Degraded Relay Channel with Non-causal State Information at the Source and Relay

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Abstract

A state-dependent discrete memoryless degraded relay channel is considered, with non-causal side information available at both the sender and the relay. The capacity of this relay model is an open problem. We improve upon the known achievable regions for this setting, in addition to proving an outer bound. The key idea in the proof of achievable region is to employ a modified decode-forward scheme. The characterization is then extended to a relay broadcast setting, where an improved inner bound over existing schemes and an outer bound are exhibited.

I. Introduction

Relay channels, introduced by van der Meulen [1], are the simplest models of multi-hop communication in networks, with a single transmitter trying to communicate to a receiver with the aid of an intermediate relay node. In their seminal paper, Cover and El Gamal [2] determined the capacity for a degraded relay model, wherein the channel from sender to relay is better than the direct link from sender to destination, in both discrete memoryless as well as additive Gaussian cases. Reference [2] also derived a general upper bound to the capacity of the relay channel, i.e. the cutset bound, and also the capacity for a relay channel with feedback. However, a single letter characterization for the capacity of the general relay channel remains unknown.

The decode-forward strategy involves the relay recovering the sender's transmissions in the current block, which is used to determine the relay transmissions in the subsequent block. The proof of the decode-forward achievability scheme in Cover and El Gamal [2] used binning and list decoding. Several alternative proof techniques have been proposed. One variation is the so called backward decoding technique, which goes back to the work of Willems and van der Meulen [3] on multiple access channels with cribbing, and Carleial [4] on the MAC with generalized feedback. The excessive delay involved in backward decoding can be alleviated using sliding window based schemes [4]. The decode-forward scheme is not suited in situations where the channel from the sender to relay is weak, and various schemes which do not require the relay to recover the entire sender transmission have been proposed, like partial decode-forward, compress forward, etc.- see [5] for a survey of different techniques.

Channels with state are used to describe communication settings where the channel laws are controlled by a state process. The case of non-causal state information known at the encoder, was solved in Gelfand-Pinsker [6] and Heegard-El Gamal [7]. This model is relevant in settings like information embedding, digital watermarking and coding for memory with defective cells. However, multi-user generalizations of the Gelfand-Pinsker scheme are usually not straightforward, and very few settings have been completely characterized. For instance, see the MAC with degraded message sets and state information at only one encoder [8], and the degraded broadcast channel with state known at both the encoder and the strong receiver [9].

The relay channel with state has attracted increasing attention in recent years. Sutivong et. al. [10] considered a state-dependent Gaussian degraded relay channel with non-causal state at both sender as well as the relay, and proved that the capacity remains unchanged by the presence of state. Zaidi et. al. [11] considered an extension of the model in [10] to the case of multiple relay nodes and derived lower bounds based on decode-forward scheme. Farsani and Marvasti [12] considered a state-dependent DM relay setting and derived an achievable rate based on partial decode-forward scheme. [12] also considered a relay setting with two state components, with only one of them known to the relay and both components known to the sender, and derived an achievable rate based on decode-forward. For a state-dependent relay channel with non-causal state only at the source, Zaidi et. al. [13] derived upper and lower bounds on the capacity. Khormuji et. al. [14] introduced a class of state-decoupled relay channels and determined the capacity for a subclass of semi-deterministic relay channels.

Notice that the capacity characterization for a relay channel with non-causal state at both sender and relay remains open. Our contributions here are improved lower bounds for this state dependent relay model over those in [10]–[12], along with an outer bound. Furthermore, the characterization is then extended to the case of a relay broadcast setting and an improved inner bound over that in [11] is shown, along with an outer bound.

The paper is organised as follows. We introduce the system model and state the main results in Section

II. The proofs of outer bound and achievable region for the relay model are given in Sections III and IV respectively. Section V deals with improved inner bound and outer bounds for the relay broadcast extension setting. Concluding remarks are given in Section VI.

II. System Model and Results

We consider a degraded relay channel with state, depicted in Figure 1. The state process S^n is IID. The sender

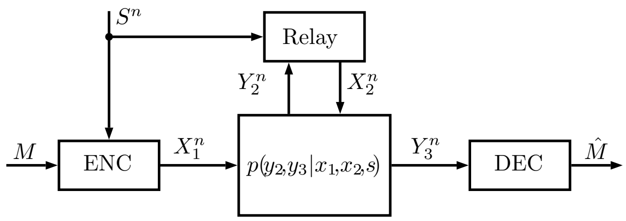

Fig. 1. Degraded Relay with Non-Causal State

as well as the relay nodes know the state-process in a non-causal fashion. The sender wishes to communicate a message $M \in [1 : 2^{nR}]$ to the receiver with the aid of the relay. The channel is described by the conditional law $p(y_2, y_3|x_1, x_2, s)$. We consider a physically degraded setting, where the conditional pmf $p(y_2, y_3|x_1, x_2, s)$ can be written as

$$p(y_2, y_3|x_1, x_2, s) = p(y_2|x_1, x_2, s)p(y_3|x_2, y_2). \tag{1}$$

In other words, the Markov condition $X_1 \to (X_2, Y_2) \to Y_3$ holds.

Let us introduce some notational conventions. We denote $U^n := U_1, \cdots, U_n$. Random variables are represented by capital letters while their their realizations are denoted by the corresponding small cases. For input symbols $x_1 \in \mathcal{X}_1$ and $x_2 \in \mathcal{X}_2$, from the sender and relay, and IID state $s \in \mathcal{S}$, the outputs (y_2, y_3) of a state dependent discrete memoryless relay channel is specified by the set of probability laws $p(y_2, y_3|x_1, x_2, s)$. Notice that the sender and the relay have non-causal knowledge of S^n, whereas the receiver's decisions are based only on Y_3^n.

Definition 1. *An (n, R, ϵ) code consists of a message set $[1 : 2^{nR}]$, two sets of encoder maps (one each at sender and relay):*

$$X_{1i}(S^n, M) : \left\{\mathcal{S}^n \times [1 : 2^{nR}]\right\} \to \mathcal{X}_1$$

$$X_{2i}(S^n, Y_2^{i-1}) : \left\{\mathcal{S}^n \times \mathcal{Y}_2^{i-1}\right\} \to \mathcal{X}_2,\ 1 \le i \le n,$$

and the decoding map at the receiver

$$\psi(Y_3^n) : \mathcal{Y}_3^n \to [1 : 2^{nR}]$$

such that for uniform choice of M, we have

$$\mathbb{P}(\psi(Y_3^n) \neq M) \le \epsilon. \tag{2}$$

We say that a rate R is achievable if an (n, R, ϵ) scheme exists for every $\epsilon > 0$, possibly by taking n large enough. Let C be the supremum of all achievable rates. Our main results are stated now. Define the rate

$$R = \min\{I(V, X_1; Y_2|U, X_2, S), I(U, V; Y_3) - I(U, V; S)\}. \tag{3}$$

Let $\mathcal{P}^i$ stand for the collection of pmfs on $\mathcal{U} \times \mathcal{V} \times \mathcal{S} \times \mathcal{X}_1 \times \mathcal{X}_2$ that satisfy

$$p(u, v, s, x_1, x_2) = p(s)p(u, v|s)p(x_1|u, v, s)\mathbb{1}_{\{X_2 = f(U,S)\}}, \tag{4}$$

where $\mathbb{1}_{\{\cdot\}}$ is an indicator function and $f(\cdot)$ is a deterministic map. Let $\mathcal{P}^o$ be the collection of pmfs on $\mathcal{U} \times \mathcal{V} \times \mathcal{S} \times \mathcal{X}_1 \times \mathcal{X}_2$ that satisfy

$$p(u, v, s, x_1, x_2) = p(s)p(u, v|s)p(x_1, x_2|u, v, s). \tag{5}$$

Now define the quantities

$$C' = \max_{p(u,v,s,x_1,x_2) \in \mathcal{P}^i} R, \tag{6}$$

$$C'' = \max_{p(u,v,s,x_1,x_2) \in \mathcal{P}^o} R. \tag{7}$$

Theorem 2. *For the discrete memoryless state-dependent physically degraded relay channel, the capacity is bounded as*

$$C' \le C \le C''. \tag{8}$$

Remark 3. *Notice that the inner and outer bounds are characterized by the same rate constraints, but the optimization in the inner bound is over deterministic maps $p(x_2|u, s) = x_2(u, s)$ while there is no such restriction in the outer bound.*

Sufficiency of deterministic maps for $p(x_2|u, s)$ would completely determine the capacity.

Before proving Theorem 2, we note that the best known inner bound for the setup under consideration is given by:

Theorem 4. *[10]–[12] For the discrete memoryless state-dependent physically degraded relay channel, the capacity is lower bounded as*

$$C \geq \max_{p(u,v,s,x_1,x_2)\in\mathcal{P}^i} \min \begin{Bmatrix} I(V;Y_2|U,X_2,S), \\ I(U,V;Y_3)-I(U,V;S) \end{Bmatrix}. \tag{9}$$

Remark 5. *On comparing theorems 2 and 4, it is clear that our contribution in Theorem 2 improves upon the lower bounds of [10]–[12] by an additive factor of $I(X_1;Y_2|U,V,X_2,S)$.*

III. Proof of Outer Bound

We first prove the outer bound. We note that by Fano's Inequality, $H(M|Y_3^n) \leq n\epsilon_n$ where $\epsilon_n \to 0$ as $n \to \infty$. Consider the following chain of inequalities for the rate concerning the receiver node:

$$\begin{aligned}
nR &= H(M) \\
&\overset{(a)}{=} H(M) - I(M;S^n) \\
&\overset{(b)}{\leq} I(M;Y_3^n) - I(M;S^n) + n\epsilon_n \\
&= H(Y_3^n) - H(Y_3^n|M) - H(S^n) + H(S^n|M) + n\epsilon_n \\
&\leq \sum_{i=1}^{n}\{H(Y_{3i}) - H(Y_{3i}|M,Y_3^{i-1}) \\
&\qquad - H(S_i) + H(S_i|M,S_{i+1}^n)\} + n\epsilon_n \\
&\overset{(c)}{=} \sum_{i=1}^{n}\{H(Y_{3i}) - H(Y_{3i}|M,Y_3^{i-1},S_{i+1}^n) \\
&\qquad - H(S_i) + H(S_i|M,Y_3^{i-1},S_{i+1}^n)\} + n\epsilon_n \\
&\overset{(d)}{=} \sum_{i=1}^{n}\{H(Y_{3i}) - H(Y_{3i}|U_i,V_i) - H(S_i) + H(S_i|U_i,V_i)\} + n\epsilon_n \\
&= \sum_{i=1}^{n}\{I(U_i,V_i;Y_{3i}) - I(U_i,V_i;S_i)\} + n\epsilon_n, \text{ where,}
\end{aligned} \tag{10}$$

where (a) follows since $M \perp\!\!\!\perp S^n$, (b) follows by Fano's inequality, (c) follows from Csiszar Sum Lemma and (d) follows by choosing $U_i = (Y_3^{i-1}, S_{i+1}^n)$ and $V_i = (M, Y_3^{i-1}, S_{i+1}^n)$. Consider the following chain of inequalities for the rate concerning the relay node:

$$nR = H(M) \overset{(a)}{=} H(M|S^n)$$

$$
\begin{aligned}
&\overset{(b)}{\leq} I(M;Y_3^n|S^n)+n\epsilon_n\\
&\leq I(M;Y_2^n,Y_3^n|S^n)+n\epsilon_n\\
&= H(Y_2^n,Y_3^n|S^n)-H(Y_2^n,Y_3^n|M,S^n)+n\epsilon_n\\
&= \sum_{i=1}^{n}\{H(Y_{2i},Y_{3i}|S^n,Y_2^{i-1},Y_3^{i-1})\\
&\qquad -H(Y_{2i},Y_{3i}|M,S^n,Y_2^{i-1},Y_3^{i-1})\}+n\epsilon_n\\
&\overset{(c)}{=} \sum_{i=1}^{n}\{H(Y_{2i},Y_{3i}|S^n,Y_2^{i-1},Y_3^{i-1},X_{2i})\\
&\qquad -H(Y_{2i},Y_{3i}|M,S^n,Y_2^{i-1},Y_3^{i-1},X_{2i})\}+n\epsilon_n\\
&\overset{(d)}{=} \sum_{i=1}^{n}\{H(Y_{2i}|S^n,Y_2^{i-1},Y_3^{i-1},X_{2i})\\
&\qquad -H(Y_{2i}|M,S^n,Y_2^{i-1},Y_3^{i-1},X_{2i})\}+n\epsilon_n\\
&\overset{(e)}{=} \sum_{i=1}^{n}\{H(Y_{2i}|S^n,Y_2^{i-1},Y_3^{i-1},X_{2i})\\
&\qquad -H(Y_{2i}|M,S^n,Y_2^{i-1},Y_3^{i-1},X_{1i},X_{2i})\}+n\epsilon_n\\
&\overset{(f)}{\leq} \sum_{i=1}^{n}\{H(Y_{2i}|Y_3^{i-1},S_{i+1}^n,S_i,X_{2i})\\
&\qquad -H(Y_{2i}|M,Y_3^{i-1},S_{i+1}^n,S_i,X_{1i},X_{2i})\}+n\epsilon_n\\
&\overset{(g)}{=} \sum_{i=1}^{n}\{H(Y_{2i}|U_i,S_i,X_{2i})-H(Y_{2i}|U_i,S_i,V_i,X_{1i},X_{2i})\}+n\epsilon_n\\
&= \sum_{i=1}^{n}\{I(V_i,X_{1i};Y_{2i}|U_i,S_i,X_{2i})\}+n\epsilon_n,\ \text{where,}
\end{aligned}
\tag{11}
$$

where (a) follows since $M \perp\!\!\!\perp S^n$, (b) follows by Fano's inequality, (c) since X_{2i} is determined by (S^n, Y_2^{i-1}), (d) since given (X_{2i}, Y_{2i}), Y_{3i} is conditionally independent of everything else by degradedness, (e) since X_{1i} is determined by (S^n, M), (f) since given (X_{1i}, X_{2i}, S_i), Y_{2i} is conditionally independent of everything else by memorylessness and (g) follows from $U_i = (Y_3^{i-1}, S_{i+1}^n)$ and $V_i = (M, Y_3^{i-1}, S_{i+1}^n)$. This completes the proof of outer bound.

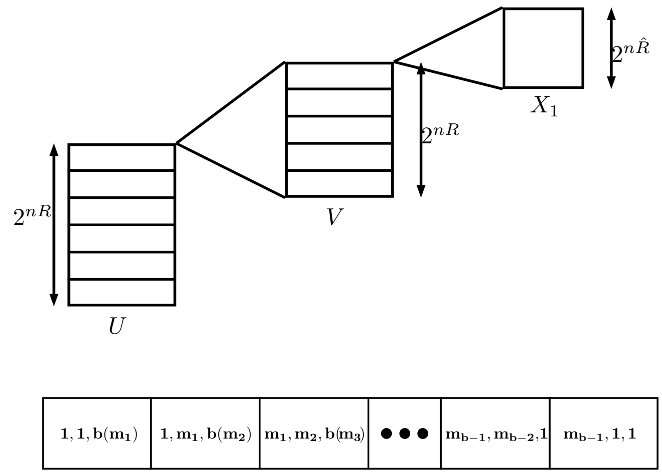

Fig. 2. Illustration of the Scheme

IV. Proof of Achievability

We use a combination of Gelfand-Pinsker binning along with a suitable modification of decode-forward scheme with a backward decoding procedure. We consider B transmission blocks and use a Block Markov coding scheme. A sequence of $B-1$ i.i.d. messages $M_j \in [1:2^{nR}]$, with $j \in [2:B-1]$ is to be sent in B blocks, with $M_0 = M_B = 1$. Each block comprises n uses of the channel.

The codebook structure and the encoding scheme are shown in Fig. 2. The message set $[1:2^{nR}]$ is binned into $2^{n\hat{R}}$ bins, with $b(m)$ representing the bin index of message m and $\mathcal{B}(b)$ representing all the codewords with bin index b. We construct a multilayer coding scheme where the transmitted codewords in block j are determined by the index set $(m_{j-1}, m_j, b(m_{j+1}))$, along with the state sequence s^n. The variables (U, V, X_1) are used to convey information about the index-set.

Codebook Generation:

Fix the pmf $p(u, v|s)$ and function $x_2(u, s)$. For $j \in [1:B]$, randomly and independently generate $2^{n(R+R_0')}$ sequences $u^n(m_{j-1}, l_{j-1})$ with $m_{j-1} \in [1:2^{nR}]$ and $l_{j-1} \in [1:2^{nR_0'}]$, each according to $\prod_{i=1}^n p_U(u_i)$. For each u^n sequence, randomly and conditionally independently generate $2^{n(R+R_1')}$ sequences $v^n(m_j, t_j, m_{j-1}, l_{j-1})$ with $m_j \in [1:2^{nR}]$ and $t_j \in [1:2^{nR_1'}]$, each i.i.d. according to $\prod_{i=1}^n p_{V|U}(v_i|u_i)$. Now for each v^n sequence, randomly and conditionally independently generate $2^{n\hat{R}}$ sequences $x_1^n(b_j, m_j, t_j, m_{j-1}, l_{j-1})$ for $b_j \in [1:2^{n\hat{R}}]$ iid according to $\prod_{i=1}^n p_{X_1|U,V,S}(x_{1i}|u_i, v_i, s_i)$. Notice that the codebook for x_1^n in each block is different, as it is generated according to the known state sequence.

Encoding:

Let $m_0 = m_B = 1$ by convention. Suppose that the relay knows m_{j-1} at the start of block j. Given the realization s^n, the relay picks a u^n sequence such that $(u^n(m_{j-1}, l_{j-1}), s^n) \in \mathcal{T}_\epsilon^n$. For the message m_j, the transmitter finds a v^n sequence such that $(v^n(m_j, t_j, m_{j-1}, l_{j-1}), u^n(m_{j-1}, l_{j-1}), s^n) \in \mathcal{T}_\epsilon^n$. The $x_1^n(b(m_{j+1}), m_j, t_j, m_{j-1}, l_{j-1})$ sequence is then sent over the channel.

Decoding at Relay:

The relay performs list decoding to recover the transmitted message. Let $\tilde{m}_0 = 1$ by convention. Assume that the relay knows $b(m_j), m_{j-1}, l_{j-1}$ at the start of block j. At the end of block j, the relay first forms a list $\mathcal{L}(y_2^n(j))$ of indices m_j such that the corresponding v^n sequences satisfy $(v^n(m_j, t_j, m_{j-1}, l_{j-1}), u^n(m_{j-1}, l_{j-1}), y_2^n(j), s^n) \in \mathcal{T}_\epsilon^n$ for some t_j. It declares $\tilde{m}_j$ was sent if it is the unique index in the intersection $\mathcal{B}(b(m_j)) \cap \mathcal{L}(y_2^n(j))$, where $\mathcal{B}(b)$ refers to set of messages in the bin corresponding to b.

The relay now finds the unique b_j, if available, such that $(x_1^n(b_j, m_j, t_j, m_{j-1}, l_{j-1}), y_2^n(j)) \in \mathcal{T}_\epsilon^n(X_1, Y_2|U, V, S)$, and declares this as $b(m_{j+1})$. Otherwise $b_j = 1$ is chosen. The relay transmissions are done according to a deterministic mapping $x_{2i}(u_i, s_i)$ for $i \in [1 : n]$.

Decoding at Destination:

The decoding at the destination employs a backward decoding procedure, which begins after all the B blocks have been received. We have taken $m_B = 1$. Assume that m_{j+1} and t_{j+1} are already known at the decoder. Then for $j = B-1, B-2, \cdots, 1$, the receiver finds the unique message $\hat{m}_j$ such that $(v^n(m_{j+1}, t_{j+1}, \hat{m}_j, \hat{l}_j), u^n(\hat{m}_j, \hat{l}_j), y_3^n(j+1)) \in \mathcal{T}_\epsilon^n$.

Notice that the x_1^n sequence is not part of the joint typicality test here, since the associated index belongs to the codeword in the subsequent block, which has already been found via backward decoding.

Analysis of Error Probability:

By the covering lemma [5], the probability of encoding errors vanish with n by choosing:

$$R_0' \geq I(U; S) \tag{12}$$

$$R_1' \geq I(V;S|U). \tag{13}$$

By the packing lemma [5], the probability of decoding errors at the receiver vanish with n by choosing:

$$R + R_0' + R_1' \leq I(U,V;Y_3). \tag{14}$$

For the decoding error probability calculation at the relay, assume that the relay has correctly decoded $b(m_j)$ at the start of block j. Notice that

$$|\mathcal{L}(y_2^n)| \approx 1 + 2^{n(R+R_1')} \times 2^{-nI(V;Y_2,S|U)} \tag{15}$$

elements. For each of the incorrect codewords, the probability that their message bin index matches the one that the relay has already decoded is $2^{-n\hat{R}}$. Thus the correct message will be chosen with probability close to one if

$$R + R_1' - \hat{R} \leq I(V;Y_2,S|U). \tag{16}$$

The error probability in decoding the index $b_j(m_{j+1})$ at the end of block j can be made arbitrarily small by choosing

$$\hat{R} \leq I(X_1;Y_2|U,V,S). \tag{17}$$

Combining the equations above, and eliminating the redundant ones via Fourier-Motzkin elimination, we arrive at the rate constraints:

$$R \leq I(V,X_1;Y_2|U,S) \tag{18}$$

$$R \leq I(U,V;Y_3) - I(U,V;S). \tag{19}$$

But since X_2 is a function of (U,S), the achievable rate can also be written as:

$$R \leq \min\{I(V,X_1;Y_2|U,S,X_2), I(U,V;Y_3) - I(U,V;S)\}.$$

This completes the proof of the inner bound.

V. Extension to Relay-Broadcast Setting

We now extend our characterization in the previous sections to a relay-broadcast channel (RBC) setting with non-causal state at both source and relay, wherein the relay node also acts as a receiver. Two independent messages $W_2 \in [1:2^{nR_2}]$ and $W_3 \in [1:2^{nR_3}]$, uniformly drawn over their respective ranges, are to be conveyed to the relay and the receiver respectively. The model is as shown in Fig. 3. An inner bound for this setting was derived in [11], however the capacity region remains open. Using similar techniques as in the proof

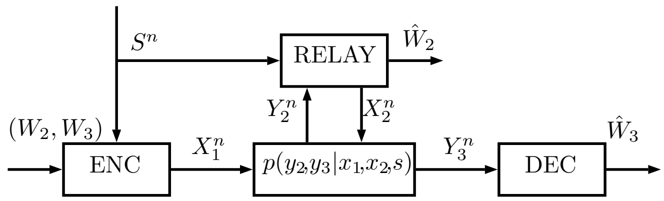

Fig. 3. Relay Broadcast setting with non-causal state

of Theorem 2, we prove the following improved inner and outer bounds to the capacity region. Suppose $\mathcal{C}$ denotes the true capacity region for the setup. Define the region $\mathcal{R}$ to be the collection of (R_2, R_3) pairs such that

$$R_2 \leq I(X_1; Y_2|U, V, X_2, S), \tag{20}$$

$$R_3 \leq \min\{I(V, X_1; Y_2|U, X_2, S), I(U, V; Y_3) - I(U, V; S). \tag{21}$$

Now let

$$\mathcal{C}' = \bigcup_{p(u,v,s,x_1,x_2) \in \mathcal{P}^i} \mathcal{R}, \tag{22}$$

$$\mathcal{C}'' = \bigcup_{p(u,v,s,x_1,x_2) \in \mathcal{P}^o} \mathcal{R}. \tag{23}$$

Theorem 6. *For the discrete memoryless state-dependent physically degraded relay-broadcast channel, the capacity region obeys the following:*

$$\mathcal{C}' \subseteq \mathcal{C} \subseteq \mathcal{C}''. \tag{24}$$

Before proving Theorem 6, we note that the best known inner bound for the setup under consideration is given by:

Theorem 7. *[11] For the discrete memoryless state-dependent physically degraded relay-broadcast channel, the capacity region is inner bounded by the union of the set of (R_2, R_3) pairs such that:*

$$R_2 \leq I(X_1; Y_2|U, V, X_2, S),$$

$$R_3 \leq \min\{I(V; Y_2|U, X_2, S), I(U, V; Y_3) - I(U, V; S)\},$$

where the union is over pmfs $p(u, v, s, x_1, x_2) \in \mathcal{P}^i$.

Remark 8. *On comparing theorems 6 and 7, it is clear that our contribution in Theorem 6 improves upon the inner bound of [11] by an additive factor of* $I(X_1; Y_2|U, V, X_2, S)$ *in the rate to the final destination.*

A. Proof of Achievability

The achievability for the rate constraints on R_3, i.e. the transmission of message W_3 to the receiver are similar to the proof of Theorem 2, using the same random codebook construction for (U, V). The difference here is that the X_1^n codewords are used to carry the message W_2 on top of W_3 using superposition coding. In particular, for each (u^n, v^n), generate 2^{nR_2} codewords by superposition coding, according to $\prod_{i=1}^n p_{X_1|U,V,S}(x_{1i}|u_i, v_i, s_i)$. The decoding rule at the relay would be to find unique x_1^n sequence such that

$$(u^n, v^n, x_2^n, s^n, x_1^n, y_2^n) \in \mathcal{T}_\epsilon^n. \tag{25}$$

The probability of decoding error at the relay goes to zero with n provided

$$R_2 \le I(X_1; Y_2|U, V, X_2, S). \tag{26}$$

This completes the proof of achievability.

B. Proof of Outer Bound

Note that the outer bounds on R_3 follow the same lines as in the proof of Theorem 2. It only remains to prove the bound on R_2. Using similar reasoning as in the proof of Theorem 2, consider the following chain of inequalities:

$$
\begin{aligned}
& nR_2 = H(W_2) = H(W_2|W_3, S^n) \\
& \le I(W_2; Y_2^n|W_3, S^n) + n\epsilon_n \\
& \le I(W_2; Y_2^n, Y_3^n|W_3, S^n) + n\epsilon_n \\
& = \sum_{i=1}^n I(W_2; Y_{2i}, Y_{3i}|W_3, S^n, Y_2^{i-1}, Y_3^{i-1}) + n\epsilon_n \\
& = \sum_{i=1}^n I(W_2; Y_{2i}, Y_{3i}|W_3, S^n, Y_2^{i-1}, Y_3^{i-1}) + n\epsilon_n \\
& = \sum_{i=1}^n I(W_2; Y_{2i}, Y_{3i}|W_3, S^n, Y_2^{i-1}, Y_3^{i-1}, X_{2i}) + n\epsilon_n \\
& = \sum_{i=1}^n I(W_2; Y_{2i}|W_3, S^n, Y_2^{i-1}, Y_3^{i-1}, X_{2i}) + n\epsilon_n
\end{aligned}
$$

$$
\begin{aligned}
&\leq \sum_{i=1}^{n} I(W_2, X_{1i}; Y_{2i}|W_3, S^n, Y_2^{i-1}, Y_3^{i-1}, X_{2i}) + n\epsilon_n \\
&= \sum_{i=1}^{n} I(X_{1i}; Y_{2i}|W_3, S^n, Y_2^{i-1}, Y_3^{i-1}, X_{2i}) + n\epsilon_n \\
&= \sum_{i=1}^{n} \{H(Y_{2i}|W_3, S^n, Y_2^{i-1}, Y_3^{i-1}, X_{2i}) \\
&\quad - H(Y_{2i}|W_3, S^n, Y_2^{i-1}, Y_3^{i-1}, X_{1i}, X_{2i})\} + n\epsilon_n \\
&\leq \sum_{i=1}^{n} \{H(Y_{2i}|W_3, S_{i+1}^n, Y_3^{i-1}, X_{2i}, S_i) \\
&\quad - H(Y_{2i}|W_3, S_{i+1}^n, Y_3^{i-1}, X_{1i}, X_{2i})\} + n\epsilon_n \\
&= \sum_{i=1}^{n} \{H(Y_{2i}|U_i, X_{2i}, S_i) - H(Y_{2i}|U_i, V_i, X_{1i}, X_{2i})\} + n\epsilon_n \\
&= \sum_{i=1}^{n} I(X_{1i}; Y_{2i}|U_i, V_i, X_{2i}, S_i) + n\epsilon_n,
\end{aligned} \tag{27}
$$

on identifying $U_i = (Y_3^{i-1}, S_{i+1}^n)$ and $V_i = (W_3, Y_3^{i-1}, S_{i+1}^n)$. This completes the proof of outer bound.

VI. Conclusion

We studied a state dependent relay channel with state non-causally known at both the sender and the relay nodes. An improved lower bound was derived using a modified decode-forward scheme. An outer bound was also shown. The characterization was then extended to give an improved inner bound and an outer bound for a relay broadcast setting. It would be interesting to explore whether the bounds presented are tight.

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