Analog Switched-Capacitor Low-Pass Filter (SC LPF) Using Different Approximations (Butterworth, Chebyshev and Elliptic)

Ara Abdulsatar Assim [1,2]

ORCID: 0000-0002-6055-9223
Email: ara.assim@su.edu.krd

[1] Department of Electrical Engineering, Salahaddin University-Erbil, 44001 Erbil, Iraq
[2] Department of Integrated Electronics, Peter the Great St. Petersburg Polytechnic University, 195251 St. Petersburg, Russia

Abstract

This work demonstrates the process of designing and simulating analog low-pass filters according to some given specifications, their conversion to active filters and switched-capacitor filters are also provided.

1. Prototype filter design

1.1 The task

It is necessary to synthesize a low-pass filter with the following characteristics by the method of elemental simulation:

$F_c = 5$ kHz – Cutoff frequency

$F_S = 6$ kHz – Stopband frequency

$a_S \geq 30$ dB – Stopband attenuation

$\Delta \leq 0.2$ dB – Passband ripple

1.2 Prototype filter design

The following are operations for normalizing the cutoff frequency and determining the reflection coefficient.

Normalization:

$$\Omega_s = F_S/F_D = 6000/5000 = 1.2$$

Determining the reflection coefficient

$$\rho = \sqrt{1 - e^{-0,23026 a_D}} = 0,212$$

$$\alpha(\rho) = 10\lg(\rho^{-2} - 1) = 13.27\ dB$$

$$a_s + \alpha(\rho) = 30 + 13,27 = 43,27\ dB$$

$$\frac{1}{\Omega_s} = \frac{1}{1,2} = 0,83333$$

Next, let us estimate the filter order according to the dependency graphs

a_S+ $\alpha(\rho)$ on Ω_s:

- For Butterworth approximation filter: filter order is 25 (maximum is 15)
- For Chebyshev approximation filter: filter order is 9
- For Zolotarev-Cauer /elliptic approximation: filter order is 5

The normalized values of ladder elements are shown in the table below:

C_P	L_P	C_T	L_T	C_C	L_C	C_{pC}
0.188043	0.555911	1.34884	1.40489	1.066663	1.059602	0.312879
0.889483	1.203743	2.274466	1.550862	1.494707	0.633204	1.026636
1.455394	1.843437	2.339126	1.550862	0.690164		
1.759654	1.798966	2.274466	1.40489			
1.759654	1.843437	1.34884				
1.455394	1.203743					
0.699483	0.555911					
0.188043						

Table 1. Normalized values of the ladder circuit

It is necessary to de-normalize the normalized values. we assume that:

$$R_s = R_l = 1K\ Ohms$$

$$n_z = 1000$$

$$n_\omega = 2\pi F_C = 31415\ rad/sec$$

An example of calculating the values (denormalization) for cauer filter:

$$C_{C\_1} = C_{kC\_1}\frac{1}{n_\omega n_z} = 1.066663 * 31.85\ nF = 33.95\ nF$$

$$C_{C\_2} = C_{kC\_2}\frac{1}{n_\omega n_z} = \ 1{,}494707 * 31{,}85\ \text{nF} = \ 47{,}56\ \text{nF}$$

$$C_{C\_3} = C_{kC\_3}\frac{1}{n_\omega n_z} = \ 0{,}690164 * 31{,}85\ \text{nF} = \ 21{,}97\ \text{nF}$$

$$L_{C\_1} = L_{kC\_1}\frac{n_z}{n_\omega} = \ 1{,}059602 * 31{,}85\ \text{mH} = \ 33{,}73\ \text{mH}$$

$$L_{C\_2} = L_{kC\_2}\frac{n_z}{n_\omega} = \ 0{,}633204 * 31{,}85\ \text{mH} = \ 20{,}16\ \text{mH}$$

$$C_{pC\_1} = C_{kpC\_1}\frac{1}{n_\omega n_z} = \ 0{,}312879 * 31{,}85\ \text{nF} = \ 9{,}96\ \text{nF}$$

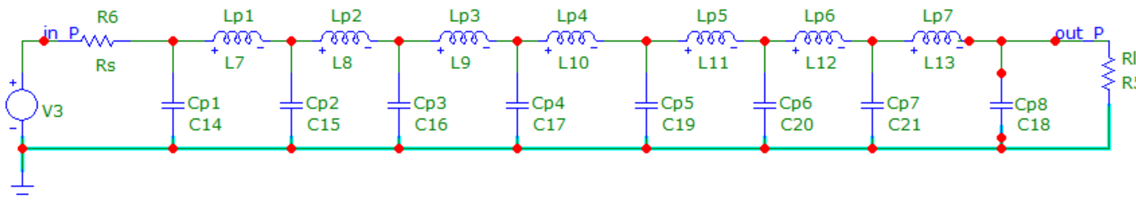
$$C_{pC\_2} = C_{kpC\_2}\frac{1}{n_\omega n_z} = \ 1{,}026636 * 31{,}85\ \text{nF} = \ 32{,}68\ \text{nF}$$

The table below shows the actual (de-normalized) values of ladder elements.

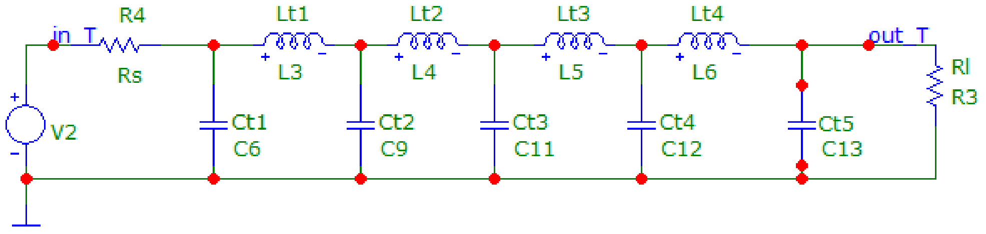
C_P. nF	L_P. mH	C_T. nF	L_T. mH	C_C. nF	L_C. mH	C_{pC}. nF
5.985594593	17.69519671	42.93491069	44.7190376	33.95293781	33.7281792	9.959247888
28.31312325	38.31632973	72.39850136	49.36547067	47.5780015	20.15550932	32.67883883
46.32662985	58.67842217	74.45669308	49.36547067	21.96860243		
56.01152645	57.26286627	72.39850136	44.7190376			
56.01152645	58.67842217	42.93491069				
46.32662985	38.31632973					
22.26523541	17.69519671					
5.985594593						

Table 2. De-normalized values of the ladder circuit

The schemes of all the three filters are shown below:



P15



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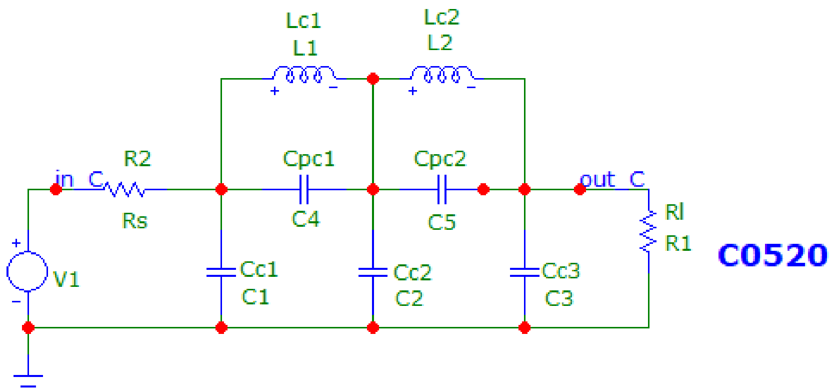

Figure 1 – Filter prototypes' schemes

And below is shown the magnitude reponse for the three types of filters given above.

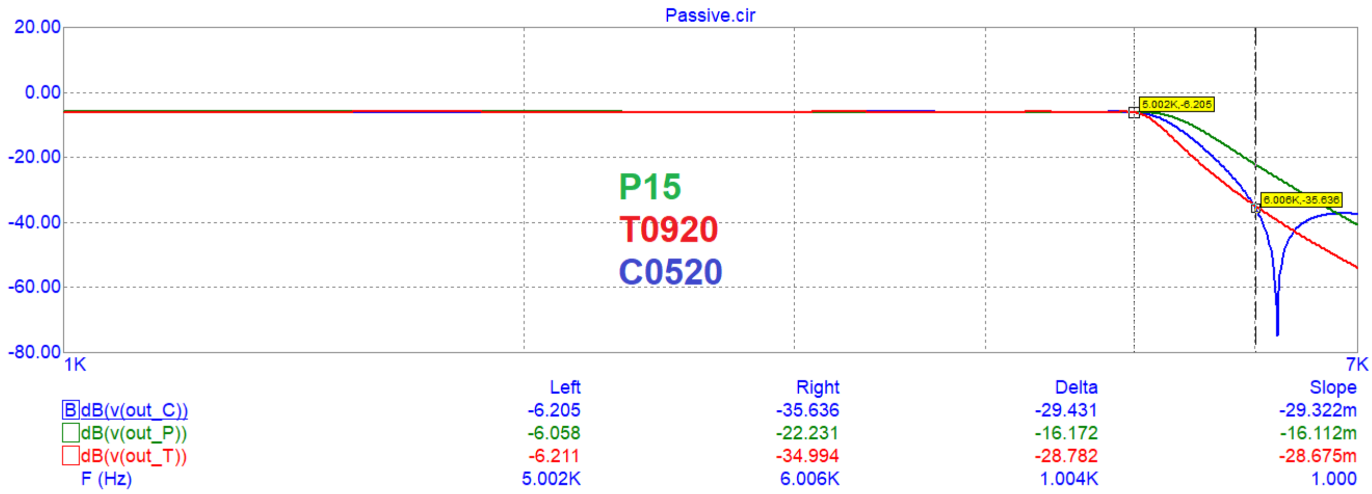

Figure 2 – Magnitude reponse of the three passive filters

Figure 3 shows the maximum values of flatness in the passband of prototype filters.

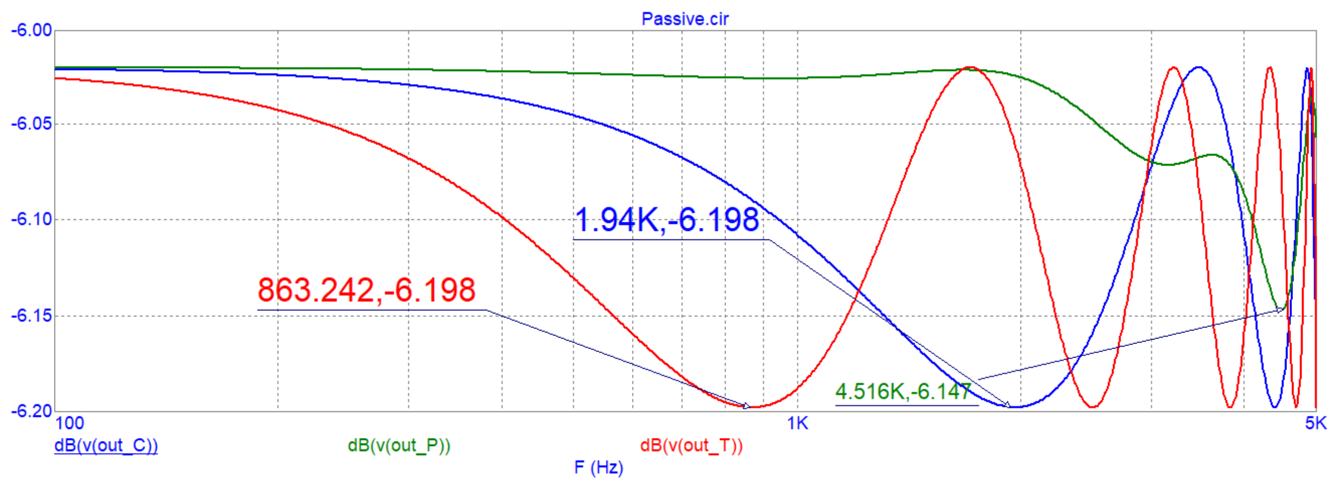

Figure 3 – Maximum values of flatness in the passband of prototype filters

The results from filters prototypes satisfy the requirement - $\Delta \leq 0.2$ dB.

Figure 4 below shows the magnitude response values of the prototype filters at the passband and stopband frequencies.

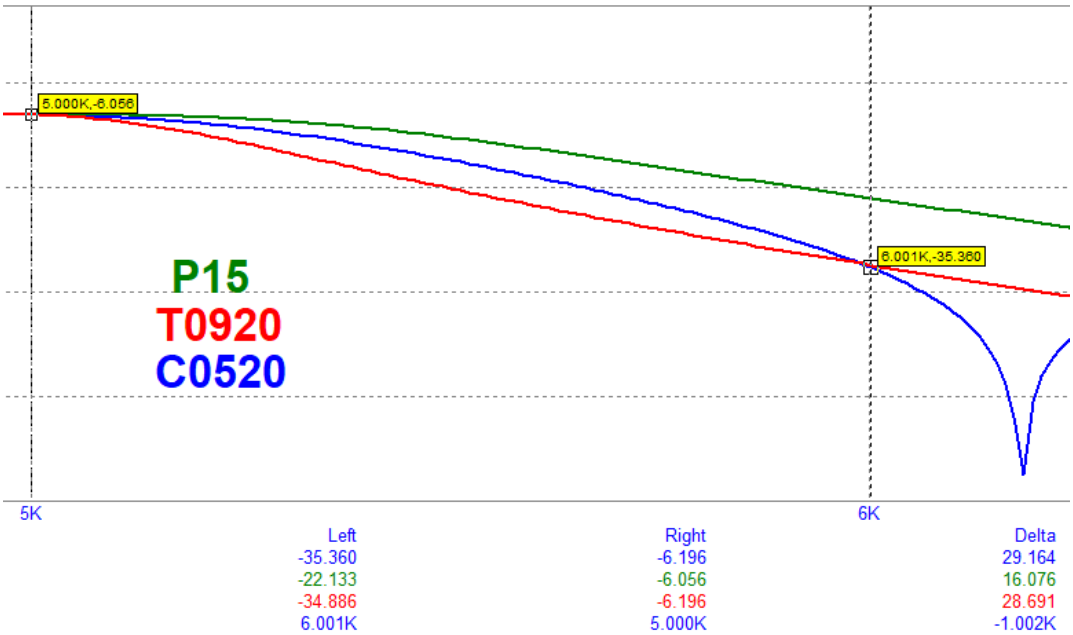

Figure 4 – Magnitude response values of the filters at pass and stop band frequencies

Two filters, namely (Chebyshev and Cauer) satisfiy the condition - $a_S \geq 30\ dB$. Butterworth doesn’t which is expected because the filter order is 15 and to achieve $a_S \geq 30\ dB$ then we need to use order 25 butterworth filter, which is not practical to implement.

The figures below (5,6 and 7) show pole-zero diagram for all the three filters (Butterworth, Chebyshev and Cauer).

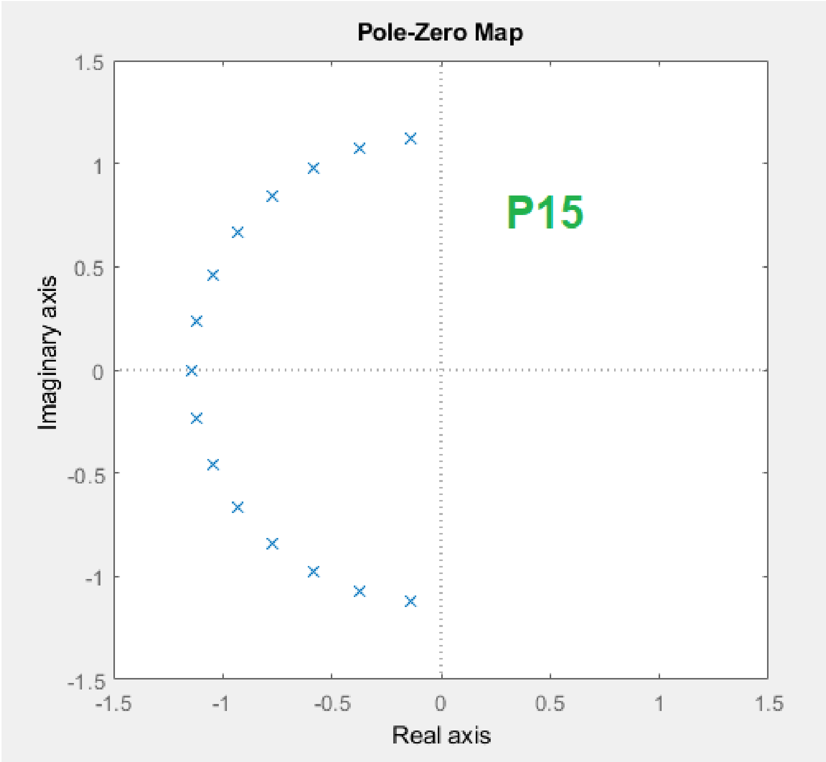

Figure 5 – Pole-zero diagram for Butterworth filter (order 15)

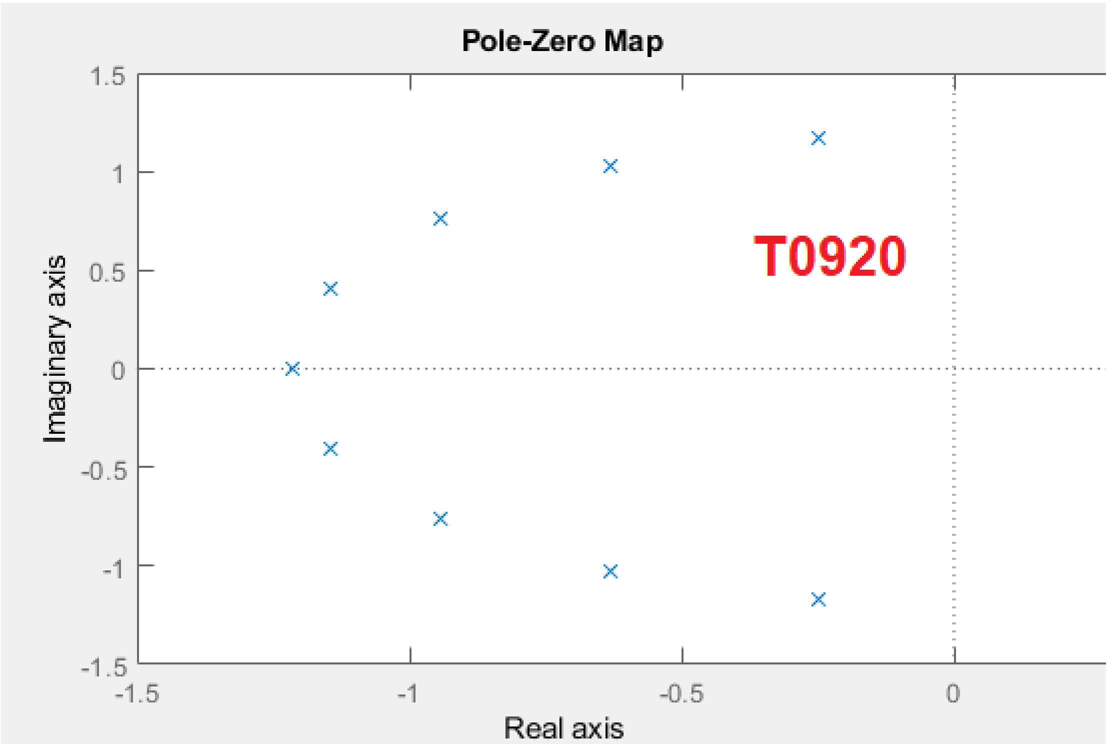

Figure 6 – Pole-zero diagram for Chebyshev filter (order 9)

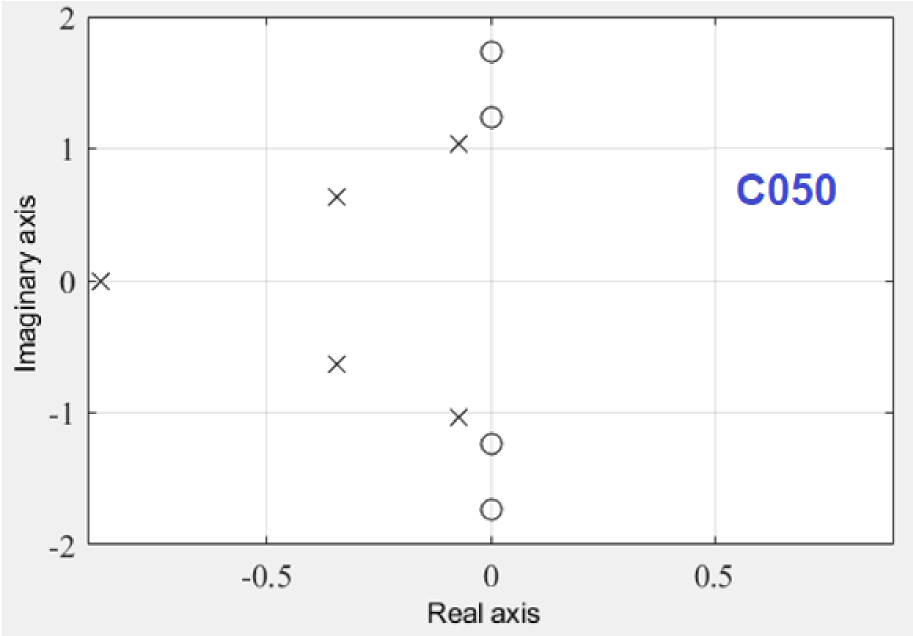

Figure 7 – Pole-zero diagram for Cauer filter (order 5)

2. Synthesis of ARC filter based on LPF prototype

2.1 Realization of active RC filter

To implement the ARC filter, we use the prototype for the Chebyshev approximation presented in Figure 9, as well as Cauer approximation presented in Figure 12.

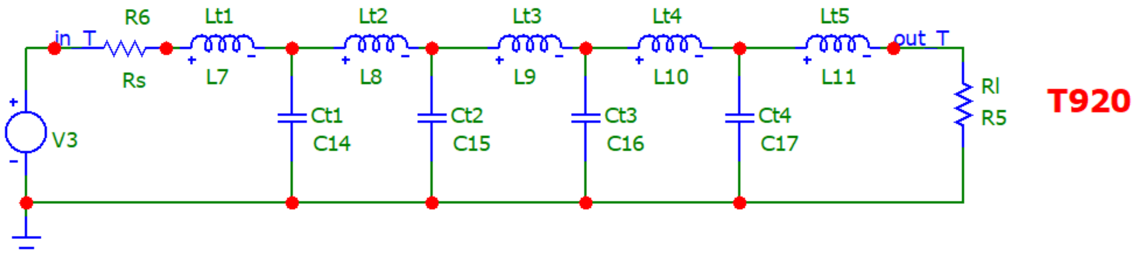

Figure 8 - Structure of the filter prototype (Chebyshev)

Transfer functions of the scheme components are given below:

$$Y_1 = \frac{1}{R_S + pL_1}$$

$$T_1 = \frac{1}{\frac{R_S}{R_n} + \frac{pL_1}{R_n}}$$

$$Z_2 = \frac{1}{pC_2}$$

$$Y_3 = \frac{1}{pL_3}$$

$$Z_4 = \frac{1}{pC_4}$$

$$Y_5 = \frac{1}{pL_5}$$

$$Z_6 = \frac{1}{pC_6}$$

$$Y_7 = \frac{1}{pL_7}$$

$$Z_8 = \frac{1}{pC_8}$$

$$Y_9 = \frac{1}{R_l + pL_9}$$

$$T_2 = -\frac{1}{pC_2R_n}$$

$$T_3 = \frac{1}{\frac{pL_3}{R_n}}$$

$$T_4 = -\frac{1}{pC_4R_n}$$

$$T_5 = \frac{1}{\frac{pL_5}{R_n}}$$

$$T_6 = -\frac{1}{pC_6R_n}$$

$$T_7 = \frac{1}{\frac{pL_7}{R_n}}$$

$$T_8 = -\frac{1}{pC_8R_n}$$

$$T_9 = \frac{1}{\frac{R_l}{R_n} + \frac{pL_9}{R_n}}$$

Using the formulas shown above, the circuit below was implemented using operational amplifiers, resistors and capacitors:

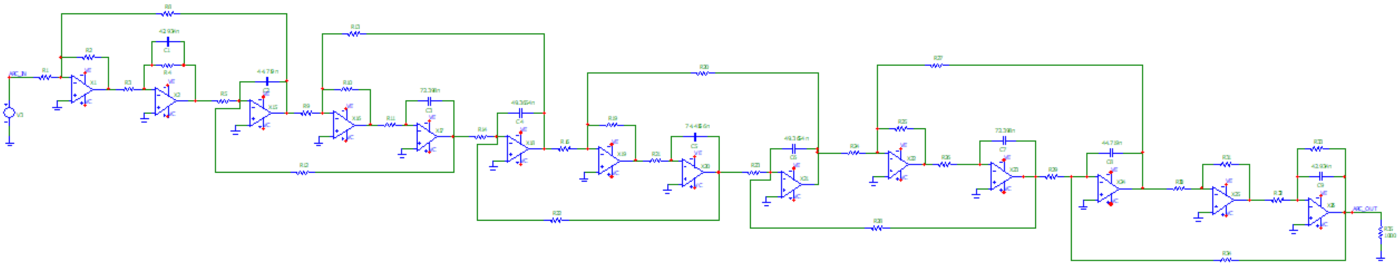

Figure 9 - Structure of the active Chebyshev RC (ARC) filter

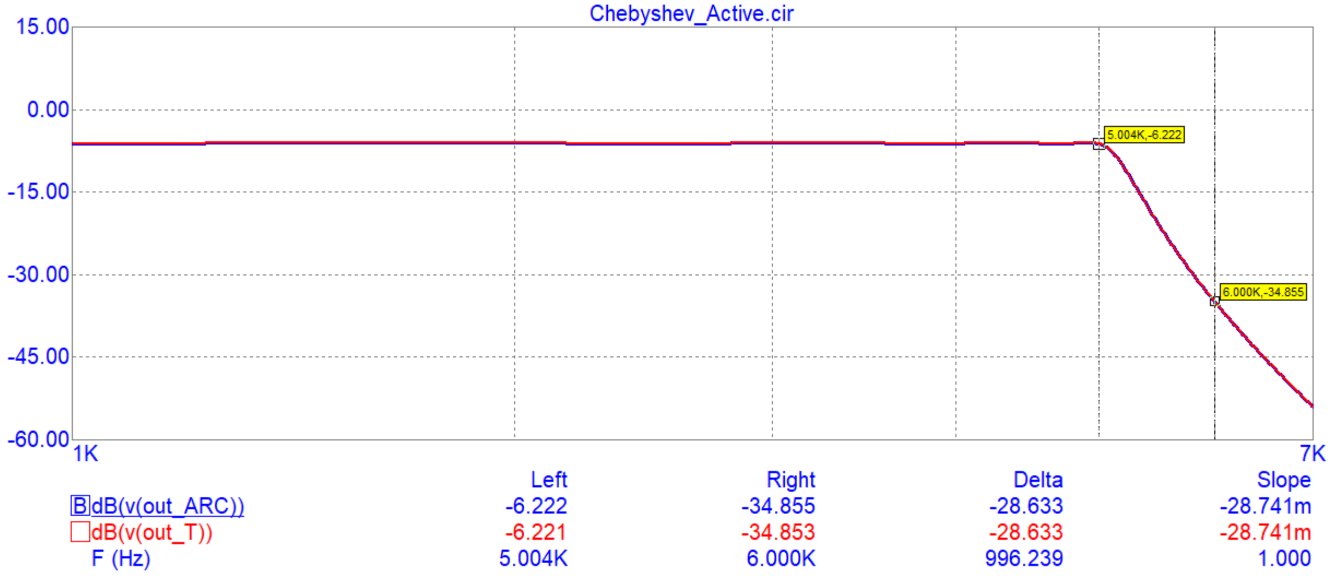

Figure 10 – Amp. Response comparison of active and passive Chebyshev filter

And below (Figure 11) shows that the amplitude-frequency characteristics completely coincide with the previous passive filter design and the given condition in the task, the condition is passband ripple $\Delta \leq 0.2$ dB, this means the design of the ARC filter is successful.

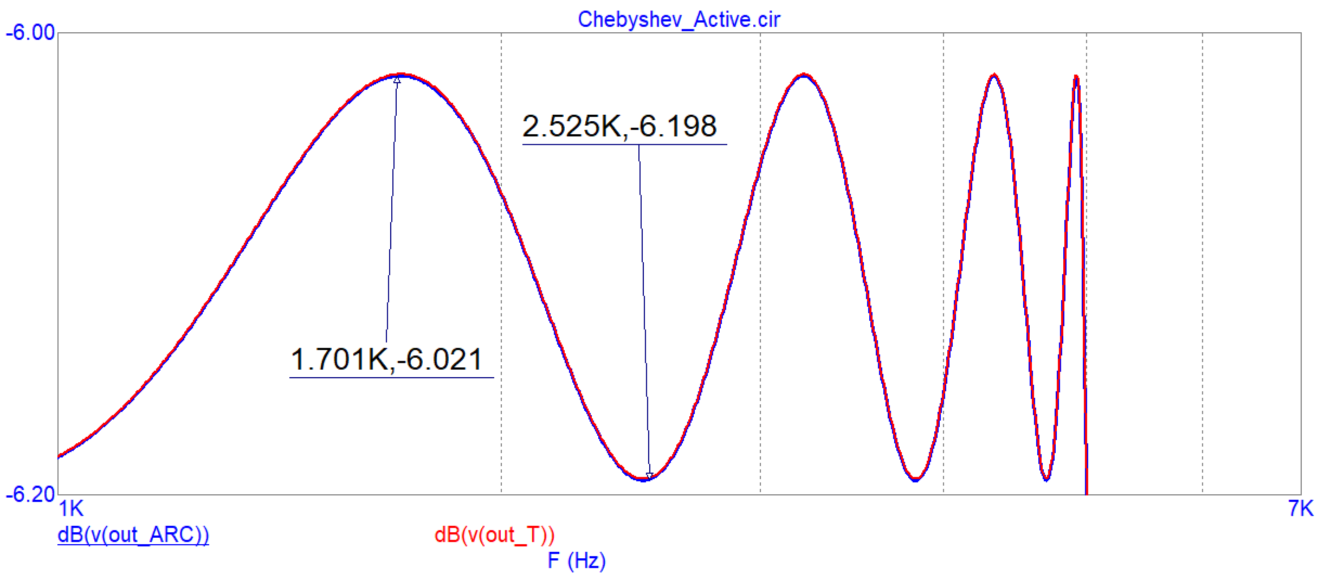

Figure 11 - Unevenness in the passband frequency range (Chebyshev)

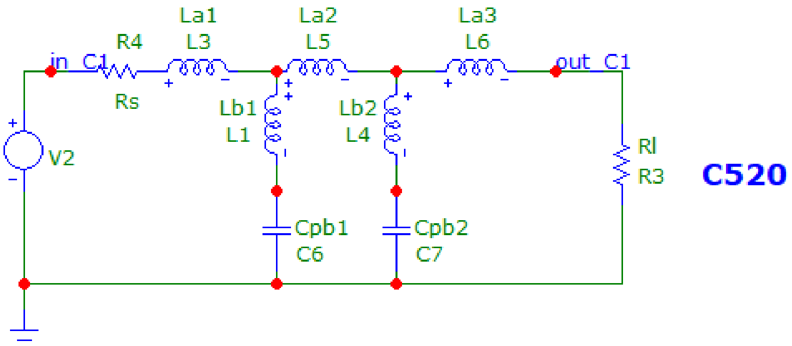

Figure 12 - Structure of the filter prototype (Cauer)

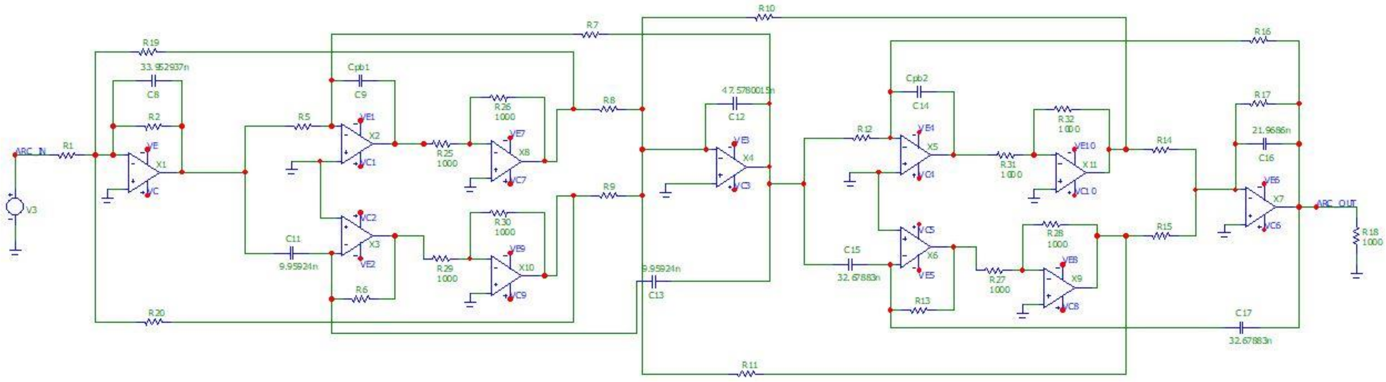

Figure 13 - Structure of the active Cauer RC (ARC) filter

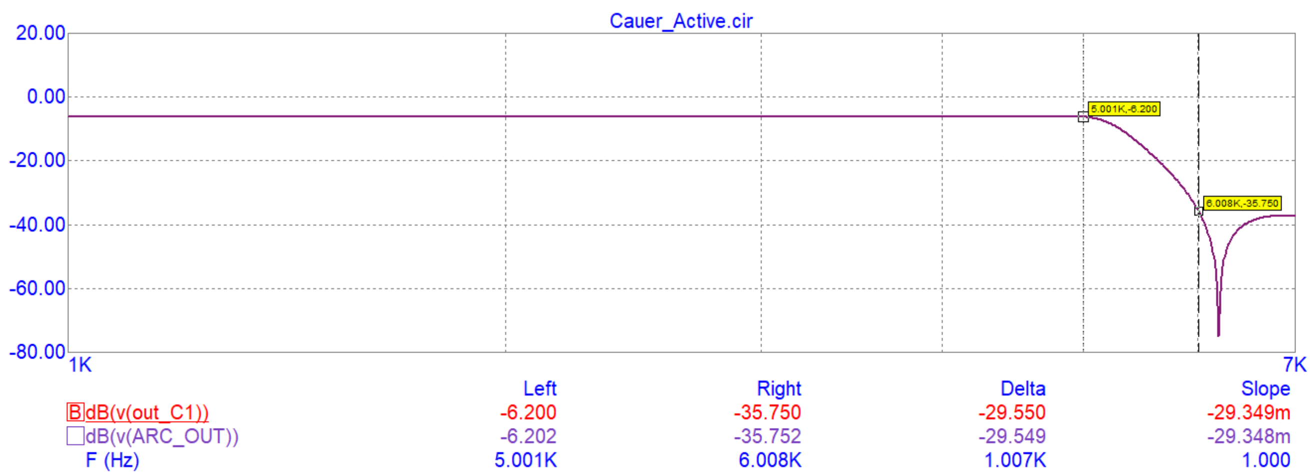

Figure 14- Amp. Response comparison of active and passive Cauer filter

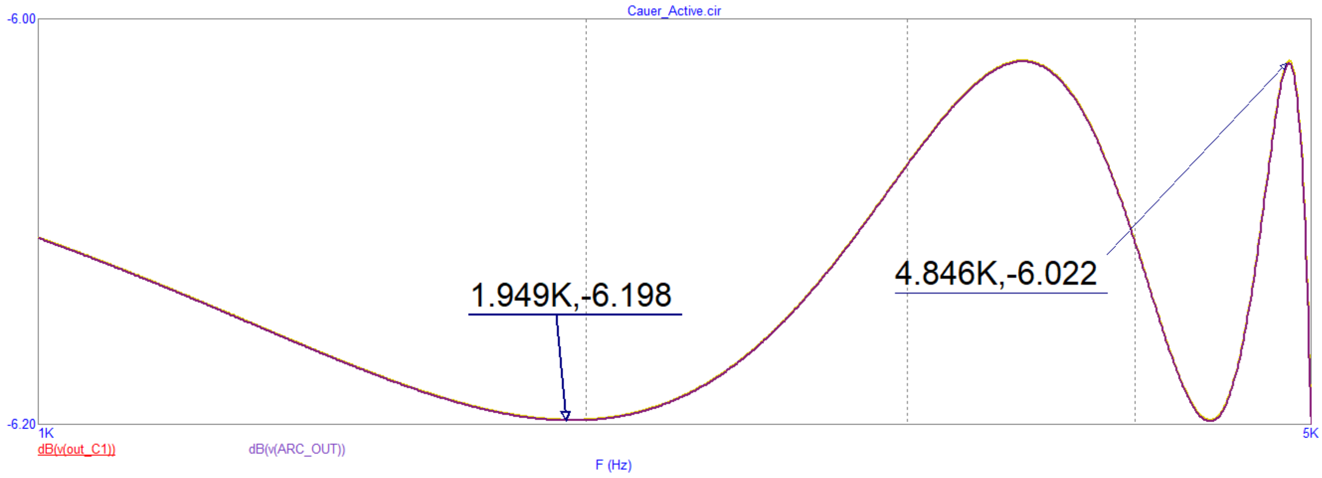

Figure 15 - Unevenness in the passband frequency range (Cauer)

3. Switched capacitor LPF

The clock frequency is chosen equal to 1 MHz. For 1 MHz, the capacitance used to replace the resistors is:

$$C = \frac{1}{fR} = \frac{1}{10^6 * 10^3} = 1 * 10^{-9} = 1\ \mathrm{nF}$$

Using Laker's models, the transfer functions of each part were obtained and the values of capacitors in the feedback of integrators were calculated:

$$H_1^{11}(z) = \frac{C_1 z^{-1}}{C_2 + C_5 - C_2 z^{-1}} \approx \frac{1}{\frac{C_5}{C_1} + T_c \frac{C_2 + C_5}{C_1} p} = T_{1arc}(p) = \frac{1}{\frac{R}{R_0} + pCR}$$

Inverting and non-inverting integrators:

$$H_{2\,3}^{11}(z) = \frac{C_3 z^{-1}}{C_4 - C_4 z^{-1}} \approx= \frac{1}{T_c \frac{C_4}{C_3} p} = T_{2arc}(p) = \frac{1}{pCR}$$

Figures 16 – 19 below show different segments of the switched capacitor filter.

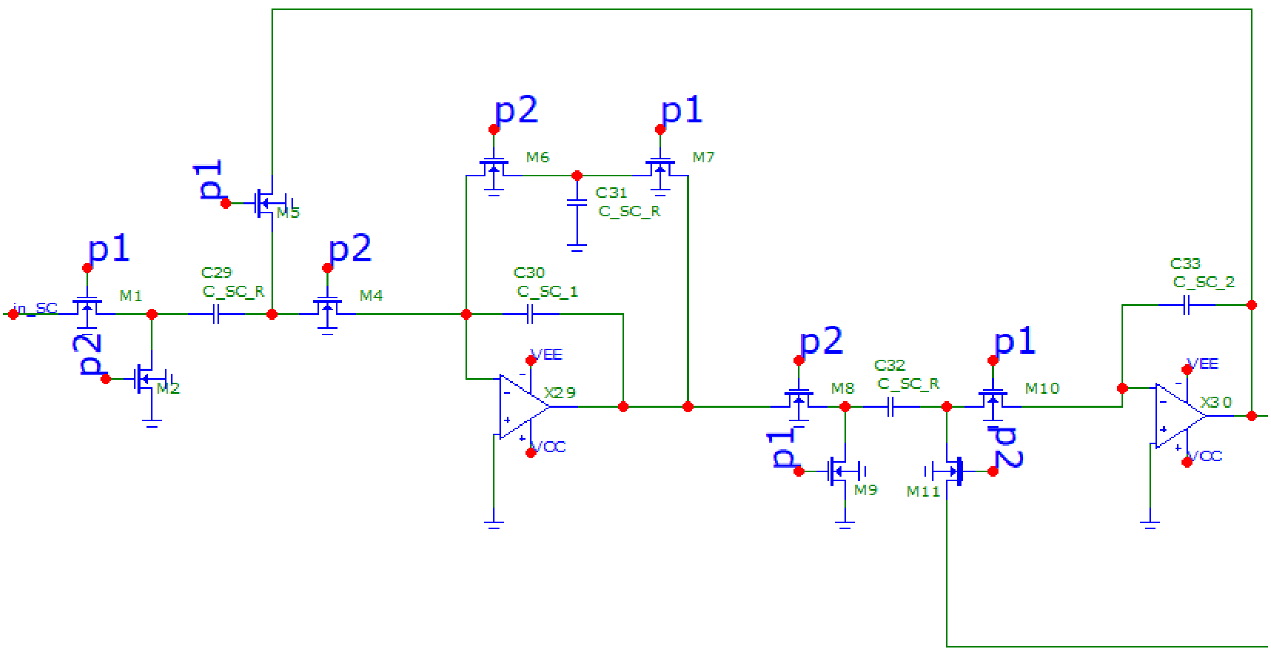

Figure 16 – First segment of the SC filter

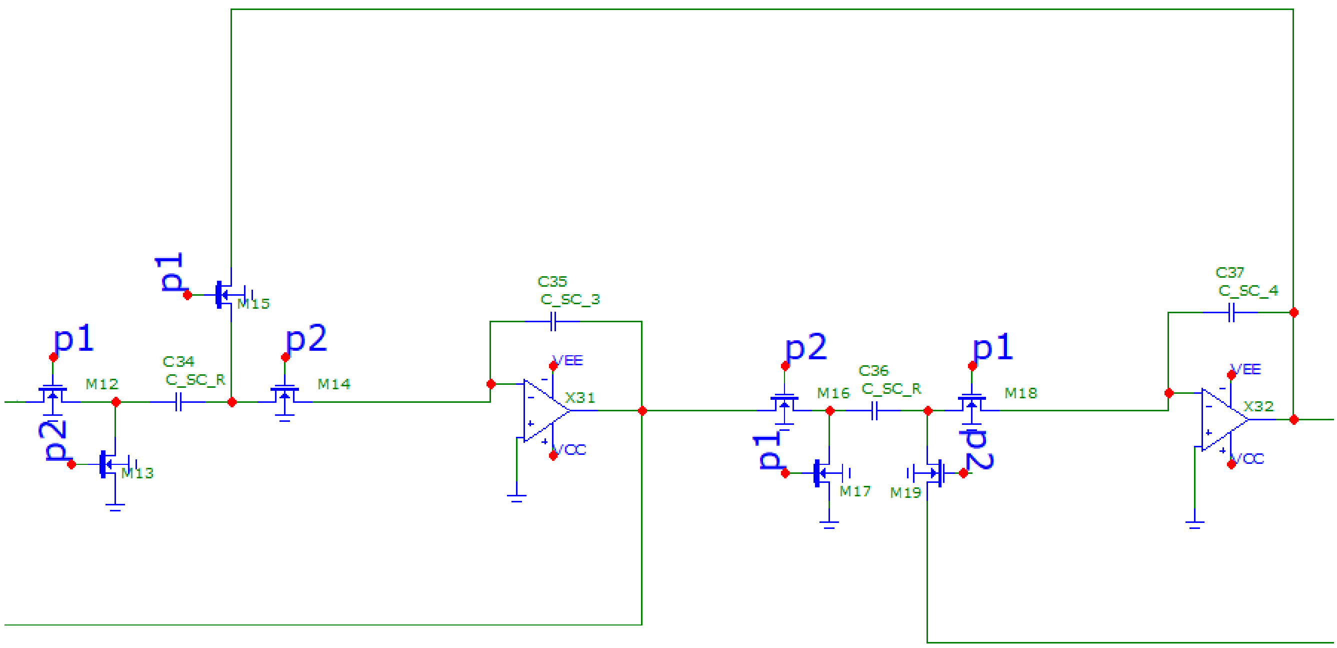

Figure 17 – Second segment of the SC filter

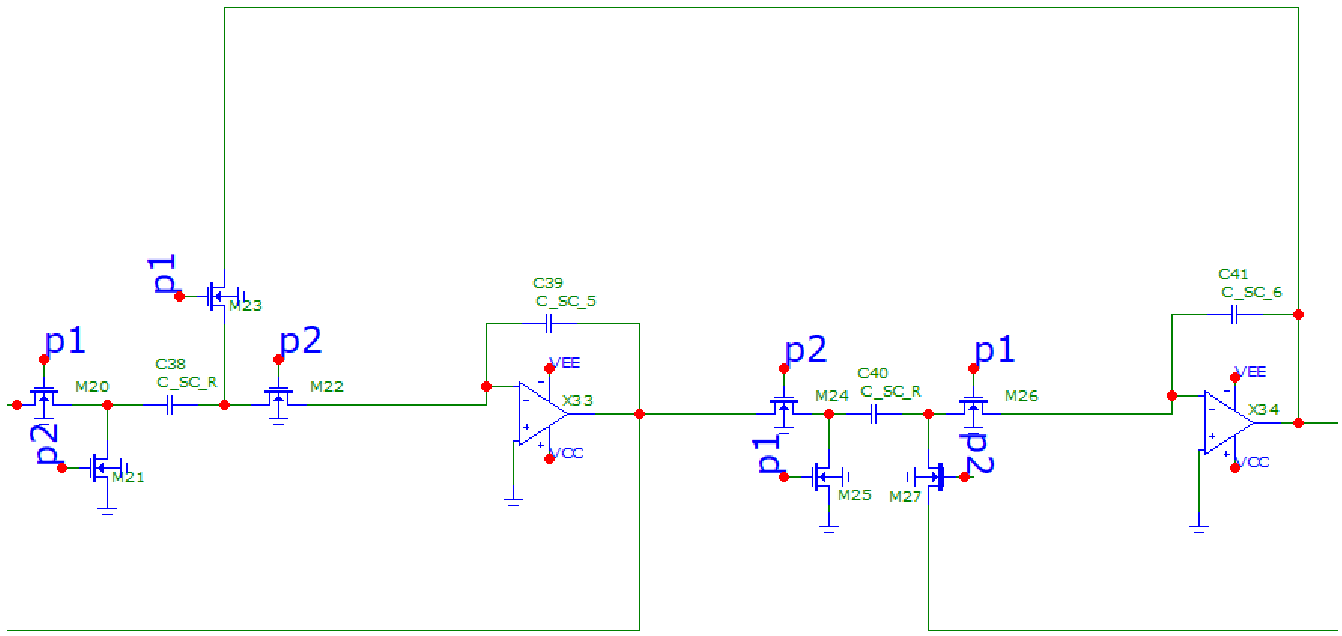

Figure 18 – Third segment of the SC filter

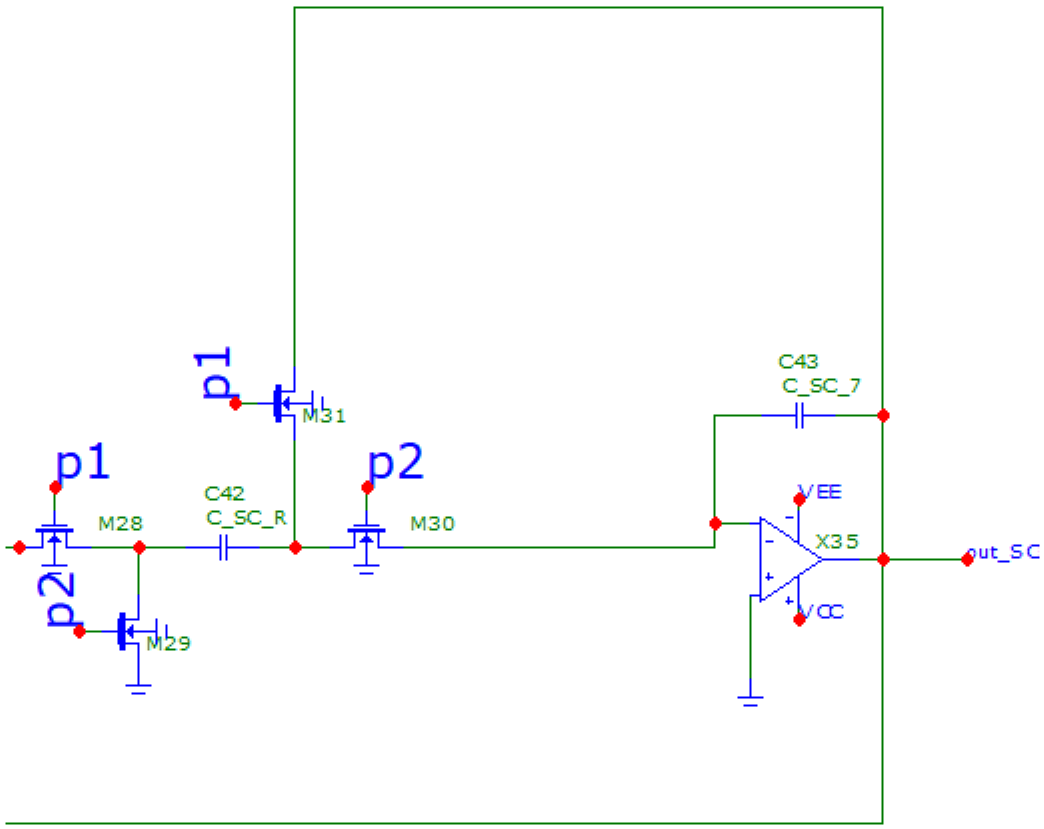

Figure 19 – Fourth (last) segment of the SC filter

To control (drive) the SC filter switches, it is necessary to generate two clock signals shifted relative to each other by a quarter of a period. Switching frequency is 1 MHz. Figure 20 shows the timing diagrams of the clock signals.

The input source is the sum of harmonic signals with frequencies from 0.5K to 8 KHz in 500 Hz steps. Figure 21 shows a schematic of the input source.

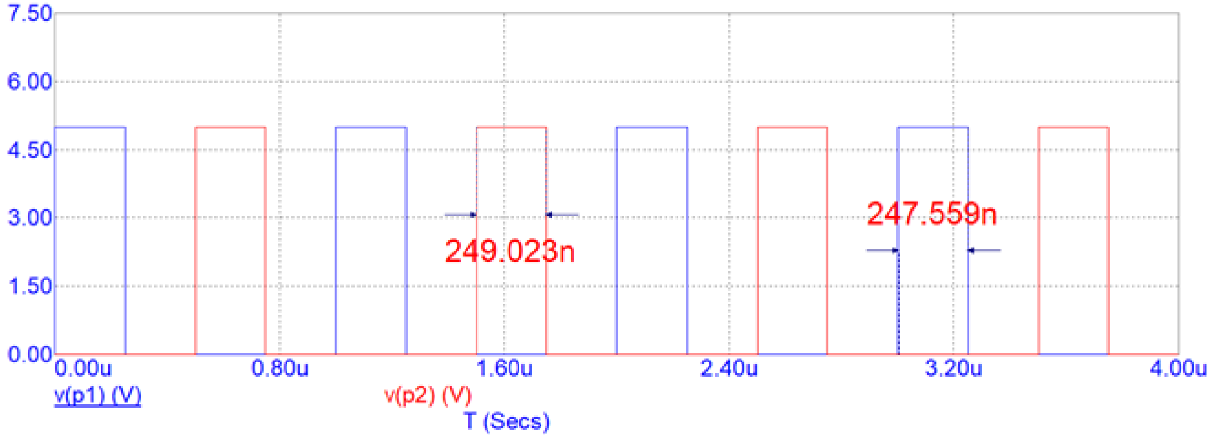

Figure 20 – The input clock signals (p1 and p2)

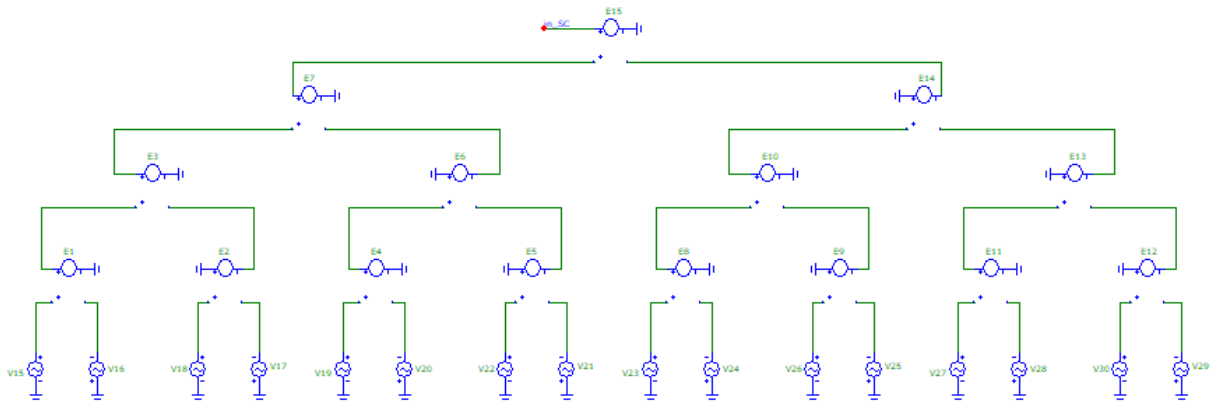

Figure 21 – The input signal sources (sum of harmonics)

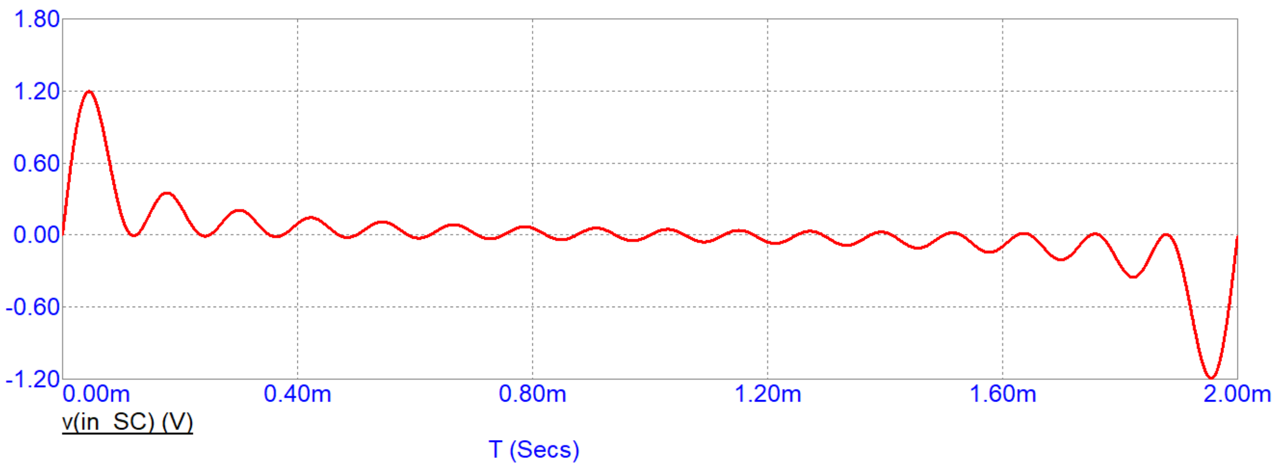

Figure 22 – The input signal (in time domain)

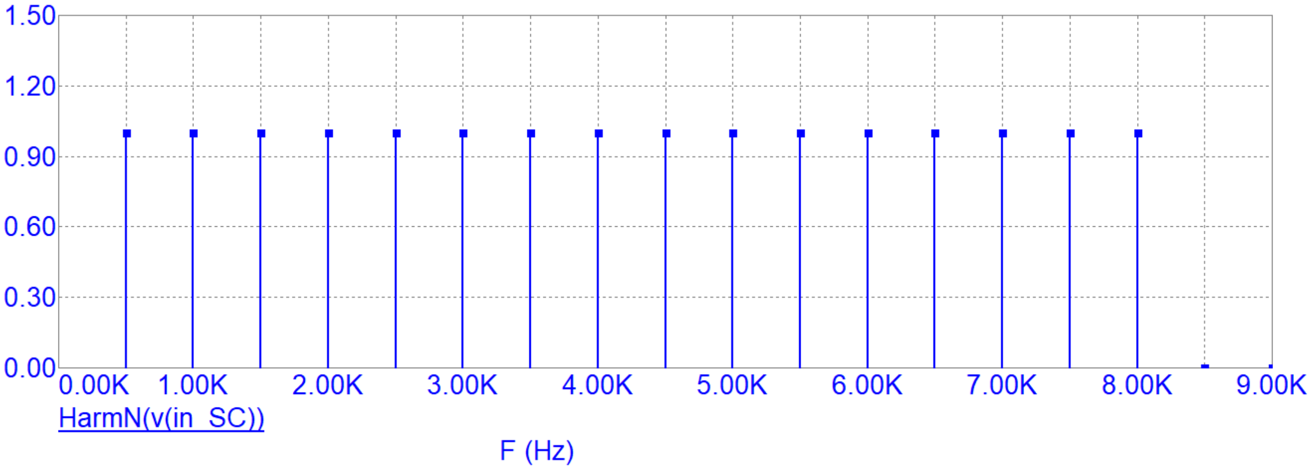

Figure 23 – Spectrum of the input signal (harmonics)

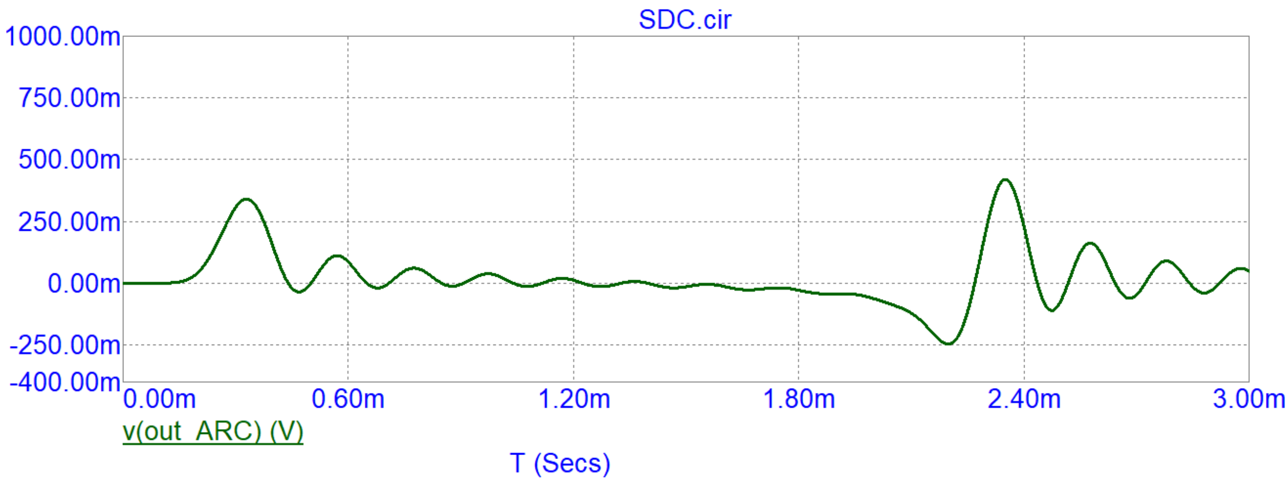

Figure 24 – Active RC filter output in time domain

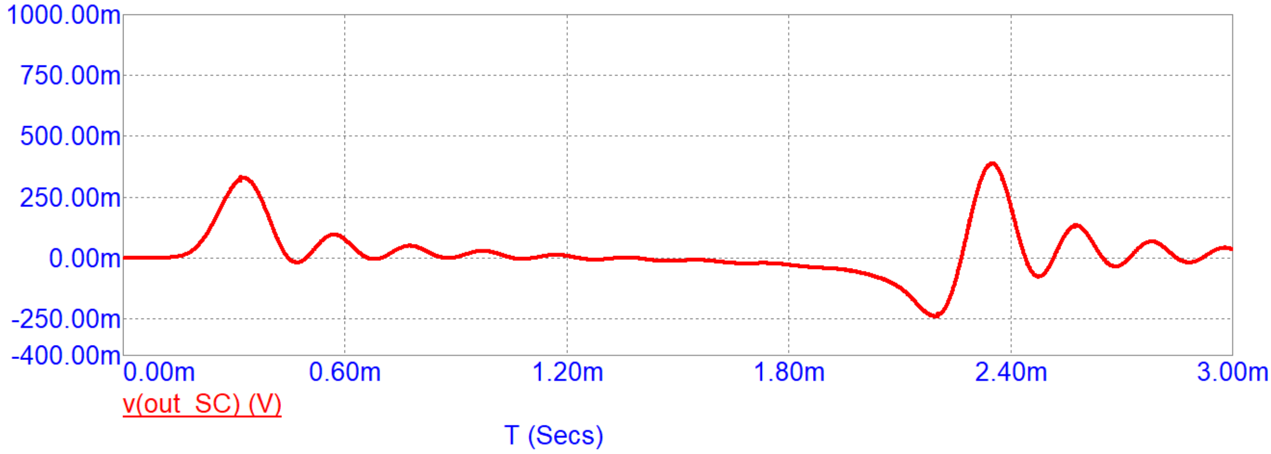

Figure 25 – Switched capacitor filter output in time domain

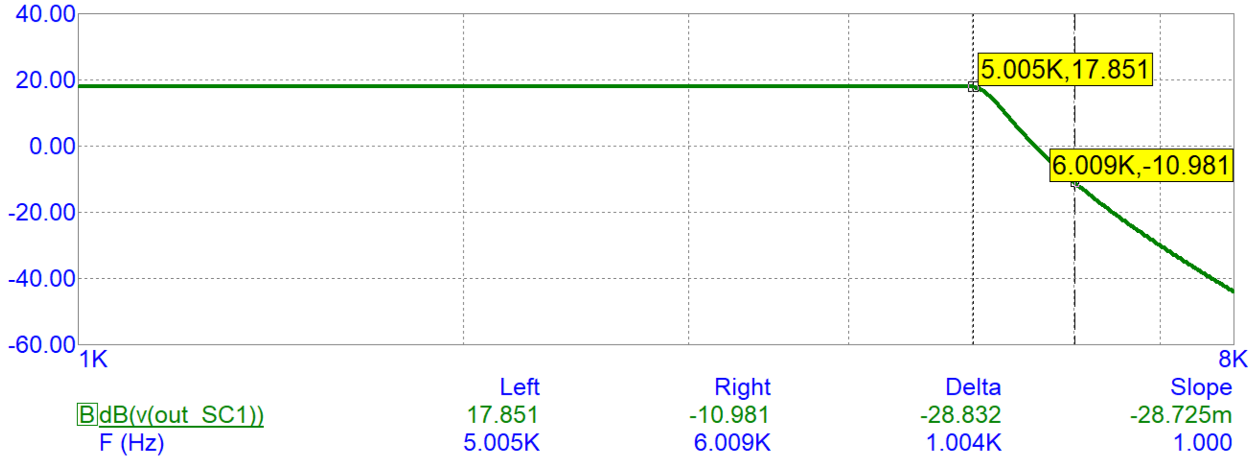

Figure 26 – Switched capacitor filter amplitude response

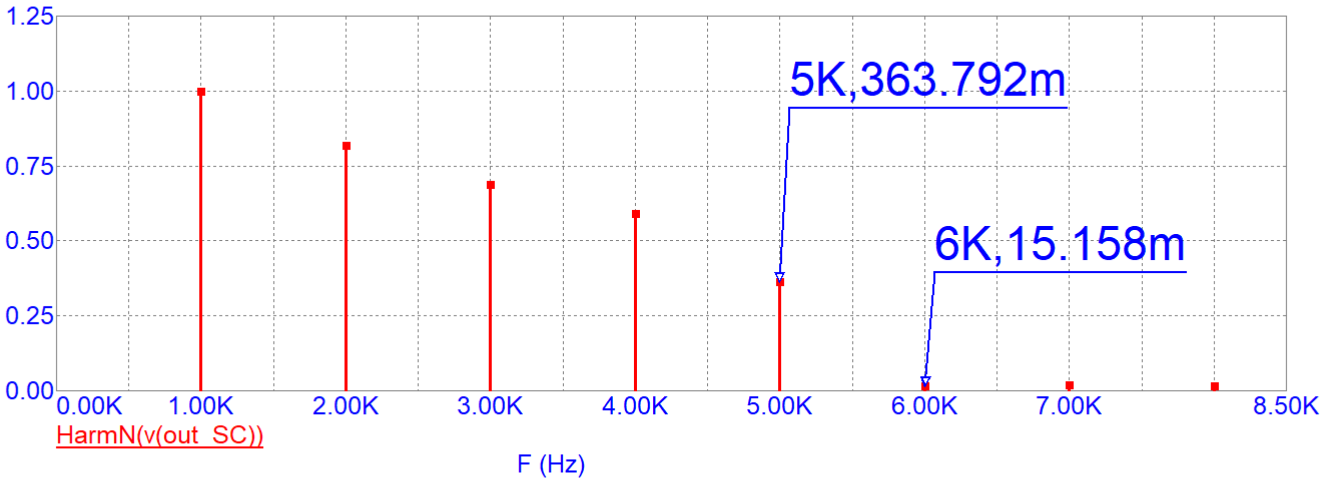

Figure 27 – Switched capacitor output spectrum (harmonics)

4. Conclusion

In conclusion, several different LPF configurations were taken into consideration in this student work, different low-pass filters (active, passive and switched capacitor LPF) were designed and implemented meeting the technical requirements given in the individual task (please refer to page 3).

5. References

1. A. S. Korotkov «Switched-capacitor filter design», «St. Petersburg Polytechnic University Publishing House», 2014.

2. Rudolf Saal, Handbook of filter design, Berlin, Germany, 1983.