integrated remanufacturing and opportunistic maintenance decision-making for leased batch production lines. *Journal of Manufacturing Science and Engineering*, 143(8), 2021. https://doi.org/10.1115/1.4049963.

[29] X. Hu, Q. Liu, B. Liu, X. Li, and L. Gu. Introducing lightweight design early in the suspension design process: A design framework. *Journal of Cleaner Production*, 279:123707, 2021. https://doi.org/10.1016/j.jclepro.2020.123707.

[30] C. M. Sonsino. Consideration of salt-corrosion fatigue for lightweight design and proof of aluminium safety components in vehicle applications. *International Journal of Fatigue*, 154:106406, 2022. https://doi.org/10.1016/j.ijfatigue.2021.106406.

[31] K. Hirano and J. H. Wright. Analyzing cross-validation for forecasting with structural instability. *Journal of Econometrics*, 226(1):139–154, 2022. https://doi.org/10.1016/j.jeconom.2020.10.009.

[32] A. Nayak and P. Singru. Effect of number of stiffening rings, their position and cross section on stress concentration near saddle support in horizontal pressure vessels. In *Advances in Industrial Machines and Mechanisms*, pages 521–530. Springer, 2021.

[33] A. V. Volyar, E. G. Abramochkin, E. V. Razueva, Y. E. Akimova, and M. V. Bretsko. Structural stability of spiral beams and fine structure of an energy flow. *Computer Optics*, 45(4):482–489, 2021. https ://doi.org/10.18287/2412-6179-CO-885.

[34] O. Zaid, F. M. Mukhtar, M. Rebeca, M. G. El Sherbiny, A. M. Mohamed, et al. Characteristics of high-performance steel fiber reinforced recycled aggregate concrete utilizing mineral filler. *Case Studies in Construction Materials*, 16:e00939, 2022. https://doi.org/10.1016/j.cscm.2022.e00939.

[35] A. Cervellon, C. J. Torbet, and T. M. Pollock. Crack initiation anisotropy of ni-based sx superalloys in the very high cycle fatigue regime. *Materials Science And Engineering: A*, 825:141920, 2021. https://doi.org/10.1016/j.msea.2021.141920.

[36] J. Yin and X. Du. High-dimensional reliability method accounting for important and unimportant input variables. *Journal of Mechanical Design*, 144(4), 2022. https://doi.org/10.1115/1.4051982.

[37] G. Capilla-Gonzá, I. Martí, D. Dí, E. Herná, V. Alcá, and A. Saldañ. Effect of the ball burnishing on the surface quality and mechanical properties of a trip steel sheet. *The International Journal of Advanced Manufacturing Technology*, 116(11):3953–3964, 2021. https://doi.org/10.1007/s00170-021-07715-x .

[38] F. Schulte, E. Kirchner, and H. Kloberdanz. Analysis and synthesis of resilient load-carrying systems. In *Proceedings of the Design Society: International Conference on Engineering Design*, volume 1, pages 1403–1412. Cambridge University Press, 2019. https://doi.org/10.1017/dsi.2019.146.

[39] Y. Yoqubjon. Methodology of optimal design of mechanical structures. *Universum*, (12-6 (93)):86–89, 2021.

[40] G. Yan, H. Zou, S. Wang, L. Zhao, Z. Wu, and W. Zhang. Bio-inspired vibration isolation: Methodology and design. *Applied Mechanics Reviews*, 73(2), 2021. https://doi.org/10.1115/1.4049946.

[41] S. Praneeth, L. Saavedra, M. Zeng, B. K. Dubey, and A. K. Sarmah. Biochar admixtured lightweight, porous and tougher cement mortars: Mechanical, durability and micro computed tomography analysis. *Science of The Total Environment*, 750:142327, 2021. https://doi.org/10.1016/j.scitotenv.2020.142327.

[42] J. S. Muñoz-Reina, M. G. Villarreal-Cervantes, and L. G. Corona-Ramírez. Integrated design of a lower limb rehabilitation mechanism using differential evolution. *Computers & Electrical Engineering*, 92:107103, 2021. https://doi.org/10.1016/j.compeleceng.2021.107103.

[43] D. Gu, X. Shi, R. Poprawe, D. L. Bourell, R. Setchi, and J. Zhu. Material-structure-performance integrated laser-metal additive manufacturing. *Science*, 372(6545):eabg1487, 2021. https://doi.org 10.1126/science.abg1487.

[44] H. Liu, H. Fu, L. Sun, C. Lee, and E. M. Yeatman. Hybrid energy harvesting technology: From materials, structural design, system integration to applications. *Renewable and sustainable energy reviews*, 137:110473, 2021. https://doi.org/10.1016/j.rser.2020.110473.

[45] H. R. R. Zaman and F. S. Gharehchopogh. An improved particle swarm optimization with backtracking search optimization algorithm for solving continuous optimization problems. *Engineering with Computers*, pages 1–35, 2021. https://doi.org/10.1007/s00366-021-01431-6.

[46] J. P. Thomas and M. A. Qidwai. Mechanical design and performance of composite multifunctional materials. *Acta materialia*, 52(8):2155–2164, 2004. https://doi.org/10.1016/j.actamat.2004.01.007.

[47] S. Reitenbach, C. Hollmann, J. Schmeink, M. Vieweg, T. Otten, J. Haessy, and M. Siggel. Parametric datamodel for collaborative preliminary aircraft engine design. In *AIAA Scitech 2021 Forum*, page 1419, 2021. https://doi.org/10.2514/6.2021-1419.

[48] W. Zhou, B. Wang, C. W. Lim, and Z. Yang. A distributed-parameter electromechanical coupling model for a segmented arc-shaped piezoelectric energy harvester. *Mechanical Systems and Signal Processing*, 146:107005, 2021. https://doi.org/10.1016/j.ymssp.2020.107005.

[49] M. Li, Y. Zhang, X. Liu, Q. Zhang, and Z. Li. Numerical investigation on the urea deposit formation process in a selective catalytic reduction system of a diesel engine based on a fluid–solid coupling method. *ACS omega*, 6(8):5921–5932, 2021. https://doi.org/10.1021/acsomega.1c00021.

[50] Y. Zhou, X. Zhang, and F. Ding. Partially-coupled nonlinear parameter optimization algorithm for a class of multivariate hybrid models. *Applied Mathematics and Computation*, 414:126663, 2022. https://doi.org/10.1016/j.amc.2021.126663.

[51] C. F. Kirschman and G. M. Fadel. Classifying functions for mechanical design. 1998. https://doi.org/10.1115/1.2829176.

[52] K. T. Ulrich and W. P. Seering. Function sharing in mechanical design. *Design Studies*, 11(4):223–234, 1990. https://doi.org/10.1016/0142-694X(90)90041-A.