Computation of Sum of Optimized Binomial Coefficients and Application in Computational Science and Engineering

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Abstract: Computational science and Engineering is a rapidly growing multi-and inter-disciplinary area where science, engineering, computation, mathematics, and collaboration uses advance computing capabilities to understand and solve the most complex real life problems. In this article, a methodological advance is developed as an application for the researchers working in computer and computational engineering.

Keywords: combinatorics, binomial coefficient, computation

1. Introduction

V_n^n is a binomial coefficient with similar number of both subscript and superscript and its result is $V_n^n = 2V_{n-1}^n$. Before proving this result, we can understand the traditional binomial coefficient and optimized binomial coefficient [1 – 6] given below:

Tradional binomial coefficient: $nC_r = \binom{n}{r} = \dfrac{n!}{r!\,(n-r)!}, n, r \in N \cup \{0\}\ \&\ N = \{1, 2, 3, \cdots\}.$

Optimized binomial coefficient: $V_r^n = \dfrac{(r+1)(r+2)\cdots\cdots(r+n)}{n!}, (\, n \geq 1\ \& r \geq 0\ \in N\,).$

$$\text{Here}, V_r^{n-r} = nC_r = nC_{n-r}\ \ and\ \ V_r^n = pC_r, \text{where } p = n + r.$$

$$V_n^0 = V_0^n = nC_{0=}\, nC_n = 1\ \&\ V_0^0 = 0C_0 = \frac{0!}{0!} = 1.$$

2. Computation of Binomial Coefficients

Proof of the result of a binomial coefficient [1 – 6] with similar number of both subscript and superscript, $V_n^n = 2V_{n-1}^n$, is

$$V_n^n = \frac{(n+1)(n+2)(n+3)\cdots 2n}{n!} = 2V_{n-1}^n\ and\ V_n^n = \prod_{i=1}^{n} \frac{(n+i)}{n!}\ \&\ V_{n-1}^n = \prod_{i=1}^{n} \frac{(n-1+i)}{n!}.$$

The following theorem is based on the computation of sum of binomial coefficients [1 -6] with similar numbers of both subscripts and superscripts:

Theorem: $\displaystyle\prod_{i=1}^{1}\frac{(1+i)}{1!} + \prod_{i=1}^{2}\frac{(2+i)}{2!} + \prod_{i=1}^{3}\frac{(3+i)}{3!} + \cdots + \prod_{i=1}^{n-1}\frac{(n-1+i)}{(n-1)!} + \prod_{i=1}^{n}\frac{(n+i)}{n!} =$

$$2\left(\prod_{i=1}^{1}\frac{(0+i)}{1!} + \prod_{i=1}^{2}\frac{(1+i)}{2!} + \prod_{i=1}^{3}\frac{(2+i)}{3!} + \cdots + \cdots + \prod_{i=1}^{n-1}\frac{(n-2+i)}{(n-1)!} + \prod_{i=1}^{n}\frac{(n-1+i)}{n!}\right)$$

$$\Rightarrow \sum_{j=1}^{n}\prod_{i=1}^{j}\frac{(j+i)}{j!} = 2\sum_{j=1}^{n}\prod_{i=1}^{j}\frac{(j-i+i)}{j!}.$$

$Proof: V_1^1 + V_2^2 + V_3^3 + V_n^n = 2(V_0^1 + V_1^2 + V_2^3 + \cdots + V_{n-1}^n) \Rightarrow \sum_{i=1}^{n} V_i^i = 2\sum_{i=1}^{n} V_{i-1}^i.$

It is understtod that $V_n^n = \prod_{i=1}^{n} \frac{(n+i)}{n!}$ and $V_{n-1}^n = \prod_{i=1}^{n-1} \frac{(n+i)}{(n-1)!}$. Now, we get

$$\prod_{i=1}^{1} \frac{(1+i)}{1!} + \prod_{i=1}^{2} \frac{(2+i)}{2!} + \prod_{i=1}^{3} \frac{(3+i)}{3!} + \cdots + \prod_{i=1}^{n-1} \frac{(n-1+i)}{(n-1)!} + \prod_{i=1}^{n} \frac{(n+i)}{n!} =$$
$$2\left(\prod_{i=1}^{1} \frac{(0+i)}{1!} + \prod_{i=1}^{2} \frac{(1+i)}{2!} + \prod_{i=1}^{3} \frac{(2+i)}{3!} + \cdots + \cdots + \prod_{i=1}^{n-1} \frac{(n-2+i)}{(n-1)!} + \prod_{i=1}^{n} \frac{(n-1+i)}{n!}\right)$$
$$\Rightarrow \sum_{j=1}^{n} \prod_{i=1}^{j} \frac{(j+i)}{j!} = 2\sum_{j=1}^{n} \prod_{i=1}^{j} \frac{(j-i+i)}{j!}.$$

Hence, theorem is proved.

3. Conclusion

In this article, a theorem has been constituted based on the computation of optimized binomial coefficients with similar numbers of both subscripts and superscripts. This theorem is a sort of methodological advance and application [7] used in theoretical computer science and computational science and Engineering.

References

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