

Axiomatic principle of multi-field coupling dynamics analysis

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Multi-field coupling dynamics analysis has always been a problem of great concern. As the basic elements of machinery, the properties of materials such as mechanics, thermodynamics, and electromagnetism play an essential role in mechanical design. On the one hand, in recent years, the material property system in a single discipline has tended to be perfect, but machinery usually works under the action of multi-field coupling, and there are often multiple design indicators that are difficult to reconcile in terms of weight and scope of action. Therefore, it is quite difficult to use the single-field performance index of the material to achieve the design goal of multi-field coupling machinery at one time. On the other hand, due to the development of science and technology, the natural balance has been greatly changed. It is foreseeable that the important direction of the next industrial revolution should be to develop in harmony with nature on the basis of meeting the needs of human survival and production. While developing new materials, we must also pay attention to the comprehensive utilization rate of materials, and cannot blindly pursue the ultimate performance in a certain aspect. Therefore, with the help of the proposed axiomatic principle of dynamic behavior of materials, a new description method of multi-field coupling index is proposed. In the future, one of these indicators can be used to develop multi-field coupling mechanical design methods for mechanical design, and the second is to use these indicators to develop evaluation criteria that can describe the environmental conditions of materials, which has certain practical significance.

I. INTRODUCTION

At present, machinery manufacturing is one of the most important production activities for human beings. German mechanic Reuleaux F [1] believed that machinery is a collection of components connected in various ways. The movement of a certain component drives other components to achieve mechanical functions, and the relative motion relationship depends on the connection state between the component units. A set of excellent mechanical design scheme can greatly improve the operation efficiency, reliability and service life of machinery [2], and can also reduce the cost of manufacturing, maintenance and energy consumption as much as possible [3]. Therefore, mechanical design is an indispensable and important part in the further development of science and technology in the future.

Usually, the mechanical design is to select reasonable modes and routes for energy transmission by comparison, and combine the functional input and output mechanism to achieve the purpose of realizing the mechanical function. Therefore, the essence of mechanical design is to choose an appropriate energy transfer method to achieve high-efficiency realization of mechanical functions. In related research, Petrescu F I et al. [4] proposed a method to determine gear efficiency, gear force, speed, and power.

Esmail E L et al. [5] proposed a two-degree-of-freedom transmission efficiency calculation formula, proving the dependence between kinematics and efficiency. Bharti S et al. [6] studied the wear performance of gears under non-lubricated and wet-lubricated conditions. Federico G et al. [7] proposed a new mathematical model to estimate the mechanical efficiency of harmonic drives.

In general, the efficiency of transmission equipment is the main evaluation index for the strength of mechanical design functions at present [8], and the improvement of mechanical efficiency means the enhancement of mechanical functions [9]. The mechanical efficiency is the percentage of the output and input work of the machine when the machine is running stably [10]. It is defined from the perspective of energy and is a key parameter in the design process of general energy power machinery [11] and transmission machinery [12], not a key parameter of general dynamic balance machinery [13]. Since efficiency cannot express the degree of action of physical quantities, when designing new machinery, it is still not possible to rely solely on mechanical efficiency to design machinery, and other important physical parameters are also considered. For example, the design of aero-engine needs to comprehensively consider the boost rate of the compressor [14]; the design of the gear transmission mechanism should comprehensively consider physical quantities such as speed [15]. Therefore, it is a very important scientific problem to clarify the design index of mechanical function. And with the help of the research on the essence of

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function, it is one of the problems to be solved in this research to clarify the design index of mechanical function.

According to the current design method, the mechanical structure design is carried out after the mechanical function design is completed [16]. In the mechanical structure design, a streamlined and suitable mechanical structure can effectively reduce the mechanical quality [17] and improve the mechanical function stability [18] and efficiency. The complicated and improper mechanical structure will improve the mechanical quality [19], and may endanger the normal operation of the machinery. When facing different usage scenarios, mechanical structural design problems can be divided into lightweight design [20]; traditional structural design based on different loads [21]; structural stability design [22] and reliable/durable design [23] problems.

The yield limit is the stress that resists a small amount of plastic deformation [24]. The strength limit is the maximum stress value that a member can withstand while maintaining the mechanical strength of the member [25]. In order to reduce the probability of plastic deformation or fracture failure in mechanical design, the more direct and simple approach is to increase the amount of material. The reduction of material utilization rate will lead to a series of economic problems such as the increase of cost [26], and even affect the development of precision [27] and batch [28] of machinery.

In order to further reduce the design weight of machinery, lightweight design has gradually become a key link in the mechanical design process [29]. In the mechanical design of transportation, aerospace and other fields, in order to keep the mechanical function unaffected, lightweight design is a recognized and necessary design process [30]. However, due to the changes in the geometric parameters of lightweight design, the current lightweight design results may lead to local instability [31], local stress concentration [32] and other phenomena of the structure, which in turn causes reliability problems such as structural stability or durability.

The stability of the structure refers to the ability of the structure to maintain the original equilibrium state under the action of load [33]. The durability of the structure refers to the long time that the structure can be used without failure [34], which is the longevity of the mechanical structure. Cracks or fractures may occur in mechanical parts under the action of alternating stress for a long time [35], thereby reducing the durability of the mechanical structure. The reliability of the structure refers to the ability of the structure to complete the function under the specified conditions and within the specified time [36].

In the mechanical structure design, the irreconcilable contradictions will occur between the structural indicators at the structural design level. Therefore, unifying the design indicators of the mechanical structure and avoiding the interference between the indicators are the problems that need to be solved in this research.

Fundamentally, the mechanical design quality deter-

mines the operational quality of a machine [37]. Schulte F et al. [38] propose a more detailed system coping strategy that includes response functions, describing system functions and their interconnections through signals, materials, and energy flow. Yoqubjon Y [39] proposed a structural optimization design method based on structural optimization and parameter optimization. Yan G et al. [40] proposed a design method of bio-inspired vibration isolation machinery from the perspective of mechanics. The development of these methods mainly relies on traditional mechanical design methods and supplements or optimizes them with industry technology.

The traditional mechanical design is mainly reflected in the serial design idea, that is, the functional design and structural design of the machine are carried out step by step [41]. With the continuous development of mechanical design research, scholars have gradually realized that there are numerous repetitive structures among the functional structures of machinery [42]. On the one hand, this problem will lead to the extension of the design cycle and the increase of the design cost. On the other hand, the simple combination of structural elements is easy to interact with each other.

Therefore, some scholars have proposed the design idea of functional structure integration. Gu D et al. [43] proposed an integrated design method of functional structure by means of additive manufacturing in *Science*. Liu H et al. [44] elaborated on the principle, structure, output performance and advantages of various hybrid energy harvesting systems. And Zaman H R R et al. [45] proposed an improved particle swarm optimization method for mechanical optimization design. Thomas J P [46] proposed a method for deriving an index. In addition, multidisciplinary fusion methods such as collaborative design [47] are also current research hotspots, but they are not essentially separated from the design idea of tandem. Researches on electromechanical coupling [48] and fluid-structure coupling [49] are widely carried out, but the accuracy of the algorithm still causes certain uncertainty in mechanical design [50].

At present, the difficulty of functional structure integrated mechanical design is mainly concentrated in the inability to establish a direct connection between the mechanical structure and the realization of functions [51][52]. Therefore, it is necessary to effectively construct a multi-field coupling design system between the structure and the function in the mechanical design.

II. MULTI-FIELD COUPLING DYNAMICS ANALYSIS

In order to further improve the quality of mechanical design, it is necessary to find a unified mechanical structure and function design index, which requires clarifying the basic principles of mechanical design.

A. Function and Structure

Definition 1. The bounded closed space composed of all points in the machine is called **the ontology space** of the machine; the bounded open space composed of all the points that the machine may act on is called **the environment space** of the machine. It can be seen from this that the ontology space of a machine is a subspace of its environment space.

Definition 2. All machines are called **design machines**, the complete set of machines is called **machine space**, machines in n -dimensional space are called n -dimensional **space machines**, and n -dimensional **space machines** must be n -dimensional design machines.

Definition 3. The set of infinite points in the mechanical environment space is $\Omega \subseteq \mathbb{R}^n$. Since the function of the machine is the effect of the mechanical part $P \subseteq \Omega$ on the environmental space Ω , and the two spaces are homeomorphic, that is, the function is the mapping between the functional part of the machine and the environmental space. Then, the **function** of the machine can be defined as

$$\varphi : p \mapsto \omega, p \in P, \omega \in \Omega.$$

If the set composed of all functions is called function set Φ , then $\varphi \in \Phi$.

As shown in Fig.1, at this time, point A in a certain area of the static boom forms a mapping relationship with point B in the corresponding area of the environment space of the boom. Assuming that the mapping relationship is displacement, then the second derivative of this mapping relationship to time is acceleraten. According to D'Alembert's principle, this acceleraten is produced by inertial forces. Therefore, this mapping can reveal the force of the boom on the object, even if the object lifts this function.

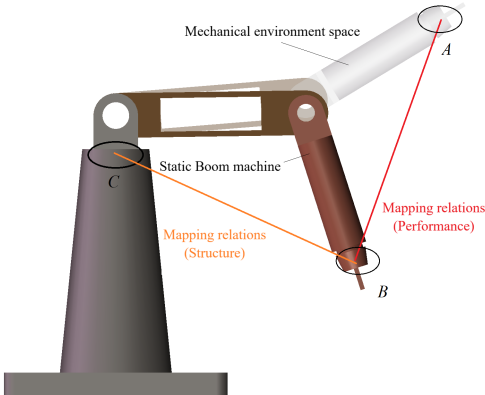


FIG. 1: Boom machinery and its environment space

Definition 4. The normed vector space composed of infinite points in the ontology space of the design machine

is $Q \subseteq \mathbb{R}^n$. The structural part $S \subseteq Q$ of the machine is to support and connect the whole machine Q and complete the function of Q . Since there must be load flow conduction in supports and connections, there is a mapping relationship between the structural part of the machine and the entire machine. Then, the **structure** of the machine can be defined as

$$\Psi_a : s \mapsto q, s \in S, q \in Q,$$

among them, a is called structural design index, then the set composed of all structural targets is called structural set Ψ , then $\psi_a \in \Psi$.

Considering that the part of the mechanically generated function is also realized by the mechanical structural part, the intersection of the part of the mechanically generated function and the structural part is called the **functional structure** part R , that is,

$$R = P \cap S.$$

B. Mass and density of physical quantity

Each physical quantity can be scalar, vector or tensor. If we just talk about the action level of physical quantities, all that is left to do is define their magnitude. Therefore, we say that a physical quantity's mass equals its magnitude, or γ . Let ϕ be a scalar, such as mass, current, charge and so on; $\gamma = a$ states that the mass of ϕ is its magnitude a . Let ϕ be any vector, including velocity, acceleration, position vector and others, its mass is represented by its modulus a , which is equal to $\gamma = |a|$. We also need to briefly consider if ϕ is a real symmetric tensor like the stress tensor. The real symmetric tensor S can be diagonalized, namely

$$C^{-1}SC = \text{diag}(\lambda_1, \lambda_2, \dots, \lambda_n),$$

where C is the matrix made up of the eigenvectors $[v_1, v_2, \dots, v_n]$ of tensor S , and their corresponding eigenvalues are represented by $\lambda_1, \lambda_2, \dots, \lambda_n$. It implies that the eigenvector is the basis vector for the fundamental transformation $S \sim \text{diag}(\lambda_1, \lambda_2, \dots, \lambda_n)$. All of the squares of a symmetric matrix's elements, or ρ , are invariants because $\rho = I_1^2 - 4I_2$, where I_1 and I_2 are well-known classical independent tensor invariants. The action of S can then be compared to that of s on a physical level

$$s = [\lambda_1, \lambda_2, \dots, \lambda_n]^T.$$

Therefore, the modulus of s is the mass of S , namely $\gamma = |s|$. Since physical quantity action often takes place in space, we can define physical quantity density as the mass of physical quantity per unit volume.

Definition 5. Let ρ be a function of n -dimensional bounded space $\Omega \subset \mathbb{R}^n$ and time $T \subset [0, T_t]$ about any vector $s \in C^m(\mathbb{R}^n \times [0, T_t])$ that changes with time and

space, when vector $\mathbf{S} \in C^m(\mathbb{R}^{n \times n} \times [0, T_t])$ that changes with time and space, when tensor $\mathbf{S}$ is a physical quantity, there are

$$\rho = \lim_{\Delta\tau \rightarrow 0} \frac{\Delta\gamma}{\Delta\tau} = \frac{d\gamma}{d\tau}, \quad (1)$$

then ρ is called the **density** of the physical quantity $\mathbf{S}$.

The action degree of a physical quantity is represented by its density. Physical quantity action is relatively concentrated when the density is high; conversely, it is comparatively sparse when the density is low. In actuality, when different materials are acted with by the same physical quantity, the properties are frequently different. The material will fail if the physical quantity exceeds a predetermined threshold value. As a result, the physical quantity densities of various materials vary. Additionally, the allowable density of material $[\rho]$ can be used to define the material's allowed density. The following criteria $\rho \leq [\rho]$ must theoretically be satisfied when choosing a material, and each substance's density varies from the same physical quantity. Meanwhile, In addition to the mass and density of physical quantities, the volume is also a very important reference factor.

Theorem 1. *The volume fraction of density can represent the effect of physical quantity formation.*

Proof. Under the framework of Euler mechanics, the change of physical quantity is realized by the distribution of the field. The function or structure of the machine, as an effect on the environment or itself, must change the field distribution under the Euler framework. Therefore, the effect formed by any physical quantity can be converted into another scalar to measure, that is, the volume fraction of density. $\square$

Definition 6. *The volume fraction of the density of a physical quantity in the environmental space is called the mass of the physical quantity, that is, for any physical quantity $\varphi_{ij} \in C^m(\mathbb{R}^{n \times n} \times [0, T_t])$, use the density of the physical quantity to construct a mapping relationship $\vartheta: \varphi_{ij} \mapsto \gamma$, γ is called the mass of the physical quantity, which can be expressed as*

$$\gamma = \int_{\tau} [\rho] d\tau, \quad (2)$$

where $[\rho]$ refers to the allowable density of the physical quantity, and refers to the volume of the space where the physical quantity acts.

Theorem 2. *When the mass of the physical quantity is a certain value, the mass of the physical quantity can be reduced to the form of the line integral of the infinite field, that is, let $[\rho]$ be any scalar in the bounded space $\Omega \subset \mathbb{R}, \tau$ is the volume of the space, then*

$$\gamma_{max} = \int_{\tau} [\rho] d\tau = \int_{-\infty}^{+\infty} [\rho] d\xi, \quad (3)$$

where γ_{max} is a constant.

Proof. After calculation and analysis, we uniquely correspond the physical quantity density $[\rho]$ with the machinery through the geometric characteristic scale ξ . At this point, the volume fraction of density is transformed into a line integral over an infinite field. Then

$$\gamma_{max} = \int_{\tau} [\rho] d\tau = \int_{-\infty}^{+\infty} [\rho](\xi) d\xi.$$

$\square$

Definition 7. *Treating the mechanical design process as a stochastic process, the probability for a single function and a single structure is*

$$\gamma_{f_{max}} P(x) = \int_{-\infty}^x [\rho_f](\xi) d\xi; \gamma_{s_{max}} S(x) = \int_{-\infty}^x [\rho_s](\xi) d\xi, \quad (4)$$

where $[\rho_f]$, $[\rho_s]$ are the density of physical quantity to realize the function and structure, and $\gamma_{f_{max}}$, $\gamma_{s_{max}}$ are constants.

As shown in Fig.2, in the process of mechanical design, there are two probability distributions of functional design and structural design. Its rate reveals the strength of mechanical function and structure.

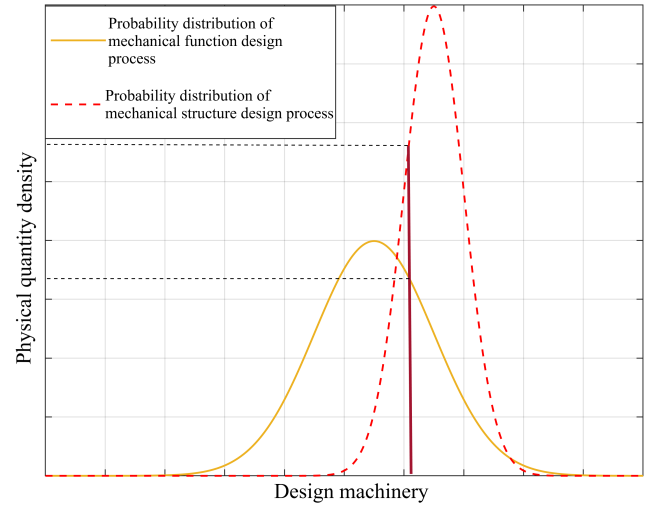


FIG. 2: Schematic diagram of the probability distribution relationship between physical quantity density and design machinery

C. Field-exchange rate (FER)

Any charged particle will experience force in the electromagnetic field, as is well known. This is as a result of a field developing around the particles. However, the force's creation is intimately tied to the chosen material's properties, including charge, mass, and others, and not just to the field. It only makes sense to expand on this

idea. We think that every force is made up of a vector made up of field variables and material characteristics. The physical quantity mass mentioned above is the relevant material property. This could be expressed as

$$\mathbf{F} = \mu\gamma\mathbf{e},$$

where μ is the strength of the field in the environment where the unit is located, γ is the physical quantity mass of unit receiving action, and $\mathbf{e}$ is unit vector along the direction of the force.

Definition 8. For the same unit, the material is determined, so ρ_m is determined. In general, the service environment of the unit is also determined. Thus, we can define a dimensionless coefficient, which is called field-exchange rate (FER)

$$z(\xi) = \frac{d\mathbf{F}}{d\mathbf{G}}; \quad z(\xi) = \frac{\mu[\rho](\xi)}{g\rho_m},$$

where $\mathbf{G}$ is the gravity, μ is the strength of the field in the environment, $[\rho](\xi)$ is the density of physical quantity, g is the acceleration of gravity, and ρ_m is the density of material.

Definition 9. The function field-exchange rate z_f or the structure field-exchange rate z_s can be defined as

$$z_f = \frac{\mu_f[\rho_f](\xi)}{g\rho_m}; \quad z_s = \frac{\mu_p[\rho_s](\xi)}{g\rho_m}, \quad (5)$$

where $[\rho](\xi)$ is the physical quantity density to achieve the function or structure; μ is the strength of the field in the environment, and ρ_m is the density of materials.

Definition 10. For a quasi-single functional machine, the rate of the density of functional physical quantities to the density of structural physical quantities is called the field-exchange rate of mechanical function to structure, and the rate of the density of structural physical quantities to the density of functional physical quantities is called the field-exchange rate of mechanical structure to function

$$z_{f,s} = \frac{\mu_f[\rho_f](\xi)}{\mu_s[\rho_s](\xi)}; \quad z_{s,f} = \frac{\mu_s[\rho_s](\xi)}{\mu_f[\rho_f](\xi)}, \quad (6)$$

where $[\rho_f]$ and $[\rho_s]$ are the density of physical quantity to realize the function and structure.

The field-exchange rate (FER) can not only intuitively describe the functional or structural strength of a quasi-single functional machine, but also directly participate in the expression of the basic dynamic equations. That is to say, the two laws can be used to design machinery.

D. Physical meaning of Field-exchange rate (FER)

From the perspective of the development of physics in the past hundred years, physics has continuously proved

the idea that the change of the microstructure of matter is one of the direct factors affecting the change of the material state and physical process of the object. Since this change usually has the complexity and synthesis of multiphysics coupling, how to sort out the state of matter and physical processes under the influence of multiphysics is an important physics topic, which involves the exploration of the essential causes of many major physical problems. We may wish to analyze the correlation of the influence of the microstructure of matter on physical quantities, which are quantities in physics that measure the properties of matter or describe the state of motion of objects and their change processes.

In an isolated system with an unchanged external environment, when the microstructure of the substance changes, the first affected physical quantity Φ can be called the matter quantity Υ , such as material density ρ_m , charge density ρ_q , current density ρ_I , momentum ρv_i or kinetic energy E_k , etc. Subsequently, the physical quantity Φ of the change of the surrounding environment caused by the change of matter quantity Υ can be called the environmental quantity μ , such as geometric parameters r , electric field intensity E , magnetic field intensity H , etc. Lastly, the physical quantity Φ that is formed by the interactions between environmental quantity μ and other matter quantity Υ can be called the action quantity Π , such as force F , torque M , or potential energy E_p , etc.

The action quantity Π represents the impact of the original object on the objects within the surrounding environment of the original object. In general, the state of matter or physical processes are mainly affected by matter quantity Υ and action quantity Π . Additionally, it is essential to follow the conservation of physical quantities, such as mass, charge, momentum, angular momentum, and energy etc., in order to establish a material state or initiate a physical process. Hence, through abstract mathematical models, we can make astonishing interpretations and unifications of the universal conservation laws, thereby discovering new laws and methods.

1. Definition of density and flux density of physical quantity

Within this framework, the magnitude of a physical quantity per unit of measurement can be defined as the density of the physical quantity ρ^Φ . Depending on the object and form of the quantity, the measure τ in the definition of the density of the physical quantity ρ^Φ can be area, volume, length or other measurement, which should be selected according to the specific object of action and the analysis process. In simple terms, the density of physical quantities can be used to visually describe the spatial distribution of a certain physical quantity Φ

$$\rho^\Phi = \frac{\delta\Phi}{\delta\tau}, \quad (7)$$

here, $\delta\tau$ is the change in spatial measurement (such as volume, area, length, etc.). The matter quantity Υ can be expressed by the density of mass quantity ρ^Υ

$$\delta\Upsilon = \rho^\Upsilon \delta\tau. \quad (8)$$

Simultaneously, considering that the action quantity Π is typically the result of the interaction between the environmental quantity μ and the matter quantity Υ , it can generally be expressed in the form of physical quantity density

$$\rho^\Pi = \mu \rho^\Upsilon. \quad (9)$$

It is worth mentioning that the correlation between the matter quantity Υ and environmental physical quantity μ may not always be present. To illustrate, there have been no findings indicating any interaction when the current intensity is exposed to a gravitational field. In general, whether the density of matter quantity ρ^Υ and the environmental quantity μ can interact to form the density of action quantity ρ^Π is determined by the experimental results. Furthermore, in line with equation (7), the change of action quantity Π can also be expressed as

$$\delta\Pi = \mu \rho^\Upsilon \delta\tau. \quad (10)$$

In general, if we designate the environmental quantity μ as 1, the action quantity P will match the matter quantity Υ in numerical terms, as the shown in equation (8).

As the matter quantity Υ and action quantity Π are physical quantities directly influencing the matter state or physical processes, we can derive a more generalized description method. In other words, when it comes to the physical quantity P that have a direct impact on the matter state or physical processes, we can describe its change as

$$\delta P = \mu \rho \delta\tau, \quad (11)$$

here, μ is the environmental quantity, ρ is the investigation object's density of matter quantity, and $\rho = \rho^\Upsilon$. When $\mu = 1$, the physical quantity is the matter quantity $P = \Upsilon$, that is, the physical quantity directly affected by microscopic structural changes; When $\mu \neq 1$, the physical quantity is the action quantity $P = \Pi$, that is, the physical quantity of the interaction between the matter quantity Υ and the environmental quantity μ .

In order to study the physical process under temporal change, it is common to treat the temporal rate of change of a physical quantity as a new physical quantity. The temporal rate of change of a physical quantity with the density of space can be called the flux density of the physical quantity ρ_f^Φ . And according to equation (7), there is

$$\rho_f^\Phi = \frac{\delta}{\delta\tau} \left(\frac{d\Phi}{dt} \right). \quad (12)$$

Similar to the analysis method of physical quantity density, according to equation (11), we can obtain a more

general description for flux density, that is, for a physical quantity P that directly affects the matter state or physical process, the change rate over time can be described as

$$\delta \frac{dP}{dt} = \mu \rho_f \delta\tau \quad (13)$$

here, μ is the environmental quantity, ρ_f is the investigation object's matter flux density, $\rho_f = \rho_f^\Upsilon$. When $\mu = 1$, the physical quantity is the matter quantity $P = \Upsilon$, that is, the physical quantity directly affected by microscopic structural changes; When $\mu \neq 1$, the physical quantity is the action quantity $P = \Pi$, that is, the physical quantity of the interaction between the matter quantity Υ and environmental quantity μ . As time and space are managed in a similar manner, there is no discernible disparity when it comes to the sequence of mathematical operations, therefore,

$$\frac{d\delta P}{dt} = \delta \frac{dP}{dt}. \quad (14)$$

Substituting equation (11) into equation (14) can obtain

$$\frac{d\delta P}{dt} = \mu \left(\frac{d\rho}{dt} + \frac{\rho}{\mu} \frac{d\mu}{dt} + \rho\theta \right) \delta\tau. \quad (15)$$

Let $\vec{v}$ represent the rate of change in measurement τ over time, then $\theta = \Delta \cdot \vec{v}$ signifies the divergence in the change of measurement τ , illustrating the degree of spatial expansion and contraction over unit time. By comparison with equation (15), we can determine the relationship between flux density and density of a physical quantity

$$\rho_f = \frac{d\rho}{dt} + \frac{\rho}{\mu} \frac{d\mu}{dt} + \rho\theta. \quad (16)$$

Moreover, the above analysis mainly considers the magnitude of the physical quantity. To fully account for the orientation of the physical quantity, it is essential to incorporate the covariant density ρ_e of that attribute in the direction perpendicular to the Cartesian coordinate system.

$$\rho_e = \rho \frac{|\vec{v}|}{r}. \quad (17)$$

Through the analysis of equations (11), (13) and (15), it becomes apparent that the density and flux density of matter are intricately linked not only to the nature of the matter itself, but also to the geometric shape of its distribution in the measurement space. Thus, by tracking the changes in the microscopic structure of matter and its reciprocal effects on the surrounding matter, and using the interdependencies between the matter quantities, the environmental quantities, and the action quantities, we can more easily articulate the essential rules of a matter state or a physical process. By providing a description that seamlessly connects the macroscopic and microscopic aspects of the physical world, it becomes more

feasible to uncover the underlying principles that govern material phenomena and microscopic structures. For example, the scenario where the minuscule composition of an object undergoes a transformation; as a result, its temperature field is altered, consequently impacting the heat field in its vicinity. The temperature of the environment plays a crucial role in shaping the behavior of other substances present, with conduction, convection, and radiation being among the ways through which this influence is manifested. As a consequence, the object undergoes modifications at the microscopic level. Additionally, looking at it from an engineering standpoint, creating a direct link between the larger and smaller scales would greatly streamline the process of selecting materials for mechanical design.

The major obstacles encountered when researching in multi-physics fields stem primarily from the considerable discrepancies in how each physics field is portrayed. In many ways, the notions of physical quantity density and flux density have effectively dealt with the task of describing multi-physics fields in single-body systems. Nonetheless, substantial discrepancies can be observed in the transfer of energy within intricate systems involving multiple bodies, owing to the fact that each individual body undergoes distinct mechanical forces and movements. To establish connections between different aspects of physics and specific elements within an intricate system, it is crucial to develop a novel dimensionless physical parameter and employ the principles of conservation laws or equilibrium equations. So, according to the above definition of FER, there is

$$z = \frac{\mu}{g} \frac{\rho_i}{\rho_m}, \quad (18)$$

where ρ_i signifies the density or flux density of the conservation physical quantity of the i -th field, μ signifies the excitation field in which the substance resides. And the FER is the ratio of a certain physical quantity to the force or torque exerted by gravity over a distance of 1 meter, in a time of 1 second, or equivalent to the torque exerted for 1 second within a 1-meter range.

2. Law of conservation of FER

When a substance is subjected to the coupled action of multi-field, the FER transfers from one physical field to another or from one part of the substance to another. The total FER remains constant throughout the space-time evolution, i.e.,

$$z_{\Sigma}(x_i, t) = \text{const} \quad (19)$$

here, x_i represents spatial parameter; t represents temporal parameter.

E. Analysis of Field-exchange rate (FER)

When functional force and structural force are applied, then the dynamic process is carried out, the dynamic behaviour can be expressed by a dynamic equation. For the simplest case, In accordance with the d'Alembert principle, the overall form of the dynamic equation can be expressed in

$$\mathbf{F}_f = \mathbf{F}_{f,1} + \mathbf{F}_{f,2} + \mathbf{F}_{f,3} + \cdots = M\ddot{x}, \quad (20)$$

where M is the mass of entire machine, $\mathbf{F}_f$ is the total functional force, and $\mathbf{F}_{f,i}$ is the i th functional force of i th field, which can be express as $\mathbf{F}_{f,i} = \mu_{f,i}\gamma_{f,i}\mathbf{e}$, when all of them are in the same direction. And this equation represents the coupling between multiple physical fields. In this case, taking into account the part $d\tau$ that realizes functions and structures with a mass of m , we can obtain the FER form of above equation

$$\theta z_{f_1} + \theta z_{f_2} + \theta z_{f_3} + \cdots = n, \quad \theta = \frac{m}{M}, \quad (21)$$

where n is the overload factor, which is determined by the machine's requirements. The structural action is an internal action, and its magnitude is related to the sum of reaction force $\mathbf{R}$. When two systems have the same reaction, then the loads of the two systems are similar, that is, the structural failures are similar. Therefore, when only structural failure is studied, we can consider that the loading is carried out by the system with the sum of reaction force $\mathbf{R}$ as the load, which is similar to the failure of the original system. We can define the structural force as $\mathbf{F}_s = \mathbf{R}$. Here it should be noted that the structural force is an imaginary external force, not an internal force, that causes structural failure. For unit, the dynamic equation is

$$\mathbf{F}_f - \mathbf{F}_s = m\ddot{x}, \quad (22)$$

where $\mathbf{F}_s$ is the structural force, which can be express as $\mathbf{F}_s = \mu_s\gamma_s\mathbf{e}$. Combined with above equations, we can obtain

$$\frac{z_f}{z_s} = \frac{1}{1 - \theta}, \quad \theta = \frac{m}{M}. \quad (23)$$

III. CONCLUSION

First of all, through analysis and calculation, we propose a new index for multi field coupling dynamics analysis, which is FER and its expression is

$$z(\xi) = \frac{\mu[\rho](\xi)}{g\rho_m},$$

where $\mathbf{G}$ is the gravity, μ is the strength of the field in the environment, $[\rho](\xi)$ is the density of physical quantity, g is the acceleration of gravity, and ρ_m is the density

of material. At the same time, for multi-field coupling machinery, the field-exchange rate dynamic equation has the simplest form

$$\frac{z_p}{z_s} = \frac{1}{1 - \theta}, \quad \theta = \frac{m}{M},$$

where M is the mass of the whole machine, and m is the mass of the research part of the whole machine. Meanwhile, when a substance is subjected to the coupled ac-

tion of multi-field, the FER transfers from one physical field to another or from one part of the substance to another. The total FER remains constant throughout the spacetime evolution, i.e.,

$$z_{\Sigma}(x_i, t) = \text{const}$$

here, x_i represents spatial parameter; t represents temporal parameter.

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