Intuitionistic Fuzzy sets and Combinatorial Techniques in Computation and Weather Analysis

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Abstract: This paper presents combinatorial techniques and intuitionistic fuzzy sets with applications in computing science and engineering. Computational science is a rapidly growing interdisciplinary area where science, engineering, computing science, mathematics and its collaboration use advance computing capabilities to solve the real-world problems. Cybersecurity is the practice of protecting the computing systems, communication networks, data and programs from cyber-attacks. The objective of these computational techniques and algorithms like RSA algorithm and Elliptic Curve Cryptography is to reduce the risk of cyber-attacks and protect against the unauthorized exploitation of systems and networks. In this connection, combinatorial techniques for cybersecurity and intuitionistic fuzzy sets for weather analysis are introduced in this article.

Keywords: cybersecurity, combinatorics, computation, factorial, fuzzy set

1. Introduction

Several mathematical and combinatorial techniques are used as applications in computational science that is a rapidly growing interdisciplinary area where science, engineering, computing science, mathematics and its collaboration use advance computing capabilities to solve the real-world problems and meet today's challenges. Also, these computing techniques are used in artificial intelligence and communication network. Wireless communication is not fully secure for transmission of information in a peer-to-peer network, that is, some information leakage through the transmission of wireless signal is unavoidable. In this case, cybersecurity is the practice of protecting the computing systems, devices, communication networks, programs and data from cyber-attacks. The aim of this article is to reduce the risk of cyber-attacks and protect against the unauthorized exploitation of systems and networks. For this purposes, we need a strong network and security algorithms like RSA algorithm and Elliptic Curve Cryptography. In this article, intuitionistic fuzzy sets [1, 2] are used for weather analysis and factorial [3, 4] and binomial theorems [5-9] can help to build a strong artificial intelligence-based cryptographic algorithm.

2. Intuitionistic Fuzzy Sets

Intuitionistic fuzzy set is a fuzzy set consisting of membership degree, hesitation degree, and non-membership degree. The hesitation degree is a part of membership degree or a part of non-membership degree or both. Let us understand the definition of an intuitionistic fuzzy set [1, 2]. Intuitionistic Fuzzy set F in a domain of discourse S where $F \subseteq S$ is defined as a non-empty set of 4-tuple elements, symbolically,

$$F = \{\langle e, \mu_F(e), \pi_F(e), \nu_F(e)\rangle | e \in S\}, \forall e \in S$$

The following examples describe the elements such as membership degree, hesitation degree, and non-membership degree in the intuitionistic fuzzy set.

Example 1: Let us consider a set of people in age group 18 years and above. The membership degree, hesitation degree, and non-membership degree of the intuitionistic fuzzy set is described as follows: If we denote the young people in age group between 18-40 years by membership degrees, then we can denote the old people in age group 50 years and above by non-membership degree. The people in the age group between 40-50 years may be considered as young or old or both. Thus, we can represent these people (age group between 40-50 years) by hesitation degrees.

Example 2: Let us consider a set of 24 hours in a single day. The membership degree, hesitation degree, and non-membership degree of the intuitionistic fuzzy set is described as follows: If we denote the day between 7 AM – 6 PM by the membership degrees, then we can denote the night between 7 PM – 5 AM by non-membership degrees. The periods between 5 AM – 6 AM and 6 PM – 7 PM may be considered as day or night or both. Thus, we may denote these periods (5 AM – 6 AM and 6 PM – 7 PM) by hesitation degrees.

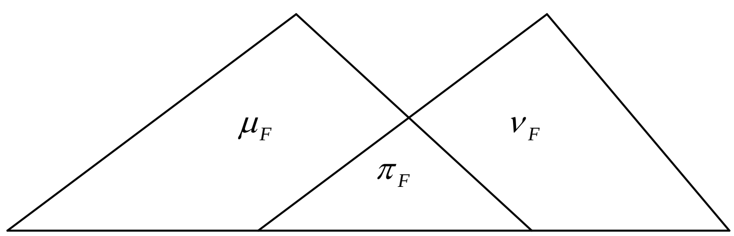

Figure 1. Illustration of the degrees μ_F, π_F, and ν_F

The notations $\mu_F, \pi_F,$ and ν_F denote the membership function $\mu_F : S \to [0,1]$, hesitation function $\pi_F : S \to [0,1]$, and non-membership function $\nu_F : S \to [0,1]$ respectively. $\mu_F(e), \pi_F(e), and\ \nu_F(e)$ quantify the membership degree, hesitation degree and non-membership degree of $e \in S$ respectively. We can represent $\mu_F, \pi_F,$ and ν_F pictorially (Figure 1).

For every $e \in S,\ \mu_F(e) + \pi_F(e) + \nu_F(e) = 1\ and\ 0 \leq\ \mu_F(e),\ \pi_F(e),\ \nu_F(e) \leq 1$.
For instance, if we know the degrees of $\mu_F(e)$ and $\nu_F(e)$, we can calculate the degree of $\pi_F(e)$, that is, $\pi_F(e) = 1 -\ \mu_F(e) - \nu_F(e), (e \in S)$.

For our convenience, we may denote each element of F as a 3-tuple element, that is,
For our convenience, we may denote each element of F as a 3-tuple element, that is,

$\langle \mu_F(e), \pi_F(e), \nu_F(e)\rangle$, and also the intuitionistic fuzzy set F as follows:

$$F = \{t_i \mid t_i = \langle \mu_F(e_i),\ \pi_F(e_i),\ \nu_F(e_i)\rangle\}\ or\ F = \{t_i \mid t_i = \langle \mu_i,\ \pi_i,\ \nu_i\rangle\},$$

where $\mu_i = \mu_F(e_i),\ \pi_i = \pi_F(e_i)$, and $\nu_i = \nu_F(e_i), \forall e \in S$.

For every $t_i, t_j \in F\ \ (i \neq j)$,

$t_i \le t_j$ if [$\mu_i \le \mu_j$, $\pi_i \circ \pi_j$ and $\upsilon_i \ge \upsilon_j$] OR [$\mu_i \le \mu_j$ and $\upsilon_i \ge \upsilon_j$] where $\circ \in \{<,=,>\}$.

$t_i = t_j$ if [$\mu_i = \mu_j$, $\pi_i = \pi_j$ and $\upsilon_i = \upsilon_j$] OR [$\mu_i = \mu_j$ and $\upsilon_i = \upsilon_j$].

If $M = \{m_i \mid m_i = \langle \mu_M(e_i), \pi_M(e_i), \upsilon_M(e_i)\rangle\ \&\ e_i \in S\}$, $\forall e_i \in S$, and $N = \{n_i \mid n_i = \langle \mu_N(e_i), \pi_N(e_i), \upsilon_N(e_i)\rangle\ \&\ e_i \in S\}$, $\forall e_i \in S$, are two intuitionistic fuzzy sets defined in the domain of discourse *S*, then

$M \subset N$ if and only if $[m_i \le n_i]$, $\forall m_i \in M\ \&\ \forall n_i \in N$ and

$M = N$ if and only if $[m_i = n_i]$, $\forall m_i \in M\ \&\ \forall n_i \in N$.

3. Intuitionistic Fuzzy Set-based Weather Analysis

For our convenience, the weather is divided into three categories: cold, warm, and hot. Let us consider that 'cold' is a membership function, 'hot' is a non-membership function, and 'warm' is a hesitation function. Here, the degree of warm may cater to either degree of hot or degree of cold or both. Let us assume that the weather is definitely cold at and below the $20°$C, it is definitely hot at and beyond the $32°$C and in between the weather is warm, i.e., the level of coldness decreases and the level of hotness increases. We represent the weather 'cold', 'warm', and 'hot' pictorially (Figure 2).

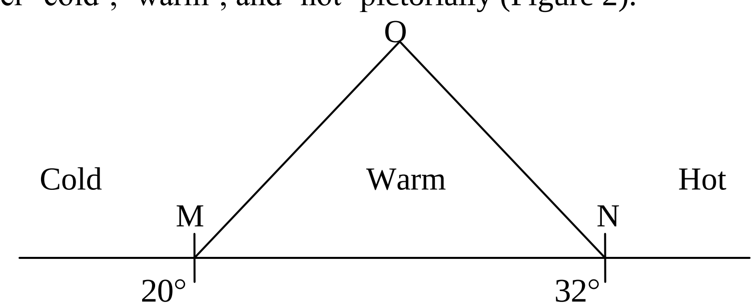

Figure 2. Illustration of the degrees of 'cold', 'warm', and 'hot'

Let *S* be the set of different values of the weather where temperature is measured in degrees Celsius °*C*, *F* the intuitionistic fuzzy set defined in the set *S*. Let $\mu_F(e)$ be the membership grade of weather 'cold' at e where $e = x^{\circ}C$. Here, x denotes a numerical value. For example, $x^{\circ}C = 20^{\circ}C$. Similarly, we denote $\pi_F(e)$ as the hesitation grade of weather 'warm' and $\nu_F(e)$ the non-membership grade of weather 'hot' at e where $e = x^{\circ}C$.

Then, $S = \{20^{\circ}C, 26^{\circ}C, 32^{\circ}C\}$ and

$$\mathrm{F} = \left\{ \begin{matrix} \langle \mu_F(20^{\circ}C), \pi_F(20^{\circ}C), \nu_F(20^{\circ}C)\rangle, \langle \mu_F(26^{\circ}C), \pi_F(26^{\circ}C), \nu_F(26^{\circ}C)\rangle, \\ \langle \mu_F(32^{\circ}C), \pi_F(32^{\circ}C), \nu_F(32^{\circ}C)\rangle \end{matrix} \right\}$$

At and below $20^{\circ}C$, there is no warm and hot, but there exists only cold.

$$\text{Hence, } \mu_F(20^{\circ}C) = 1,\ \pi_F(20^{\circ}C) = 0,\ \nu_F(20^{\circ}C) = 0, i.e., \langle 1, 0, 0\rangle.$$

At $26^{\circ}C$ (at the point O), there is no cold and hot, but there exists only warm.

$$\text{Hence, } \mu_F(26^{\circ}C) = 0,\ \pi_F(26^{\circ}C) = 1,\ \nu_F(26^{\circ}C) = 0, i.e., \langle 0, 1, 0\rangle.$$

At and beyond $32^{\circ}C$, there is no cold and warm, but there exists only hot.

$$\text{Hence, } \mu_F(32^{\circ}C) = 0,\ \pi_F(32^{\circ}C) = 0,\ \nu_F(32^{\circ}C) = 1, i.e., \langle 0, 0, 1\rangle.$$

Therefore, $\quad F = \{\langle 1, 0, 0\rangle, \langle 0, 1, 0\rangle, \langle 0, 0, 1\rangle\}.$

Cold decreases and warm increases in between M and O, i.e., $1 > \mu > 0\ and\ 0 < \pi < 1.$

Warm decreases and hot increases in between O and N, i.e., $1 > \pi > 0\ and\ 0 < \nu < 1.$

4. Artificial Intelligence

The artificial intelligence uses the binomial theorem and discrete binomial distribution for data analysis. The following binomial theorem (binomial expansion or binomial series) [10-16] is newly constituted for application in computational science and engineering.

$$\sum_{i=0}^{r} V_i^{n+1}x^i = \sum_{i=0}^{r} V_i^n x^i + \sum_{i=1}^{r} V_{i-1}^n x^i + \sum_{i=2}^{r} V_{i-2}^n x^i + \cdots + \sum_{i=r-1}^{r} V_{i-(r-1)}^n x^i + \sum_{i=r}^{r} V_{i-r}^n x^i.$$

Proof: Let’s proof that the computation of addition of binomial series (right-hand side) is equal to the sum of binomial series for upper limit $r+1$(left-hand side).

$$\sum_{i=0}^{r} V_i^{n+1}x^i = \sum_{i=0}^{r} V_i^n x^i + \sum_{i=1}^{r} V_{i-1}^n x^i + \sum_{i=2}^{r} V_{i-2}^n x^i + \cdots + \sum_{i=r-1}^{r} V_{i-(r-1)}^n x^i + \sum_{i=r}^{r} V_{i-r}^n x^i$$

$$= (V_0^n + V_1^n x + V_2^n x^2 + V_3^n x^3 + \cdots + V_r^n x^r) + (V_0^n x + V_1^n x^2 + V_2^n x^3 + V_3^n x^4 + \cdots + V_{r-1}^n x^r)$$

$$+(V_0^n x^2 + V_1^n x^3 + V_2^n x^4 + V_3^n x^5 + \cdots + V_{r-2}^n x^r) + \cdots + (V_0^n x^{r-1} + V_1^n x^r) + V_0^n x^r$$

$$= V_0^n + (V_0^n + V_1^n)x + (V_0^n + V_1^n + V_2^n)x^2 + \cdots + (V_0^r + V_1^r + V_2^r + V_3^r + \cdots + V_n^r)x^r$$

$$\text{(Note that } V_0^p + V_1^p + V_2^p + \cdots + V_r^p = V_r^{p+1} \text{ and } V_0^p = V_0^{p+1} = 1)$$

$$= V_0^{n+1} + V_1^{n+1}x + V_2^{n+1}x^2 + V_3^{n+1}x^3 + V_4^{n+1}x^4 + \cdots + V_{r-1}^{n+1}x^{r-1} + V_r^{n+1}x^r$$

$$= \sum_{i=0}^{r} V_i^{n+1}x^i.$$

Hence, it is proved.

Several mathematical and combinatorial techniques are used as an application in the artificial intelligence-based cybersecurity and these results also help to create a strong security algorithm like RSA algorithm and Elliptic Curve Cryptography in the field of cybersecurity in order to reduce the risk of cyber-attacks and protect against the unauthorized exploitation of systems and networks.

We have already understood the below factorial identity and detailed proof in the previous section. Now, we find how to use this result in another way.

$(n_1 + n_2 + n_3 + \cdots + n_k)! \;=\; (a_1 \times a_2 \times a_3 \times \cdots \times a_{k-1}) \times n_1! \times n_2! \times n_3! \times \cdots \times n_k!$,

That is, $(n_1 + n_2 + n_3 + \cdots + n_k)! = T \times n_1! \times n_2! \times n_3! \times \cdots \times n_k!$.

Here, $n_1, n_2, n_3, \cdots, n_k$ are possitive integers.

An alternative way for finding the positive integer T is given below:

$$T = \frac{(n_1 + n_2 + n_3 + \cdots + n_k)!}{n_1! \times n_2! \times n_3! \times \cdots \times n_k!}, \qquad where\ T, n_i \in N = \{1, 2, 3, \cdots\}\ \&\ i = 1,2,3, \cdots$$

For our convenience, we rearrange the positive integers that are equal to the product T.

Let $T = 64$. $T = 1 \times 64$; $T = 2 \times 32$; $T = 2 \times 4 \times 8$; $T = 4 \times 4 \times 4$; etc.

Similarly, we rearrange for $T = 21$, that is, $T = 1 \times 21 \;\; or \;\; T = 3 \times 7$.

5. Conclusion

Mathematical and combinatorial techniques and its applications [17] for an artificial intelligence-based cybersecurity and weather analysis have been introduced in this article. Computational algorithm on artificial intelligence-based cybersecurity is used to protect the computing systems, devices, networks, programs and data from cyber-attacks, that is, to reduce the risk of cyber-attacks and protect against the unauthorized exploitation of systems, programs, and networks. The factorial and binomial identities and expansions can be used as artificial intelligence-based methodological advance in cybersecurity.

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