Computation of Binomial Expansions and Application in Science and Engineering

Chinnaraji Annamalai
School of Management, Indian Institute of Technology, Kharagpur, India
Email: anna@iitkgp.ac.in
https://orcid.org/0000-0002-0992-2584

Abstract: This paper presents a computational technique for the multiple summations of binomial expansions with the binomial coefficient and binomial identity. This computational technique is a sort of methodological advance that will help to the researchers who are working in computational science and Engineering, cybersecurity, and information technology.

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1. Introduction

Computational science is a rapidly growing multi-and inter-disciplinary area where science, engineering, mathematics, and collaboration uses advance computing capabilities to understand and solve the most complex real life problems. In this article, we provide the sum of summations of coefficients with detailed proofs. The results of binomial coefficients and identities [1- 6] are useful to solve the scientific problems in computational science and engineering.

For basic understanding, the traditional coefficient and optimized coefficient are given below:

Tradional coefficient: $nC_r = \binom{n}{r} = \frac{n!}{r!\,(n-r)!}$, where $\quad n, r \in N \cup \{0\}\ \&\ N = \{1, 2, 3, \cdots\}$.

Optimized coefficient: $V_r^n = \frac{(r+1)(r+2)\cdots\cdots(r+n)}{n!}, \qquad (n \geq 1, r \geq 0\ \&\ n, r \in N)$.

Here, $V_r^{n-r} = nC_r = nC_{n-r}$ *and* $V_r^n = pC_r$, where $p = n + r$.

$$V_n^0 = V_0^n = nC_{0=} nC_n = 1\ \&\ V_0^0 = 0C_0 = \frac{0!}{0!} = 1.$$

2. Summations of Binomial Coefficients

Theorem: $\sum_{i=0}^{r} V_i^0 + \sum_{i=0}^{r} V_i^1 + \sum_{i=0}^{r} V_i^2 + \sum_{i=0}^{r} V_i^3 + \cdots + \sum_{i=0}^{r} V_i^{n-1} = \sum_{i=1}^{r} V_i^n, \qquad$ that is,

$$\sum_{i=0}^{r} \frac{1}{0!} + \sum_{i=0}^{r} \frac{(i+1)}{1!} + \sum_{i=0}^{r} \frac{(i+1)(1+2)}{2!} + \sum_{i=0}^{r} \frac{(i+1)(i+2)(i+3)}{3!} + \cdots$$

$$\cdots + \sum_{i=0}^{r} \frac{(i+1)(i+2)(i+3)\cdots(i+n-1)}{(n-1)!} = \sum_{i=1}^{r} \frac{(i+1)(i+2)(i+3)\cdots(i+n)}{n!}.$$

Proof: The theorem is provided using the following binomial identity [1 - 6], that is,

$$V_0^n + V_1^n + V_2^n + V_3^n \cdots + V_{r-1}^n + V_r^n = \sum_{i=0}^{r} V_i^n = V_r^{n+1}.$$

Let us write the binomial expansions separately for $n = 1, 2, 3, \cdots, n-1$ to prove the theorem. Accordingly, we can revise the binomial identity as follows:

$$V_0^{n-1} + V_1^{n-1} + V_2^{n-1} + V_3^{n-1} \cdots + V_{r-1}^{n-1} + V_r^{n-1} = \sum_{i=0}^{r} V_i^{n-1} = V_r^n.$$

For $n = 0, \sum_{i=0}^{r} V_i^0 = V_0^0 + V_1^0 + V_2^0 + \cdots + V_r^0 = \dfrac{r+1}{1!} = V_r^1$ $(\because \mathrm{V}_\mathrm{n}^0 = V_0^n = V_0^0 = 1)$.

For $n = 1$, $\quad \sum_{i=0}^{r} V_i^1 = 1 + 2 + 3 + \cdots + \dfrac{r+1}{1!} = \dfrac{(r+1)(r+2)}{2!} = V_r^2.$

For $n = 2$, $\quad \sum_{i=0}^{r} V_i^2 = 1 + 3 + 6 + \cdots + \dfrac{(r+1)(r+2)}{2!} = \dfrac{(r+1)(r+2)(r+3)}{3!} = V_r^3.$

For $n = 3$, $\quad \sum_{i=0}^{r} V_i^3 = \sum_{i=0}^{r} \dfrac{(i+1)(i+2)(i+3)}{3!} = \dfrac{(r+1)(r+2)(r+3)(r+4)}{4!} = V_r^4$

For $n = 4, \sum_{i=0}^{r} V_i^4 = \sum_{i=0}^{r} \dfrac{(i+1)(i+2)(i+3)(i+4)}{4!} = \dfrac{(r+1)(r+2)\cdots(r+5)}{5!} = V_r^5.$

Similarly, we can continues this process upto $n-1$, that is,

$$\sum_{i=0}^{r} V_i^{n-1} = \sum_{i=0}^{r} \frac{(i+1)(i+2)(i+3)\cdots(i+n-1)}{(n-1)!} = \frac{(r+1)(r+2)(r+3)\cdots(r+n)}{n!} = V_r^n.$$

By adding to both sides, we get

$$\sum_{i=0}^{r} V_i^0 + \sum_{i=0}^{r} V_i^1 + \sum_{i=0}^{r} V_i^2 + \sum_{i=0}^{r} V_i^3 + \cdots + \sum_{i=0}^{r} V_i^{n-1} = \sum_{i=1}^{r} V_i^n, \qquad \text{that is,}$$

$$\sum_{i=0}^{r} \frac{1}{0!} + \sum_{i=0}^{r} \frac{(i+1)}{1!} + \sum_{i=0}^{r} \frac{(i+1)(1+2)}{2!} + \sum_{i=0}^{r} \frac{(i+1)(i+2)(i+3)}{3!} + \cdots$$

$$\cdots + \sum_{i=0}^{r} \frac{(i+1)(i+2)(i+3)\cdots(i+n-1)}{(n-1)!} = \sum_{i=1}^{r} \frac{(i+1)(i+2)(i+3)\cdots(i+n)}{n!}.$$

Hence, theorem is proved.

3. Conclusion

In this article, a computational technique has been used to computer the multiple summation of binomial expansions with binomial coefficients and application [7]. This computational technique is a sort of methodological advance that will help to the researchers who are working in computational science and engineering, cybersecurity, and information technology.

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