# A Two-Stage Cooperative Game-Theoretic Based Approach for Distributed & Model-Free Control of Gas Transmission Networks

Habibollah Safari[a] [1]

[a] Chemical and Petroleum Engineering Department, Sharif University of Technology, Tehran, Iran

## Abstract

Model dependency in the majority of control approaches makes them costly "if not infeasible" to get implemented for large-scale and cyclic gas network control. This work, hence, introduces a distributed, model-free, and game-theoretic approach comprising a simultaneous game followed by a sequential game. The two-stage cooperative approach is adopted in the control structure of gas transmission networks. The humans supervising the compressor stations in the network are considered as game players (agents), the rationality (utility function) of which is emulated via a designed fuzzy inference system such that decisions made by the agents closely match those made by the human operators. At the first stage of the game, players prevent the network from collapsing due to pressure drop in deliveries. In the subsequent stage, all players strive to enhance their utilities until the desired condition is met. Contrary to previous studies on gas network control, controllers learn appropriate action in the proposed strategy rather than calculating it using specific mathematical models. The performance and robustness of the proposed algorithm are assessed by its utilization to control a cyclic and interdependent gas transmission network. The results evidence that supervisory controllers operate the gas network in the permissible range in the presence of various loads; thus, such a comprehensive model-free control approach is a pragmatic solution in the systems whose models are not applicable for controller design purposes or exact utility evaluation in their game-based model is almost impossible.

Keywords:

Game-Theoretic based Control, Model-Free Control, Learning Algorithm, Cooperative System, Gas Transmission Network, Fuzzy Inference System

[1] * Corresponding author.
E-mail address: Habibollah.Safari@alum.sharif.edu

## 1. Introduction

Increasing gas consumption worldwide directs industries to utilize gas pipelines for transportation purposes; however, the operation and control of such complex networks are costly and challenging[1],[2]. Given the infeasibility of model-based control methods in interdependent complex systems such as gas networks due to considerable computational costs, developing a novel model-free and distributed solution strategy is inevitable. From the control structure point of view, as large-scale transmission networks are interdependent due to the cyclic topology[1], a distributed control approach has to be applied to the gas network operation in which utilization of a decentralized or centralized control configuration is inefficient and unaffordable if not impractical[3],[4].

From a different perspective, the control approach should fulfill the requirements of gas network operation; firstly, a transmission network is an infinite-dimensional system whose manipulated variables (e.g., power of compressor stations) are widely distributed throughout the network, the model of which is infinite-dimensional state model (the system's states depend on both time and location) and its real-time simulation requires enormous computational demands[5]. Secondly, there are on-off valves changing their states in gas networks and transforming network topology and governing models[3]. Besides, uncertainties in various model parameters restrict the interval at which the model can effectively simulate the network operation. All of the above encourage the development of a new model-free control approach for gas transmission network operation.

While prior investigations have implemented various methods to control the gas network operation in an optimum or near an optimal condition, most of them depend on a particular mathematical model. Ahmadian and Boozarjomehry used an unscented transform for optimizing the gas network[6]. Zlotnik and Chertkov outlined a control system for dynamic of distributed pipelines, which reduces the network PDE equation to a set of Ordinary Differential Equations (ODE)[7]. Mak, T.W. et al. applied a two-stage optimization method for compressors operations in a large-scale gas network, although its topology was not cyclic[8]. Aßmann, Liers, and Stingl developed a new single-stage robust approach reformulating a two-stage optimization problem in a gas network under both pipe physical parameters and demand uncertainty[9]. Notably, the model-dependency of nearly all previous studies restricts them to networks with low-scale or at least simple topology; applications of such methods are inevitably limited in practical cases.

In the context of model-free and distributed control, game theory and learning-based control have gained widespread currency[10]. However, game-theoretic based algorithms have been rarely used for the operation of gas transmission networks. Tang, Lie, and Wang contrive a game-theoretic based decentralized strategy for operation and control of power demand management in Hong Kong University[11]. Mie et al. utilized the cooperative game theory concept to improve the efficiency of microgrids while each microgrid has its own preferences[12]. Marden et al. substantiated that there is no feasible model for controlling a network of wind turbines[13] and proposed two model-free methods[14], [15]to operate the wind farm in two different conditions. Marden and Arslan introduced a particular class of games called weakly acyclic games[16] and applied this class to dynamic sensor coverage and sensor deployment problem[16].

[1]Generally speaking, gas transmission networks are classified in three types of topology: Cyclic, Tree and Linear networks. For many reasons including increasing short-term storage and lowering fuel consumption in Compressor stations, transmission networks are usually designed in a cyclic type.

| Nomenclature | | | |
|---|---|---|---|
| | | $P_{effective,i}$ | effective pressure for player i |
| $CS$ | compressor station | $w_{i,j}$ | impact factor of consumer j on player i |
| $CS_i$ | compressor station i | $Q_{Norm,i}$ | normalized power in station i |
| $GCR$ | gas compression ratio | $Q_{Max,i}$ | maximum power in station i |
| $GCR_i$ | gas compression ratio $CS_i$ | $Q_{Min,i}$ | minimum power in station i |
| $EC_i$ | energy consumption in station i | $S_i$ | decision space of player i |
| $N$ | players set (compressor stations) | $n_c$ | number of consensuses |
| $M$ | consumers set | $n_r$ | number of required consensus |
| $P$ | pressure | $k$ or $K$ | sample time index |
| $P_j$ | pressure in consumer j | $LS$ | lower bound of the safe horizon |
| $P_{MA}$ | minimum allowable pressure | $US$ | upper bound of the safe horizon |
| $P_{Rel}$ | Relief valve pressure in the regulatory valve | $m_i(k)$ | mood of player i at t=k |
| $P_{Norm,j}$ | normalized pressure in consumer j | $u_i(k)$ | utility of player i at t=k |
| $P_{Max}$ | maximum pressure in the network | $T_s$ | settling time of system |
| $P_{Min}$ | minimum pressure in the network | $\tau$ | time constant of system |
| MPa | Megapascal | | |
| MMCFD | million cubic feet per day | | |

Regardless, in all learning algorithms developed so far for model-free and distributed control in the game theory framework, it has been assumed that exact utility is accessible in every sample time. It implies that systems in which controlled by the algorithms have fast dynamics and their dynamics are negligible in operation. Accordingly, all system states observed by the algorithm, are in a steady-state condition. This presumption makes these algorithms infeasible on systems with slow dynamics or time delay in response, including gas transmission networks.

Considering the above, the main contribution of this work is the promotion of gas transmission operation to a distributed model-free structure that is not dependent on any kind of model, including mechanistic, empirical, or intuitionistic. This approach outlines a learning-based algorithm considering the system as a black-box and deal with it based on random decision-making to observe the system response, which is appropriate for systems requirements without an appropriate controller design model. The assimilation of the proposed strategy into a game theory concept transforms the system model to a new domain called game space, which is not only free of the previous complexities such as time-demanding governing equations solving but also accommodates to gas network operation in real cases[17], [18].

The rest of this paper is organized as follows. Section 2 describes the control structure of a gas transmission network and its representation as a cooperative game. This section includes utility function design, which is a crucial part of game-based models. Section 3 presents the main limitation of distributed learning algorithms in cooperative games and removes it by proposing a new one. Section 4 contains simulation of a real gas transmission network for assessment of the proposed control strategy. Section 5 concludes the paper and discusses it.

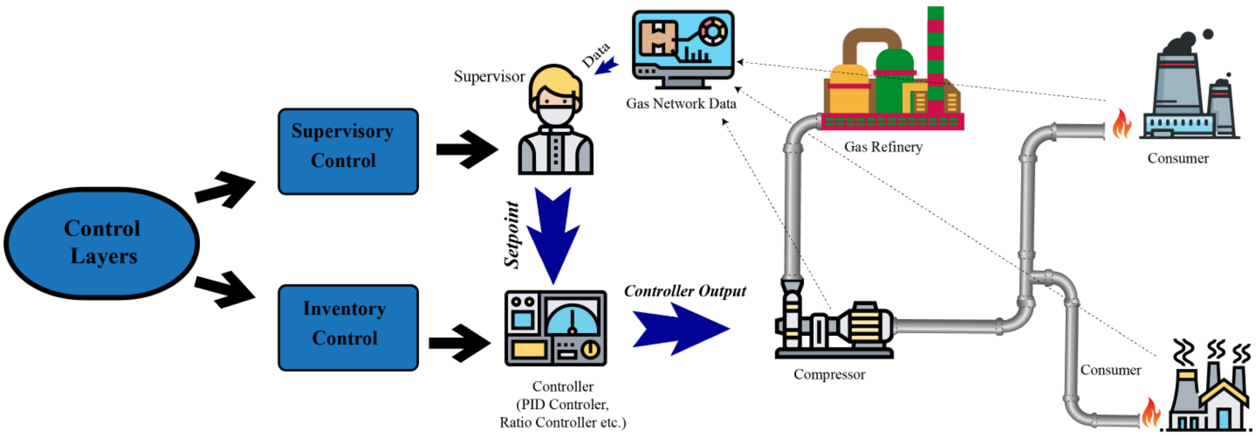



**Fig. 1.** A schematic scheme of control structure in the gas transmission network.

## 2. Reformulation of gas network control structure as a game

In a gas transmission network, each compressor station (CS) is often operated by a human observer (player), which determines an appropriate setpoint (Gas Compression Ratio or Power of Various Compression Stage) based on the network condition. This setpoint is eventually applied by a conventional controller such as PID or ratio controller to keep the station Gas Compression Ratio (GCR) in a desired amount. Fig. 1 represents a schematic of the aforementioned control structure in gas transmission networks[19].

In the context of game-theoretic control, problem reformulation as a game includes three steps: players definition and their rationality during the game, determination of decision space for each player (this part is equivalent to defining manipulating variables in control theory), and finally assigning an algorithm in which all players adhere to it during the game[18]. In the formulated game of network operation, supervisors considered as players collaborating for a safe operation to assure that all consumers receive natural gas in a permissible range of pressure. The players who are responsible for the supervisory control layer in the network communicate with each other to decide that network operation is acceptable or not. The key factor for assessing operation quality in decision-making is the rationality of players emulated by a utility function. The utility function design is the most crucial part of the game theory engineering applications for distributed control since it has to be designed somehow that system's overall behavior becomes desirable while players access game information locally[20], [21].

### 2.1 Design of utility function

Utility function design in the game theory has the same role as the control law has in the control theory[18]. After specification of the players, their preferences should be investigated based on the principles of real cases. In this context, a network operation's desirability depends on two main factors: energy consumption in compressor stations and delivery pressures. Higher pressure in delivery points enhances the network robustness and reliability in the presence of various loads and uncertainties even though it increases energy consumption in compressor stations. Accordingly, players have to compromise between these two factors, which have an inverse effect on network operation efficiency. A player operating a compressor station tries to minimize its energy consumption and at the same time maintain the delivery pressures in an acceptable range. Eq. 1. Represents the mathematical formulation of the problem that player i tries to solve based on his/her decision-making scenario:

$$\begin{cases} \text{minimize} & EC_i \\ \text{subject to } P_j > P_{MA} & \forall\, j \in M \end{cases} \quad (1)$$

Where $EC_i$ is the energy consumption of i'th compressor station (which is operated by player i), and M is a set of consumers in the network. $P_{MA}$ refers to minimum allowable pressure in the city gates or industrial consumers, which is composed of a relief threshold in the regulatory valves with their corresponding safe margin:

$$P_{MA} = P_{Rel} + \text{Safe Margin} \quad (2)$$

In industrial consumers or city gates in transmission networks, regulatory valves have a role in cutting off the gas flow if delivery pressure falls below the specific amount ($P_{Rel}$). Considering a network never should be allowed to operate near the $P_{Rel}$ in the delivery points, supervisors consider a safe margin which guarantees gas network operation far enough from $P_{Rel}$. $P_{MA}$ is usually set to 4.82 MPa (~700 psia) due to the fact that such a value fulfills almost all practical considerations in the operation of a gas transmission network.

Eq. 1. can be interpreted as players' rationality in the game of network operation. As previously explained, the gas network operation desirability depends on two general criteria, including energy consumption and pressure of delivery points, which are qualitative expressions of Eq. 1. Utility function (rationality) design in the game theory accompanies by the mathematical formalism complexity. Meanwhile, the gas network operation encompasses the qualitative expressions. In this regard, using a rule-based structure such as a Fuzzy Inference System (FIS) incorporating qualitative principles in the gas transmission network can be appropriate in utility function design[21],[22]. Considering a FIS as a utility function for a player eliminates the necessity to develop a mathematical utility function satisfying Eq. 1.and also its complexity.

In fuzzy utility function design, it is assumed that three main factors affect utility function; the pressure of delivery points, station's instant power, and history of station's power representing average power of compressor station since the start of the game (Eq. 3.). With these three factors, a player weighs his/her options during the game. Fig. 2. illustrates the FIS designed to get used as a utility function.

$$Q_{average} = \frac{\int_{t_{ref}}^{t} Q\, dt}{t - t_{ref}} \quad (3)$$

The Fuzzy system comprises 5, 5, and 3 linguistic values for pressure, instant power, and average power, respectively, as inputs, and includes seven linguistic values representing the utility desirability as the output. As the whole possible condition has to be covered in the FIS, mathematical permutation represents seventy-five rules constituting the fuzzy system (Table 1). The designed FIS is a Mamdani Fuzzy Inference System that is more discussed in Appendix A.

**Table 1**
**Rules governing fuzzy utility function**

| Instant Power \ Effective-Pressure | Critical | Dangerous | Normal | Safe | Absolutely Reliable |
|---|---|---|---|---|---|
| Very Low | Low | Normal | Very high | Very Very High | Very Very High |
| Low | Low | Normal | Very high | Very Very High | Very Very High |
| Normal | Very Low | Low | High | Very High | Very Very High |
| High | Very Low | Low | High | Very High | Very High |
| Very High | Very Low | Very Low | Normal | High | High |
| Average power is Low | | | | | |
| Very Low | Low | Normal | High | Very High | Very Very High |
| Low | Very Low | Low | High | Very High | Very High |
| Normal | Very Very Low | Very Low | Normal | Very High | Very High |
| High | Very Very Low | Very Low | Normal | High | Very High |
| Very High | Very Very Low | Very Low | Normal | Normal | High |
| Average power is Normal | | | | | |
| Very Low | Very Low | Low | High | Very High | Very High |
| Low | Very Low | Very Low | High | Very High | Very High |
| Normal | Very Very Low | Very Very Low | Low | High | High |
| High | Very Very Low | Very Very Low | Low | High | High |
| Very High | Very Very Low | Very Very Low | Low | Normal | Normal |
| Average power is High | | | | | |

In order to simplify fuzzy system implementation, inputs variables are normalized according to the maximum and minimum quantities as formulated in Eq. 4. and Eq. 5.:

$$P_{Norm,j} = \frac{P_j - P_{Min}}{P_{Max} - P_{Min}} \qquad j \in M \tag{4}$$

$$Q_{Norm,i} = \frac{Q_i - Q_{Min,i}}{Q_{Max,i} - Q_{Min,i}} \qquad i \in N \tag{5}$$

At Minimum power condition, all of the station's compressors[1] operate in the free by-pass mode, which means $Q = 0$ and $GCR = 1$. $P_{Max}$ and $P_{Min}$ represent the maximum and minimum pressure in the network and generally change by type of the networks but in a transmission network $P_{Max}$ may reach up to 9.65 MPa (~1400 psia). $P_{Min}$ is also considered relief pressure in regulatory valves, which is assumed 3.47 MPa (~500 psia) in the current work (section 4).

[1] In gas transmission network, compressor stations are often composed of three or four centrifugal compressors and each compressor can operate in three modes, including free by-pass, half load operation, and full load operation.

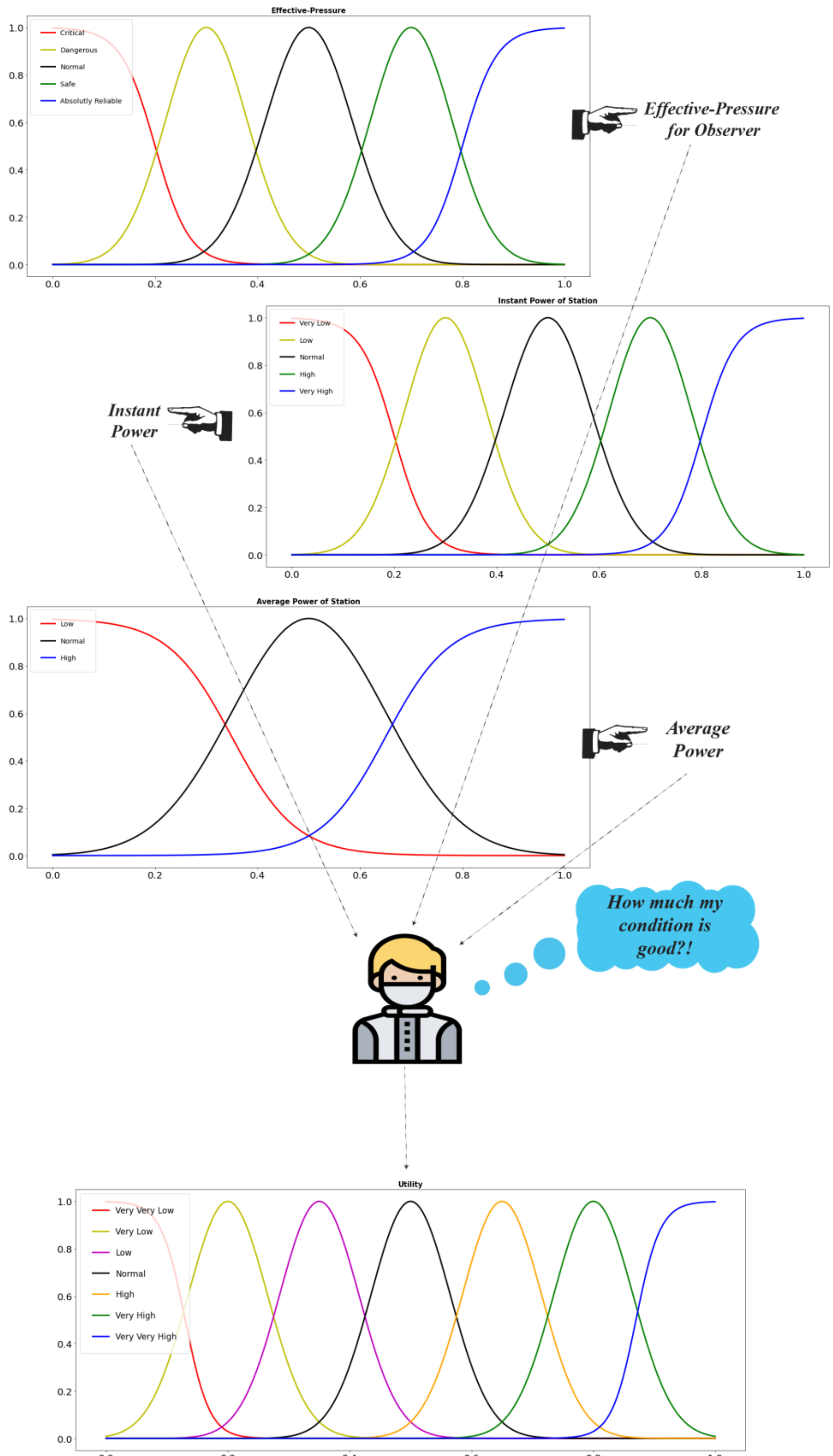



**Fig.2.** Schematic Scheme of Fuzzy Utility Function.

## 2.2 Effective-pressure

Given the interdependency of a cyclic and complex gas transmission network, each compressor station affects all consumers. In this regard, all pressures of delivery points must be considered in utility function design. With the intention of simplification, we use one factor, called **effective-pressure**, to represent the desirability of end-points pressures in the fuzzy inference system. Using effective-pressure prevents the input increment in FIS and further simplification. This factor entails all delivery pressures multiplied by different weight coefficients for each player. These weight coefficients indicate how much a compressor station affects consumers:

$$P_{effective,i} = \sum_j w_{i,j} * P_j \qquad \forall\, i \in N,\ j \in M \tag{6}$$

In real cases, each player considers gas network topology and network characteristics and estimates $w_{i,j}$s based on hydraulic resistance between Compressor Station (CS) and consumers. In Section 4, we investigate $w_{i,j}$s by sensitivity analysis of the designed network. It should be noted that in transmission networks existing in practice, there might be more than dozens of consumers in the network, while a few CSs perform to deliver natural gas to the consumers. Therefore, using the effective-pressure concept is inevitable since it is impossible to pair each CS to a special delivery point.

## 2.3 Decision Space

Another stage of the game formulation is determining players' decision space, which strictly depends on the nature of the system operation. Knowingly that all stations in the gas network comprise four centrifugal compressors, and each compressor has three modes of operation, the decision space is:

$$S_i = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\} \times \frac{Q_{Max,i}}{12} \qquad i \in N \tag{7}$$

$S_i$ indicates the decision space in which players can choose one of the 13 choices and evaluate outcomes. Option {0} means all of the compressors in the stations are in the free by-pass mode, while {12} represents that all compressors operate in full load operation.

## 3. Two-stage cooperative algorithm in the game of gas network operation

Previous studies proposed numerous algorithms for players' decision strategy in the cooperative game context; nevertheless, none can be applied to systems with a slow dynamic (high settling time) or delay in response. The key assumption in most game theory algorithms is that all players access the exact utility value at each sampling time and evaluate it. Although this supposition simplifies algorithm implementation and is mandatory to guarantee game convergence to a particular point, such as Nash equilibrium[1] or Pareto optimality, it restricts algorithm applications in the study of systems described above. We can exemplify this challenge with the cooperative game simulating transmission network operation. In a gas transmission network, supervisors in compressor stations decide upon appropriate setpoints and apply them to the network. We suppose that at the sample time $k$, a consumption load is imposed on a delivery point in the network, and at sample time $k + 1$, players revise decisions based on their utility. The utility function is highly dependent on effective-pressure, and each fluctuation in network consumption changes the player's effective-pressure; consequently, the player's utilities. Since the interval between two sample

[1] Nash Equilibrium is a solution in non-cooperative games in which no player can improve its utility with changing strategy.

times is much less than the network settling time ( i. e., $T_S \gg \Delta T$ or $\tau \gg \Delta T$), the instant pressures observed by supervisors are far from the steady-state pressures in which the network is going to settle. In this regard, supervisors cannot access steady-state pressures at each sample time (i.e., they observe transient utilities rather than the exact utilities value), and they have to either decide in a transitional state or somehow predict the eventual steady state of the system, which is impossible in a model-free approach. Besides, the supervisors should not wait for the network to settle since it might lead the network to a critical condition in which delivery pressures fall below the regulatory valves' relief pressures. Considering the above and the notion that prevalent algorithms are promoted in the context of complete information games, such methods are unimplementable in distributed control of slow dynamic systems since players have incomplete information and limited access to game data.

The literature in algorithmic game theory evidence that most of the proposed learning algorithms lead the system to a particular point, such as Nash equilibrium, correlated equilibria, and Pareto optimality. However, in incomplete information games, such as network operation game, convergence investigation to a particular point is impossible[23], [24]. These games are more realistic than others because players' access to all data is an insubstantial assumption in real benchmarks.

### 3.1 First stage; simultaneous consensus game

In the first stage, players do not access their exact amount of utility, so they try to find the increasing direction of utility. If all players find the direction in which all utilities increase, the system will converge to an appropriate point. Fig. 3. stretches a flow diagram of the algorithm in the first stage of the game in which each player observes its own utility and recalls it in the next sample time. If any of the players notice a change in its utility, the first stage of the game starts. At each sample time, players declare their mode. While players are not aware of others' information, they have limited communication and announce their status to others (see section 2). Each player compares its utility with its last gained utility, and if the utility increases, the player is **content** with the game. If not, the player declares its status as **discontent**. All players start to search randomly in their decision space if any of the players’ statuses become discontent. If all players become content unanimously, players' consensus happens, and all players have to keep their decision for the next sample time. The first stage of the game ends if the number of consecutive consensuses ($n_c$) becomes more than a specific value, which is regarded as a designed parameter ($n_r$).

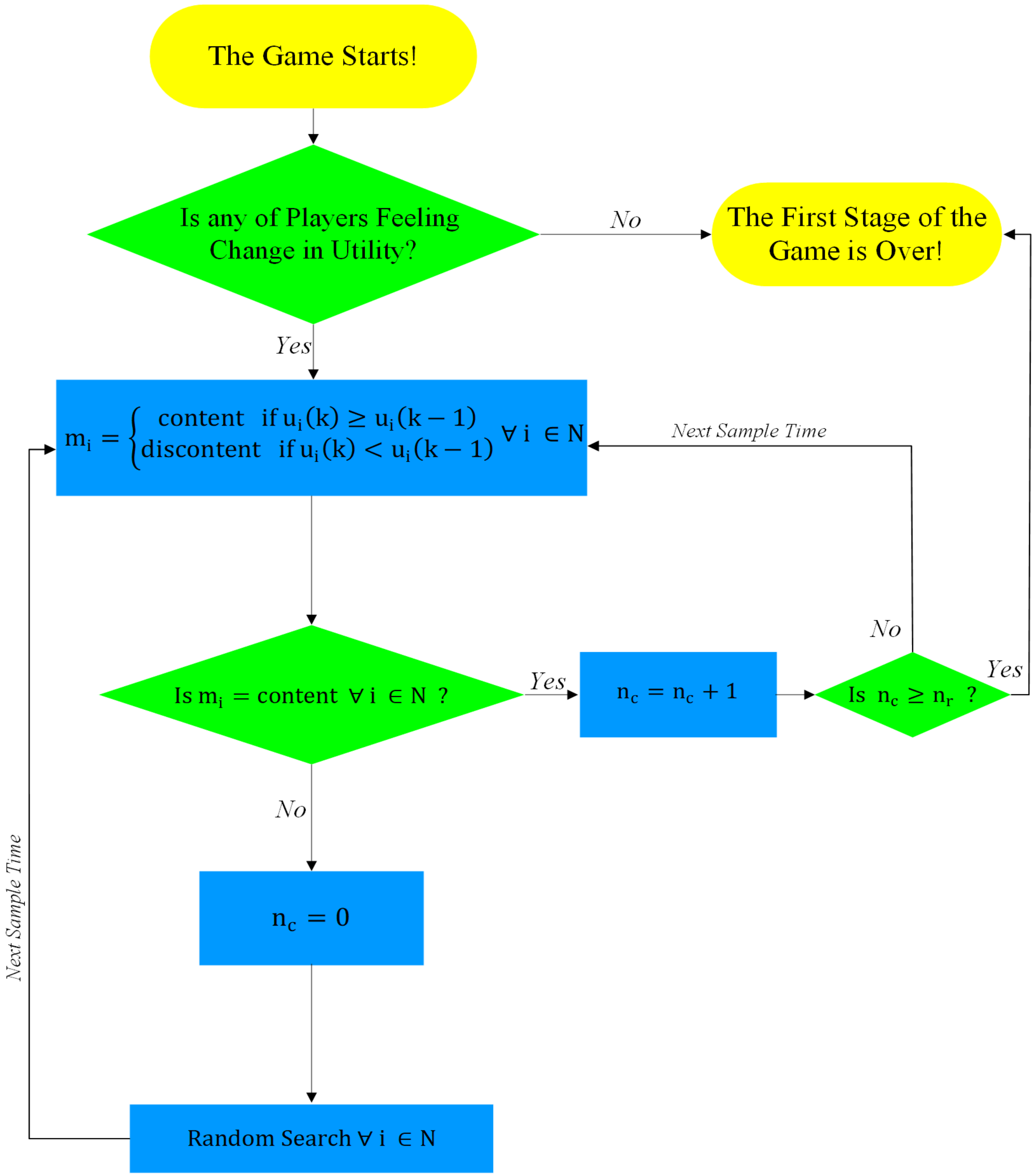



**Fig. 3.** Flow diagram of the first stage of the game.

### 3.2 Second stage; sequential amelioration game

Termination of the first stage of the game settles the transmission network in a reliable and appropriate condition; however, players may find some opportunity to enhance the network operation. The key factor in which players revise their decision in the second stage is effective-pressure. As the delivery point pressures (effective-pressure) increase, robustness and reliability in the network are improved though it incurs energy consumption costs in CSs. Therefore, every player considers a pressure span called **safe area or safe horizon** for its effective-pressure in which the network has an appropriate energy consumption and reliable endpoints pressure. This span (**safe area** or **safe horizon)** strictly depends on the design and class of networks (gathering, transmission, or distribution). Supervisors in compressor stations intuitively consider this safe area, and it is usually about $4.82 - 5.51$ MPa ( ~ 700 - ~800 psia) in real cases(transmission networks). Therefore, the lower bound of the safe horizon (LS) is $4.82$ MPa (~700 psia) and the upper bound of the safe horizon is $5.51$ MPa (~800 psia).

When the first stage finishes, three different conditions, may occur, which eventually lead to different scenarios and actions in the game:

I. If all effective-pressures are on the safe horizon; therefore, no one starts the second stage of the game, and the game is over!

II. If at least one of the players' effective-pressure is positioned lower than the LS, players start the second stage of the game. Each player who observes that its effective-pressure is lower than the safe area switches its power to the next level in the domain of the decision variable. For example, if for player i $(\mathrm{i} \in \mathrm{N})$ $\mathrm{P}_{\mathrm{effective,i}} < \mathrm{LS}$ and $\mathrm{S_i} = \{4\}$ player i changes its decision to $\mathrm{S_i} = \{5\}$. Since energy consumption has to be minimized, the player who has minimum capacity should start the game. This priority is rational because one stage increasing in CS with low capacity is lower than one stage increasing in CS with high capacity, and this ensures that for improvement of reliability in network operation (putting effective-pressures is the safe horizon), players increase energy consumption minimally.
It is notable that just one player can play the game in each term (each sample time), and if one player did its action, no one could act until the next sample time.

III. The third situation is when at least one of the effective-pressures is higher than the upper bound of the safe horizon (US). As higher effective-pressure for a player is equivalent to higher energy consumption, players should try to reduce it. In this situation player who has maximum capacity starts the game.

The second stage of the game is not only to ensure that stations' energy has been almost minimized but also to assure the robustness of network operation in the presence of unpredictable loads. Fig. 4 shows the flow diagram for the second stage of the game. After the termination of the second stage, network monitoring continues to measure the states in order to start the first stage of the game if it is necessary (utility change for any of the players).

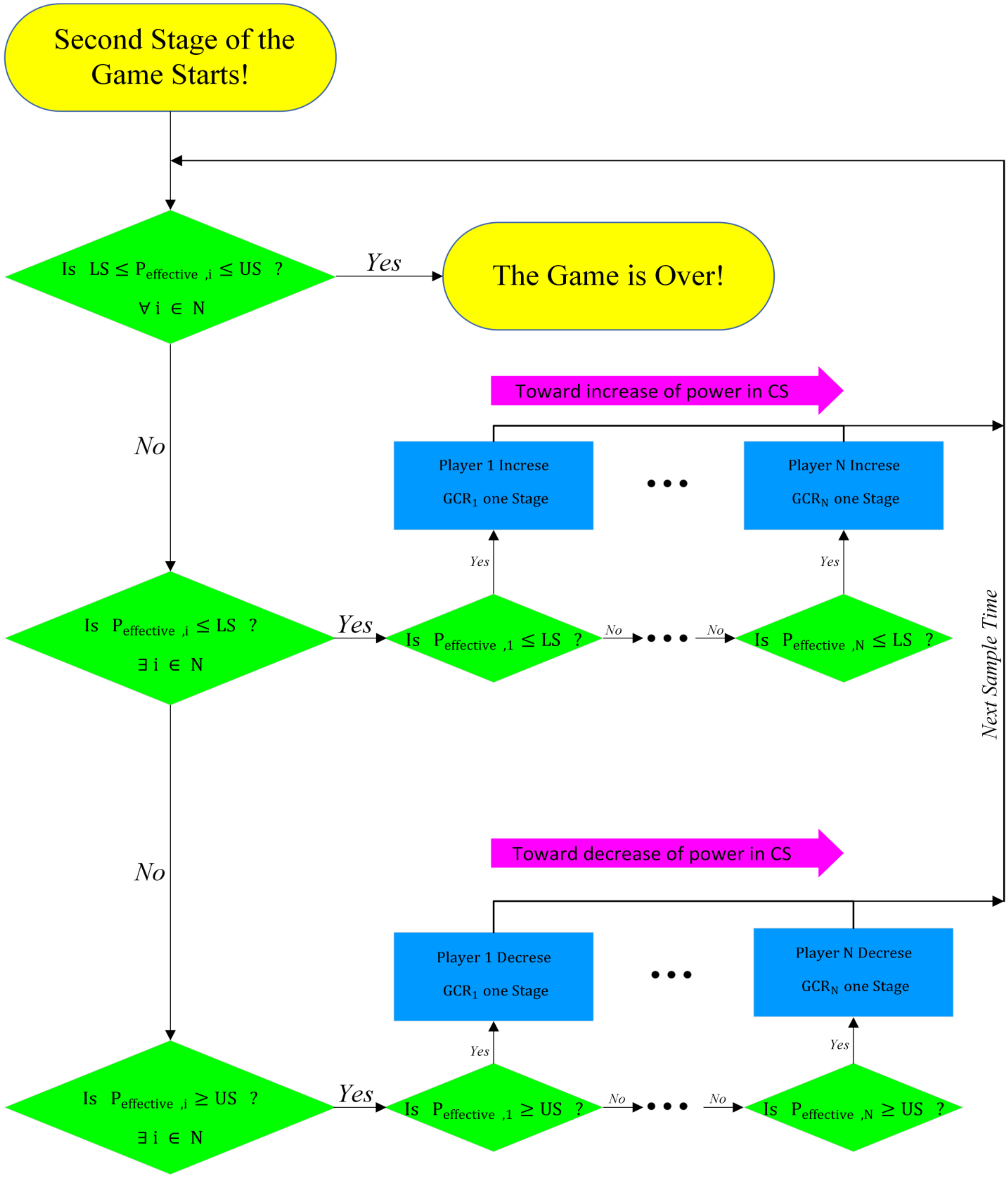



Fig. 4. Flow diagram of the second stage of the game.

## 4. Results and discussion

In this section, we have used a benchmark that is almost similar to a part of the gas transmission network in the South Part of Iran simulated in order to evaluate the performance of the proposed algorithms to control a typical realistic gas transmission network with all its intricate behavior. Fig. 5 shows an overview of this gas network, which includes two gas suppliers (gas refinery), four consumers, and three compressor stations (3 players). The network has a cyclic topology since it includes at least one CS in the loop. Table 2 shows the network's physical characteristics, and Table 3 represents the network nominal conditions. The gas network has been simulated by a commercial software which supports Component Object Model (COM) technology. COM technology is one of the standard technologies for remote procedure call between two or more software application, enabling programming languages to use various software in Windows platform. We also implemented the cooperative game, including fuzzy utility, and the proposed learning algorithm in the Python programming language. In each time step of two-stage game, players receive data from the gas simulator (commercial software) and calculate their utilities to revise their decisions and then apply their decisions to their corresponding CS in the network (Fig. 6).

**Table 2**
Physical characteristic of the designed gas network

| Physical characteristic | Value |
|---|---|
| Network Pipelines length | $6 \times 10^5$ m (600 km) |
| Pipeline diameter | 1.42 m (56 in) |
| Pipeline roughness | $1.2 \times 10^{-5}$ m (0.012 mm) |
| Maximum compressor station capacity 1 (player 1) | $1.34 \times 10^8$ Watt (~ 180000 hp) |
| Maximum compressor station capacity 2 (player 2) | $5.97 \times 10^7$ Watt (~ 80000 hp) |
| Maximum compressor station capacity 3 (player 3) | $5.21 \times 10^7$ Watt (~ 70000 hp) |

**Table 3**
Nominal condition of the designed gas network

| Boundary node | Pressure in MPa (Psia) | Volumetric flow rate in $m^3/s$ ( Million cubic feet per day ) |
|---|---|---|
| Supplier 1 | 8.82 (1266) | 2.96 ( ~ 9.032) |
| Supplier 2 | 8.82 (1266) | 3.07 (~ 9.376) |
| Consumer 1 | 5.52 (800) | 0.92 (~ 2.802) |
| Consumer 2 | 5.34 (775) | 1.43 (~ 4.360) |
| Consumer 3 | 5.17 (750) | 2.86 (~ 8.744) |
| Consumer 4 (Load) | 5.77 (837) | 0.82 (~ 2.5) |

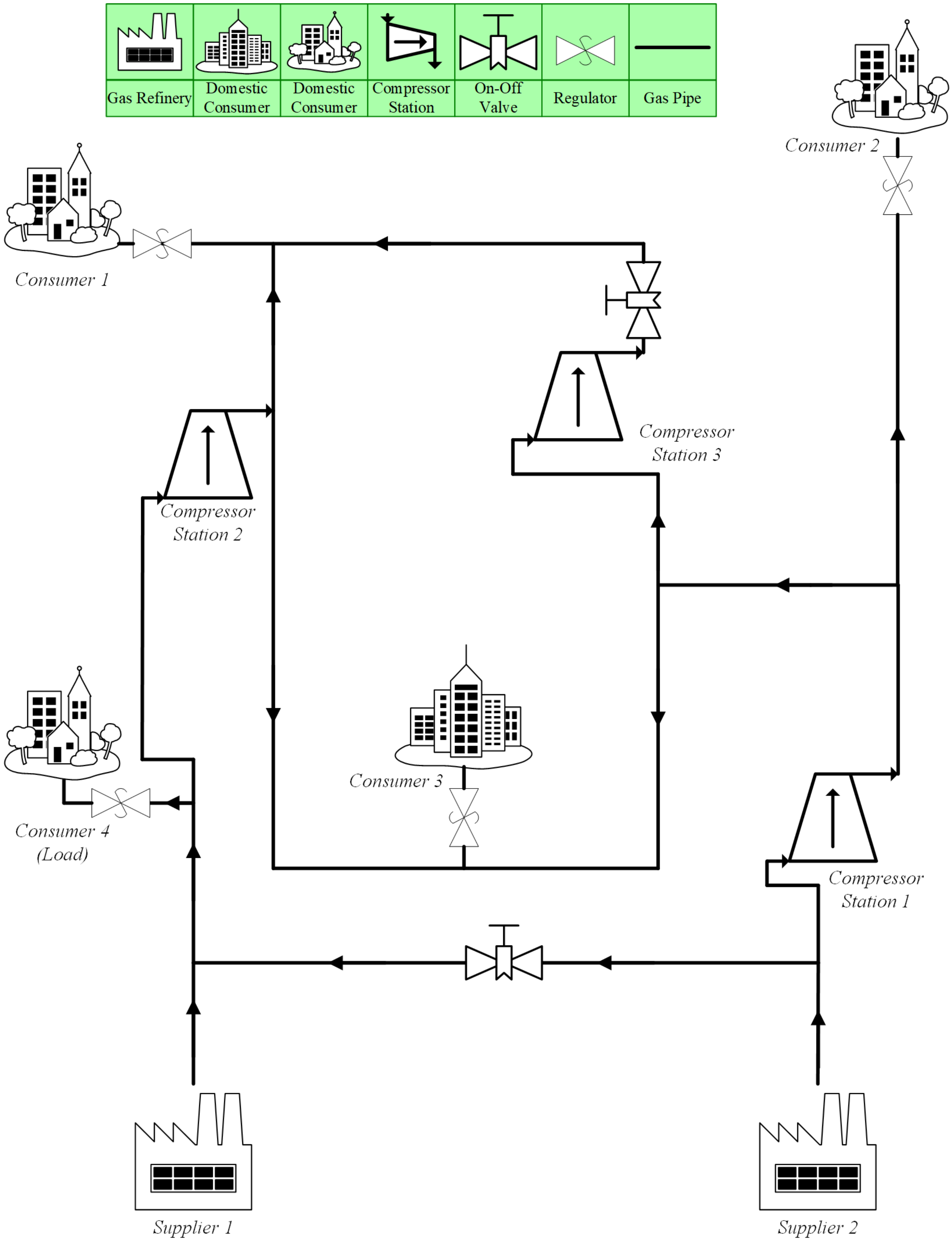



**Fig. 5.** Overview of the designed gas network.

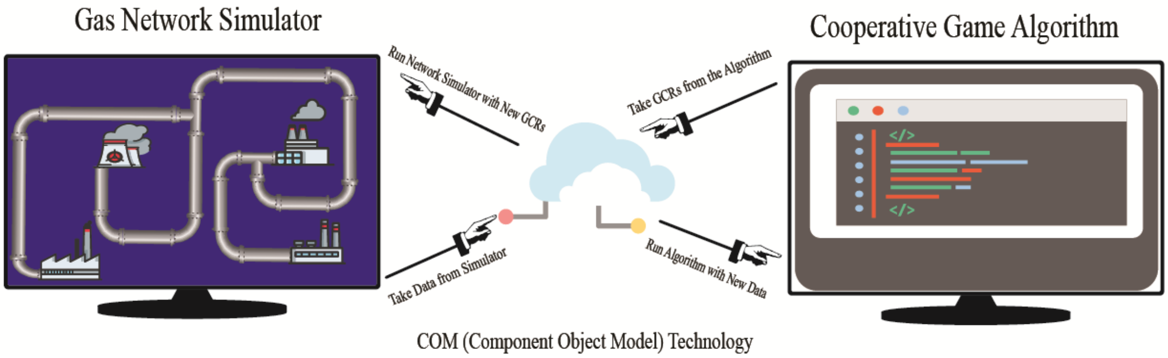



**Fig. 6.** The connection between gas network simulator and learning algorithm.

## 4.1 Effective-pressure of the gas transmission network and sensitivity analysis

Since each player requires its effective-pressure for utility evaluation, it needs to access $w_{i,j}$ (see section 2). We obtain $w_{i,j}$ for each player by sensitivity analysis in which random inputs (random GCRs or power consumption in CS) are imposed on the network in nominal condition, and the pressure changes in consumption points are investigated. Given the pressure changes, $w_{i,j}s$ (Table 4) are calculated according to equations 8-a - 8-c. Notably, the higher value of $w_{i,j}$ denotes that $CS_i$ exert more effect on consumer j:

$$P_{effective,i} = \sum_j w_{i,j} * P_j \qquad \forall\, i \in N,\; j \in M \tag{8-a}$$

$$\Delta P_j = \mathrm{Max}\big(P_j \text{ during random input}\big) - \mathrm{Min}(P_j \text{ during random input}) \tag{8-b}$$

$$w_{i,j} = \frac{\Delta P_j}{\sum_j \Delta P_j} \tag{8-c}$$

Fig. 7, Fig. 8, and Fig. 9 display CS random inputs and pressure change in consumers, which have been used to calculate the $w_{i,j}s$. These figures also demonstrate the effects of various compressor stations on all consumers, this in fact reflects the network interdependency. It is notable that in the investigation of $w_{i,j}s$ and network analysis (investigating different scenarios) we only consider three consumers which are positioned after the CSs in the network since the pressure in consumer 4 is almost unaffected by the CSs. This pressure is almost influenced by refineries and this is out of the context in this work.

**Table 4**
Impact factors of effective-pressure for players

| | Consumer 1 | Consumer 2 | Consumer 3 |
|---|---|---|---|
| Compressor station (player) 1 | 0.3 | 0.36 | 0.34 |
| Compressor station (player) 2 | 0.47 | 0.23 | 0.3 |
| Compressor station (player) 3 | 0.72 | 0.15 | 0.13 |

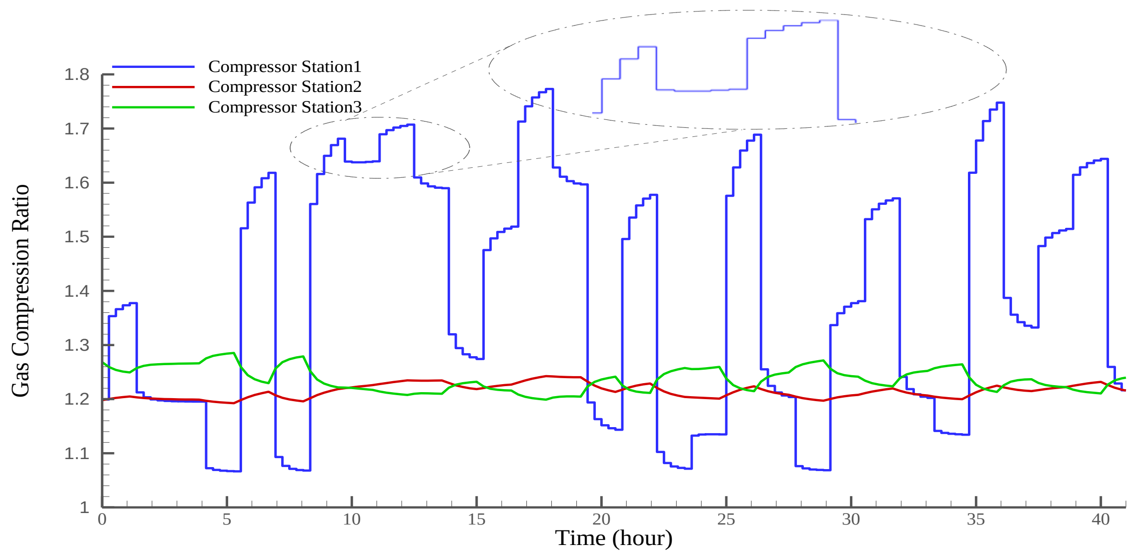



Fig. 7-a. Random inputs imposed by player 1.

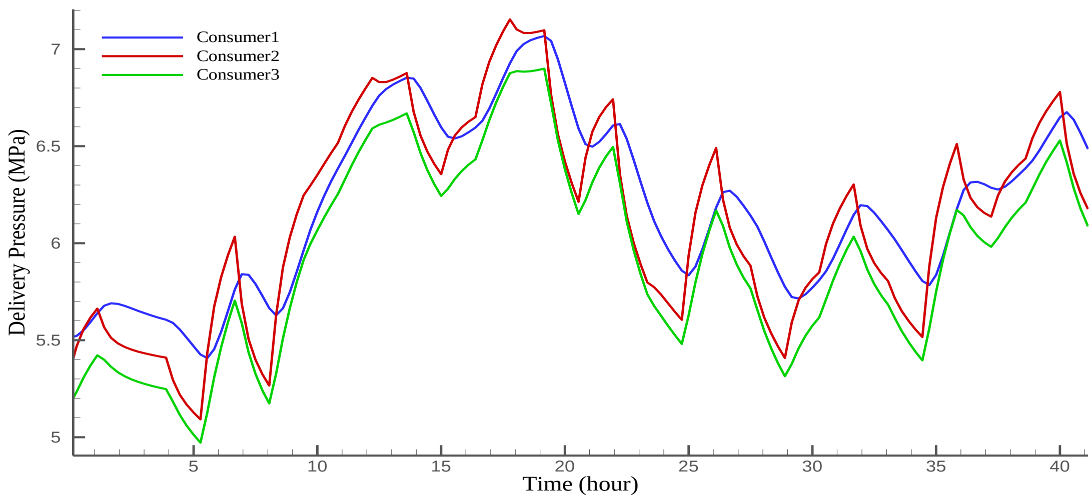



Fig. 7-b. Pressure change in consumers due to random inputs imposed by player 1.

Fig. 7 evidences all end-points pressure in consumers are strictly dependent on the decisions made by player 1. This dependency is also comprehendible from network topology in which there is a path between $CS_1$ and all consumers in the transmission network graph. This is also the case for other CSs.

Fig. 7-a, Fig. 8-a, and Fig. 9-a show the inputs to the network which are imposed from CSs. The input variables displayed in these figures are gas compression ratio in compressor stations which is common to show manipulating variables in CSs since they are often between 1 ~ 2. Therefore it is more convenient to show CSs' inputs by GCR rather than compressor power. Fig. 7-b, Fig. 8-b, and Fig. 9-b also represent the pressure in the consumption points due to GCR change in CSs.

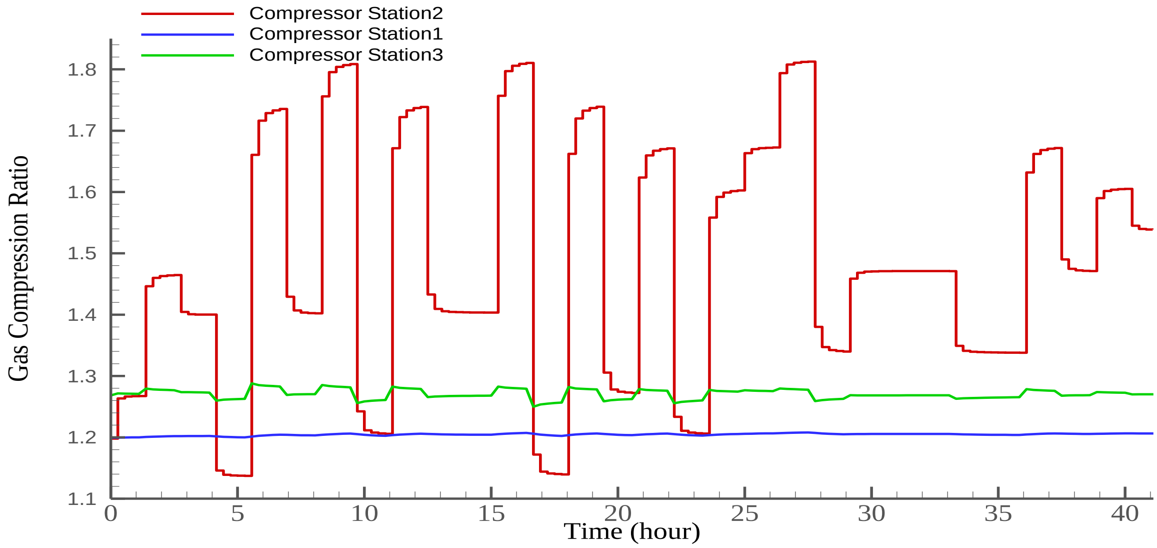



Fig. 8-a. Random inputs imposed by player 2.

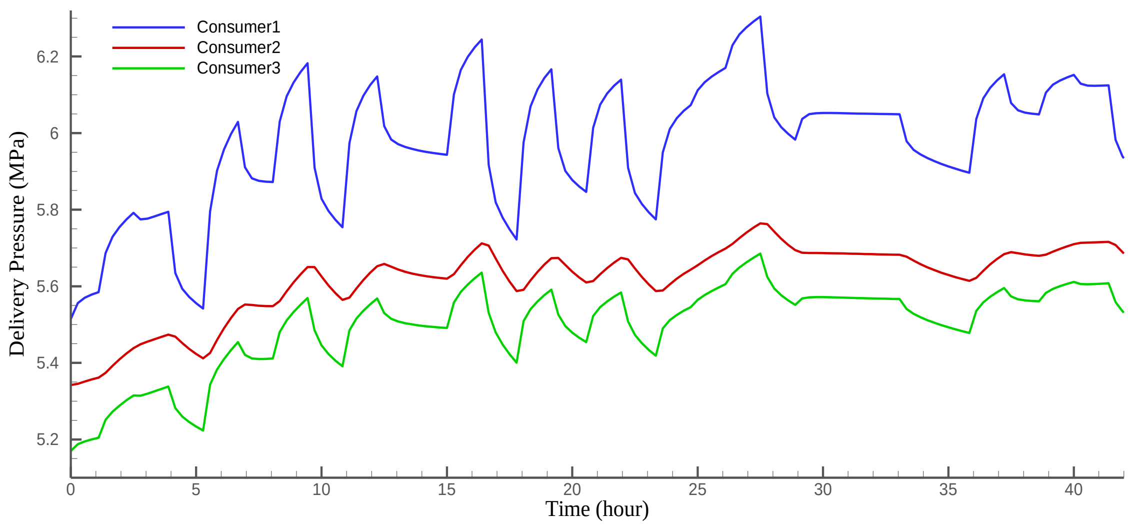
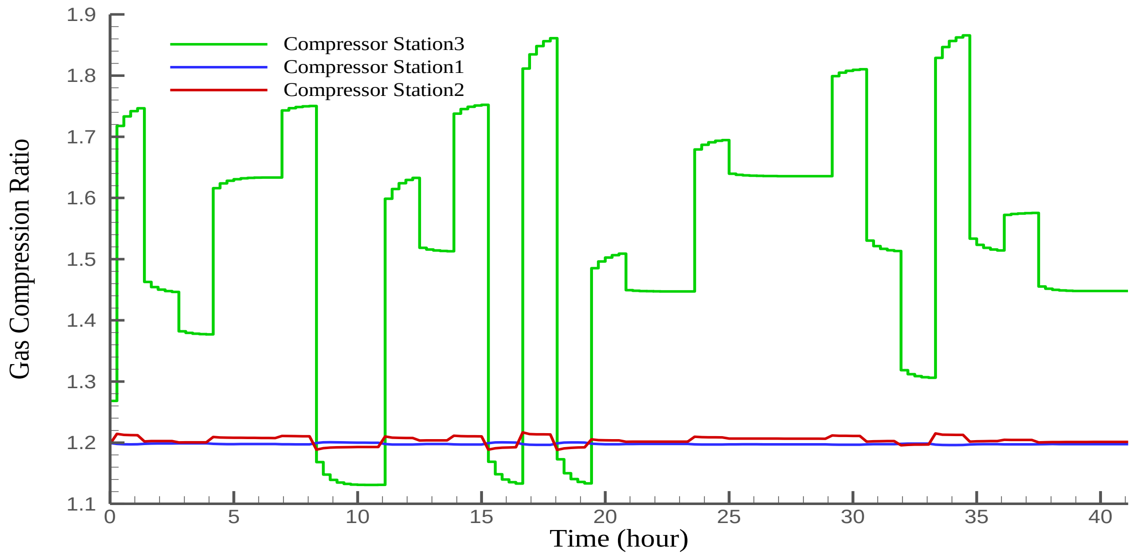



Fig. 8-b. Pressure change in consumers due to random inputs imposed by player 2.




` Fig. 9-a. Random inputs imposed by player 3

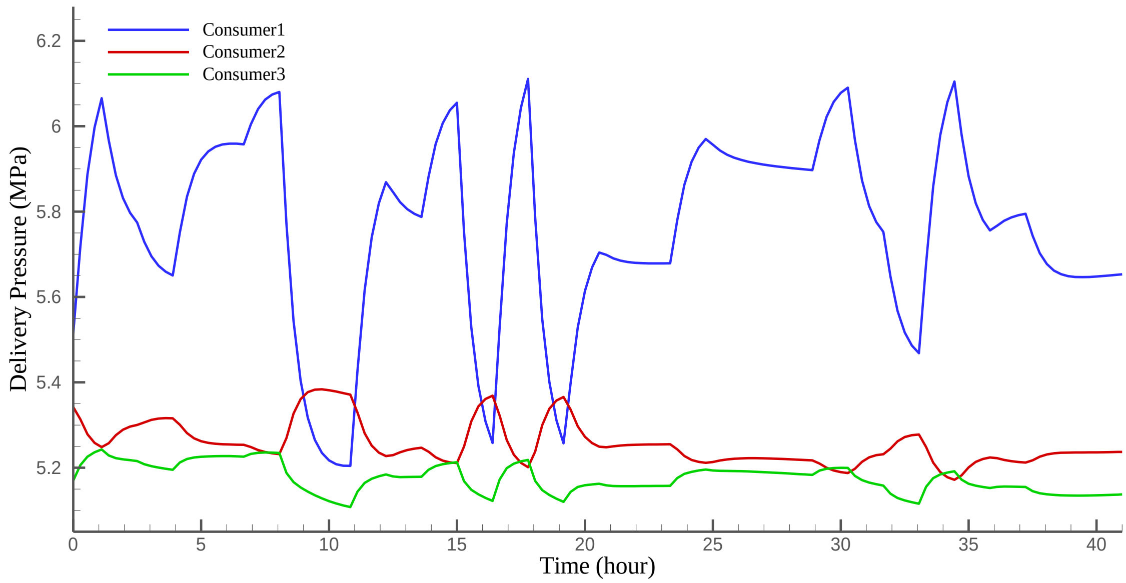



Fig. 9-b. Pressure change in consumers due to random inputs imposed by player 3.

### 4.2 Analysis of the proposed algorithm in different scenarios

To evaluate the performance of the proposed algorithm, we study two different scenarios in the operation of the gas transmission network. These scenarios correspond to consumption change, which is a typical load imposed on gas transmission networks. In the first scenario, the gas network is operating in its nominal condition, and the demand of delivery point having the largest gas consumption increases.

At t=10 hr, the gas demand of consumer 3 increases from 2.86 $m^3/sec$ to 3.60 $m^3/sec$ (~8.74 to ~11 MMCFD), which is about a 25% increase in its demand and is a high disturbance in nominal condition (Fig. 10). This event changes delivery pressure in consumer 3, and consequently effective-pressure and the utility for all players. Since players comprehend utility change in the game, they start the first stage for finding inputs (GCRs in their compressor stations) in which the network would settle in a safe condition.

The first stage of the game begins at t ~ 10 hr and ends at t ~ 24 hr at which players find appropriate actions, the choosing of which lead the network to a safe operation. At t=18 hr, players find the direction in which all utilities increase. In this regard, they keep their decisions to achieve the consensus in the first stage. At t ~ 24 hr, the consensus in the game occurs, and the first stage of the game is over. Afterward, players start the second stage of the game in which player 3 considers its effective-pressure and detects that it is higher from the US of the safe horizon. Considering the principles in the second stage of the game, player 3 starts the second stage (although in this condition, priority is for player 1 and then player 2, none of them find effective-pressure deviation from the safe horizon ). At t ~ 40 hr, the network is almost settled; however, the second stage of the game is not over yet, and at t ~ 70 hr, player 2 performs its action in the second stage, and the game is over (Fig. 11).

In the following, non of the players perform an action, and after about t ~ 100 hr, the gas network is at its new steady state. Fig. 12 and Fig. 13 display the delivery pressures in consumers and effective-pressure for the players during the game. Even though there is a deviation from the nominal condition in the delivery-points pressure, players successfully put their effective-pressure in the safe horizon.

Fig. 12 and Fig. 13 demonstrate that the proposed algorithm controls the transmission network in the presence of load and evidence in the condition that all supervisors of compressor stations adhere to the aforementioned collaboration, network is operated neither to need predictions obtained by any model nor does it require any centralized controller.

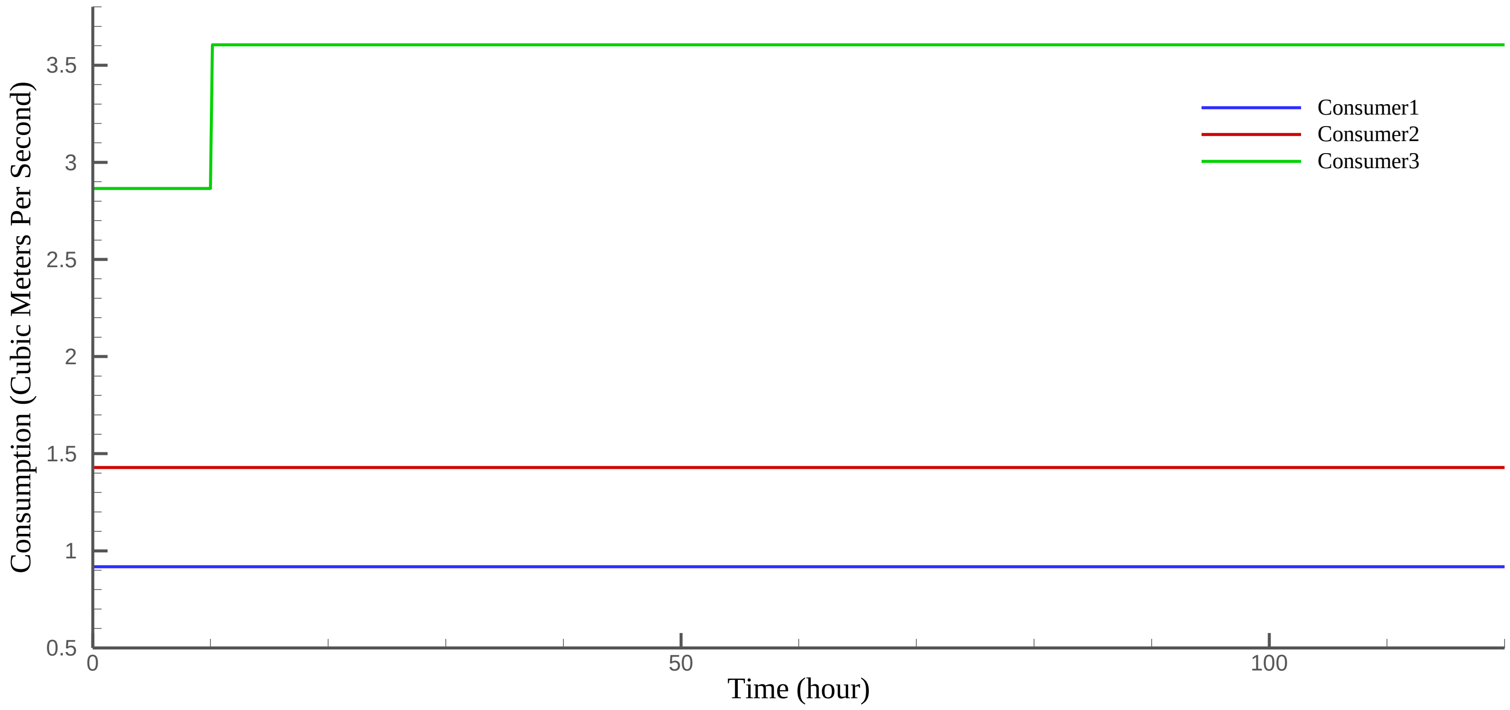



Fig. 10 natural gas consumption in deliveries in the first scenario

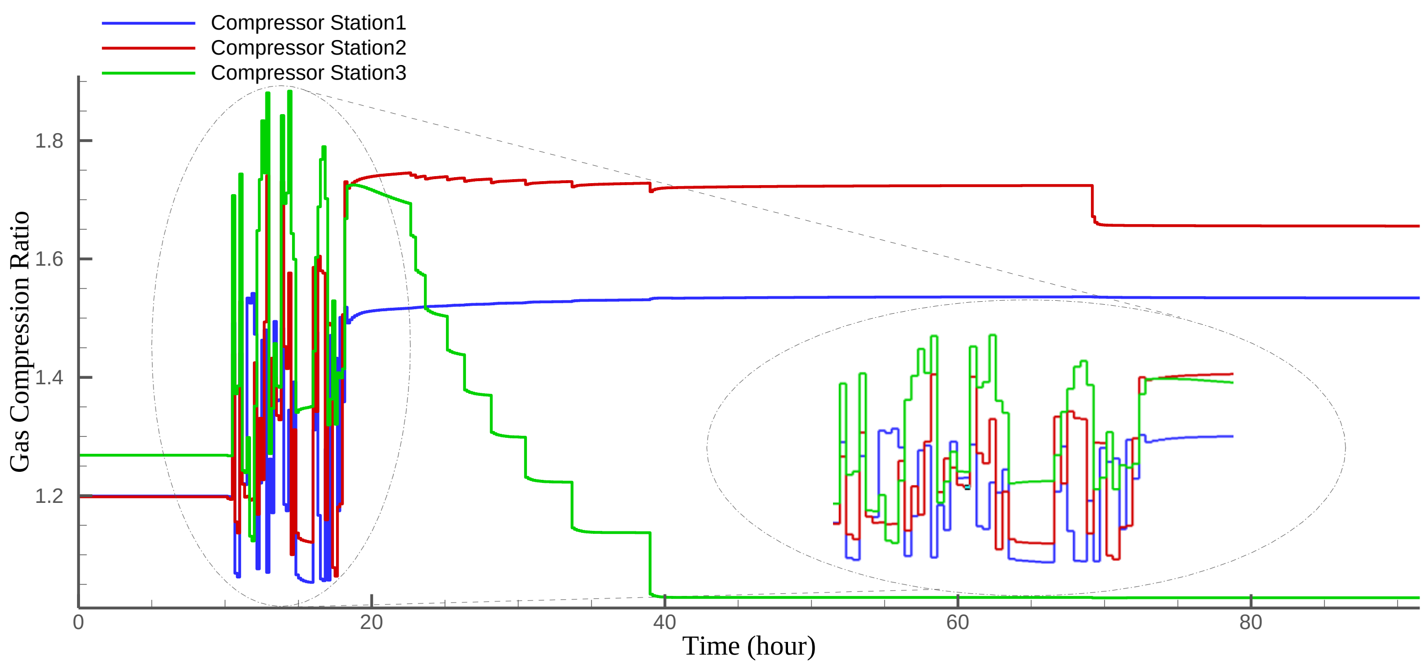



Fig. 11 gas compression ratio in compressor stations (decisions of players) in the first scenario during the game

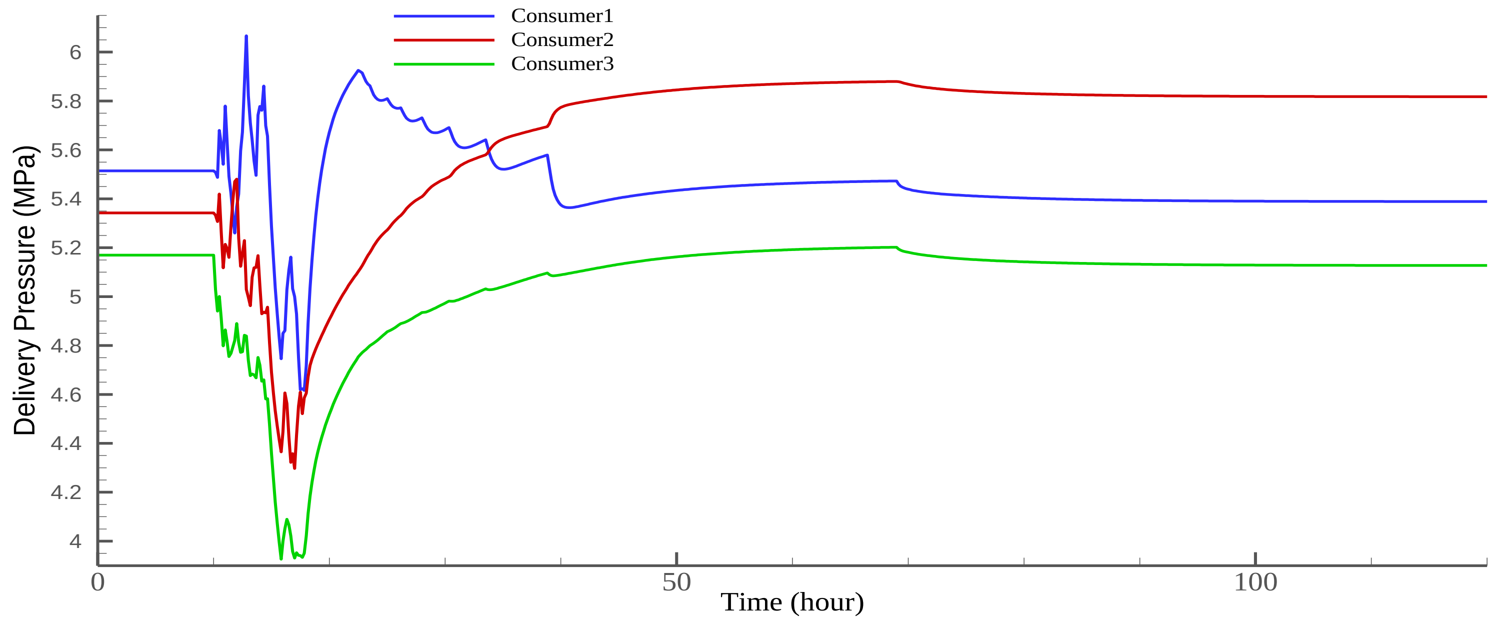



Fig. 12 delivery pressures in consumers in the first scenario during the game

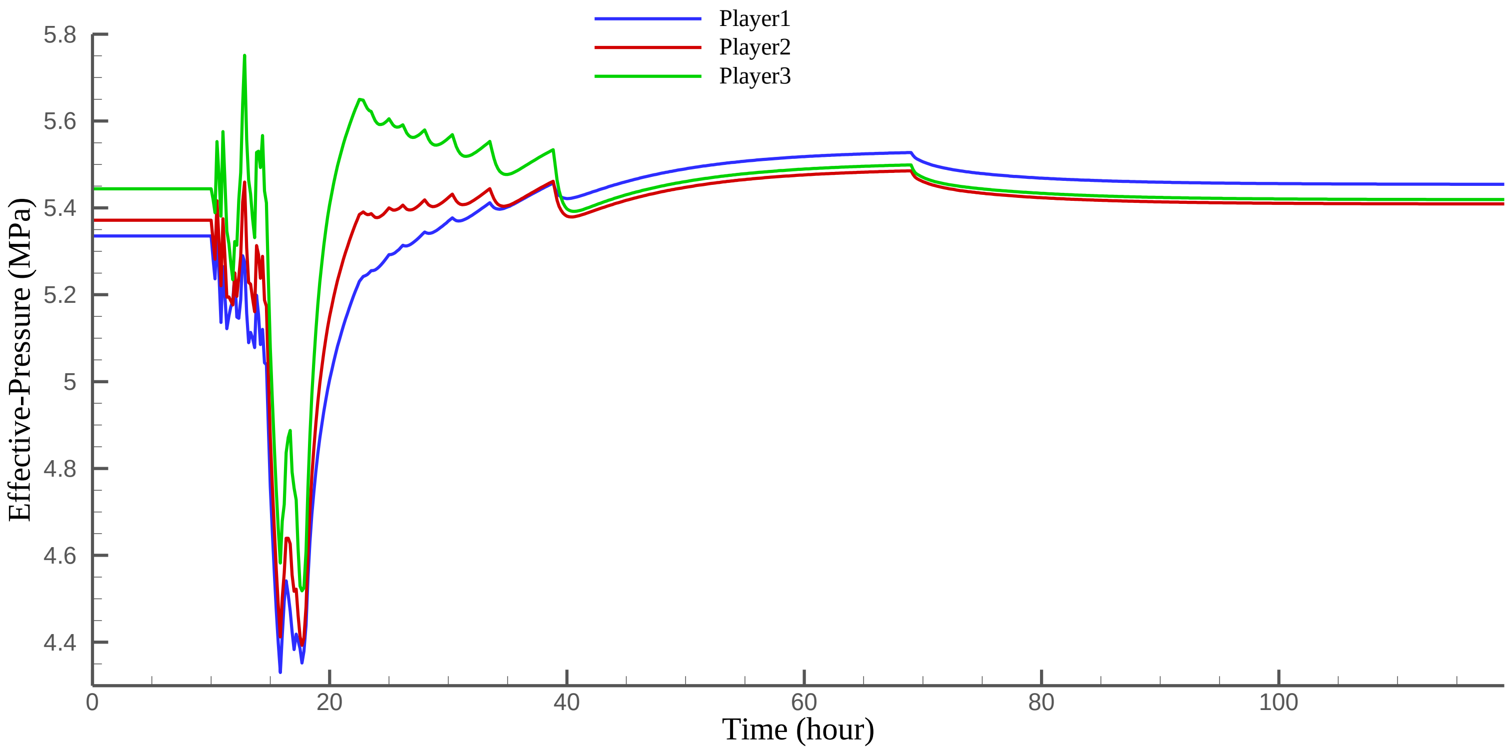



Fig. 13 effective-pressures for players in the first scenario during the game

The second scenario imposes more severe disturbance on the network. The network gas demand in two consumption points increases at t = 18 hr and t = 110 hr. The total increase in gas demand is about 14% which is an intense load in the transmission networks. The events that occur sequentially in the second scenario are as follows.

1. At t= 10 hr the gas demand in consumer 3 increases from 2.86 $m^3/sec$ to 3.27 $m^3/sec$ ( ~8.74 to ~10 MMCFD), which is about a 15% increase in its demand.
2. The consumption change in consumer 3 results in effective-pressure change and consequently utility, so the first stage of the game starts.
3. Players start to find the appropriate decisions in which unanimously all utilities increase during the Time. At t ~28 hr, this takes place and the network is in a safe and reliable steady-state condition.
4. Regarding there is no condition to galvanize the second stage of the game, all players adhere to their decisions and the game is over until other events stimulate players.
5. At t = 110 hr, consumer 1 increases its gas demand from 0.92 $m^3/sec$ to 1.31 $m^3/sec$ ( ~2.80 to ~ 4 MMCFD), which is about a 42% increase in gas consumption.
6. Considering players feel a change in their utilities, they start the first stage of the game and at t ~120 hr they find appropriate decisions in which all utilities increase.
7. This time also corresponds with a condition in which all effective-pressure are on the safe horizon, the game is over.

Fig. 14, Fig. 15, Fig. 16, and Fig. 17 display the gas consumption of consumers, players' decisions, delivery pressures in consumers, and effective-pressures for players respectively, in the second scenario, during the time and represent that all players strive to operate the gas network in a reliable condition despite the various load imposed on the network.

It is notable that due to the stochastic nature of the proposed algorithm, players do not necessarily behave in the same way for two identical perturbations. However, these behaviors lead the network to the almost same final condition that in addition to pressure delivery in consumers are reliable, the power of the compressor stations are almost minimized. This is interpretable from the rules governing the utility function in the first stage and the principles of the second stage.

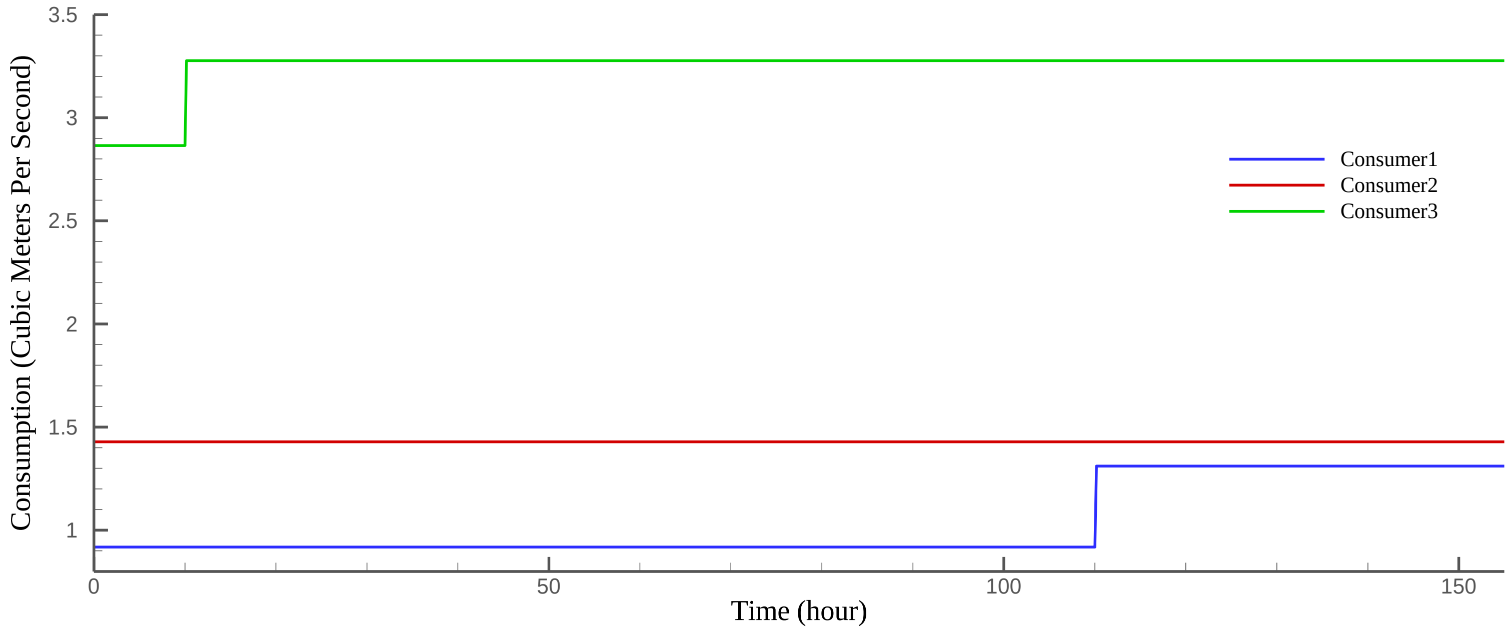



Fig. 14 natural gas consumption in deliveries in the second scenario

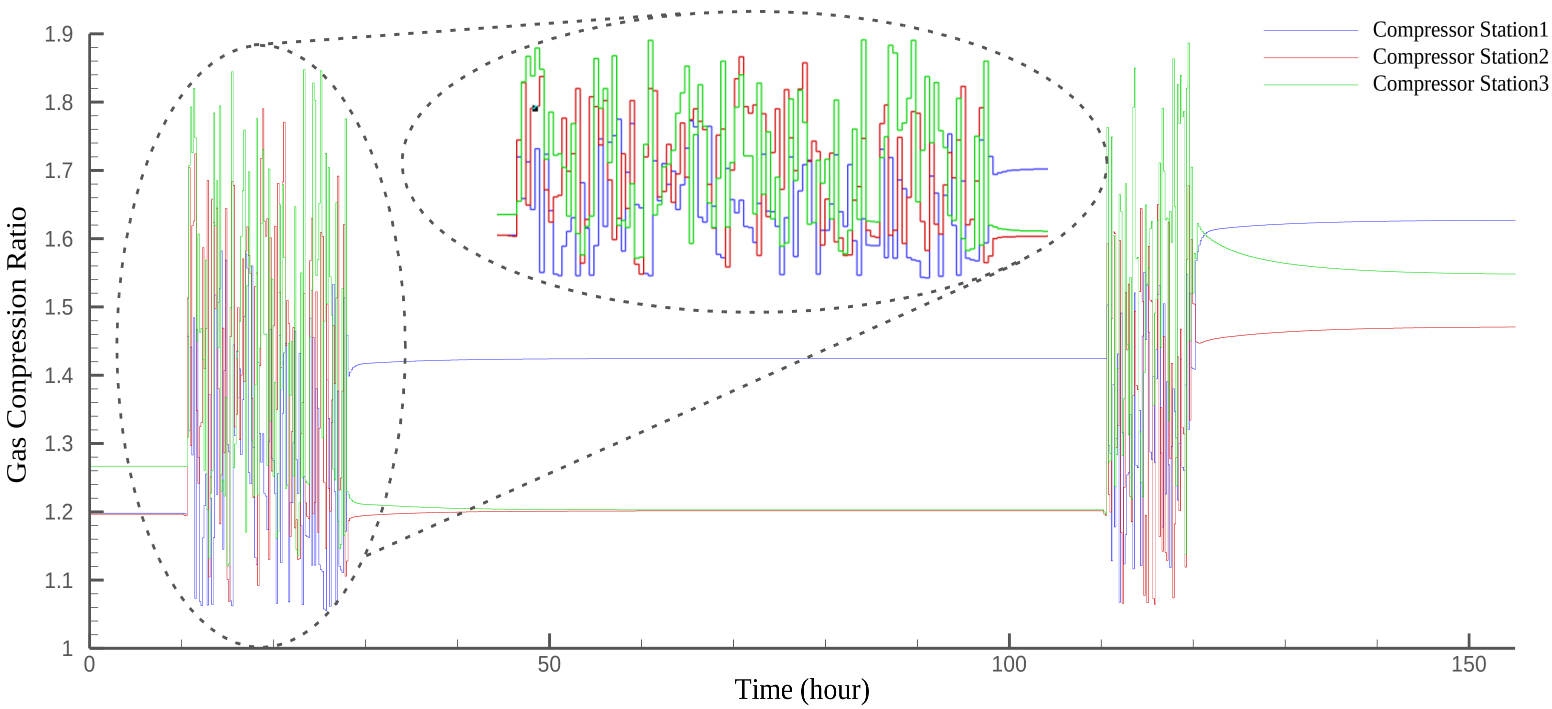



Fig. 15 gas compression ratio in compressor stations (decisions of players) in the second scenario during the game

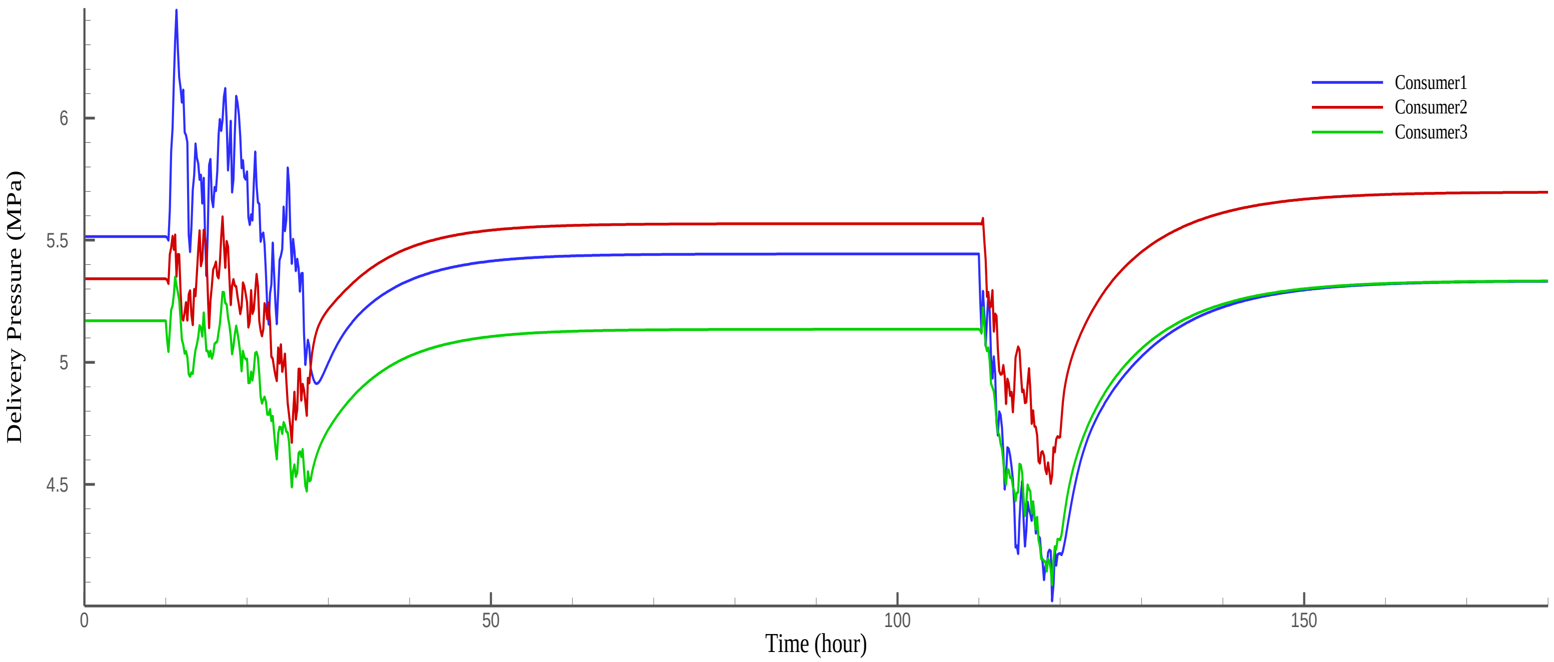



Fig. 16 delivery pressures in consumers in the second scenario during the game

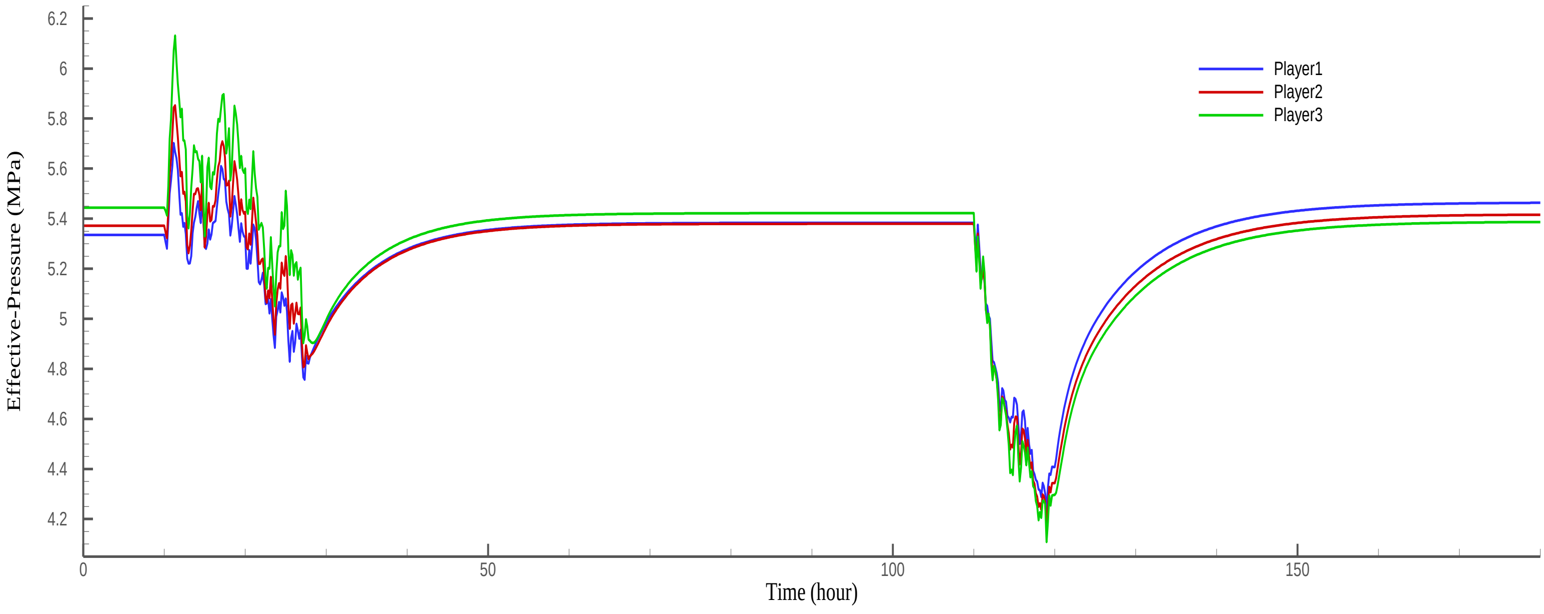



Fig. 17 effective-pressures for players in the second scenario during the game

## 5. Conclusion

In this work, we contextualize a novel two-stage cooperative algorithm in the area of game theory, which is a solution strategy for model-free and distributed control of systems whose main characteristics are their high settling time and/or time delay in their response (e.g., gas transmission networks). This paradigm utilizes the incomplete information game concept for collaborative learning in the operation of the transmission network whose control structure is reformulated as a cooperative game. In this research, a Fuzzy Inference System has been designed and implemented to emulate players' rationality, which is consistent with the nature of qualitative thinking of human supervisors in compressor stations of gas transmission networks. The implementation of the new approach on a cyclic and large-scale gas transmission network manifests that in the presence of an intense change in network demand or under uncertain loads, supervisors find appropriate setpoints in which the network settles downs to a safe horizon, and all consumers receive natural gas in a permissible range of pressure. Reconsidering their decisions concerning effective-pressure, players also promote the efficiency of network operation. From the game theory perspective, this research contemplates games without exact utility evaluation which opens a new line of thinking versus conventional game-theoretic algorithms in which exact utility evaluation is a substantial assumption. The study also ushers in the utilization of game theory notion in the control of interconnected systems with slow dynamics whose exact model which can be used in controller design is unaffordable if not impossible to obtain.

Future studies include the development of a utility function based on the combined FIS and data-driven approaches (e.g., Adaptive Neuro-Fuzzy Inference System) to be more accommodated with the rationality in human supervisors. Addiotnaly other types of game theoretic-based algorithms such as the competitive approaches may be investigated to market gas demand analysis in transmission networks.

**Appendix A.**

To design the utility function as a mathematical formalism, precise rules are required. However, there are many descriptions in the real world which are difficult to be defined clearly[25]. The preference of humans supervising CSs in the gas transmission network is a notable example of such descriptions. The vague human assessments can be represented by a Fuzzy Logic System.

Fuzzy Systems use Fuzzy set theory to process data. Fuzzy set theory is a mathematical expression in which everything is a matter of degree. A Fuzzy Inference System includes Fuzzification, Implication, Aggregation, and Defuzzification. The t-norm and s-norm which are used in the designed Fuzzy utility function are Zadeh standard minimum and Zadeh standard maximum respectively. The method which is used for Defuzzification is based on calculating the centroid of the area[26],[27]. The membership functions in the fuzzy utility which are represented in Fig. 2. are Gaussian or Sigmoid. The Gaussian function is a function that is widely used in Fuzzy set theory to express a degree of membership of a value to a particular set:

$$P(X) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2} \qquad (9)$$

P is a value that denotes the degree of membership to a particular set and X is an independent variable. $\sigma$ is the standard deviation and $\mu$ is the mean of the Gaussian function.

The Sigmoid function is also generally used to indicate the degree of membership in the beginning and end of the variable span:

$$P(X) = \frac{1}{1+ e^{-a*(X-C)}} \qquad (10)$$

The $a$ and C are constant values in the Sigmoid function.

Considering these two types of functions the Fuzzy utility is designed based on the parameters represented in tables 5, 6, 7, and 8.

**Table 5**
The function's parameters in Effective-Pressure for Observer (Input 1 to FIS)

| Name of Set | Type of Membership | Parameters |
|---|---|---|
| Critical | Sigmoid | $a$ = -30 , C = 0.2 |
| Dangerous | Gaussian | $\sigma$ = 0.08 , $\mu$ = 0.3 |
| Normal | Gaussian | $\sigma$ = 0.085, $\mu$ = 0.5 |
| Safe | Gaussian | $\sigma$ = 0.08 , $\mu$ = 0.7 |
| Absolutely Reliable | Sigmoid | $a$ = +30 , C =0.8 |

**Table 6**
The function's parameters in Instant Power (Input 2 to FIS)

| Name of Set | Type of Membership | Parameters |
|---|---|---|
| Very Low | Sigmoid | $a$ = -30 , C = 0.2 |
| Low | Gaussian | $\sigma$ = 0.08 , $\mu$ = 0.3 |
| Normal | Gaussian | $\sigma$ = 0.085, $\mu$ = 0.5 |
| High | Gaussian | $\sigma$ = 0.08 , $\mu$ = 0.7 |
| Very High | Sigmoid | $a$ = +30 , C =0.8 |

**Table 7**
The function's parameters in Average Power (Input 3 to FIS)

| Name of Set | Type of Membership | Parameters |
|---|---|---|
| Low | Sigmoid | $a$ = -16 , C = 0.35 |
| Normal | Gaussian | $\sigma$ = 0.15, $\mu$ = 0.5 |
| High | Sigmoid | $a$ = 16 , C = 0.35 |

**Table 8**
The function's parameters in Calculated Utility for Players (Output from FIS)

| Name of Set | Type of Membership | Parameters |
|---|---|---|
| Very Very Low | Sigmoid | $a$ = -50 , C = 0.13 |
| Very Low | Gaussian | $\sigma$ = 0.065 , $\mu$ = 0.2 |
| Low | Gaussian | $\sigma$ = 0.065 , $\mu$ = 0.35 |
| Normal | Gaussian | $\sigma$ = 0.065, $\mu$ = 0.5 |
| High | Gaussian | $\sigma$ = 0.065 , $\mu$ = 0.65 |
| Very High | Gaussian | $\sigma$ = 0.065 , $\mu$ = 0.8 |
| Very Very High | Sigmoid | $a$ = 50 , C = 0.87 |

## Reference


[1] Anual Energy Outlook 2018 from eia. available at https://www.eia.gov.

[2] Z. Taheri, M. R. Shabani, K. Nazari, and A. Mehdizaheh, "Natural gas transportation and storage by hydrate technology: Iran case study," *Journal of Natural Gas Science and Engineering,* vol. 21, pp. 846-849, 2014.

[3] E. S. Menon, *Gas pipeline hydraulics*. Crc Press, 2005.

[4] P. D. Christofides, R. Scattolini, D. M. de la Pena, and J. Liu, "Distributed model predictive control: A tutorial review and future research directions," *Computers & Chemical Engineering,* vol. 51, pp. 21-41, 2013.

[5] R. Z. Ríos-Mercado and C. Borraz-Sánchez, "Optimization problems in natural gas transportation systems: A state-of-the-art review," *Applied Energy,* vol. 147, pp. 536-555, 2015.

[6] H. A. Behrooz and R. B. Boozarjomehry, "Dynamic optimization of natural gas networks under customer demand uncertainties," *Energy,* vol. 134, pp. 968-983, 2017.

[7] A. Zlotnik, M. Chertkov, and S. Backhaus, "Optimal control of transient flow in natural gas networks," in *2015 54th IEEE conference on decision and control (CDC)*, 2015, pp. 4563-4570: IEEE.

[8] T. W. Mak, P. V. Hentenryck, A. Zlotnik, and R. Bent, "Dynamic compressor optimization in natural gas pipeline systems," *INFORMS Journal on Computing,* vol. 31, no. 1, pp. 40-65, 2019.

[9] D. Aßmann, F. Liers, and M. Stingl, "Decomposable robust two-stage optimization: An application to gas network operations under uncertainty," *Networks,* vol. 74, no. 1, pp. 40-61, 2019.

[10] D. Bauso, *Game theory with engineering applications*. SIAM, 2016.

[11] R. Tang, H. Li, and S. Wang, "A game theory-based decentralized control strategy for power demand management of building cluster using thermal mass and energy storage," *Applied Energy,* vol. 242, pp. 809-820, 2019.

[12] J. Mei, C. Chen, J. Wang, and J. L. Kirtley, "Coalitional game theory based local power exchange algorithm for networked microgrids," *Applied energy,* vol. 239, pp. 133-141, 2019.

[13] J. R. Marden, S. D. Ruben, and L. Y. Pao, "A model-free approach to wind farm control using game theoretic methods," *IEEE Transactions on Control Systems Technology,* vol. 21, no. 4, pp. 1207-1214, 2013.

[14] J. R. Marden, H. P. Young, G. Arslan, and J. S. Shamma, "Payoff-based dynamics for multiplayer weakly acyclic games," *SIAM Journal on Control and Optimization,* vol. 48, no. 1, pp. 373-396, 2009.

[15] J. R. Marden, H. P. Young, and L. Y. Pao, "Achieving pareto optimality through distributed learning," *SIAM Journal on Control and Optimization,* vol. 52, no. 5, pp. 2753-2770, 2014.

[16] J. R. Marden, G. Arslan, and J. S. Shamma, "Cooperative control and potential games," *IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics),* vol. 39, no. 6, pp. 1393-1407, 2009.

[17] E. Ben-Porath, "Game Theory, Michael Maschler, Eilon Solan, Shmuel Zamir, Cambridge University Press (2013)," ed: Elsevier, 2014.

[18] J. R. Marden and J. S. Shamma, "Game theory and control," *Annual Review of Control, Robotics, and Autonomous Systems,* vol. 1, pp. 105-134, 2018.

[19] S. Skogestad, "Control structure design for complete chemical plants," *Computers & Chemical Engineering,* vol. 28, no. 1-2, pp. 219-234, 2004.

[20] J. R. Marden and J. S. Shamma, "Game theory and distributed control," in *Handbook of game theory with economic applications*, vol. 4: Elsevier, 2015, pp. 861-899.

[21] J. R. Marden and A. Wierman, "Overcoming the limitations of utility design for multiagent systems," *IEEE Transactions on Automatic Control,* vol. 58, no. 6, pp. 1402-1415, 2013.

[22] P. De Wilde, "Fuzzy utility and equilibria," *IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics),* vol. 34, no. 4, pp. 1774-1785, 2004.

[23] S. Lasaulce and H. Tembine, *Game theory and learning for wireless networks: fundamentals and applications*. Academic Press, 2011.

[24] S. Ross and B. Chaib-draa, "Satisfaction equilibrium: Achieving cooperation in incomplete information games," in *Conference of the Canadian Society for Computational Studies of Intelligence*, 2006, pp. 61-72: Springer.

[25] Y. Han, Q. Zeng, Z. Geng, and Q. Zhu, "Energy management and optimization modeling based on a novel fuzzy extreme learning machine: Case study of complex petrochemical industries," *Energy conversion and management,* vol. 165, pp. 163-171, 2018.

[26] H.-J. Zimmermann, *Fuzzy set theory—and its applications*. Springer Science & Business Media, 2011.

[27] T. J. Ross, *Fuzzy logic with engineering applications*. John Wiley & Sons, 2005.