Thevenin equivalent circuit components of the silver oxide-zinc battery in energy generation and absorption phases

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***Abstract*— The paper concerns the determination of the Thevenin equivalent circuit components of the 1.55V, 24mAh Ag_2O-Zn button battery using a transient circuit analysis method. This battery is used to power small profile packaged microcircuits in medical devices and wearable bioelectronics. For a typical battery at full charge, the series resistance, contact resistance and double layer capacitance are respectively, 28.6Ω, 10.5Ω and 0.06F in the power absorbing phase, and 35Ω, 10.5Ω and 0.08F in the power generating phase. For a given battery, the internal series resistance in the generating phase is greater than the corresponding resistance in the absorbing one. This applies to batteries at both 100% and 70% state of charge (SOC). At 70% SOC the series resistances in both absorbing and generating phases fall to about half of their respective values at 100% SOC. The power dissipated in the battery during generation is greater than during absorption. The electrode double layer capacitance is in the 0.06F to 0.1F range. Transient time constants vary from ~0.3 to ~0.9 seconds. The resulting equivalent model is useful in the analysis of battery operated microcircuits in providing insight into their dc and transient responses.**

***Index Terms*—battery circuit components, transient circuit analysis, Ag_2O-Zn battery**

1. Introduction

The aim of the present work is the determination of the Thevenin equivalent circuit components of the Ag_2O-Zn button battery. With its small size and steady voltage this battery is a good choice to power small profile microcircuits in medical devices and wearable bioelectronics. Battery specification sheets give the equivalent circuit as an open circuit voltage in series with a resistor. A more detailed description of the battery is given in the present work where the battery equivalent circuit consists of the open circuit battery voltage in series with a resistor (the series resistor, representing the internal resistance of the battery) in series with the parallel combination of a resistor (the contact resistance, representing the resistance of the plate-electrolyte contact) and a capacitor (representing the electrode double layer capacitance) [1]-[4]. This model enables a more detailed design and analysis of battery powered circuits, and results in additional insight into the dc and transient response of the battery circuit under load. The relevance of the Thevenin model to the circuit time constant during the transient response, and to the electrical energy dissipated in the cell in the subsequent steady state response, is discussed here. Moreover, in battery powered circuits, the battery, whether primary or secondary, can, depending on load conditions, act as a generator or an absorber of energy. Equivalent circuit components of the battery were determined here for both the generator and absorber phases. The components were determined by measuring the battery voltage as a function of time, as the battery was switched from the absorbing to the generating mode, with constant current flowing, respectively, into or out of the positive terminal.

2. Theory of Transient Analysis Method

The batteries studied were 1.55V, 24mAh silver oxide-zinc button cells used in watches and hearing aids. The cathode is monovalent silver oxide Ag_2O to which graphite is added to increase conductivity. The anode is zinc powder mixed with a gelled alkaline electrolyte solution. Separators between the anode and cathode, such as cellulose, allow the electrolyte to pass through but minimize the transport of silver ions to the anode. These are primary batteries. Therefore, the terms absorbing and generating were used here, rather than the analogous charging and discharging ones used for secondary batteries. This is because although generating and discharging apply synonymously to both primary and secondary cells, no charging takes place in primary ones where the input power is absorbed and dissipated only as heat.

The temporal variation of the battery voltage for the battery under constant current, in generation and absorption, was described using the method of transient circuit analysis [5]. The resulting equations were used to calculate the electrical resistance of the bulk ionic electrolyte, the contact resistance and the double layer capacitance in generation and absorption.

2i. The Power Absorbing Phase

The equivalent circuit of the battery before and during the absorbing phase appears to the left of terminals *a* and *b* in Fig. 1. It consists of the open circuit battery voltage V_{oc}, the series resistor R_1, the contact resistor R_2 and the capacitor C_1.

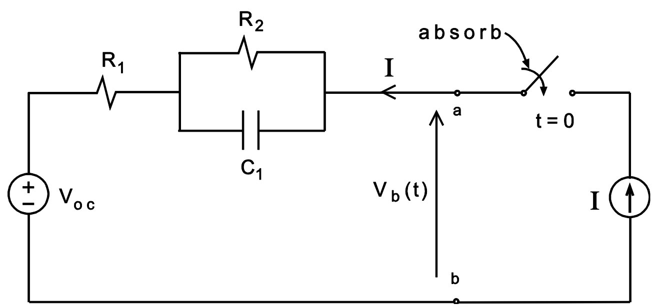

Fig. 1. The battery equivalent circuit before and during the energy absorbing phase. In the latter case the constant current I enters the positive terminal of the battery. $v_b(t)$ is the battery voltage.

$v_b(t)$ is the battery voltage at time *t*. Before the switch is closed $v_b(t \le 0) = V_{oc}$. The switch is then closed at time *t*=0 and the absorption phase starts as conventional constant current I starts to flow into the positive terminal of the battery. The temporal variation of the battery voltage, following the closing of the switch, is a transient given by

$$v_b(t) = V_{oc} + IR_1 + IR_2[1 - \exp(-t / R_2C_1)] \quad (1)$$

and is shown as the absorption phase in Fig 2.

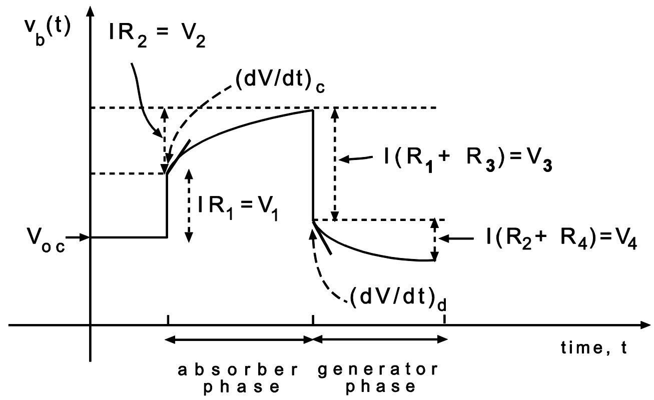

Fig. 2. The battery terminal voltage as a function of time during energy absorption and generation phases.

The abrupt change in the battery voltage at the start of the absorption phase, at t=0, is

$$IR_1 = V_1 \tag{2}$$

from which R_1 is calculated.

The change in the battery voltage from t=0 to when the transient reaches its steady state is

$$IR_2 = V_2 \tag{3}$$

from which R_2 is calculated.

The capacitance C_1 is derived from (1) as

$$C_1 = \frac{I}{(dv_b / dt)_c} \tag{4}$$

where $(dv_b / dt)_c$ is the gradient of the battery voltage at the start of the absorption transient.

2ii. The Power Generating Phase

The absorbing battery voltage will reach 99% of its final steady state voltage in $5R_2C_1$ seconds. At or after this time, the switch is thrown to start the battery generating (discharge) phase with a constant current of I (see Fig.3).

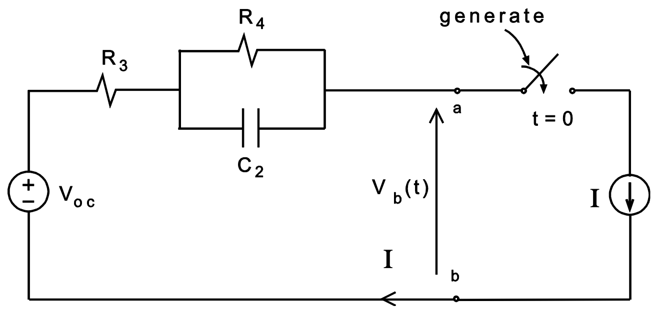

Fig. 3. The battery circuit during the generating phase.

The battery is represented by R_4 and C_2 for the components in the parallel circuit, and a series resistance R_3. The voltage source V_{oc} acts as the energy generator. Starting with a new time origin at t=0 at the start of the generating phase, the battery voltage is a transient given by

$$v_b(t) = V_{oc} - IR_3 - IR_4 + (IR_2 + IR_4)\exp(-t / R_4C_2) \tag{5}$$

The battery voltage changes abruptly by V_3 at the start of generation (see Fig.2) where

$$V_3 = (IR_1 + IR_3) \tag{6}$$

from which R_3 is calculated.

The decay in the generation transient is V_4, (see Fig. 2), and is given by

$$V_4 = IR_2 + IR_4 \tag{7}$$

from which R_4 is calculated.

The capacitance C_2 is derived from (5) as

$$C_2 = -\frac{I(R_2 + R_4)}{R_4(dv_b / dt)_d} \tag{8}$$

where $(dv_b / dt)_d$ is the gradient of the battery voltage at the start of the generation transient.

3. Results

The battery voltage–time measurements were made with the circuit of Fig 4. The battery is to the left of terminals a and b. The constant current source I is switched with the double-pole-double-throw switch to drive the battery first into the absorbing phase and then into the generating one.

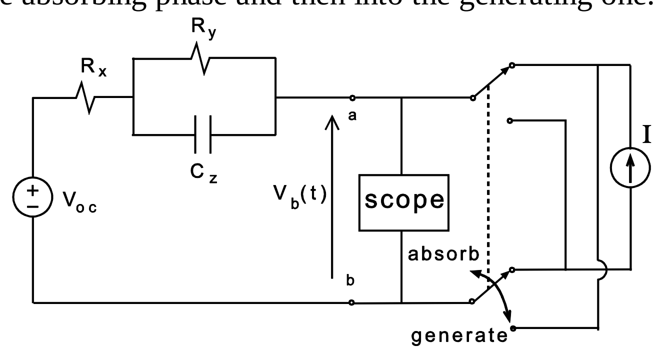

Fig. 4. The measurements circuit. The battery voltage is acquired by the oscilloscope.

A typical result is shown in Fig. 5. The equivalent circuit parameters calculated from such results for batteries at 100% state of charge (SOC) and at 70% SOC appear in Table I.

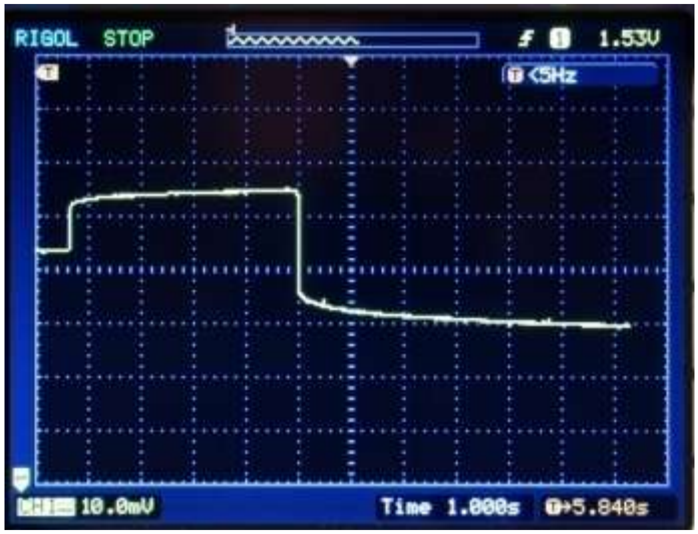

Fig. 5. Typical oscilloscope trace showing the variation of the battery voltage V_b (10mV/div) as a function of time (1sec/div) for a 1.55V, 24mAh Ag_2O-Zn battery at full state of charge.

SOC /%	B1/100	B2/100	B3/100	B4/100	B5/100	B5/70
R_1 / Ω	21.905	28.570	20.952	20.000	20.000	11.429
R_2 /Ω	9.524	10.476	5.714	6.667	9.523	5.952
C_1 /F	0.056	0.056	0.084	0.084	0.056	0.056
R_3 /Ω	26.666	35.240	20.953	25.714	22.857	12.381
R_4 /Ω	6.667	10.476	10.476	7.619	10.477	7.381
C_2 /F	0.014	0.084	0.043	0.012	0.064	0.101
(R_1+R_2)/Ω	31.429	39.046	26.667	26.667	29.523	17.381
(R_3+R_4)/Ω	33.333	45.716	31.429	33.333	33.334	19.762
$\tau=R_2C_1$/s	0.533	0.587	0.480	0.560	0.533	0.333
$\tau=R_4C_2$/s	0.093	0.880	0.450	0.091	0.672	0.745

TABLE I

Component values and time constants for batteries at 100% and 70% SOC.

4. Discussion and Conclusions

The first five columns of Table I refer to batteries at 100% SOC. The results in the sixth column are typical of batteries at 70% SOC.

For a given battery, the internal series resistance R_3 in the generating phase is greater than the corresponding resistance R_1 in the absorbing phase. This applies to the batteries at both 100% and 70% SOC. For instance, in column 4, R_3 equals 25.7Ω while R_1 is 20Ω.

At 70% SOC, the internal series resistances during absorption R_1, and during generating R_3, fall to about half of their respective values at 100% SOC. For instance $R_1$and R_3 are 11.1Ω and 12.4Ω in column 6, while corresponding resistances are 20Ω and 22.9 Ω in column 5.

After the transient phase, when steady state conditions ensue, the power dissipated in the battery is proportional to the series combination of the series resistance and the contact resistance (R_1+R_2) in absorption, and (R_3+R_4) in generation. The results show that $(R_1+R_2) < (R_3+R_4)$, indicating that the I^2R power dissipated in the battery is greater when the battery is generating power.

The electrode double layer capacitance in the absorbing phase, C_1, is in the 0.06F to 0.08F range for both 100% and 70% SOC. In the generating phase C_2 is in the 0.01F to 0.08F range for batteries with full charge, while at SOC of 70% C_2 shows an order of magnitude increase compared to fully charged batteries.

For the absorbing phase the transient constant is $\tau_c = R_2 C_1$. Therefore, τ_c~0.5 seconds at 100% SOC and τ_c~0.3 seconds at 70% SOC, and the respective transients last $5\tau_c$~2.5 and $5\tau_c$~1.5 seconds.

In the generating (discharging) phase the time constant $\tau_d = R_4 C_2$ is in the 90ms to 880ms range.

The results on the equivalent circuit components of the Ag_2O-Zn battery are important in research and design involving battery powered biomedical microcircuits. In future work the characterization method can be extended to other electrochemical cells.

5. References

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