

Simultaneous Aircraft Sizing and Multi-Objective Optimization Considering Off-Design Mission Performance during Early Design

Abstract

Traditional aircraft conceptual design primarily involves determination of top-level sizing parameters, resulting in an initial design which satisfies specified point-performance constraints while flying the so-called design mission. In practical scenarios, commercial aircraft are also expected to operate optimally in the actual missions they fly which may drastically differ from the design mission. To improve performance and reduce operating costs, optimization shall be performed using specific objectives from the on-design mission and from one or more representative off-design reference mission(s). Such multi-mission optimizations may result in different designs for the same performance constraints. Moreover, the size of aircraft and choice of reference mission(s) may also have an effect in the difference resulting from such multi-mission optimizations. This paper solves a series of multi-objective on-design and multi-mission optimization problems in the conceptual design phase on aircraft spanning a range of size-classes, using the Non-dominated Sorting Genetic Algorithm II (NSGA-II). Results shed light on the design differences between formulations that exclusively consider design mission metrics of interest from the ones that consider metrics of interest from disparate, as-flown, i.e., off-design missions. In addition, results also reveal the impact of off-design mission weightings on the designs obtained from the optimization problems.

Keywords: Conceptual Design, Commercial Aircraft Sizing, Off-design Optimization, Multi-objective Optimization, Payload-range Characteristics

1. Introduction

The primary goal of aircraft sizing in conceptual design phase is to use point performance and mission performance requirements to obtain an initial estimate of the geometric scale, the propulsion characteristics, and the design gross weight, which are typically represented by the wing planform area, the rated sea-level static thrust, and the maximum takeoff weight, (MTOW), respectively. Traditionally, this process yields a solution which satisfies the performance requirements for the so-called design mission. Due to airlines' concerns with regard to operating and maintenance costs and environmental factors such as noise and emissions, the design is optimized for objectives such as MTOW, operating empty weight (OEW), takeoff field length (TOFL), noise, and/or fuel consumption of the mission for which the aircraft is sized, i.e., the design mission.

While commercial aircraft are sized to fly the design mission, in real-world operations, the aircraft operate a variety of off-design missions with diverse payload and range combinations. In some cases, airlines sacrifice payload capacity in order to fly certain ultra-long-range missions. For example, Singapore Airlines used to operate non-stop flights between Singapore and New York at a great circle distance of 8300 nmi using the Airbus A340-500 aircraft in a special configuration carrying only 100 passengers instead of the typical 293-seat configuration [1]. In other cases, many airlines operate long-range aircraft on much shorter routes due to airport slot constraints, capacity demand, and/or fleet utility, etc. For example, the long-range Airbus A350, A380, Boeing 787, and 777 are frequently used on short-haul high-capacity flights within Asia.

From the airlines' perspective, design optimization considering off-design performance offers the prospect of high mission flexibility along with the promise of operating the actual missions as efficiently as possible. This can be achieved by choosing off-design metrics such as specific air range (SAR) instead of the design mission performance such as the block fuel (BF). However, one, such formulations may not shed light on whether and if so, how the choice of ob-

31 jective mission performance metric impacts the payload-range capability and
32 performance of other off-design missions under identical point performance and
33 design mission requirements; two, the distribution of off-design missions relative
34 to the design mission in the payload-range space may also affect the trade-off
35 (if present) mentioned above.

36 The primary goal of this work is to investigate the aforementioned issues
37 through several multi-mission multi-objective sizing and optimization studies
38 to assess the differences in top-level aircraft design parameters and off-design
39 mission performance metrics among the Pareto optimal designs across a few size-
40 classes, specifically, a Regional Jet (RJ), a Small Single-aisle Aircraft (SSA), and
41 a Large Twin-aisle Aircraft (LTA).

42 Researchers have attempted off-design optimization using a single-objective
43 or “a-priori” multi-objective optimization problem formulations [2, 3, 4, 5, 6],
44 where multiple objectives were aggregated into a single objective function by
45 defining a “utility function” or “value function” instead of investigating a set
46 of Pareto optimal solutions. A-posteriori multi-objective optimization problem
47 formulations have also been leveraged [7, 8, 9, 10] to obtain Pareto frontiers
48 to investigate the trade-off between aircraft gross weight, design mission fuel
49 burn, noise, and emissions, etc., but these studies do not explicitly factor-in
50 off-design considerations. This work, on the other hand, highlights the insights
51 gained from a multi-mission a-posteriori multi-objective optimization performed
52 on both on-design and off-design mission performance metrics.

53 The appropriate size of an aircraft’s design parametrization is a largely ne-
54 glected issue in aircraft conceptual design and optimization studies. Typically,
55 the extent of parametrization is constrained by both the lack of information
56 and the semi-empirical nature of the component weight build-up, aerodynam-
57 ics, and propulsion models. Notable studies [3, 4, 5, 6, 8, 9, 10] on aircraft
58 sizing and optimization decide the number of parameters describing an aircraft
59 based on either a widely accepted set of significant inputs and subject matter
60 expertise, or by considering all the variables availed by their respective analysis
61 tools. It is a well-recognized and widely accepted fact that a majority of the

62 metrics of interest in conceptual design are largely affected by a few key scaling
63 factors such as the geometric and the propulsive scale, i.e., the wing loading
64 and thrust loading respectively. However, other geometric aircraft parameters
65 cannot simply be neglected because they may have significant secondary effects
66 on the metrics of interest through their appearance in equations for compo-
67 nent weight build-ups which affect the weight, and therefore, the aerodynamics
68 and propulsion characteristics. On the other hand, considering all the possible
69 parameters exposed by respective analyses may introduce variables that affect
70 the outputs insignificantly. Consequently, including such variables leads to rela-
71 tively complex and hard-to-solve optimization problem formulations while only
72 adding marginal value. Moreover, the effect of input variables on the metrics of
73 interest also depends on the fidelity of the underlying analysis. To address the
74 aforementioned issues, this work proposes beginning with the full set of relevant
75 inputs and down-selecting the most important ones for the specific metrics of
76 interest by rank-ordering them based on the significance of their effect on the
77 outputs. Such a screening procedure ensures a parsimonious optimization prob-
78 lem while ensuring that a majority of the variation in the outputs is retained.

79 The primary goal of this work is accomplished by proposing and leveraging
80 a methodology that: one, formulates and compares the insights from on-design
81 and off-design optimization problems. A practically meaningful off-design opti-
82 mization problem is constructed by performing a thorough assessment of several
83 off-design objective functions and identifying conflicting ones; two, provides a
84 principled method to construct a parsimonious optimization problem by down-
85 selecting the most significant inputs via sensitivity analysis, considering an ex-
86 haustive set of conceptual phase aircraft design parameters. The developed
87 methodology is design tool agnostic and is easily extensible to a wide range of
88 aircraft concepts and mission types. The salient novel features of the study
89 are: 1) both on-design and off-design quantities of interest are considered in the
90 a-posteriori multi-objective optimization setting, 2) a thorough study, with a
91 large set of design variables, is performed across major aircraft size classes to
92 give an insight into the trade-offs involved within them with regard to on-design

93 and off-design considerations.

94 The remainder of this paper is organized as follows: Sec. 2 presents the
95 formulation of both the on-design and off-design optimization problems; Sec. 3
96 discusses the methods used in major disciplinary analyses performed in concep-
97 tual aircraft design; Sec. 4 describes the mission profile and the tool to perform
98 off-design mission performance evaluation; Sec. 5 briefly describes the char-
99 acteristics of NSGA-II, the optimization algorithm used in this paper; Sec. 6
100 presents the optimization results along with discussions; finally, Sec. 7 draws
101 the conclusion.

102 **2. Problem Formulation**

103 This section describes the objectives considered for the on-design (single-
104 mission) optimization and the off-design (multi-mission) optimization in Sec. 2.1,
105 the constraints considered for both optimization problems in Sec. 2.2, the set
106 of aircraft-level design variables in Sec. 2.3, the necessity for down-selecting the
107 design variables through screening in Sec. 2.4, the complete problem statement
108 in Sec. 2.5, and finally, the reference vehicles' data and the corresponding design
109 variable ranges in Sec. 2.6.

110 *2.1. Objective Functions*

111 The problem of optimizing the performance of an aircraft for multiple mis-
112 sions simultaneously while sizing the aircraft for a single design mission is nat-
113 urally posed as a multi-objective optimization problem. In the literature con-
114 cerning aircraft design optimization at the conceptual stage, researchers have
115 considered a number of objective functions motivated mainly by the intended
116 objectives of the specific study [8, 11]. Because the goal of this work is to
117 compare differences in designs when on-design and off-design metrics are taken
118 into account together, formulations with a mix of mission-invariant and mission-
119 dependent metrics are considered. The mission-invariant metrics imply the ca-
120 pability of a sized aircraft, regardless of the actual mission being operated,

121 whereas the mission-dependent metrics measure the performance of a sized air-
 122 craft which depends on the actual mission being flown.

123 For the on-design optimization, two mission-invariant objectives are consid-
 124 ered: 1) the maximum ramp weight, MRW, and 2) the nominal takeoff field
 125 length, TOFL, computed at standard sea level conditions and at maximum
 126 takeoff weight. A third objective for the on-design optimization is the block
 127 fuel of the design mission, $W_{BF,des}$, which is a good measurement of the direct
 128 operating cost when the aircraft flies the design mission.

129 For the off-design optimization, the on-design objectives to minimize MRW
 130 and TOFL remain. A third objective to be minimized is selected among the
 131 following three candidate functions involving mission-dependent variables, i.e.
 132 functions of payload (W_P) and range (R):

$$O_1 = \iint_M f(W_P, R) W_{BF}(W_P, R) dW_P dR \quad (1)$$

$$O_2 = - \iint_M f(W_P, R) \frac{R}{W_{BF}(W_P, R)} dW_P dR \quad (2)$$

$$O_3 = - \iint_M f(W_P, R) \frac{W_P R}{W_{BF}(W_P, R)} dW_P dR \quad (3)$$

133 where M represents the feasible mission space as dictated by a domain bounded
 134 by the maximum takeoff weight, maximum fuel capacity, and maximum pay-
 135 load weight constraints in mission range and payload space for a sized vehicle;
 136 $f(W_P, R)$ is a weighting function for the off-design missions, and to serve its
 137 purpose, this work chooses the mission frequency distribution function, i.e. nor-
 138 malized frequency or expected probability density that an aircraft of a single
 139 type will operate the mission specified by a given payload and range combi-
 140 nation; finally, W_{BF} is the mission-dependent block fuel which is assumed to
 141 depend solely on payload and still-air range for a sized aircraft. Before pro-
 142 ceeding, listed below are some noteworthy points regarding the three candidate
 143 off-design objectives mentioned above:

- 144 • In O_1 , the mission block fuel W_{BF} directly relates to fuel cost, which is
 145 the most significant contributor to the operating cost

- 146 • In O_2 , the term R/W_{BF} is the mission-averaged specific air range, an
147 indicator of fuel efficiency for a given mission
- 148 • In O_3 , the term $W_P R$ is a good representative for airline revenue when
149 carrying the payload of W_P over a distance of R ; the fraction $W_P R/W_{BF}$
150 measures the flight productivity for a given mission

151 From the operators' perspective, the flight productivity is of most interest
152 among the three discussed, simply because it meaningfully combines payload,
153 range, and fuel consumption simultaneously. Similar to O_3 , Hileman et al. [12]
154 proposed a metric named payload fuel energy efficiency (PFEE) as a measure
155 of flight productivity. For a single mission, PFEE is defined as [12]

$$\text{PFEE} = \frac{\text{Total payload carried} \times \text{Great-circle distance}}{\text{Fuel energy consumed}} \quad (4)$$

156 This work adopts the original definition of PFEE and modifies it based on the
157 goal of the optimization problem in question. For the on-design optimization,
158 minimizing the block fuel of the design mission is equivalent to minimizing the
159 negative on-design PFEE, defined as

$$-\text{PFEE}_0 = -\frac{W_{P,des} R_{des}}{W_{BF,des} q_{fuel}/g} \quad (5)$$

160 where g is the gravitational acceleration and $q_{fuel} = 42.80$ MJ/kg is the specific
161 energy of Jet-A fuel [13], which essentially serves as a normalization factor
162 to non-dimensionalize the metric. For the off-design optimization, the third
163 objective to minimize is the negative average off-design PFEE, defined as

$$-\text{PFEE}_1 = -\iint_M f(W_P, R) \frac{W_P R}{W_{BF}(W_P, R) q_{fuel}/g} dW_P dR \quad (6)$$

164 Besides the flight productivity implied by PFEE_1 , another metric to consider
165 in the off-design scenario is the mission coverage index (MCI), defined as the
166 cumulative mission frequency or mission probability density within the payload-
167 range envelope of a sized vehicle. MCI must be regarded as an important metric

168 since potential aircraft customers may have specific requirements on the pay-
 169 load and range capability of the aircraft. Note that large MCI values imply that
 170 the aircraft can optimally operate for larger number of payload-range combina-
 171 tions within the envelope for a prescribed mission frequency distribution. In
 172 certain cases, a higher MCI can only be achievable by up-sizing the aircraft, i.e.
 173 increasing the MRW and/or design fuel capacity, which is accompanied by an
 174 increase in the operating empty weight, an overall increase in fuel consumption
 175 within the payload-range envelope, and a decrease in PFEE_1 . Therefore, MCI
 176 and PFEE_1 are conflicting in nature, implying the existence of a Pareto frontier
 177 when optimized simultaneously. In this work, the negative MCI is included as
 178 the fourth and last objective to be minimized for the off-design optimization, as
 179 shown in Eq. (7):

$$-\text{MCI} = - \iint_M f(W_P, R) dW_P dR \quad (7)$$

180 2.2. Constraint Functions

181 The optimization problem statement is subject to performance, regulatory,
 182 and operational constraints, each of which is cast as a nonlinear inequality
 183 constraint. A total of five constraints are considered in this work: the positive
 184 second segment climb thrust (SSFOR) and positive missed approach gradient
 185 thrust (AMFOR) constraints [14] place lower bounds on permissible thrust-to-
 186 weight ratio; the upper-bound on permissible landing approach speed (VAPP)
 187 constrains the maximum feasible wing loading; the upper-bound on wingspan
 188 (SPAN) constrains the feasible wing aspect ratio given a wing area; and the
 189 upper-bound on vertical tail height (VTH) constrains the feasible vertical tail
 190 aspect ratio for a given vertical tail area, which is governed by the wing geometry
 191 through constant vertical tail volume coefficient. Finally, the bounds for VAPP,
 192 SPAN, and VTH are determined based on the specifications of FAA Aircraft
 193 Design Category [15].

194 *2.3. Design Variables and Bounds*

195 As mentioned in Sec. 1, any trade-off in terms of optimality is expected to
196 be revealed by using a combination of both on-design and off-design metrics.
197 Accessibility to designers and an ability to significantly impact the aerodynam-
198 ics, propulsion, and weights disciplines motivate the choice of design variables
199 in the conceptual phase.

200 The thrust-to-weight ratio (TWR) and the wing loading (WSR) are scal-
201 ing parameters that affect top-level aircraft point performance such as takeoff
202 field length, approach speed, rate of climb, etc. [16] The wing and tailplane
203 geometries are expected to have significant impact on component weights and
204 aerodynamic characteristics, therefore affecting the vehicle gross weight, per-
205 formance, and fuel consumption. Consequently, this work considers the aspect
206 ratio, taper ratio, average thickness-to-chord ratio, and quarter-chord sweep an-
207 gle of the wing, the horizontal tail, and the vertical tail as the geometric design
208 variables. To maintain invariant stability and control characteristics when siz-
209 ing the vehicle for different design variables, the tail volume coefficients are held
210 constant.

211 TWR, WSR, and the geometric design variables are continuous variables,
212 which may take any value within defined upper and lower bounds. Typical
213 ranges for the design variables are chosen such that the resulting aircraft remains
214 within its size-class. Given a baseline vehicle in a certain size-class, this work
215 uses a $\pm 10\%$ variation about the baseline values for TWR, WSR, aspect ratios,
216 taper ratios, and thickness-to-chord ratios, a ± 5 deg variation for wing sweep
217 angle, and a ± 10 deg variation for tail sweep angles.

218 In traditional on-design aircraft sizing methods, the design mission is of-
219 ten regarded as an equality constraint, i.e. the aircraft is sized to have a range
220 capability of exactly the design range when carrying the design payload. How-
221 ever, in the case of off-design optimization, depending on the distribution of
222 mission weighting function $f(W_P, R)$ and the relative location of design mis-
223 sion $(W_{P,des}, R_{des})$ in the mission space, it may be necessary to up-size the
224 vehicle such that its feasible mission space M covers a larger region where

225 $f(W_P, R) > 0$, thus potentially increasing PFEE₁ and/or MC. In this study,
 226 an increase in the aircraft’s range capability when carrying the design payload
 227 is chosen to represent the up-sizing. Therefore, the design range capability
 228 (DESRNG) is included as the final design variable with only a lower bound
 229 constraint equal to the required design range (R_{des}).

230 A summary of the design variables and their bounds relative to the reference
 231 vehicles (to be discussed in Sec. 2.6) is presented in Table 1.

Table 1: Design Variables and Bounds Relative to Baseline Values

Group	Design Variable	Min	Max
Vehicle-level parameters	Design range capability, DESRNG	+0%	∞
	Thrust-to-weight ratio, TWR	−10%	+10%
	Wing loading, WSR	−10%	+10%
Wing geometry	Aspect ratio, AR	−10%	+10%
	Taper ratio, TR	−10%	+10%
	Thickness-to-chord ratio, TCA	−10%	+10%
	Quarter-chord sweep angle, SWEEP	−5°	+5°
Horizontal tail geometry	Aspect ratio, ARHT	−10%	+10%
	Taper ratio, TRHT	−10%	+10%
	Thickness-to-chord ratio, TCHT	−10%	+10%
Vertical tail geometry	Quarter-chord sweep angle, SWPHT	−10°	+10°
	Aspect ratio, ARVT	−10%	+10%
	Taper ratio, TRVT	−10%	+10%
	Thickness-to-chord ratio, TCVT	−10%	+10%
	Quarter-chord sweep angle, SWPVT	−10°	+10°

232 *2.4. Design Space Reduction via Sensitivity Analysis*

233 Several challenges need to be tackled especially when performing many-
 234 query exercises with computationally expensive, black-box functions in high-
 235 dimensional parameter spaces. Wang et al. [17, 18] classify techniques to re-

236 duce the input space dimensionality in the context of surrogate modeling into
237 decomposition-, mapping-, screening-, and visualization-based approaches.

238 Decomposition-based techniques partition the original problem into smaller,
239 more manageable sub-problems; the choice of decomposition is often subjective,
240 when and if a given problem is decomposable. The goal of mapping-based
241 approaches is to find a transformation that maps a set of correlated variables
242 into a new, smaller set of uncorrelated variables that retain most of the original
243 information. While mapping aids both optimization and modeling by alleviating
244 the *curse of dimensionality*, the optimization must occur in the low dimensional
245 space. Moreover, it must be assumed that a mapping from the low dimensional
246 space to the original high dimensional space exists and lies within the feasible
247 design space. While these challenges can be tackled, this work relies on a simpler
248 screening-based approach to manage the size of the input space.

249 Screening methods reduce dimensionality by exploiting sampled points to
250 recognize and retain the most important inputs and their interactions while re-
251 moving noise and other insignificant contributors to variability in the outputs.
252 Analysis of variance [19], weighted average of local sensitivity, partial rank cor-
253 relation coefficient, multi-parametric sensitivity analysis, and Fourier amplitude
254 sensitivity analysis and Sobol’s [20] method are common techniques to perform
255 screening. Once the significant variables and interactions are identified, the
256 other variables are either simply dropped from consideration (for instance, in
257 the case of surrogate modeling), or held constant at some nominal baseline
258 value (for instance, in the case of optimization). In multi-objective optimiza-
259 tion, fixing the insignificant contributors to nominal baseline values allows for an
260 easier-to-solve optimization problem at the cost of convergence to a marginally
261 different Pareto optimal set.

262 In this work, the most important set of inputs for formulating the optimiza-
263 tion problem is chosen using Sobol’s [20] method; a global sensitivity analysis
264 approach which decomposes a model output’s variance into summands of vari-
265 ances of the input parameters to determine the contribution of each input and
266 their interactions. The set of objectives and constraints is first evaluated for

267 the set of optimal experiments generated using Saltelli’s sampling scheme [21]
 268 using the aircraft sizing and mission analysis code (described in Sec. 4) Then,
 269 the Sobol’s method is applied to each objective and constraint function indepen-
 270 dently to determine the effective set of input variables to capture a prescribed
 271 normalized output variance threshold. Finally, the optimization problem is for-
 272 mulated by considering the inputs obtained by taking a superset of significant
 273 contributors to each objective and constraint function.

274 *2.5. Formal Optimization Problem*

275 Considering the objective functions, constraints, and design variables de-
 276 scribed earlier, the on-design and off-design optimization problems may now be
 277 formally stated as

$$\underset{\mathbf{x}}{\text{minimize}} \quad \mathbf{f}(\mathbf{x}) = \begin{bmatrix} \text{MRW}(\mathbf{x}) \\ \text{TOFL}(\mathbf{x}) \\ -\text{PFEE}_0(\mathbf{x}) \end{bmatrix} \quad \text{or} \quad \mathbf{g}(\mathbf{x}) = \begin{bmatrix} \text{MRW}(\mathbf{x}) \\ \text{TOFL}(\mathbf{x}) \\ -\text{PFEE}_1(\mathbf{x}) \\ -\text{MCI}(\mathbf{x}) \end{bmatrix}$$

$$\text{subject to} \quad \text{SPAN}(\mathbf{x}) - \text{SPAN}_{\max} \leq 0,$$

$$\text{VTH}(\mathbf{x}) - \text{VTH}_{\max} \leq 0,$$

$$\text{VAPP}(\mathbf{x}) - \text{VAPP}_{\max} \leq 0,$$

$$- \text{AMFOR}(\mathbf{x}) \leq 0,$$

$$- \text{SSFOR}(\mathbf{x}) \leq 0,$$

$$\mathbf{x} : [\text{Effective Design Variables}] \in [\text{Table 1 intervals}]$$

278 where $\mathbf{f}(\mathbf{x})$ and $\mathbf{g}(\mathbf{x})$ are the on-design and off-design objectives, respectively,
 279 and the set of effective design variables is determined with the following steps:

- 280 1. Perform screening tests on each objective and constraint considered;
- 281 2. Given a threshold between 0 and 100%, for each response, identify the
 282 minimal set of design variables which captures a cumulative percentage
 283 total variation greater than the threshold;

284 3. The final set of effective design variables is chosen as the superset of all
285 sets identified in step 2.

286 2.6. Aircraft Size-Classes Considered and Reference Vehicles

287 This work considers the assessment of three aircraft size-classes, each repre-
288 sented by a Regional Jet (RJ), a Small Single-aisle Aircraft (SSA), and a Large
289 Twin-aisle Aircraft (LTA), respectively. The RJ has a T-tail and tail-mounted
290 nacelle configuration, whereas the SSA and LTA have a conventional tail and
291 under-wing nacelle configuration. The baseline aircraft specifications are shown
292 in Table 2 and the constraints for each size-class are presented in Table 3. The
293 mission frequency distribution function $f(W_P, R)$ is obtained by collecting data
294 from the U.S. Department of Transportation Bureau of Transportation Statis-
295 tics Form 41 Schedule T-2 database [22]. Data on passenger-carrying flights
296 between January 2014 and September 2018 for the Bombardier CRJ900, Boe-
297 ing 737-800, and Boeing 777-200 (all variants) are used to represent the mission
298 frequency distribution of the RJ, SSA, and LTA size-classes, respectively. Before
299 proceeding, the readers must be cautioned about the following caveats:

- 300 • The historical data only include flights departing from and/or arriving in
301 the United States, thus merely serving as examples of potential represen-
302 tative mission distributions for this work. By no means are they intended
303 to represent actual worldwide operations.
- 304 • The reported flight distance in the database is the great circle distance
305 between the origin and the destination airports. The actual still-air flight
306 distance depends on routing and en-route wind considerations, which is
307 unique to each flight. In this work, the reported flight distance is divided
308 by an average horizontal flight efficiency of 0.93 [23] in order to obtain an
309 estimate of still air distance to be used in off-design mission analysis.

Table 2: Baseline aircraft specifications

Parameter	RJ	SSA	LTA
Thrust-to-weight ratio	0.338	0.312	0.296
Wing loading, lb/ft ²	113	124.1	133.3
Passenger capacity	86	160	305
Design payload weight, lb	17 200	33 600	64 050
Design range, nmi	1900	2900	7500
Cruise Mach number	0.780	0.785	0.840
Maximum cruise altitude, ft	41 000	41 000	43 000
Maximum payload weight, lb	22 750	47 000	125 500
Wing quarter-chord sweep, deg	27.0	25.7	30.9
Wing aspect ratio	8.29	9.74	8.81
Wing taper ratio	0.281	0.312	0.176
Wing thickness-to-chord ratio	0.109	0.109	0.109
Horizontal tail quarter-chord sweep, deg	29.5	29.9	34.8
Horizontal tail aspect ratio	4.59	6.27	4.62
Horizontal tail taper ratio	0.461	0.203	0.330
Horizontal tail thickness-to-chord ratio	0.094	0.109	0.088
Vertical tail quarter-chord sweep, deg	43.2	35.0	40.0
Vertical tail aspect ratio	1.11	1.92	1.84
Vertical tail taper ratio	0.644	0.276	0.299
Vertical tail thickness-to-chord ratio	0.110	0.115	0.093

Table 3: Size-dependent constraints for the RJ, SSA, and LTA size-classes

Constraint	RJ	SSA	LTA
FAA Design Category	C-II	C-III	D-V
$SPAN_{\max}$ (ft)	79	118	214
VTH_{\max} (ft)	30	45	66
$VAPP_{\max}$ (kts)	140	140	165

3. Major Disciplinary Analyses in Conceptual Aircraft Design

The aircraft design process is multidisciplinary in nature due to the inherent complexity of an aircraft as a system. Figure 1 lists a few major disciplines that play a significant role at various different stages in the aircraft design process. Due to the uneven distribution of knowledge throughout the aircraft design process [24], emphases are primarily placed on aerodynamic analysis, weight estimation, and preliminary propulsion system sizing in conceptual design stage.

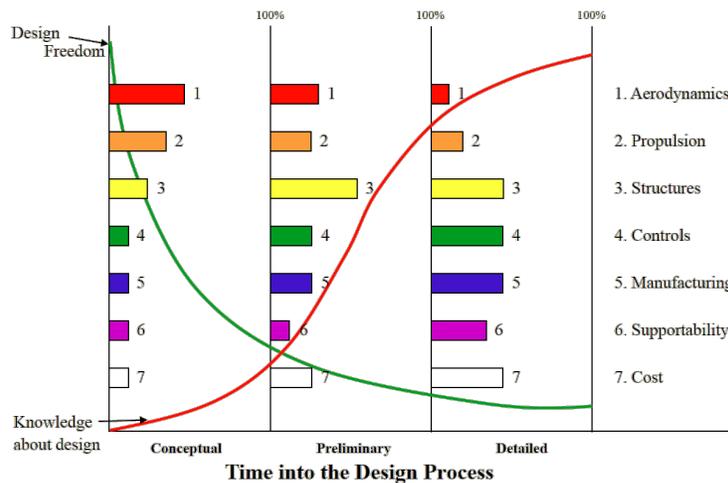


Figure 1: Uneven distribution of knowledge in the aircraft design process [24]

317 *3.1. Aerodynamics Analysis*

318 The aerodynamic characteristics are estimated using the semi-empirical drag
319 build-up method used by NASA software Flight Optimization System (FLOPS) [25].
320 FLOPS internally computes the drag of wing, empennage, fuselage, nacelles, and
321 other miscellaneous elements dynamically as the size of the components changes
322 during sizing iterations. The aircraft drag coefficient is assumed to be a function
323 of flight altitude, Mach number, and aircraft lift coefficient.

324 *3.2. Weights Estimation*

325 In conceptual design, aircraft empty weight estimation typically employs
326 semi-empirical regression-based methods rather than fully physics-based meth-
327 ods due to the need for rapid design space exploration, which requires evaluation
328 of a large amount of design candidates with limited computational resources and
329 time [26]. Existing regression-based methods include the Raymer method [27],
330 the Roskam method [28], and the FLOPS method [29], all of which compute
331 the weight of each component and take their sum to obtain the aircraft empty
332 weight. In this paper, the FLOPS method is used for weight estimation, where
333 the component weights are dynamically computed based primarily on geomet-
334 ric parameters, rated thrust, and extreme flight conditions with the assumption
335 that a conventional subsystem architecture is installed in the aircraft.

336 *3.3. Propulsion System Scaling*

337 The engine model of each baseline aircraft is sized using Numerical Propul-
338 sion System Simulation (NPSS) [30] such that the rated thrust matches the sea
339 level static thrust based on the baseline design gross weight and thrust-to-weight
340 ratio. NPSS also generates an engine deck containing the variation of thrust and
341 fuel flow as functions of Mach number, altitude, and power code (a surrogate for
342 the throttle setting). The baseline engine weight and dimensions are computed
343 using the Object-Oriented Weight Analysis of Turbine Engines computer code
344 (WATE++) [31]. For a design candidate, if the desired rated thrust is different
345 from that of the baseline engine, the engine weight and dimensions are scaled

346 using the FLOPS method [29]. The thrust specific fuel consumption as a func-
347 tion of power code, altitude, and Mach number of the scaled engine is assumed
348 to be identical to that of the baseline engine. The drag of the scaled engine
349 nacelles is recalculated in FLOPS based on the actual dimensions.

350 4. Mission Analysis

351 4.1. Mission Profile and Fuel Requirements

352 A generic commercial transport mission profile, as shown in Fig. 2, is as-
353 sumed for all missions studied in this work. This work assumes a generic com-
354 mercial transport mission profile, as shown in Fig. 2, for all the missions con-
355 sidered. All climb segments use a minimum fuel-to-climb profile and the cruise
356 segments of the regular mission are flown at a constant design Mach number as
357 specified in Table 2. Actual missions operated by commercial aircraft are simu-
358 lated by assuming that a step cruise is performed at the design Mach number at
359 altitudes between 29 000 ft and the service ceiling with an increment of 2000 ft,
360 based on the rules of Reduced Vertical Separation Minimum (RVSM) [32]. Be-
361 tween the altitudes mentioned above, the specific air range (SAR) is maximized
362 to obtain the initial cruise altitude, i.e., the altitude at which the aircraft begins
363 its cruise segment. The aircraft climbs to the next available higher altitude if
364 the SAR at the new altitude is larger (due to a decrease in the gross weight)
365 than the current SAR. As an additional requirement, the aircraft is constrained
366 to perform a step climb in the last step cruise segment before descending only if
367 a minimum remaining cruise distance of 300 nmi is available. The descent seg-
368 ment is flown at the optimum lift-to-drag ratio. The reserve mission consists of
369 a missed approach, followed by a climb to the reserve altitude, a cruise segment
370 at the optimum Mach number for SAR, a descent to 1500 ft, and a hold (loiter)
371 for 30 min at the optimum Mach number for endurance.

372 As shown in Fig. 2, the mission fuel is defined as the required amount of
373 fuel on-board at the start of the mission, including both the block fuel and the
374 reserve fuel. The block fuel is the amount of fuel consumed between engine start

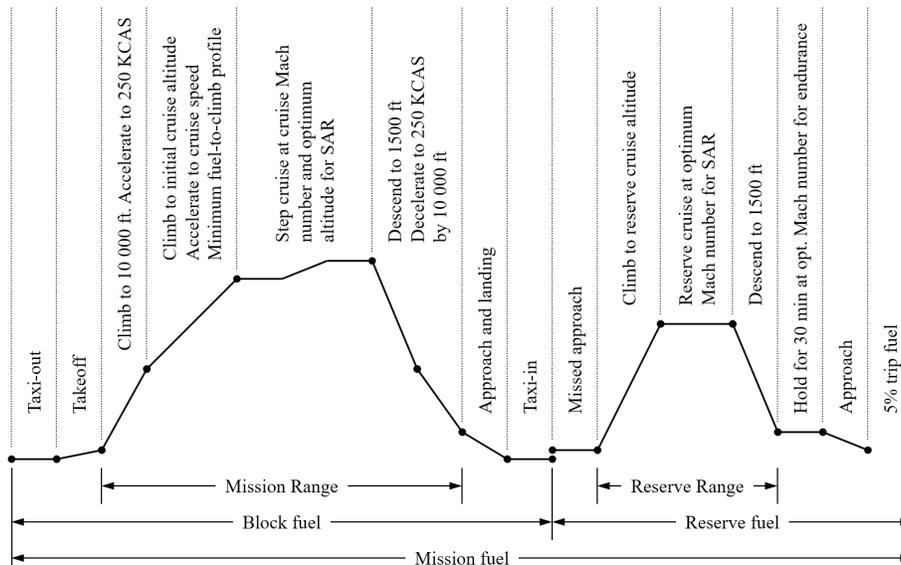


Figure 2: Generic mission profile for commercial aircraft

375 and engine shutdown for a regular mission. The reserve fuel includes the fuel
 376 required to fly the reserve mission, plus an additional 5% on top of the regular
 377 and the reserve mission fuel.

378 For the vehicles considered in this work, fuel is assumed to be stored entirely
 379 in wing fuel tanks, including wing center fuel tanks, without fuselage tanks.
 380 The wing fuel tank volume is estimated and scaled based on wing geometric
 381 parameters in the FLOPS method [29], as shown in Eq. (8):

$$V_{wt} = k_{wt} z_t \frac{S_w^2}{b} \left(1 - \frac{\lambda}{(1 + \lambda)^2} \right) \quad (8)$$

382 where k_{wt} is a non-dimensional tank volume coefficient, S_w is the wing planform
 383 area, z_t is the wing average thickness-to-chord ratio (equivalent to TCA), b is the
 384 wingspan (equivalent to SPAN), and λ is the wing taper ratio (equivalent TR).
 385 The value of k_{wt} for each size-class is calibrated based on the corresponding
 386 baseline aircraft in Table 2, and is held invariant during sizing for all design
 387 candidates of that size-class.

388 *4.2. Mission Performance Evaluation with Gross Weight and Fuel Capacity*
389 *Constraints*

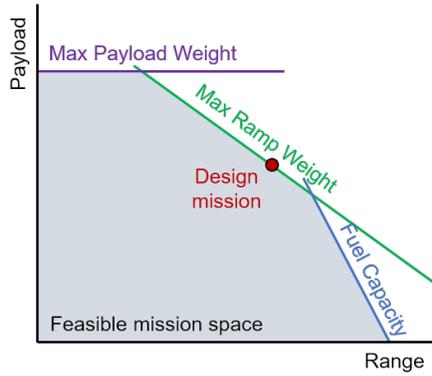
390 The mission performance is evaluated in FLOPS, which accepts climb, cruise,
391 and descent schedule definitions as inputs, and performs internal optimization
392 to find the optimum vertical trajectory satisfying the mission profile described
393 in Sec. 4.1, subject to the constraints of gross weight, zero fuel weight, and
394 mission range [25].

395 Traditionally, when a design candidate is being sized and the design mission
396 is being evaluated, the mission analysis assumes that the aircraft departs at the
397 same gross weight used to scale the geometry and the engines and to perform
398 weight estimation [27, 16], which is typically referred to as the maximum ramp
399 weight (MRW). This approach implicitly assumes that the difference between
400 the MRW and the zero fuel weight (sum of operating empty weight and payload
401 weight) is the weight of available fuel which can be achieved by filling the fuel
402 tanks; the resulting payload-range envelope is expected to be similar to Fig. 3(a).

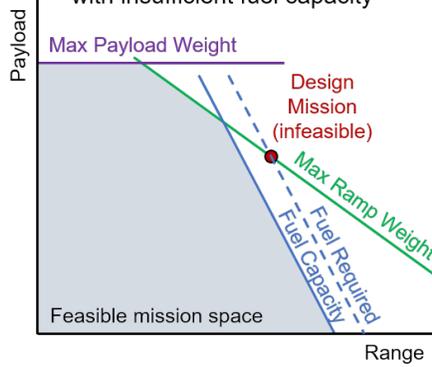
403 However, such an assumption does not always hold true. When a vehicle
404 is to be sized for a relatively high design wing loading (WSR), it is likely that
405 the fuel tanks in the small wing would fail to contain the fuel required for
406 the design mission, thus invalidating the resulting design even if the vehicle
407 sizing converges, as shown in Fig. 3(b). In this case, the design mission is
408 *fuel-constrained* instead of being *weight-constrained*.

409 To address this gap, in this work, a local optimizer is added to the existing
410 mission analysis. The vehicle size is iteratively adjusted, meaning that a new
411 value of MRW is assigned, followed by re-evaluation of geometry (including fuel
412 tank capacity), aerodynamic characteristics, component weights, and engine
413 scaling, while TWR and WSR are held constant. The mission analysis is then
414 repeated to obtain the updated fuel requirement for the design mission. The
415 optimization converges when the fuel capacity matches the fuel required for the
416 design mission within a specified tolerance. The payload-range envelope of the
417 resulting vehicle notionally resembles the envelope shown in Fig. 3(c). In this
418 work, a combination of the secant method and the bi-section method is used to

(a) Weight-constrained design mission



(b) Infeasible design mission with insufficient fuel capacity



(c) Design mission is fuel-constrained

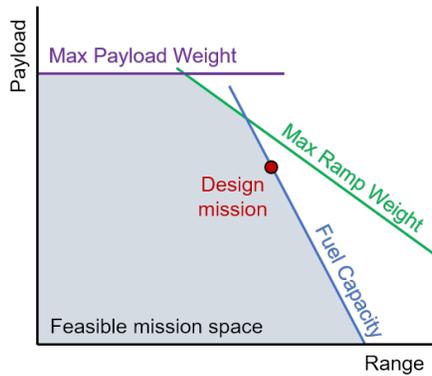


Figure 3: Notional payload-range diagrams for weight-constrained and fuel-constrained sizing

419 find the minimum feasible MRW in the case of fuel-constrained sizing.

420 *4.3. Off-Design Mission Evaluation Scheme*

421 The off-design mission evaluation involves determination of the payload-
 422 range envelope and the performance of off-design missions. Aircraft Sizing and
 423 Off-Design Mission Analyzer (SODA), a MATLAB program developed by the
 424 authors, automatically generates additional input data specifying the range,
 425 payload, and/or fuel available of each off-design mission, based on the output
 426 of the vehicle sizing module. In this work, a FLOPS interface is implemented
 427 in SODA, which translates the inputs to each off-design mission into relevant
 428 FLOPS namelists. SODA can be configured in six modes which evaluate three
 429 types of off-design missions, in addition to the design mission, as shown in
 430 Table 4 and Fig. 4. What follows is a concise description of the off-design
 431 evaluation modes.

Table 4: Mission evaluation modes in SODA

Mission Type	Mode					
	A	B	C	D	E	F
Design mission	✓	✓	✓	✓	✓	✓
Payload-range envelope		✓	✓	✓	✓	
Sample payload-range grid			✓		✓	
Specified off-design mission(s)				✓	✓	✓

432 *4.3.1. Payload-Range Envelope*

433 The payload-range envelope defines the boundary of the feasible mission
 434 space within which the aircraft is capable of flying for a given combination of
 435 payload and range. The envelope consists of three segments corresponding to
 436 three constraints:

- 437 1. **Maximum structural payload constraint.** The maximum structural

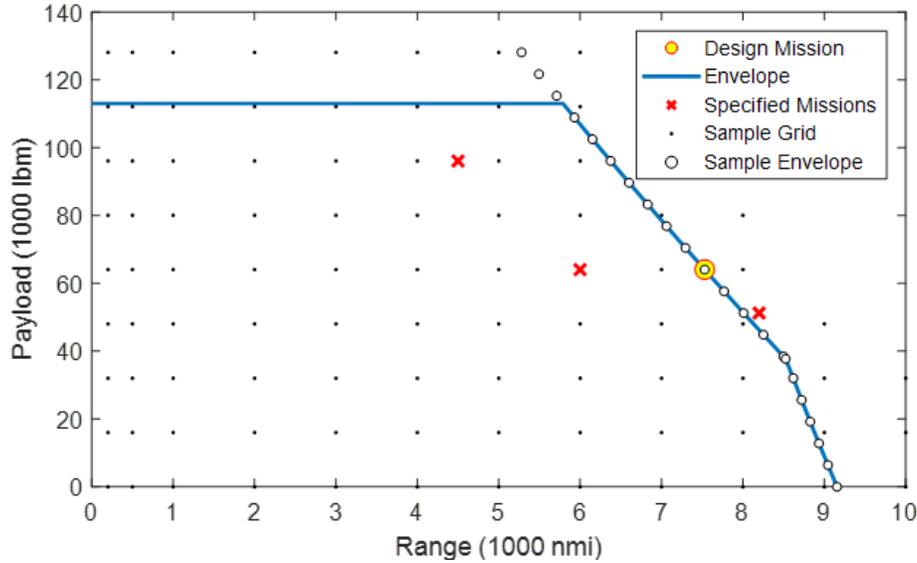


Figure 4: Notional payload-range diagram showing all mission types evaluated

438 payload weight (MPW) is the difference between the operating empty
 439 weight (OEW) and the maximum zero fuel weight (MZFW). MZFW and
 440 MPW are commonly used in detailed structural analysis, but not explicitly
 441 used for conceptual design in this work. Instead, MPW is estimated based
 442 on data for existing aircraft in the same size-class, and is assumed to
 443 remain constant for all design candidates in each size-class.

444 **2. Maximum ramp weight constraint.** The maximum ramp weight
 445 (MRW) is the maximum gross weight the aircraft can achieve when parked
 446 on the ground. In early design, the MRW is an important parameter used
 447 in sizing of major aircraft components, and specifically, in many compo-
 448 nent weight estimation equations in regression-based methods. The inter-
 449 section of MRW and MPW constraints indicates the harmonic range, i.e.
 450 the maximum range the aircraft can reach when carrying full payload. It
 451 is assumed that MRW is the upper limit for gross weight when the aircraft
 452 starts a mission.

453 **3. Maximum fuel weight constraint.** The maximum fuel weight (MFW)

454 is the maximum amount of fuel which can be carried on-board. MFW
455 is determined based on wing and fuselage geometry, and is used in fuel
456 system sizing. The intersection of MRW and MFW constraints indicates
457 the mission on which the aircraft departs at both MRW and MFW. It
458 is assumed in this paper that MFW depends solely on the fuel capacity,
459 while in reality, there may be weight and balance considerations which
460 impose a more restrictive constraint on fuel allowed on-board based on
461 allocation of payload even when the gross weight is smaller than MRW.

462 Since each candidate is constrained by the same design mission, the payload-
463 range envelope always encloses the point representing the design mission in
464 mission space. However, different design variables may result in different aero-
465 dynamic and propulsion characteristics and OEW, which affect the trade-off
466 between payload and range, implied by different slopes of the MRW and MFW
467 constraints. When assessing the impact on mission performance metrics of two
468 candidates such as takeoff gross weight, block fuel, and PFEE, etc., comparison
469 is valid only in the intersection of the two feasible mission spaces, i.e. the mis-
470 sion space containing missions that are common to both the candidates being
471 compared.

472 4.3.2. *Sample Grid*

For a given aircraft, any mission performance metric strongly related to
range and payload can be represented using a surrogate model [33, 34, 35] as
shown in Eq. (9):

$$Q = h(W_p, R) \quad (9)$$

473 Examples of metric Q include takeoff gross weight, block fuel, and PFEE
474 (Eq. (4)), etc. Once the mission space is sampled at multiple points, such
475 surrogate modeling methods allow for fast evaluation of any off-design mission
476 without actually running the physics-based mission analysis code. In addition,
477 the surrogate models are also useful for creating contour plots and performing
478 other assessments involving a large number of off-design mission evaluations.

479 To balance accuracy and execution time, the sample grid typically covers the
480 interior of the envelope and extends beyond the envelope by a certain margin
481 (as shown in Fig. 4), but can be user-defined for special cases.

482 *4.3.3. Specified Missions*

483 In order to eliminate model representation errors, off-design missions may
484 be evaluated directly using true models (e.g. FLOPS) instead of using surrogate
485 model(s). These missions are meant to represent payload-range combinations
486 which are most commonly or typically flown by the aircraft. Without evaluating
487 the envelope, the feasibility of each mission can be determined by comparing
488 the payload weight, ramp weight, and mission fuel weight against MPW, MRW,
489 and MFW from the sizing results.

490 *4.4. Verification of the Analysis Environment*

491 To verify the capability of SODA, a notional SSA is sized using the geome-
492 try and performance data as published in the Boeing 737-800 Airport Planning
493 Document [36]. The resulting payload-range characteristics are compared in
494 Fig. 5 and Table 5. It can be observed that, the payload-range envelope gener-
495 ated by SODA closely matches the reference envelope, and the off-design mission
496 gross weights are within small errors from the reference values. Additionally,
497 Fig. 5 also demonstrates SODA’s capability to perform aircraft sizing using the
498 fuel-constrained sizing algorithm as described in Sec. 4.3.

499 **5. Optimization Algorithm**

500 Most approaches that deal with multiple objectives solve a set of single-
501 objective problems to compute the Pareto frontier [37, 38, 39]. Typically, in such
502 approaches, a weighted aggregation of the objectives defines a single-objective
503 problem. Multiple such instances with different weights are solved to obtain the
504 complete Pareto frontier. Success of such methods relies on strong assumptions
505 such as convexity of the frontier, among others. Moreover, the distribution of
506 points obtained on the frontier heavily relies on the series of single-objective

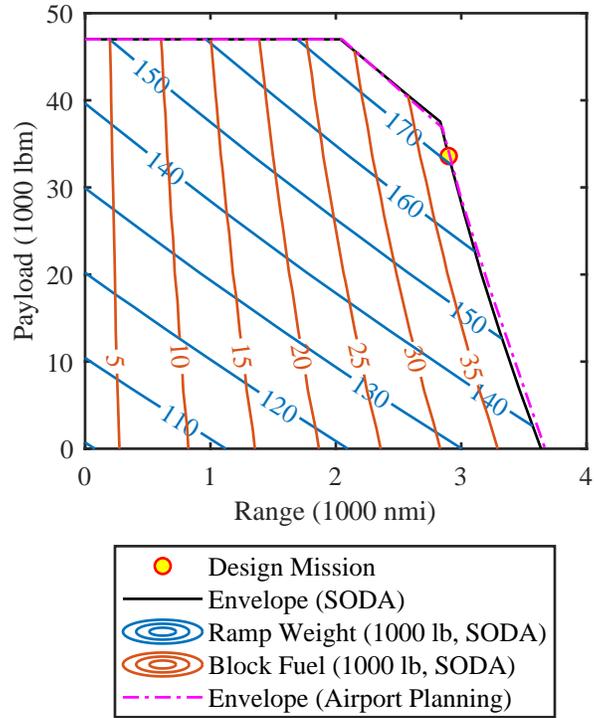


Figure 5: Comparison of payload-range characteristics between SODA outputs and Airport Planning data for a notional SSA resembling the Boeing 737-800

Table 5: Comparison of off-design mission gross weight computed by SODA and as published in the Airport Planning data for a notional SSA resembling the Boeing 737-800

Payload (lb)	Range (lb)	Gross Weight (lb)		Error
		SODA	Reference	
0	1500	113.8×10^3	112.7×10^3	+0.98 %
10 000	3000	142.6×10^3	142.0×10^3	+0.42 %
20 000	1000	130.8×10^3	129.2×10^3	+1.20 %
30 000	2500	161.0×10^3	160.5×10^3	+0.29 %
40 000	2000	166.1×10^3	165.7×10^3	+0.24 %

507 optimization problems solved. Therefore, in general, these methods are inap-
508 propriate candidates when an even distribution of points on the Pareto frontier
509 is desired.

510 On the other hand, the NSGA-II [40, 41] operates on a population of de-
511 signs (much like genetic algorithms) and successively refines it through meta-
512 heuristic operators like crossover, mutation, and selection until the members in
513 the so-called evolved populations stop improving. The notion of improvement is
514 handled through the concept of non-domination level. A member is said to be
515 non-dominated when it is better than all the other members in the population
516 in at least one objective. The NSGA-II converges by retaining the most non-
517 dominated members which by definition lie on the Pareto frontier. In addition,
518 the concept of non-domination level lends itself naturally to penalize members
519 that violate constraints. In this paper, constraints are handled by artificially pe-
520 nalizing the non-domination level adversely to ensure that constraint violating
521 designs get discarded as the algorithm progresses. It is known that in compari-
522 son to other algorithms that use gradient information, NSGA-II usually requires
523 a higher number of function calls. However, a favorable feature of NSGA-II is
524 that multiple designs can be evaluated simultaneously in parallel to alleviate
525 the relatively higher computational costs it demands.

526 **6. Results and Discussions**

527 *6.1. Sensitivity Analysis*

528 A total of 12 000 samples are generated using the SALib [42] in Python and
529 evaluated in SODA in MATLAB. The objective and constraint responses are
530 extracted and analyzed via SALib’s implementation of Sobol’s method, which
531 reports the individual effects of each design variable on each response. To con-
532 clude the sensitivity analysis, the design variables are ranked by their individual
533 effects and then filtered by their cumulative effects on each objective and con-
534 straint response for each aircraft. The filtered effective design variables and
535 their effects are summarized in Fig. 6. In this work, the threshold for normal-

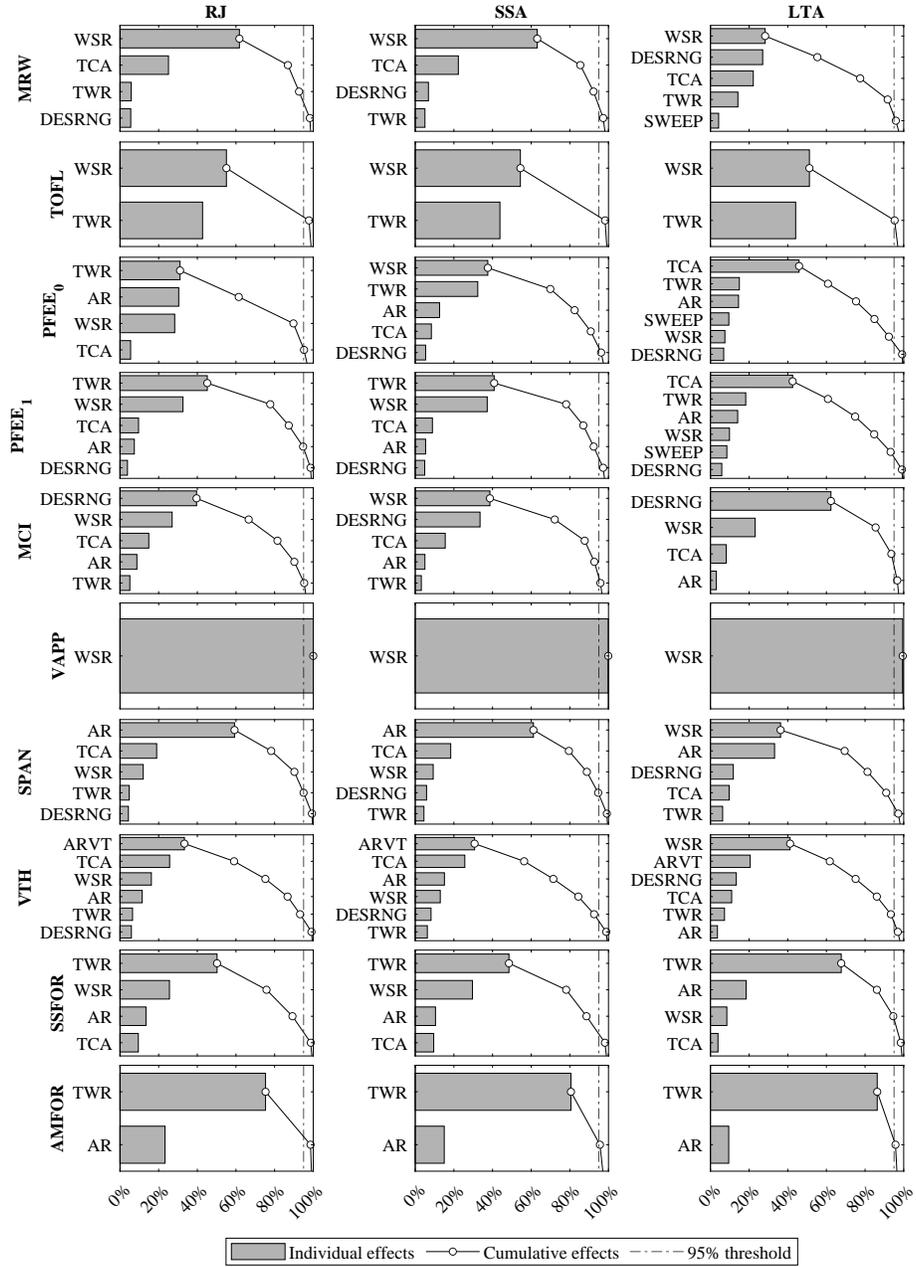


Figure 6: Normalized individual and cumulative effects of filtered effective design variables for individual responses

536 ized cumulative effects to determine the set of effective design variables is set to
537 95 %. Listed below in no particular order is the superset of all effective design
538 variables for each response, members of which are used as the design variables
539 for the optimization environment:

- 540 • TWR: sea-level static thrust-to-weight ratio
- 541 • WSR: design wing loading
- 542 • DESRNG: design range capability
- 543 • AR: wing aspect ratio
- 544 • TCA: wing average thickness-to-chord ratio
- 545 • SWEEP: wing quarter-chord sweep angle
- 546 • ARVT: vertical tail aspect ratio

547 As expected, observe that given the range of the design variables, the set
548 of most effective design variables primarily include the vehicle-level param-
549 eters (TWR, WSR, and DESRNG) and wing geometry parameters (AR, TCA,
550 and SWEEP). While ARVT does not have significant impact on any objective
551 function, it has the most impact on the vertical tail height (VTH) which, as a
552 constraint function, determines the design feasibility. Also note that the sensi-
553 tivity depends on the range of the design variables and the design requirements
554 which affect the function for each response. Therefore, the set of effective de-
555 sign variables may change based on the range of the design variables, the design
556 requirements, and the selected threshold.

557 *6.2. Summary of Optimization*

558 The optimization environment is set up in MATLAB, with a wrapper trans-
559 ferring design variables and vehicle information to and from FLOPS. The MAT-
560 LAB function `gamultiobj` is used to configure the NSGA-II algorithm. The
561 population size is set to 150 for both on-design and off-design optimizations
562 for each size-class. Some preliminary experiments indicate that these settings

563 generate a sufficiently dense Pareto frontier while keeping the computational
564 cost manageable. A scattered crossover function is used to form children arising
565 from the crossover operator. The scattered crossover function selects members
566 from the mating pool and randomly exchanges design variables between them to
567 create off-springs for future generations. An adaptive feasible mutation function
568 is used to randomly mutate selected individuals in directions and steps that are
569 adapted based on the results in the past generations. As formulated in Sec. 2, a
570 total of six optimization runs are performed, which are summarized in Table 6.

Table 6: Summary of the optimization runs for each aircraft

Case	Population Size	Generations	Spread
RJ, on-design	150	165	7.605×10^{-2}
RJ, off-design	150	102	6.283×10^{-2}
SSA, on-design	150	110	8.022×10^{-2}
SSA, off-design	150	119	8.061×10^{-2}
LTA, on-design	150	205	9.283×10^{-2}
LTA, off-design	150	150	6.925×10^{-2}

571 6.3. Comparison of Payload-Range Characteristics

572 Figure 7 presents a parallel coordinate plot for each size-class, showing the
573 distribution of each design variable and objective of the final generation in
574 both the on-design optimization and the off-design optimization, while high-
575 lighting the characteristics of the single-objective optimal designs. The bounds
576 of each design variable (based on Table 1) are also marked in the plot, where for
577 DESRNG, an artificial upper bound is placed at 110% of the lower bound value
578 for consistent axis scaling between the vehicles. Figure 8 presents the payload-
579 range envelopes of the single-objective optimal designs overlaid on the off-design
580 mission weighting function for each aircraft size-class. Since the mission weight-
581 ing function is discrete and bounded on the payload-range plane, there exists

582 no unique design that maximizes MCI; among all designs which maximize MCI,
583 the one with the highest PFEE₁ is highlighted in Figs. 7 and 8.

584 *6.3.1. Observations from On-Design Optimization*

585 For all size-classes, in the on-design optimization, the mission weighting
586 has no effect on the Pareto optimality. In this case, sizing for a higher range
587 capability (DESRNG) than the required design range will cause an increase in
588 the MRW. Therefore, the lower bound constraint on DESRNG is active for the
589 entire population as seen in Fig. 7. In other words, the payload-range envelopes
590 of all on-design optima pass through the design mission in each chart of Fig. 8.

591 For all on-design Pareto optima, the design mission is also always constrained
592 by MRW for the entire population. As implied in Sec. 4.2, the fuel-constrained
593 sizing algorithm requires upsizing the aircraft from the initial weight-constrained
594 sizing output with all design variables held constant, which leads to an increase
595 in MRW and fuel burn for all feasible missions, while not benefiting TOFL.

596 According to Fig. 7, the single-objective optimal designs for MRW and
597 PFEE₀ have very similar performance, implying little trade-off between MRW
598 and PFEE₀. It is also observed that, for these two designs, the design mission
599 is both weight- and fuel-constrained as shown in Fig. 8, resulting in the worst
600 mission capability (i.e. lowest MCI) among all Pareto optimal designs, as shown
601 in Fig. 7.

602 *6.3.2. Observations from Off-Design Optimization*

603 In the off-design optimization, the mission weighting has an impact on the
604 shape of payload-range envelope of Pareto-optimal designs: the shape of feasible
605 mission space enclosed by the envelope affects both PFEE₁ and MCI by their
606 definitions in Eqs. (6) and (7).

607 For the RJ, based on Fig. 8(a), a majority of the missions are within 1000 nmi
608 while longer missions come with reduced payload. Therefore, weight-constrained
609 designs with DESRNG at its lower bound, such as the optimal designs for MRW
610 and PFEE₀ are already able to achieve high values for PFEE₁ and MCI, as

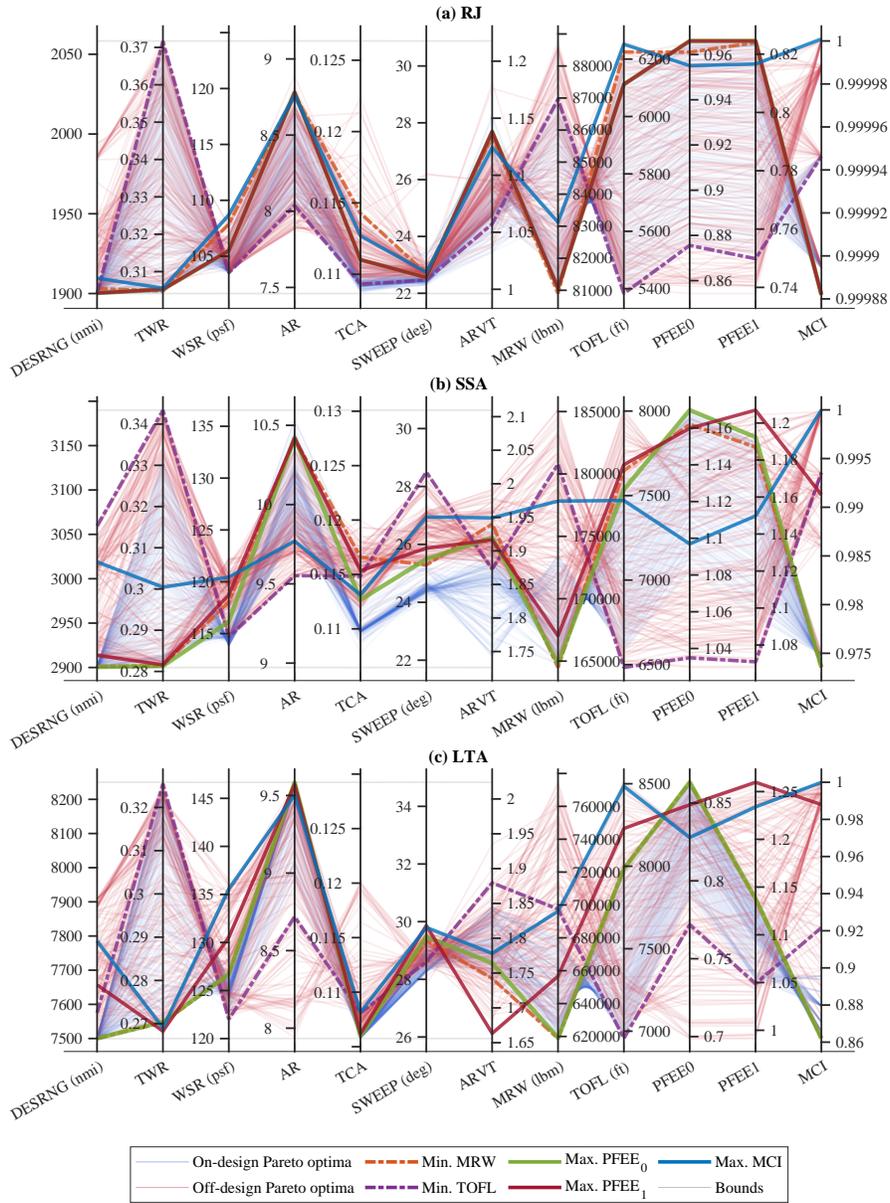


Figure 7: Design variables and objectives for Pareto optimal designs

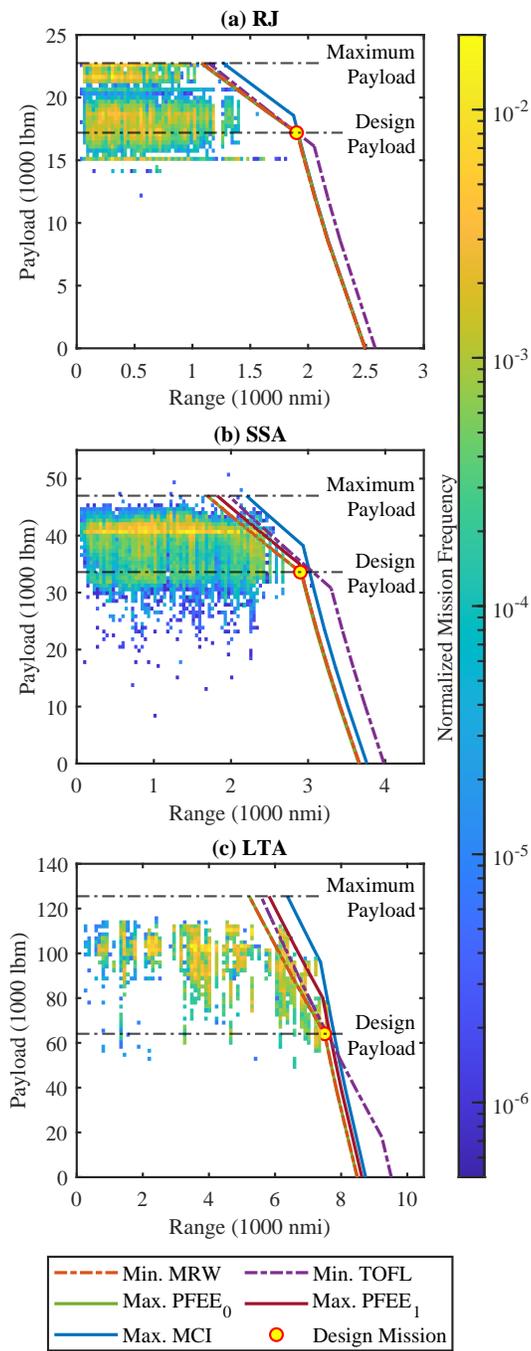


Figure 8: Payload-range envelopes of the single-objective optima

611 shown in Fig. 7(a). While it slightly improves MCI to further upsize the vehicle
612 by increasing DESRNG and/or switching to fuel-constrained designs as indi-
613 cated in Fig. 7(a), the marginal benefit of a higher mission coverage is negated
614 by an overall increase in fuel consumption within the feasible mission space,
615 thus worsening PFEE₁. Therefore, in the example of RJ, designs optimized
616 for PFEE₀ and PFEE₁, respectively, have similar values for all performance
617 metrics.

618 As the weighting function covers more missions in the long-and-heavy region
619 on the payload-range plane, it may be necessary to upsize the vehicle to improve
620 the flight productivity (PFEE₁) and mission capability (MCI).

621 In the examples of SSA and LTA, Figs. 7 and 8 show that the MCIs of
622 the on-design Pareto optima are farther from 1 compared to that for the RJ,
623 which makes PFEE₁ of the on-design optima sub-optimal. To achieve optimal
624 PFEE₁, the off-design optimization attempts different values of the design vari-
625 ables. An increase in the design wing loading (WSR) as shown in Fig. 7(b)(c)
626 reduces the wing area and the fuel capacity for a given MRW, thus triggering
627 the fuel-constrained sizing algorithm to increase MRW to satisfy the fuel capac-
628 ity constraint, as indicated by a rightward movement of the weight-constrained
629 segment of the payload-range envelope in Fig. 8(b)(c). An increase in DESRNG
630 may also be necessary to achieve optimal PFEE₁ by balancing the mission capa-
631 bility and fuel consumption. In the extreme case where MCI is maximized, the
632 vehicle may be oversized for a noticeably longer range than the design range:
633 according to Fig. 7(b) and (c), when optimized for MCI, both SSA and LTA are
634 sized for a range of approximately 4% longer than the required design range,
635 at the cost of significantly larger MRW and smaller PFEE₁.

636 6.4. Comparison of Objectives and Pareto Frontiers

637 The scatterplot matrices in Figs. 9 through 11 plot the MRW, TOFL, PFEE₀,
638 PFEE₁, and MCI of the final population in both on-design and off-design opti-
639 mization for each aircraft size-class, where the arrow in each cell indicates the
640 direction of improvement on each projection plane.

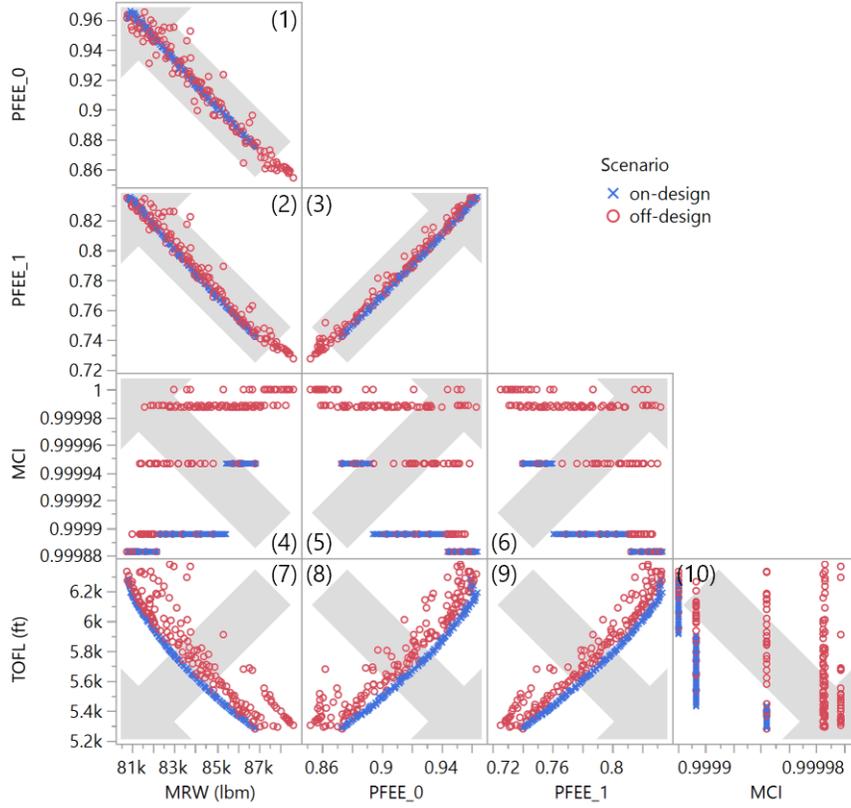


Figure 9: Projected Pareto frontiers for RJ

641 *6.4.1. Observations from On-Design Optimization*

642 For each size-class, in the on-design optimization, the Pareto frontier appears
 643 to be a curve in the three-dimensional space formed by MRW, TOFL, and
 644 $PFEE_0$. When considering only two objectives at a time, the projected on-
 645 design 3-D Pareto frontier in subplots (7) and (8) form 2-D Pareto frontiers for
 646 MRW vs TOFL and $PFEE_0$ vs TOFL, while in subplot (1), MRW and $PFEE_0$
 647 simultaneously reach their optimal values. Note that, based on the definition in
 648 Eq. (5), $PFEE_0$ is only related to the design mission block fuel. The results are
 649 therefore consistent with the positive correlation between gross weight and fuel
 650 consumption for a given mission. This also explains why the design mission,
 651 the only mission that concerns the on-design optimization, is always weight-

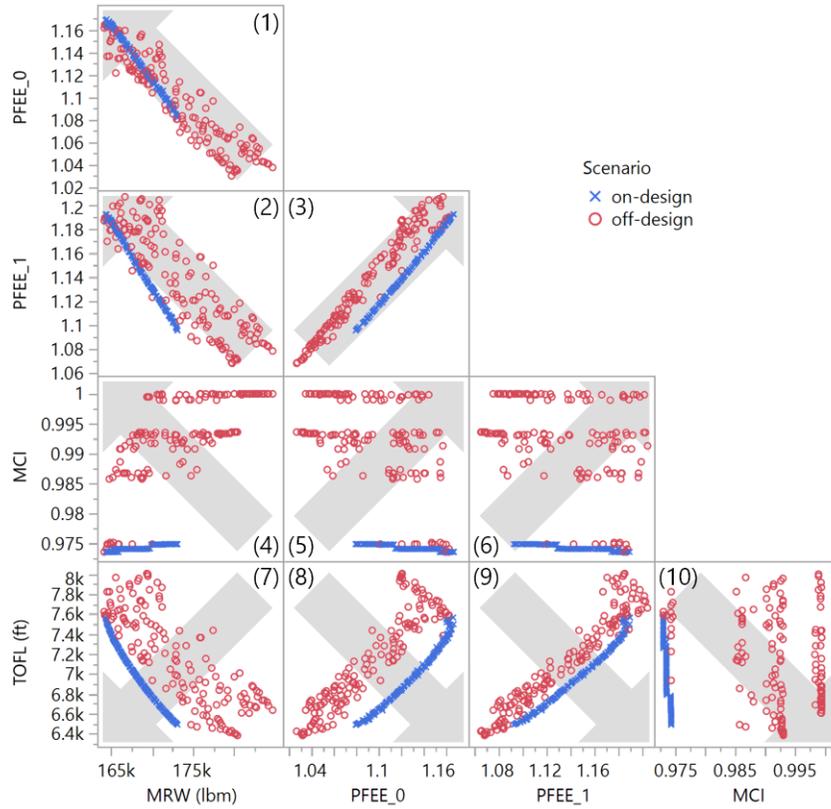


Figure 10: Projected Pareto frontiers for SSA

652 constrained for the entire Pareto optimal population.

653 On the other hand, the goal to minimize TOFL conflicts with minimizing
 654 MRW and maximizing PFEE₀ (i.e. minimizing design mission block fuel), since a
 655 shorter TOFL requires more powerful engines for the same take-off gross weight,
 656 which translates to a higher TWR (as shown in Fig. 7) and in turn increases
 657 the size of engines and fuel flow at cruise, resulting in higher fuel consumption
 658 and MRW.

659 6.4.2. Observations from Off-Design Optimization

660 For the RJ, given the mission weighting function focusing on short missions,
 661 the off-design Pareto frontier is generally aligned with the on-design Pareto fron-

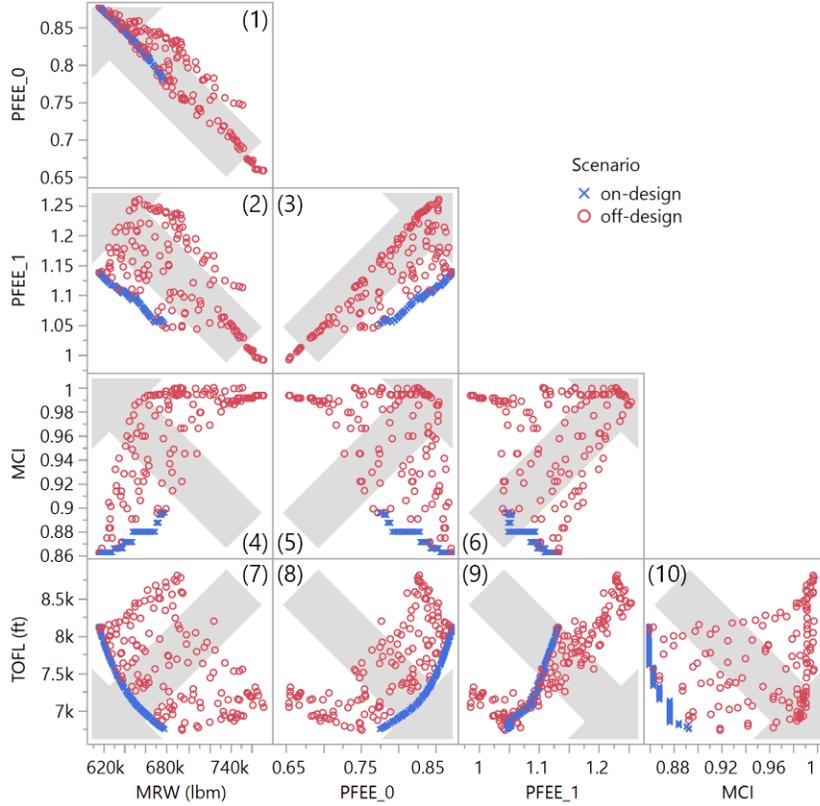


Figure 11: Projected Pareto frontiers for LTA

662 tier. As discussed in Sec. 6.3, there is no motivation to trade MRW for PFEE₁,
 663 therefore MRW and PFEE₁ can reach their optimal values simultaneously as
 664 shown in Fig. 9 subplot (2). When comparing on-design and off-design optimiza-
 665 tion, the patterns of the projected PFEE₀ vs PFEE₁ values in subplot (3) clearly
 666 align with each other, indicating that the Pareto optimal designs have similar
 667 values for the pair PFEE₀ and PFEE₁ regardless of the goal of optimization.

668 For the SSA, Fig. 10 subplot (2) implies a trade-off between MRW and
 669 PFEE₁ when considering these two objectives only, as discussed in Sec. 6.3.2.
 670 Note that the Pareto frontier of MRW vs PFEE₁ in subplot (2) only spans over
 671 a smaller interval of MRW compared to the Pareto frontier of MRW vs TOFL
 672 in subplot (7). The reason is that, as MRW increases beyond a certain thresh-

673 old (approximately 167×10^3 lbm in this case), the disadvantage of higher fuel
674 consumption offsets the benefit gained by covering more missions, resulting in a
675 decrease in PFEE_1 from the single-objective maximum; this is consistent with
676 the discussions made in Sec. 6.3.2. In subplot (3), towards higher PFEE_0 val-
677 ues, a few designs optimized for off-design missions interestingly appear superior
678 than the on-design candidates optimized for the same PFEE_0 value, indicating a
679 marked benefit in productivity over a purely on-design performance centric de-
680 sign optimization. This observation clearly highlights the benefit in optimizing
681 for off-design performance (or productivity as indicated by PFEE_1).

682 For the LTA, as shown in Fig. 11, the trade-off between MRW and PFEE_1 is
683 even more significant in subplot (2), where the single-objective maximum value
684 of PFEE_1 is approximately 11 % greater than the PFEE_1 value for the minimum-
685 MRW design. The wide-spread pattern of the off-design Pareto optima above
686 the on-design Pareto optima in subplot (3) implies that designs optimized merely
687 for the design mission may be suboptimal in actual airline operations than the
688 designs optimized for the given off-design missions. Also note that the PFEE_0
689 value of the single-objective optimum design for PFEE_1 is approximately 2.1 %
690 lower than the maximum value of PFEE_0 , indicating a trade-off between PFEE_0
691 and PFEE_1 .

692 7. Conclusion

693 This paper presents a multi-objective constrained optimization approach
694 for aircraft sizing, taking into account both on-design and off-design mission
695 performance metrics. Parsimonious optimization problems are formulated by
696 down-selecting inputs from a large set of design variables that captured most
697 of the variance in the objective and constraint functions. An off-design mission
698 weighting function is used to transform mission-dependent variables such as
699 payload, range, fuel consumption into a scalar through an integration on the
700 feasible mission space identified by the payload-range envelope of a sized vehicle.
701 In this work, historical mission data are used to obtain the mission weighting

702 function for each vehicle size-class being studied, which simulates the market in
703 which a new design of the same size-class is expected to operate. Results show
704 that the mission weighting function has a strong impact on the Pareto optimal
705 designs when off-design mission performance metrics such as the payload fuel
706 energy efficiency and the mission coverage index are explicitly considered as
707 objectives along with mission-invariant characteristics such as the maximum
708 ramp weight and maximum takeoff field length at standard sea-level condition.
709 When the long-and-heavy missions are given higher weighting, increasing the
710 design range capability and/or switching to fuel-constrained sizing modifies the
711 shape of payload-range envelope and may increase the flight productivity with
712 a penalty on the maximum ramp weight, as shown in the examples of the Small
713 Single-aisle Aircraft and the Large Twin-aisle Aircraft.

714 Some avenues for future work include: 1) employ more accurate aerodynam-
715 ics, weights, and propulsion system analysis, which still exploit high-level vehicle
716 design parameters to obtain better estimation of vehicle performance metrics;
717 2) consider the impact of subsystem architectures which may impact vehicle
718 empty weight, fuel burn, and drag characteristics in different manners; 3) con-
719 sider and quantify the impact of different choices of mission weighting function
720 and mission-dependent metrics on the Pareto optimal solutions; 4) establish a
721 process which facilitates making decisions regarding whether a family of aircraft
722 (instead of a single type) should be designed to split the market share in order
723 to improve flight productivity given the set of Pareto optimal designs based on
724 the selected metrics and mission weighting function.

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