

Modeling dynamic formability of porous ductile sheets subjected to biaxial stretching: Actual porosity versus homogenized porosity

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Abstract

This paper investigates the effect of porous microstructure on the necking formability of ductile sheets subjected to dynamic in-plane stretching. We have developed an original approach in which finite element calculations which include actual void distributions obtained from additively manufactured materials are compared with simulations in which the specimen is modeled with the Gurson-Tvergaard continuum plasticity theory (Gurson, 1977; Tvergaard, 1982) which considers porosity as an internal state variable. A key point of this work is that in the calculations performed with the continuum model, the initial void volume fraction is spatially varied in the specimen according to the void distributions included in the simulations with the actual porous microstructure. The finite element computations have been carried out for different loading conditions, with biaxial strain ratios ranging from 0 (plane strain) to 0.75 (biaxial tension) and loading rates varying between 10000 s^{-1} and 60000 s^{-1} . We have shown that for the specific porous microstructures considered, the necking forming limits obtained with the Gurson-Tvergaard continuum model are in qualitative agreement with the results obtained with the calculations which include the actual void distributions, the quantitative differences for the necking strains being generally less than $\approx 25\%$ (the calculations with actual voids systematically predict greater necking strains). In addition, the spatial distribution of necks formed in the sheets at large strains is very similar for the actual porosity and the homogenized porosity models. The obtained results demonstrate that the voids promote plastic localization, acting as preferential sites for the nucleation of fast growing necks. Moreover, the simulations have provided individualized correlations between void volume fraction, maximum void size and necking formability, and highlighted the influence of the heterogeneity of the spatial distribution of porosity on plastic localization.

Keywords:

Formability, Necking, Inertia, Porous microstructure, Finite element simulations

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1. Introduction

Ductile fracture of metals and alloys commonly occurs by plasticity-mediated growth and coalescence of voids, which are generally nucleated from inclusions, second phase particles and micro-cracks formed during materials manufacturing and processing (Tipper, 1949; Goods and Brown, 1979; Marino et al., 1985; Pineau et al., 2016). For instance, solidification defects, such as porosity and hot cracking, commonly observed in metal-based additive manufacturing processes (e.g. Aboulkhair et al. (2014), Gong et al. (2015), Samei et al. (2019) and Marvi-Mashhadi et al. (2021)), are customarily assumed to be responsible for the inferior ductility and fatigue life of 3D-printed metals as compared to their counterparts manufactured by traditional processes (casting, forging, extrusion, etc.), e.g., Zhao et al. (2018), Voisin et al. (2018), Li et al. (2019) and Laursen et al. (2020). In fact, inconsistent quality and densification problems characteristic of additively-manufactured metallic materials are considered limiting factors for the application of 3D-printed parts and components in high loading environments (Slotwinski et al., 2014; Ngo et al., 2018; Yuan, 2019).

Nevertheless, papers published so far assessing the performance of printed structures subjected to severe mechanical solicitations are quite scarce. It has to be highlighted the recent work of Kristoffersen et al. (2020), who carried out the tensile characterization and ballistic performance evaluation of additively-manufactured AlSi10Mg. The printed samples contained a substantial number of voids, with the largest pores having diameter of $\approx 200 \mu\text{m}$, and displayed severe plastic anisotropy. Tensile and high-velocity perforation tests were repeated using traditionally die-cast specimens having the same chemical composition as the powder used in the 3D printing. The die-cast specimens were very dense, with only a few pores of diameter smaller than $2 \mu\text{m}$, and displayed less pronounced plastic anisotropy than the printed samples. In addition, the flow stress was found to be lower and the fracture strain higher for the cast material. However, despite the differences in the microstructure and in the macromechanical properties of printed and cast specimens, the perforation experiments showed that the ballistic perforation resistance was almost identical. Moreover, the ballistic experiments performed on the printed specimens were simulated using ABAQUS/Explicit, modeling the target material with the isotropic Hershey-Hosford yield criterion (Hershey, 1954; Hosford, 1972) with an exponent equal to 8, associated flow rule, a relationship for the yield stress evolution which included strain hardening, strain rate hardening and thermal softening effects, and the Cockcroft and Latham fracture criterion. The discrepancy between numerical and experimental results for the ballistic limit varied between $\approx 15\%$ and $\approx 35\%$, depending on the set of parameters used for the yield stress evolution, with the calculations repeatedly underestimating the experimental measurements. While it is difficult

to determine the specific reason for this disagreement –which may not be considered particularly large within the context of ballistic simulations–, the authors pointed out the influence of the constitutive model parameters in the numerical predictions. In addition, no special measures were undertaken to account for the fact that the target was produced by additive manufacturing, so that no information was provided regarding the effect that porosity has on the impact behavior of printed AlSi10Mg.

A physically sound constitutive framework to model the macromechanical behavior of porous metals subjected to high strain rate impacts was developed by Molinari and Mercier (2001). This approach, which followed the works of Carroll and Holt (1972) and Wang (1997), among others, was derived using a dynamic **homogenization** procedure which assumes that the total stress is additively decomposed into a static part and a dynamic part. The static contribution is generally computed using a Gurson-type flow potential (Gurson, 1977) and the dynamic term –which assumes that the pores are initially spherical and remain spherical during loading– accounts for the effect that the local acceleration fields that develop around the voids at high loading rates have on the macroscopic material response. The dynamic stress, customarily referred to as microinertia, brings out that the initial void size is a natural length scale that modulates the overall dynamic response of the material (Molinari and Mercier, 2001; Czarnota et al., 2006, 2008; Jacques et al., 2015). This key feature is not captured by standard damage models in which the porosity stands for the single damage parameter with no contribution of the void size. Czarnota et al. (2008) and Jacques et al. (2010) implemented the constitutive framework of Molinari and Mercier (2001), coupled with a specific void nucleation model, into the finite element code ABAQUS/Explicit, and simulated the spall fracture observed in the plate impact experiments performed by Roy (2003) and Lin et al. (2004). The models of Czarnota et al. (2008) and Jacques et al. (2010) provided accurate predictions for the velocity in the rear side of the impacted plate, and for the spatial and size distribution of the voids in the spall plane. The finite element analysis also brought out the stabilizing effect of microinertia, which controls the level of the maximum stress and the growth of porosity in the spall plane. Moreover, Jacques et al. (2012) investigated the role of microinertia in dynamic ductile crack growth. Finite element calculations of notched bars and edge cracked specimens subjected to dynamic tension showed that microinertia effects refrain void growth, delaying the strain localization process that precedes fracture, lowering the crack speed and increasing the dynamic fracture toughness. Recently, Czarnota et al. (2017, 2020) employed the constitutive framework of Molinari and Mercier (2001) to investigate the effect of microinertia in the propagation of shock waves in porous ductile metals. For low shock pressures, microinertia effects were shown to be negligible and the shock structure is mostly determined by the material strain rate sensitivity and the

initial void volume fraction, with no effect of the distribution of void sizes. The increase of the shock amplitude was shown to boost the role of microinertia, so that the rate of collapse of the voids and the structure of the shock front become determined by the acceleration fields that develop around the void boundaries. Specifically, microinertia effects increased the shock width –which was scaled by the initial void radius– and refrained the voids compaction. However, to this day, the structural problems that have been addressed using the microinertia-based approach are limited to the three applications discussed above: spall fracture, dynamic ductile crack growth and shock wave propagation. It is therefore evident that much research still needs to be done to determine the specific loading conditions and material behaviors for which microinertia has an important effect on the plastic flow and fracture of porous metals. In fact, most of the papers available in the literature modeling localization and fracture of porous metals under dynamic loading do not consider microinertia effects (e.g., Guduru and Freund (2002), Becker (2002), Osovski et al. (2015) and N’souglo et al. (2018)), which raises the question of whether the standard Gurson-type porous plasticity theories yield sound results for dynamic problems, provided that the acceleration of the material particles in the vicinity of the voids is not critically large.

The present paper addresses the question of the influence of the voids distribution on the dynamic formability of porous materials. For this purpose, the necking of metal sheets subjected to **biaxial** stretching is modeled with (i) finite element simulations in which the porous microstructure is explicitly represented and (ii) the Gurson-Tvergaard continuum plasticity theory (Gurson, 1977; Tvergaard, 1982). Different porous microstructures for which the void volume fraction varies from $\approx 0.003\%$ to $\approx 0.03\%$, and the size of the pores between $6 \mu\text{m}$ and $100 \mu\text{m}$, have been considered (Marvi-Mashhadi et al., 2021). The calculations have been carried out in ABAQUS/Explicit (2019) for loading rates varying between 10000 s^{-1} and 60000 s^{-1} . The forming limit diagrams obtained with the continuum plasticity theory find qualitative resemblance with the calculations which include the actual voids, the quantitative differences for the limit necking strains being generally less than $\approx 25\%$, so that the calculations containing explicitly resolved pores systematically predict greater formability limits. In addition, selected calculations have been performed with the constitutive model proposed by Jacques et al. (2012), which extends the Gurson-Tvergaard theory to consider microinertia effects using the dynamic **homogenization** procedure developed by Molinari and Mercier (2001). The stabilizing effect of microinertia has been shown to be relevant, only, after necking localization, so that the necking strains obtained with the Gurson-Tvergaard model and with the formulation of Jacques et al. (2012) are virtually the same. Moreover, the calculations with explicitly resolved voids have provided individualized correlations between initial porosity, maximum voids size and specimen formability,

bringing to light that the void volume fraction distribution is the main factor controlling the limit strains and the spatial distribution of the necks in the localization pattern that emerges in the sheets at large strains. Note that numerical simulations of specimens containing a large population of voids was performed by Becker and Callaghan (2018, 2020) to assess the load path and mean stress dependence of void growth, by Marvi-Mashhadi et al. (2021) to identify the role of material defects in multiple necking and fragmentation of metal rings subjected to dynamic expansion, by Vishnu et al. (2022a) to investigate the influence of porosity on the formation of multiple shear bands in thick-walled cylinders subjected to rapid radial collapse, and by Vishnu et al. (2022b) to study the effect of porous microstructure on the formation of shear localization bands in thin-walled tubes subjected to dynamic torsion. However, to the authors' knowledge, this is the first paper ever that (i) computes dynamic forming limit diagrams for specimens with actual distributions of porosity and (ii) provides a systematic comparison with the results obtained with the Gurson-Tvergaard continuum plasticity theory.

2. Constitutive modeling

The mechanical behavior of the material is modeled as elastic-plastic, with yielding defined by the von Mises criterion (Mises, 1928) in the simulations with explicitly resolved porosity, see Section 2.2, and by the Gurson-Tvergaard criterion (Gurson, 1977; Tvergaard, 1982) in the calculations with homogenized porosity, see Section 2.3. The Gurson-Tvergaard criterion considers the material to display a periodic porous microstructure approximated by an array of representative volume elements idealized as hollow spheres with a central hole and with matrix described by the von Mises criterion.

2.1. General equations for elastic/plastic materials

We assume the additive decomposition of the total rate of deformation tensor $\mathbf{d}$ into an elastic part $\mathbf{d}^e$ and a plastic part $\mathbf{d}^p$:

$$\mathbf{d} = \mathbf{d}^e + \mathbf{d}^p \quad (1)$$

where the elastic part of the rate of deformation tensor is related to the rate of the stress by the following linear elastic law:

$$\overset{\nabla}{\boldsymbol{\sigma}} = \mathbf{L} : \mathbf{d}^e \quad (2)$$

141 where $\overset{\nabla}{\boldsymbol{\sigma}} = \dot{\boldsymbol{\sigma}} + \boldsymbol{\sigma}\boldsymbol{\Omega} - \boldsymbol{\Omega}\boldsymbol{\sigma}$ is the Green-Naghdi objective derivative of the Cauchy stress tensor used by ABAQUS/Explicit
 142 (in order to achieve incremental objectivity of the constitutive equations), where $\boldsymbol{\Omega} = \dot{\mathbf{R}}\mathbf{R}^T$ with $\mathbf{R}$ being the polar
 143 rotation tensor. Note that $(\dot{})$ denotes differentiation with respect to time. Moreover, $\mathbf{L}$ is the tensor of isotropic
 144 elastic moduli given by:

$$\mathbf{L} = 2G\mathbf{I}' + K\mathbf{1} \otimes \mathbf{1} \quad (3)$$

145 with $\mathbf{1}$ and $\mathbf{I}'$ being the unit second-order tensor and the unit deviatoric fourth-order tensor, respectively. Moreover,
 146 G and K are the shear modulus and the bulk modulus.

147

148 Assuming an associated plastic flow rule, the plastic part of the rate of deformation tensor is:

$$\mathbf{d}^p = \dot{\lambda} \frac{\partial \Phi}{\partial \boldsymbol{\sigma}} \quad (4)$$

149 where $\dot{\lambda}$ is the rate of plastic multiplier and Φ is the flow potential.

150

151 The formulation of the constitutive model is completed with the Kuhn-Tucker loading-unloading conditions:

$$\dot{\lambda} \geq 0, \quad \Phi \leq 0, \quad \dot{\lambda}\Phi = 0 \quad (5)$$

152 and the consistency condition during plastic loading:

$$\dot{\Phi} = 0 \quad (6)$$

153

154 2.2. Actual porosity: von Mises yield criterion

155 Specialization of the flow potential for the von Mises (1928) yield criterion leads to:

$$\Phi = \frac{\bar{\sigma}}{\sigma_Y} - 1 \quad (7)$$

where $\bar{\sigma} = \sqrt{\frac{3}{2} \mathbf{s} : \mathbf{s}}$ is the effective von Mises stress, with $\mathbf{s} = \boldsymbol{\sigma} - \sigma_h \mathbf{1}$ being the deviatoric part of the Cauchy stress tensor, and $\sigma_h = \frac{1}{3} \boldsymbol{\sigma} : \mathbf{1}$ is the hydrostatic stress. Moreover, σ_Y is the flow strength of the material:

$$\sigma_Y = \sigma_Y^0 + \sigma_K (\bar{\varepsilon}^p)^n \left(\frac{\dot{\bar{\varepsilon}}^p}{\dot{\varepsilon}_{ref}} \right)^m \left(\frac{T}{T_{ref}} \right)^\nu \quad (8)$$

where σ_Y^0 represents the initial flow strength, σ_K is the plastic modulus, n is the strain hardening exponent, m and ν are the strain rate and temperature sensitivity parameters, and $\dot{\varepsilon}_{ref}$ and T_{ref} are the reference strain rate and temperature, respectively. The effective plastic strain is $\bar{\varepsilon}^p = \int_0^t \dot{\bar{\varepsilon}}^p d\tau$, with $\dot{\bar{\varepsilon}}^p$ being the effective plastic strain rate. Moreover, T is the current temperature.

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The effective plastic strain rate is obtained from the work conjugacy relation:

$$\dot{\bar{\varepsilon}}^p = \frac{\boldsymbol{\sigma} : \mathbf{d}^p}{\sigma_Y} \quad (9)$$

164

Assuming adiabatic conditions of deformation (no heat flux) and considering that plastic work is the only source of heat, the evolution of the temperature is given by:

166

$$\dot{T} = \beta \frac{\sigma_Y \dot{\bar{\varepsilon}}^p}{\rho^0 C_p} \quad (10)$$

where ρ^0 is the initial material density, C_p the specific heat and β the Taylor-Quinney coefficient.

2.3. Homogenized porosity: Gurson-Tvergaard yield criterion

Specialization of the flow potential for the Gurson-Tvergaard yield criterion (Gurson, 1977; Tvergaard, 1982) leads to:

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$$\Phi = \left(\frac{\bar{\sigma}}{\sigma_Y} \right)^2 + 2q_1 f \cosh \left(\frac{3q_2 \sigma_h}{2\sigma_Y} \right) - 1 - (q_1 f)^2 \quad (11)$$

where q_1 and q_2 are material parameters, and f is the porosity (void volume fraction). Recall from Section 2.2 that $\bar{\sigma}$ and σ_h are the effective von Mises stress and the hydrostatic stress, respectively. The flow strength of the matrix material σ_Y is defined by equation (8).

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The effective plastic strain rate in the matrix material is obtained assuming that the rate of macroscopic plastic work is equal to the rate of effective plastic work in the matrix material:

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$$\dot{\bar{\epsilon}}^p = \frac{\boldsymbol{\sigma} : \mathbf{d}^p}{(1 - f) \sigma_Y} \quad (12)$$

177

Assuming the incompressibility of the matrix material, the evolution of the void volume fraction is defined as:

178

$$\dot{f} = (1 - f) \mathbf{d}^p : \mathbf{1} \quad (13)$$

We consider that the evolution of the porosity is due only to the growth of preexisting voids (void nucleation is neglected). If the initial material porosity f^0 is set to zero, the Gurson-Tvergaard model reduces to the von Mises yield criterion. Moreover, the evolution of temperature is computed using equation (10).

2.4. Material parameters

The values of the material parameters used in the simulations reported in Section 4 are given in Table 1, and they correspond to AISI 430 Steel (Vaz-Romero et al., 2015), except the parameters β , q_1 and q_2 which take standard values used in the literature. Additional calculations with constitutive parameters corresponding to Copper and Tantalum are included in Appendix A. The idea is to substantiate the conclusions of this paper with different material behaviors.

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Symbol	Property and units	Value
ρ^o	Initial density (kg/m ³), Eq. (10)	7740
C_p	Specific heat (J/kg K), Eq. (10)	460
K	Bulk modulus (GPa), Eq. (3)	196
G	Shear modulus (GPa), Eq. (3)	75
σ_Y^0	Initial yield strength (MPa), Eq. (8)	175.67
σ_K	Strain hardening modulus (MPa), Eq. (8)	530.13
n	Strain hardening exponent, Eq. (8)	0.167
$\dot{\epsilon}_{ref}$	Reference strain rate (s ⁻¹), Eq. (8)	0.01
m	Strain rate sensitivity exponent, Eq. (8)	0.0118
T_{ref}	Reference temperature, Eq. (8)	300
ν	Temperature sensitivity exponent, Eq. (8)	-0.51
q_1	Material parameter, Eq. (11)	1.25
q_2	Material parameter, Eq. (11)	1
β	Taylor-Quinney coefficient, Eq. (10)	0.9

Table 1: Material parameters corresponding to AISI 430 Steel used in the finite element simulations reported in Section 4. Data after Vaz-Romero et al. (2015), except the parameters β , q_1 and q_2 which take standard values used in the literature.

3. Finite element modeling

The problem addressed is that of a plate subjected to in-plane biaxial stretching. In order to reduce the computational time of the calculations, we have only modeled a representative volume element consisting of a strip of initial length $L_x^0 = 8$ mm, and square cross-section of initial thickness $L_y^0 = L_z^0 = 0.5$ mm, with symmetry boundary conditions (corresponding to a plate thickness of 1 mm), see Fig. 1. The Lagrangian Cartesian coordinate system associated to the applied loading is denoted by (X, Y, Z) . The origin of coordinates is located at the rear bottom left corner of the finite element model (i.e., at the center of mass of the specimen if no symmetry boundary conditions are applied). The finite element calculations are performed with ABAQUS/Explicit (2019) under the following imposed initial and boundary conditions:

$$\begin{aligned}
 V_x(X, Y, Z, 0) &= \dot{\epsilon}_{xx}^0 X; & V_y(X, Y, Z, 0) &= \dot{\epsilon}_{yy}^0 Y; & V_z(X, Y, Z, 0) &= -(\dot{\epsilon}_{xx}^0 + \dot{\epsilon}_{yy}^0) Z \\
 T(X, Y, Z, 0) &= T^0 = T_{ref}
 \end{aligned}$$

$$\begin{aligned}
V_x(L_x^0, Y, Z, t) &= \dot{\varepsilon}_{xx}^0 L_x^0; & V_y(X, L_y^0, Z, t) &= \dot{\varepsilon}_{yy}^0 L_y^0 \\
U_x(0, Y, Z, t) &= 0; & U_y(X, 0, Z, t) &= 0; & U_z(X, Y, 0, t) &= 0
\end{aligned}$$

197 with $\dot{\varepsilon}_{xx}^0$ and $\dot{\varepsilon}_{yy}^0$ being the imposed initial strain rates, see Fig. 1. The loading condition is determined by
 198 the ratio $\chi = \dot{\varepsilon}_{yy}^0 / \dot{\varepsilon}_{xx}^0$, which is varied between 0 (plane strain tension) and **0.75** (near equibiaxial stretching)
 199 in the calculations presented in this paper (Z-direction is parallel to the thickness of the plate). Hereinafter,
 200 $\dot{\varepsilon}_{xx}^0$ and $\dot{\varepsilon}_{yy}^0$ will be referred to as the imposed initial major and minor strain rate, respectively, and χ as the
 201 loading path ($\dot{\varepsilon}_{xx}^0$ will be also called loading rate). We have investigated imposed initial major strain rates varying
 202 from 10000 s^{-1} to 60000 s^{-1} . These strain rates are the highest values generally attained in dynamic forming
 203 processes (e.g. see Dariani et al. (2009), Golovashchenko et al. (2013) and Li et al. (2017), among others). For
 204 these strain rates, the dimensionless parameter customarily used to represent the inertial resistance to motion
 205 $\hat{H}^{-1} = 2\sqrt{\frac{\rho^0(\dot{\varepsilon}_{xx}^0 L_x^0)^2}{\sigma_Y^0}}$, see Zhou et al. (2006) and N'souglo et al. (2021), varies from ≈ 0.07 to ≈ 0.4 . Note that
 206 finite element simulations reported by N'souglo et al. (2020) and Zheng et al. (2020) indicated that dynamic effects
 207 start to play an important role in the inception of necks for $\hat{H}^{-1}$ in the range $0.06 - 0.1$. Moreover, the imposed
 208 initial conditions allow to minimize the propagation of stress waves within the specimen caused by the abrupt
 209 motion of the strip at $t = 0$. Otherwise, the waves generated due to the application of the velocity boundary
 210 conditions could lead to instantaneous plastic localization (Needleman, 1991; Xue et al., 2008). Moreover, note
 211 that considering a thin plate subjected to dynamic loading with these initial conditions leads to the formation
 212 of multiple necks at large strains (Zaera et al., 2015). Note also that for isotropic materials subjected to biaxial
 213 stretching, the preferential orientation of the necks is perpendicular to the major loading direction (Zaera et al.,
 214 2015; Rodríguez-Martínez et al., 2017), leading to local thinning of the thickness of the strip. **Notice that the initial**
 215 **and boundary conditions of the finite element model are consistent with a thin plate subjected to in-plane loading,**
 216 **so that modeling a wider plate does not affect the results. Moreover, since for the considered loading conditions**
 217 **and isotropic material behavior the orientation of the necks is perpendicular to the major loading direction (Stören**
 218 **and Rice, 1975), the specimen width does not affect the necking pattern.**

219 3.1. Actual porosity

220 The specimen includes the actual porous microstructure of three additively manufactured metallic materials:
 221 stainless steel 316L, titanium alloy Ti6Al4V and Inconel 718, see Fig. 2. The mechanical behavior of the matrix

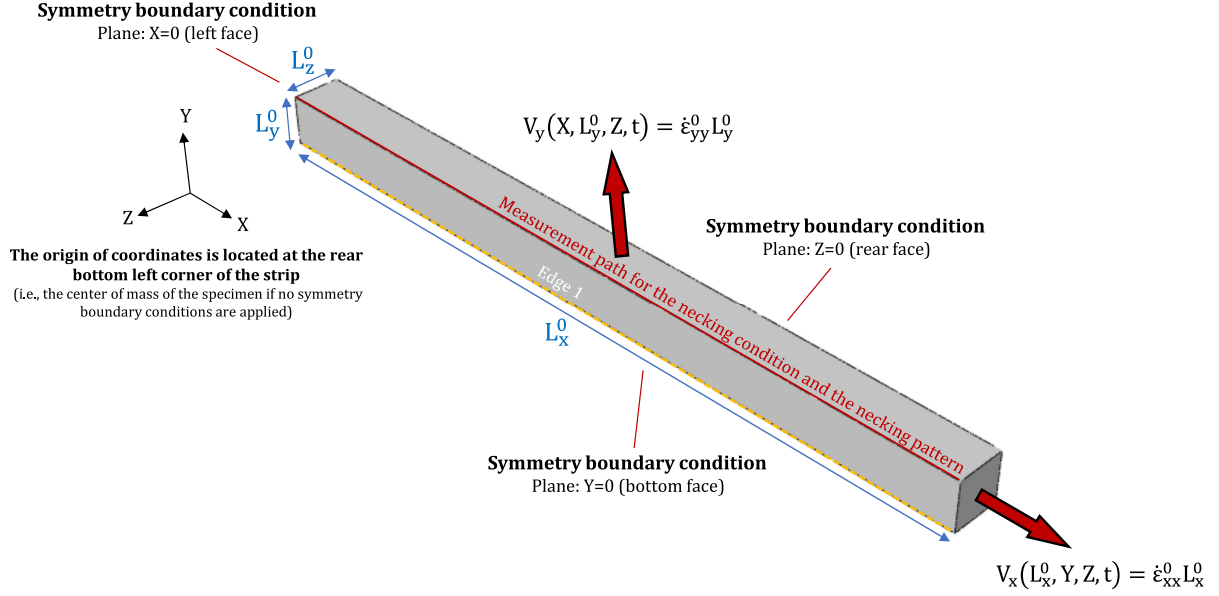


Figure 1: Schematic of the geometry and boundary conditions of the problem addressed: a strip of initial length $L_x^0 = 8$ mm and square cross-section of initial thickness $L_y^0 = L_z^0 = 0.5$ mm, subjected to biaxial stretching. The Lagrangian Cartesian coordinate system associated to the applied velocity field is denoted by (X, Y, Z) . The origin of coordinates is located at the rear bottom left corner of the finite element model.

material is modeled with the constitutive framework described in Section 2.2, which has been implemented in ABAQUS/Explicit (2019) through a user subroutine VUMAT. As in Marvi-Mashhadi et al. (2021) and Vishnu et al. (2022a,b), the additively manufactured materials are solely employed to obtain representative porosity distributions to be used in the finite element models with actual porosity (no attention is paid to the specific mechanical response of the printed materials from which the microstructures are taken).

The voids size distributions obtained from the X-ray tomography analysis carried out by Marvi-Mashhadi et al. (2021) are imported to the finite element model following the methodology developed by Marvi-Mashhadi et al. (2021) and Vishnu et al. (2022a,b). The voids are taken to be spherical, and they are randomly distributed in the strip. The voids size distribution of the three porous microstructures –referred to as SS5Z, Ti0.5Z and INC1XY in Table 1 of Marvi-Mashhadi et al. (2021)– is approximated by a Log-normal statistical function with parameter values given in Table 2. For each of the three porous microstructures, we have generated three realizations of voids size and positions distribution (R1, R2, R3) which fulfill the Log-normal probability function. The goal is to assess the scatter in the finite element results induced by the random spatial distribution of pores and the distribution of void sizes. Moreover, Table 3 shows the number of voids, the initial void volume fraction and the number of voids per mm^3 included in the strips, for the 3 microstructures investigated, and for the 3 realizations that we have generated per specimen. The differences between the void volume fractions and the void densities obtained from the experimental measurements, Table 2, and included in the finite element models, Table 3, are attributed to

the random nature of the position of the voids and to the statistical distribution of void sizes in the experimental specimen (see Marvi-Mashhadi et al. (2021)), which are carried over to the computational model, leading to deviations from the number of voids, the maximum size of the voids and the initial void volume fraction measured experimentally (note that the X-ray tomography analysis is performed over a greater volume, see Marvi-Mashhadi et al. (2021)).

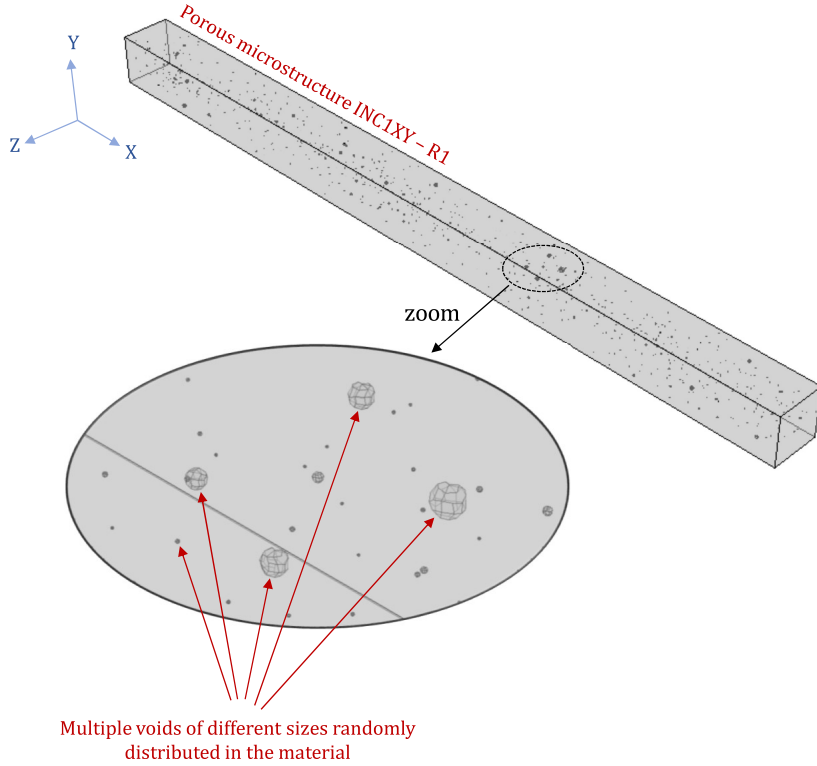


Figure 2: Finite element model with actual porosity. Strip of initial length $L_x^0 = 8$ mm and square cross-section of initial thickness $L_y^0 = L_z^0 = 0.5$ mm with porous microstructure INC1XY and realization R1. The spherical voids are randomly distributed in the specimen. The distribution of void sizes fulfills the Log-normal statistical function with parameter values given in Table 2.

	SS5Z	Ti0.5Z	INC1XY
Initial void volume fraction, f^0 (%)	0.0290	0.0033	0.0203
Voids density, N_v (num./mm ³)	18	18	390
Maximum diameter of voids, $d_{\max}$ (μm)	100.00	31.44	45.52
Minimum diameter of voids, $d_{\min}$ (μm)	7.44	7.44	6.36
Mean of Log-normal distribution, μ (μm)	18.45	14.30	6.41
Standard deviation of Log-normal distribution, dev (μm)	9.85	9.31	4.56

Table 2: Summary of the measurements obtained from the X-ray tomography analysis: initial void volume fraction f^0 (%), voids density N_v (num./mm³), maximum diameter of voids $d_{\max}$ (μm), minimum diameter of voids $d_{\min}$ (μm), and mean μ (μm) and standard deviation dev (μm) values of fitted Log-normal distribution. Results taken from Marvi-Mashhadi et al. (2021).

The finite element models were discretized using C3D4 linear tetrahedral elements (ABAQUS/Explicit, 2019),

	SS5Z			Ti0.5Z			INC1XY		
	R1	R2	R3	R1	R2	R3	R1	R2	R3
Number of voids, N_t (num.)	30	31	32	38	34	37	719	748	760
Initial void volume fraction, f^0 (%)	0.0378	0.0423	0.0117	0.0018	0.0033	0.0029	0.0193	0.0267	0.0190
Voids density, N_v (num./mm ³)	15	15.5	16	19	17	18.5	359.5	374	380

Table 3: Number of voids N_t (num.), initial void volume fraction f^0 (%) and voids density N_v (num./mm³) included in the finite element models corresponding to the 3 microstructures investigated, and the three realizations (R1, R2, R3) that we have generated per microstructure.

see Fig. 3. For the microstructures SS5Z, Ti0.5Z and INC1XY the average number of elements and the standard deviation are 710428 ± 213135 , 1303415 ± 9903 and 2248399 ± 225540 , respectively. Following Marvi-Mashhadi et al. (2021), the meridian of the smallest pore was meshed with 5 elements, such that the number of elements surrounding the voids increases with the size of the pores. The calculations were performed using 48 cores of a workstation with processor Dual Intel Xeon Gold 5220R @ 2.2 ~ 4.0 GHz. The computational time of each simulation varied between 4 hours and 6 days, depending on the microstructure considered, such that the computational cost increases with the number of elements in the mesh. We have performed a mesh sensitivity analysis, increasing the number of elements, and checked that the actual porosity results presented in this paper are little dependent on the discretization (see Appendix B). For instance, increasing the number of elements by a factor of 7 for the microstructure SS5Z and realization R1, the loading rate 60000 s^{-1} , and the loading path $\chi = 0$, does not modify the position and the number of necks, while the necking strain only decreases $\approx 9\%$. Moreover, we have also carried out the same calculation using C3D10 elements, obtaining very similar necking pattern, while the necking strain drops $\approx 14\%$.

3.2. Homogenized porosity

The strip does not contain explicitly resolved voids. The mechanical behavior of the material is described with the constitutive framework presented in Section 2.3, which has been implemented in ABAQUS/Explicit (2019) through a user subroutine VUMAT. The porous microstructure is represented by the void volume fraction, a scalar parameter resulting from the porosity homogenization, which acts as an internal variable of the constitutive model, see equations (11) and (13).

The spatial and size distribution of the voids is captured using an original methodology which consists of dividing the strip into several cubic cells of size L_c , see Fig. 4, with initial void volume fraction computed by equation (14) from the positions and the sizes of the voids included in the finite element models with actual porosity:

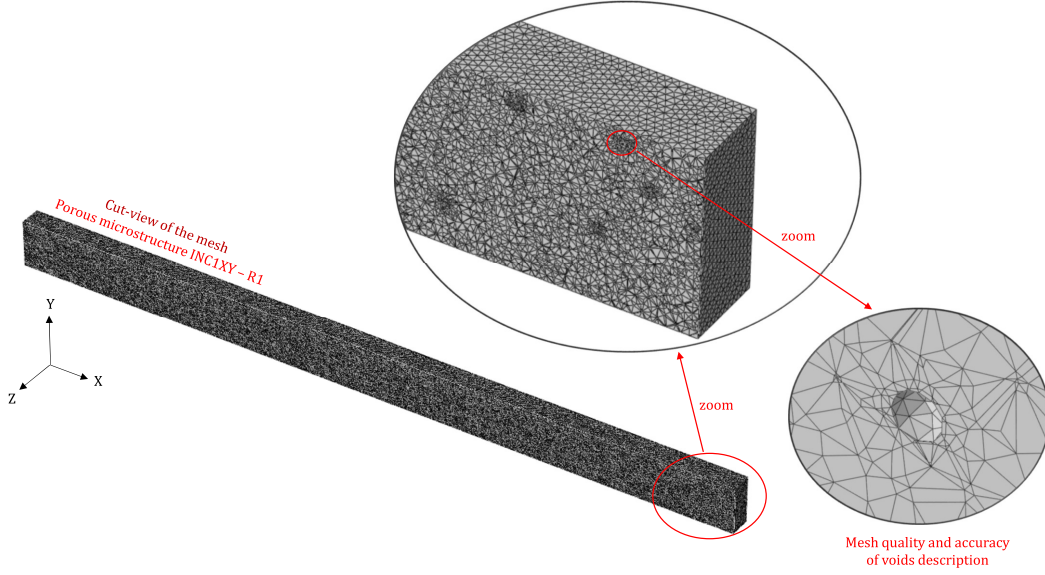


Figure 3: Finite element model with actual porosity. Cut-view of the mesh. Strip of initial length $L_x^0 = 8$ mm and square cross-section of initial thickness $L_y^0 = L_z^0 = 0.5$ mm with porous microstructure INC1XY and realization R1.

$$f_i^0 = \frac{\sum_{j=0}^{N_i} \frac{1}{6} \pi d_j^3}{V_c} \quad (14)$$

where f_i^0 and N_i are the initial void volume fraction and the number of voids in the cell i , respectively, V_c denotes the volume of the cells, and d_j is the diameter of a given void. The cell size is taken to be a sub-multiple of the dimensions of the strip, and it has to be larger than the mean spacing between voids, so that the void volume fraction in the cell is different from zero (the whole specimen domain is considered porous). For the microstructures SS5Z and Ti0.5Z, the average spacing between voids is ≈ 0.4 mm, which is close to the thickness of the strip, such that the cell size matches the cross-section of the specimen ($L_c = 0.5$ mm). For the microstructure INC1XY, the average void spacing is smaller (≈ 0.14 mm), so that the cell size can be taken to be either $L_c = 0.5$ mm or 0.25 mm. We have carried out simulations for both values of the cell size and have checked that the results are similar (see Appendix C). All the calculations reported in Section 4 correspond to $L_c = 0.5$ mm (for the three microstructures investigated).

Figures 5(a) and 5(b) compare the evolution of the initial void volume fraction along the normalized axial coordinate of the strip with the initial diameter of the voids of the actual porous microstructure for SS5Z-R1 and INC718XY-R1, respectively. The cells with large void volume fraction correspond to big voids of the resolved porosity microstructure, showing the connection between the void volume fraction heterogeneity in the homogenized

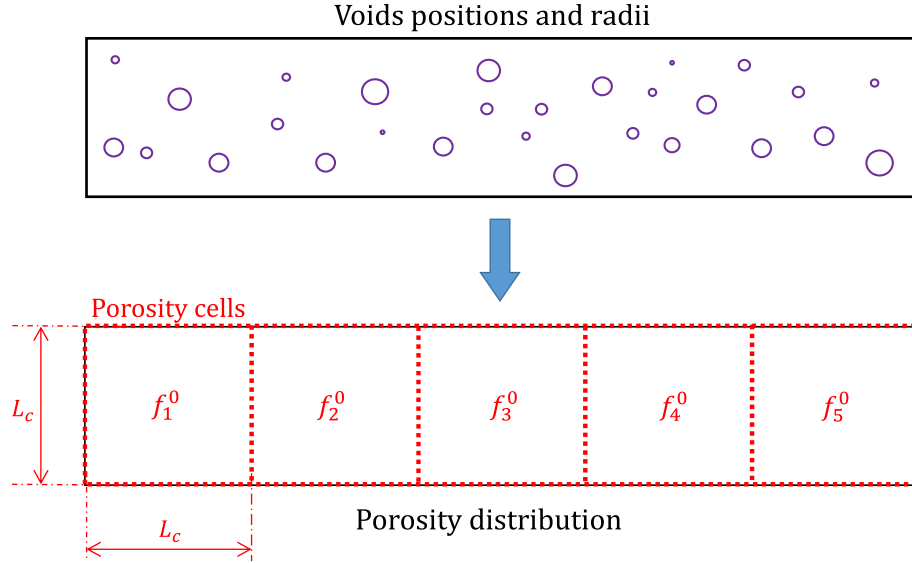


Figure 4: Illustration of the methodology used to define the initial void volume fraction distribution in the homogenized porosity finite element model. The strip domain is divided into several cells of size L_c with initial void volume fraction computed from the positions and sizes of the voids included in the finite element models with actual porosity.

porosity model and the void size distribution in the actual porosity model.

The finite element model was discretized using solid elements with tri-linear interpolation function, reduced integration and hourglass control (C3D8R in ABAQUS/Explicit (2019) notation). The dimensions of the elements are $25 \times 50 \times 50 \mu\text{m}^3$ (the smallest value corresponding to the axial direction of the strip), so that the total number of elements is 32000. The calculations were performed using a single core of a laptop computer with a processor Intel(R) Core(TM) i5-8350U CPU @ 1.70 GHz. The computational cost of a simulation varied between 15 and 60 minutes, depending on the loading path and the strain rate, i.e., these calculations are significantly faster than the simulations with the actual porous microstructure. We have performed a mesh sensitivity analysis, increasing the number of elements, and checked that the homogenized porosity results presented in this paper are hardly dependent on the discretization (see Appendix B). Note that previous papers of Marvi-Mashhadi et al. (2021) and Vishnu et al. (2022a,b) did not provide a comparison between calculations performed with actual porous microstructures and simulations with homogenized porosity.

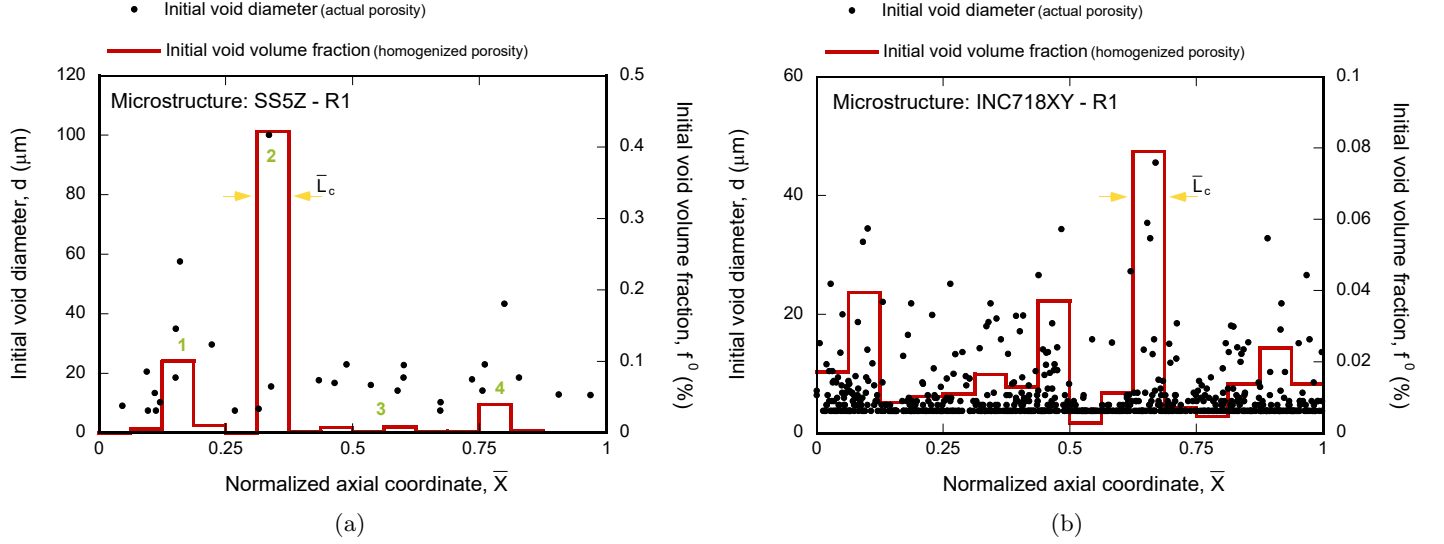


Figure 5: Examples of initial microstructures: (a) SS5Z and realization R1, and (b) INC718XY and realization R1. Initial diameter of the voids d along the normalized axial coordinate $\bar{X} = \frac{X}{L_x^0}$ in the actual porosity model. Comparison with the initial void volume fraction f^0 (%) in the homogenized porosity model. The cell size in the homogenized porosity model is $L_c = 0.5$ mm. Note that $\bar{L}_c = \frac{L_c}{L_x^0}$ is the normalized cell size.

4. Results

The presentation of results is split up into four sections: the procedure for comparison of the localization patterns obtained from the calculations with actual porosity and homogenized porosity is described in Section 4.1, Section 4.2 investigates the influence of the porous microstructure on the necking formability of the strip, Section 4.3 shows the stabilizing effect of the loading rate on the necking strain, and Section 4.4 presents forming limit diagrams for the three microstructures considered and different loading rates.

4.1. Salient features

The results presented in this section correspond to the three realizations generated for SS5Z. This microstructure displays the greater void volume fraction and the larger voids, see Table 2, which helps to enlighten the role of porosity in necking formation. The calculations correspond to $\chi = 0$ and $\dot{\varepsilon}_{xx}^0 = 60000 \text{ s}^{-1}$, this combination of loading path and loading rate being chosen because plane strain is the loading path that most favors the nucleation of necks (see Section 4.4), and increasing the loading rate increases the number of necks (see Section 4.3).

Fig. 6 shows contour plots of effective plastic strain $\bar{\varepsilon}^p$ corresponding to realization R1 for both the actual porosity model and the homogenized porosity model. The color coding of the isocontours for the calculation with explicitly resolved pores is such that effective plastic strains ranging from 0.55 to 0.8 correlate with a color scale

that goes from blue to red. Effective plastic strains above 0.8 remain red and below 0.55 remain blue. In the case of the calculation with homogenized porosity, the threshold values for the effective plastic strain are 0.5 and 0.7, respectively. The snapshots are taken at the necking time t^c , i.e., at the loading time leading to necking formation. The necking pattern is assumed to form when a material point of the path indicated in Fig. 1 first meets the condition $\frac{d\varepsilon_{xx}}{dt} < 10^{-3} \text{ s}^{-1}$, so that outside the necks the plastic strain rate becomes negligible. To apply the necking criterion, we track the evolution of the major strain ε_{xx} for all material points in the path along time and identify the first one reaching the necking condition. The necking criterion corresponds to the specimen unloading, indicating that plastic strain has localized, giving rise to the formation of necking instabilities. As an illustration, Figure D.27 in Appendix D shows that the major strain outside the necks saturates, so that the material is unloaded (see also Sections 2 and 3 in Xue et al. (2008)). Moreover, note that, while 10^{-3} s^{-1} is an arbitrary value for the necking condition, we have checked that the necking strain is largely independent of this threshold value, provided that it is small enough so that the sections outside the necks are unloaded (we have checked that the results obtained with $\frac{d\varepsilon_{xx}}{dt} < 10^{-4} \text{ s}^{-1}$, 10^{-2} s^{-1} and 10^{-1} s^{-1} , are virtually the same). Similar criterion and methodology have been used by N'souglo et al. (2021) and Jacques and Rodríguez-Martínez (2021) to determine the formation of necks in plates and rings subjected to dynamic stretching. The saturation value of the major strain outside the necks is referred to as the major necking strain ε_{xx}^c , and the minor strain measured at the same material point, and at the same loading time, is called the minor necking strain ε_{yy}^c (note that $\varepsilon_{yy}^c = 0$ for $\chi = 0$, and that major and minor necking strains will be used in Section 4.4 to construct forming limit diagrams). For this realization, the major necking strain and the necking time for the actual porosity model, $\varepsilon_{xx}^c = 0.45$ and $t^c = 11.3 \text{ } \mu\text{s}$, respectively, are greater than for the calculation with homogenized porosity, $\varepsilon_{xx}^c = 0.34$ and $t^c = 8.9 \text{ } \mu\text{s}$, which predicts earlier plastic localization. On the other hand, the spatial distribution of the necks in the strip is very similar for both modeling approaches (4 necks indicated in Fig. 6 at the locations numbered in Fig. 5(a)). The correspondence between the position of the necks, Fig. 5(a), the location of large voids when the strip is modeled with explicitly resolved porosity, Fig. 6(a), and the cells with greater void volume fraction when the strip is modeled with the Gurson-Tvergaard plasticity theory, Fig. 6(b), makes apparent that the porous microstructure plays a key role in the necks spacing. Note that we have not included contours of effective plastic strain rate in the paper as they do not provide additional information to the contours of effective plastic strain shown in Figs. 6 and 10 (the strain rate in the necked sections is large, and outside the necks, after the necking criterion is met, it becomes negligible).

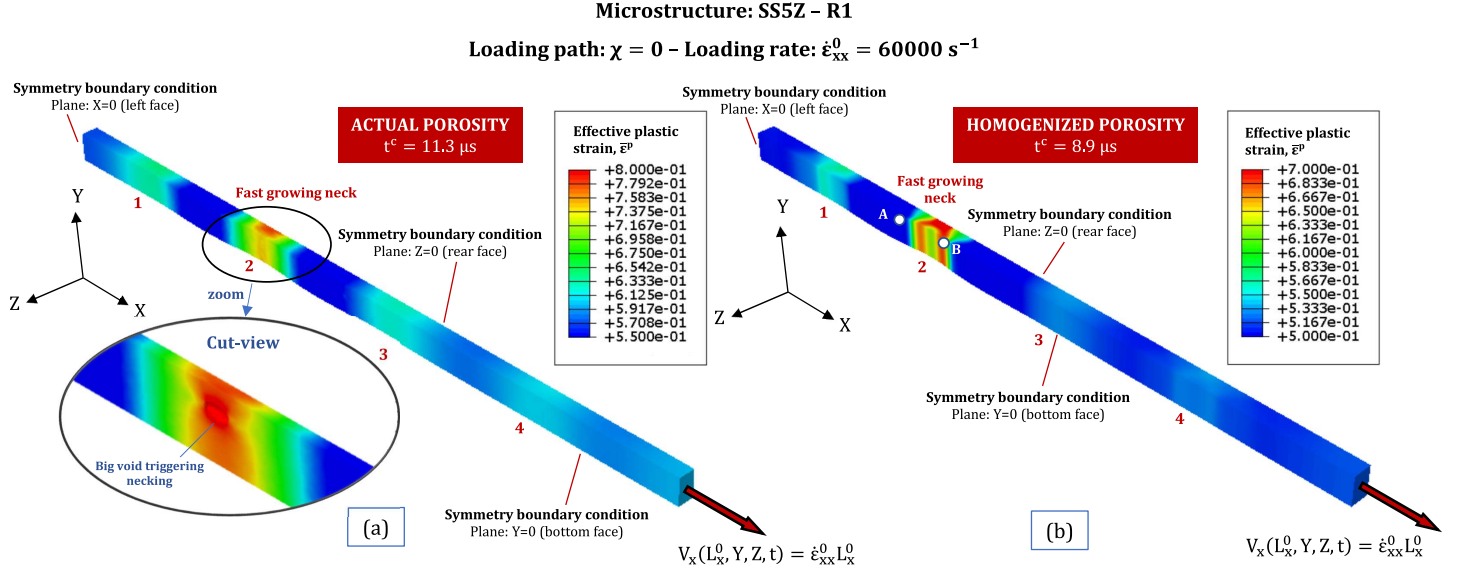


Figure 6: Microstructure SS5Z and realization R1. Contours of effective plastic strain $\bar{\epsilon}^p$ corresponding to calculations performed for loading path $\chi = 0$ (plane strain stretching) and loading rate $\dot{\epsilon}_{xx}^0 = 60000 \text{ s}^{-1}$: (a) actual porosity model and (b) homogenized porosity model. The loading time corresponds to the necking condition. For interpretation of the references to color in this figure, the reader is referred to the web version of this article.

The correlation between the location of the necks and the porous microstructure is further illustrated in Fig. 7, which shows the evolution of the effective plastic strain $\bar{\epsilon}^p$ and the initial void volume fraction f^0 (%) along the normalized axial coordinate $\bar{X}$ for realizations R2 and R3. The effective plastic strain is measured at the necking time along the path indicated in Fig. 1. We have checked that the quantitative values for the plastic strain are very similar when the measurements are taken on the opposite longitudinal edge of the model (Edge 1 in Fig. 1), located on the symmetry plane of the sheet. Recall that the initial void volume fraction distribution is computed by dividing the sample into 16 cells with $L_c = 0.5 \text{ mm}$ (see Section 3.2). Notice that the results obtained with the actual porosity model and the homogenized porosity model are qualitatively very similar, being the $\bar{\epsilon}^p - \bar{X}$ curves computed with the Gurson-Tvergaard theory shifted downwards because the homogenized porosity approach predicts earlier necking formation (as mentioned before). The evolution of the effective plastic strain shows a succession of peaks and valleys, the peaks corresponding to the necks (the greater the peak, the more developed the neck), and the valleys to the unloading sections of the strip. Note that despite the dynamic loading conditions, and the stress wave intervention within the specimen during loading, the material points outside the necked sections after localization of plastic deformation are eventually unloaded, see Appendix D (we have also checked that the elastic stiffness of the material plays negligible role on the necking strain and the necking pattern). Fig. 7(a) shows a large excursion of strain which represents the fastest growing neck at $\bar{X} \approx 0.9$ (number 4), where the initial void volume fraction is greater due to a large pore located at that section of the strip. Additional smaller excursions of

strain –corresponding to secondary necks with slower growth rate–, emerge at other sections of the specimen with large values of the initial void volume fraction (numbers 1, 2 and 3). Recall that the axial length of the specimen must be significantly greater than the cross-section size to enable the formation of multiple necks. Moreover, the results for realization R3 pictured in Fig. 7(b) also show that the fastest growing neck in the homogenized porosity model (number 3) is located at the section of the strip with the greatest initial void volume fraction, near to a large void in the actual porosity model. In fact, the main difference between the localization patterns obtained from both modeling approaches is that necks 5 and 6 in the homogenized porosity model are replaced by a single neck in the actual porosity model (number 5), most likely because the compartmentalization into cells of the initial void volume fraction in the calculation with the Gurson-Tvergaard theory leads to some sort of rearrangement of the preferential locations for the nucleation of necks. All in all, these results illustrate the remarkable capacity of the homogenized porosity approach to predict the necking pattern obtained in the calculations with explicitly resolved pores. This is an important outcome of this work, as the computational cost of the homogenized porosity calculations is significantly lower (see Section 3). The same conclusion has been obtained from calculations in which the material behavior corresponds to Tantalum and Copper, with parameters taken from Mercier et al. (2010), see Appendix A. Moreover, note that the Gurson-Tvergaard model does not account for void coalescence. On the other hand, we have performed some simulations with the Gurson-Tvergaard-Needleman model (Tvergaard and Needleman, 1984), which describes the effects of void coalescence (for a coalescence porosity of 0.12). We have found that both models yield the same results in terms of necking strain (results not shown for the sake of brevity). The reason is that, for the microstructures considered in this paper, which all have relatively low values of initial porosity, coalescence takes place after the occurrence of localized necking. This result is consistent with the finding of Zheng et al. (2020), who showed (for a bar loaded in uniaxial tension) that fracture precedes necking only for materials having very large initial porosity levels. Note, however, that using a fixed value of the coalescence porosity is questionable, as this quantity can be affected by the distribution of the voids within the material (Hure, 2021; Cadet et al., 2021) and the mechanisms of void coalescence under dynamic loading may be different from those observed in quasi-static conditions (Thomason, 1999; Jacques et al., 2012; Molinari et al., 2015).

Fig. 8 displays the temperature increase ΔT along the normalized axial coordinate $\bar{X}$ for the same calculations shown in Fig. 7. The temperature increase is measured at the necking time along the path indicated in Fig. 1. Notice that the profiles of temperature are qualitatively the same than the profiles of effective plastic strain. The temperature increases with the effective plastic strain in adiabatic conditions of deformation, so that the

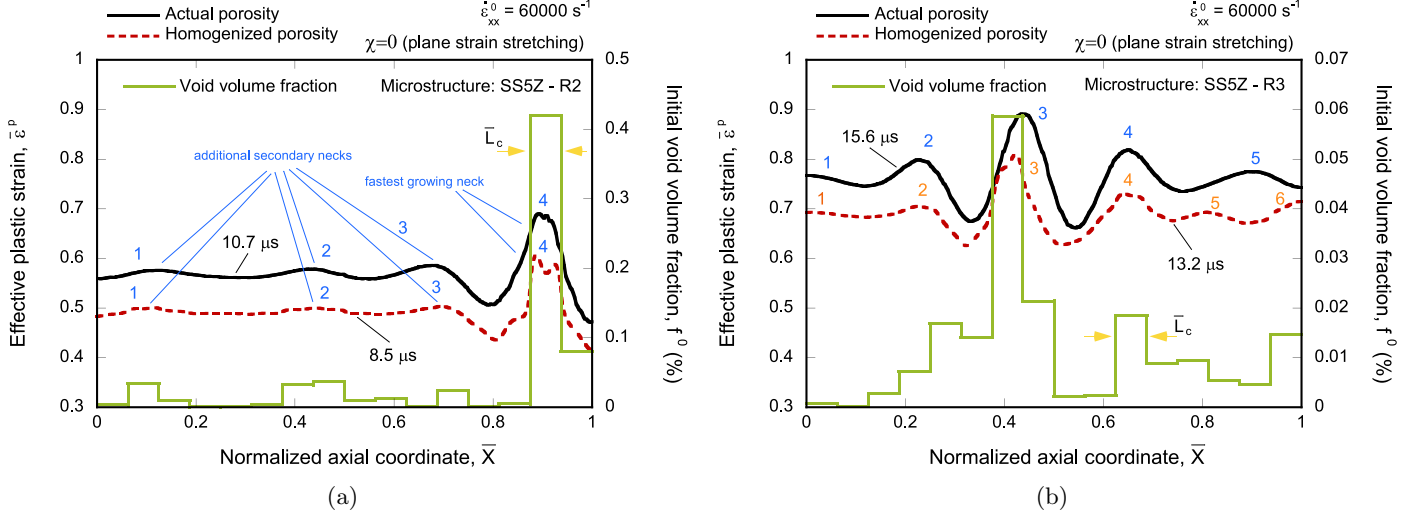


Figure 7: Microstructure SS5Z. Evolution of effective plastic strain $\bar{\epsilon}^p$ and initial void volume fraction f^0 (%) along the normalized axial coordinate $\bar{X} = \frac{X}{L_x^0}$ (see Fig. 1). Comparison between results obtained with the actual porosity model and the homogenized porosity model for loading path $\chi = 0$ (plane strain stretching) and loading rate $\dot{\epsilon}_{xx}^0 = 60000 \text{ s}^{-1}$: (a) R2 and (b) R3. The loading time corresponds to the necking condition. The void volume fraction is computed dividing the sample into 16 cells with $L_c = 0.5 \text{ mm}$ (the strategy used in the homogenized porosity model). Note that $\bar{L}_c = \frac{L_c}{L_x^0}$ is the normalized cell size.

maximum increase of temperature corresponds to the fastest growing neck. For the actual voids simulations, inside the specimen, the temperature increase is even greater, reaching 175 K and 159 K for the calculations with actual porosity corresponding to realizations R2 and R3, respectively. The peak temperature values are located at the surface of the main void which triggers the fastest growing neck.

4.2. The influence of porous microstructure

The results presented in this section correspond to the three realizations of the three microstructures investigated, see Tables 2 and 3. Two different loading paths, $\chi = 0$ and 0.4, and two different loading rates, $\dot{\epsilon}_{xx}^0 = 10000 \text{ s}^{-1}$ and 60000 s^{-1} (lowest and highest strain rate considered, respectively) are considered for the analysis. The goal is to generalize the results and conclusions derived in Section 4.1.

Fig. 9 shows the evolution of the effective plastic strain and the initial void volume fraction along the normalized axial coordinate for microstructure INC1XY and realization R1. The $\bar{\epsilon}^p - \bar{X}$ curves for both the actual porosity model and the homogenized porosity model correspond to the necking time. The imposed initial major strain rate is $\dot{\epsilon}_{xx}^0 = 10000 \text{ s}^{-1}$. Recall that the void volume fraction is calculated dividing the sample into 16 cells with $L_c = 0.5 \text{ mm}$ (all the $f^0 - \bar{X}$ curves shown in this paper are computed following this procedure).

The results for plane strain stretching ($\chi = 0$) are shown in Fig. 9(a). The necking patterns for both modeling approaches are very similar (as in Fig. 7), with four necks developing at different locations of the strip,

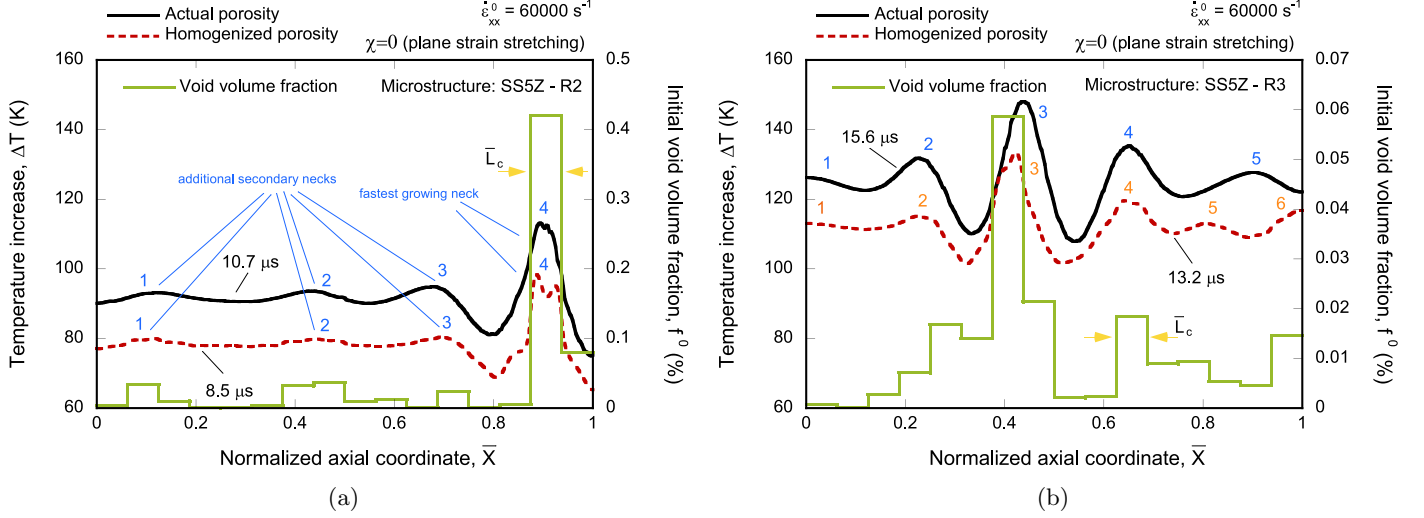


Figure 8: Microstructure SS5Z. Evolution of temperature increase ΔT and initial void volume fraction f^0 (%) along the normalized axial coordinate $\bar{X} = \frac{X}{L_c}$ (see Fig. 1). Comparison between results obtained with the actual porosity model and the homogenized porosity model for loading path $\chi = 0$ (plane strain stretching) and loading rate $\dot{\epsilon}_{xx}^0 = 60000 \text{ s}^{-1}$: (a) R2 and (b) R3. The loading time corresponds to the necking condition. The void volume fraction is computed dividing the sample into 16 cells with $L_c = 0.5 \text{ mm}$ (the strategy used in the homogenized porosity model). Note that $\bar{L}_c = \frac{L_c}{L_0}$ is the normalized cell size.

as illustrated in the isocontours of effective plastic strain of Fig. 10. The fastest growing neck (number 3) is nucleated at $\bar{X} \approx 0.65$, which corresponds to the section of the specimen with greater initial void volume fraction due to the cluster of voids featured in Fig. 10(a). Necks 1, 2 and 4 are also located near sections of the strip with large values of the initial void volume fraction, reinforcing the idea that for the porous microstructures considered, there is an interplay between the necking pattern and the spatial and size distribution of voids in the specimen (these results are consistent with the calculations with explicitly resolved voids carried out by Marvi-Mashhadi et al. (2021) for ductile porous rings subjected to rapid radial expansion, see Fig. 16 therein). Moreover, note that the necking time is smaller for the homogenized porosity approach (as in Fig. 7), showing (again) that replacing the actual voids by the Gurson-Tvergaard theory leads to earlier localization.

The results for $\chi = 0.4$ are shown in Fig. 9(b). The necking time is less for the homogenized porosity approach, so that the corresponding $\bar{\epsilon}^p - \bar{X}$ curve is below the results obtained with the actual porosity model. On the other hand, in comparison with the calculations for $\chi = 0$ –Fig. 9(a)– the effective plastic strain in the strip is larger for both, actual porosity model and homogenized porosity model, showing that when the loading path moves away from plane strain towards biaxial stretching, the occurrence of localized necking is delayed (as it has been often observed in the literature, e.g., Kuroda and Tvergaard (2000), Brunet and Morestin (2001) and Banabic et al. (2010)). In addition, the sequence of peaks and valleys shaping the necking pattern for plane strain stretching –Fig. 9(a)– turns into a necking pattern with a very large excursion of strain in the case of $\chi = 0.4$ (number 3), which

is located at the section of the strip with greater initial void volume fraction ($\bar{X} \approx 0.65$). These results suggest that increasing χ increases the effect of material imperfections on necking formation (the weakest link of the plate triggers a fast growing neck which develops much quicker than any other neck), bearing a definite resemblance with the results of N'souglo et al. (2021), who showed that the effect of geometric imperfections on necking formation increases as the loading path moves away from plane strain stretching and approaches equibiaxial tension –see Fig. 14 therein–. Note that the secondary necks indicated with numbers 1, 2 and 4 are much less developed. The fact that the necking pattern becomes less regular –increasing differences in the growth rate of the necks– as the loading path moves away from plane strain is consistent with the results of Rodríguez-Martínez et al. (2017) –see Fig. 12 therein–, who showed that the stabilizing effect of inertia on necking localization decreases as χ increases, leading to the development of less necks which display larger differences in their speed of grow.

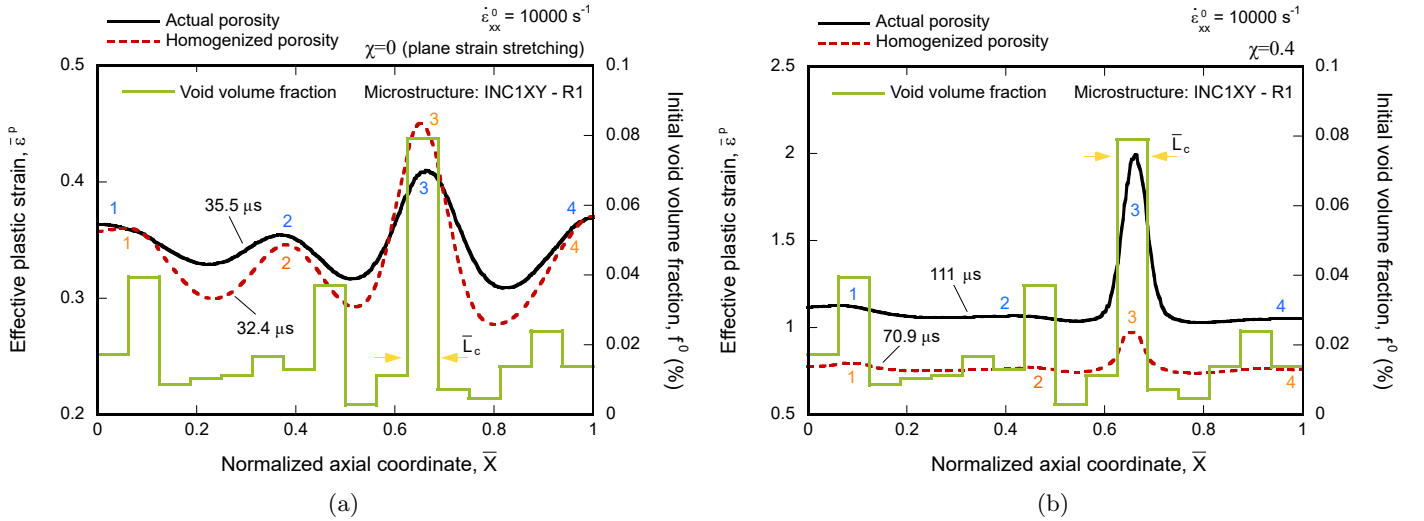


Figure 9: Microstructure INC1XY and realization R1. Evolution of effective plastic strain $\bar{\epsilon}^P$ and initial void volume fraction f^0 (%) along the normalized axial coordinate $\bar{X} = \frac{X}{L_x^0}$ (see Fig. 1). Comparison between results obtained with the actual porosity model and the homogenized porosity model for loading rate $\dot{\epsilon}_{xx}^0 = 10000 \text{ s}^{-1}$. Loading path: (a) $\chi = 0$ (plane strain stretching) and (b) $\chi = 0.4$. The loading time corresponds to the necking condition. The void volume fraction is computed dividing the sample into 16 cells with $L_c = 0.5 \text{ mm}$ (the strategy used in the homogenized porosity model). Note that $\bar{L}_c = \frac{L_c}{L_x^0}$ is the normalized cell size.

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Fig. 11 shows the evolution of the effective plastic strain and the distribution of initial void volume fraction along the normalized axial coordinate for microstructure Ti0.5Z and realization R3. Results for $\chi = 0$ (plane strain stretching) and $\chi = 0.4$ are shown in Figs. 11(a) and 11(b), respectively. The goal is to broaden the general trends and conclusions derived from the results presented in Fig. 9, as for this microstructure the porosity is significantly smaller (≈ 10 times less than for INC1XY, see Table 2) and the loading rate considered in the calculations is greater $\dot{\epsilon}_{xx}^0 = 60000 \text{ s}^{-1}$.

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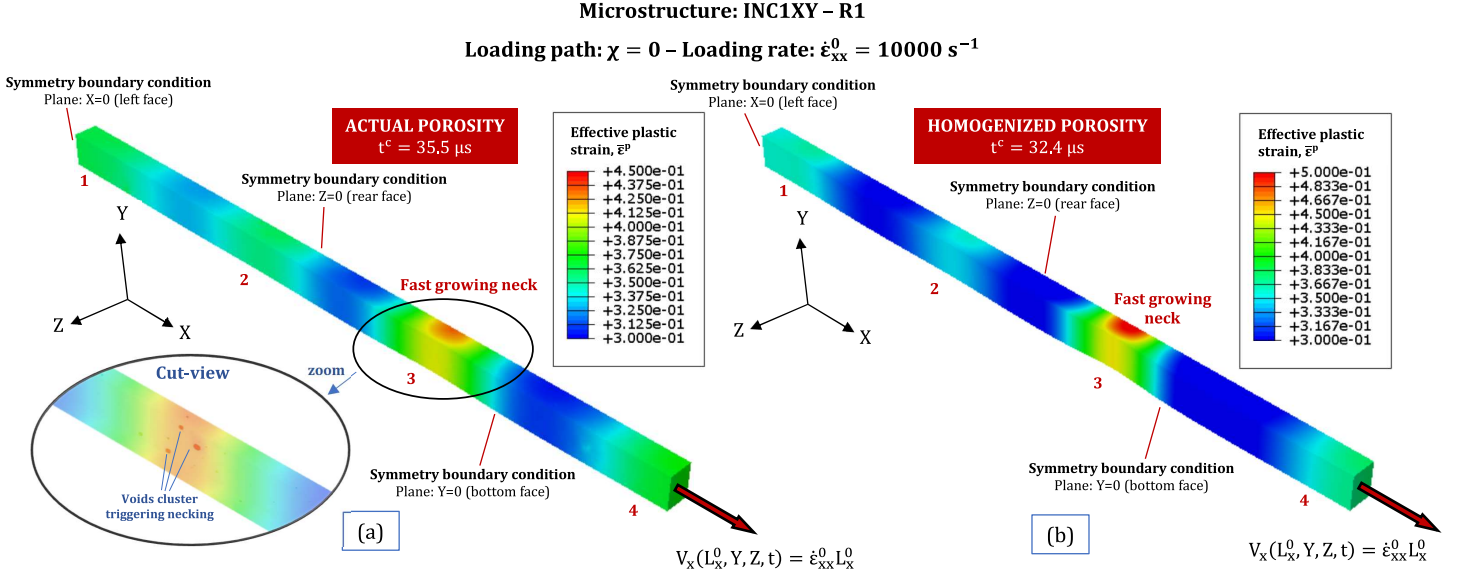


Figure 10: Microstructure INC1XY and realization R1. Contours of effective plastic strain $\bar{\epsilon}^p$ corresponding to calculations performed for loading path $\chi = 0$ (plane strain stretching) and loading rate $\dot{\epsilon}_{xx}^0 = 10000 \text{ s}^{-1}$: (a) actual porosity model and (b) homogenized porosity model. The loading time corresponds to the necking condition.

The necking time is less for the calculations performed with the Gurson-Tvergaard theory, substantiating the conclusion that homogenization of the porosity leads to a decrease in the necking strain (for all microstructures and realizations considered, as later illustrated in Fig. 12). There are six necks at locations of the bar with large values of the initial void volume fraction, and the main two necks –numbers 5 and 6– are developed at the two sections of the specimen with greater porosity, $\bar{X} \approx 0.8$ and $\bar{X} \approx 1$, making apparent that large voids and clusters trigger fast growing necks. Again, increasing the loading path from $\chi = 0$ to $\chi = 0.4$ increases the necking time, delays plastic localization, and leads to a more irregular necking pattern in which the two main necks –numbers 5 and 6– clearly stand out from the secondary necks –numbers 1, 2, 3 and 4–, consistently with the calculations of Fig. 9. These results reinforce the idea that the *weakest link effect* is exacerbated as the loading path moves away from plane strain towards biaxial stretching.

Fig. 12 collects results for the 3 microstructures and the 3 realizations (R1, R2, R3) to provide a quantitative measure of the effect that initial void volume fraction (in the whole specimen) and maximum void diameter have on the necking strain (according to Marvi-Mashhadi et al. (2021) these are the two main features of the porous microstructure affecting the necking formation). The calculations are performed for $\chi = 0$ (plane strain stretching) and two different loading rates $\dot{\epsilon}_{xx}^0 = 10000 \text{ s}^{-1}$ and 60000 s^{-1} . The results are obtained with both actual porosity and homogenized porosity models (black markers and red markers, respectively). Note that, in addition to the calculations performed with the porous microstructures of Tables 2 and 3, we have carried out 4

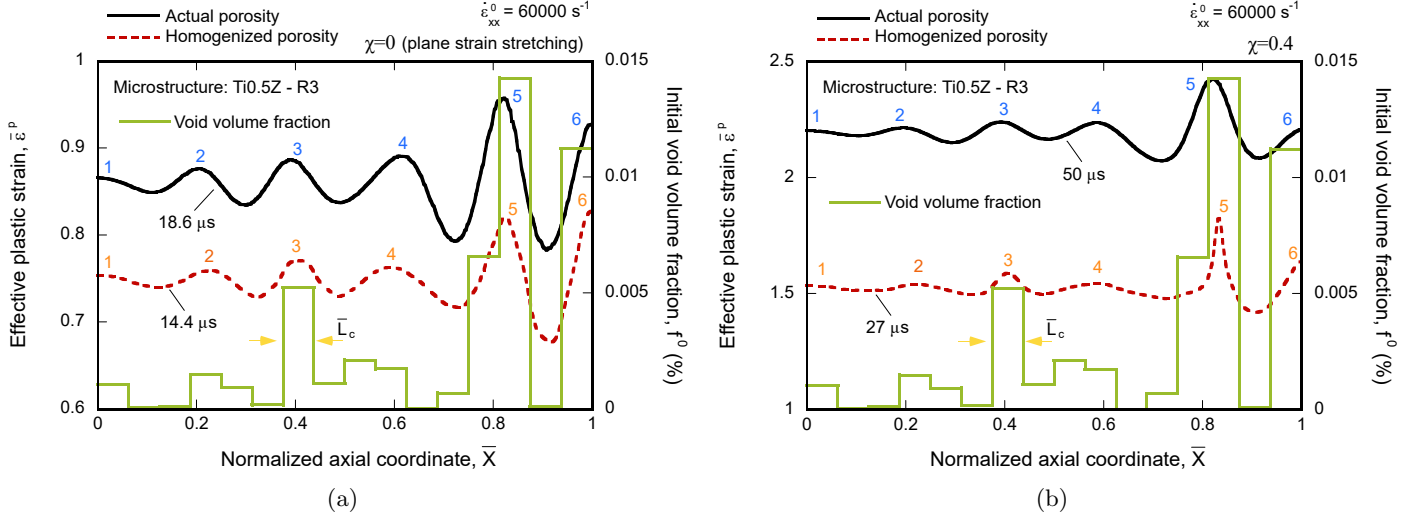


Figure 11: Microstructure Ti0.5Z and realization R3. Evolution of effective plastic strain $\bar{\epsilon}^p$ and initial void volume fraction f^0 (%) along the normalized axial coordinate $\bar{X} = \frac{X}{L_x^0}$ (see Fig. 1). Comparison between results obtained with the actual porosity model and the homogenized porosity model for loading rate $\dot{\epsilon}_{xx}^0 = 60000 \text{ s}^{-1}$. Loading path: (a) $\chi = 0$ (plane strain stretching) and (b) $\chi = 0.4$. The loading time corresponds to the necking condition. The void volume fraction is computed dividing the sample into 16 cells with $L_c = 0.5 \text{ mm}$ (the strategy used in the homogenized porosity model). Note that $\bar{L}_c = \frac{L_c}{L_x^0}$ is the normalized cell size.

numerical simulations with actual voids of the same diameter. These *virtual* microstructures (V1, V2, V3, V4) have the same initial void volume fraction and the same number of voids located at the same positions than SS5Z-R1, SS5Z-R3, Ti0.5Z-R3 and INC1XY-R1, respectively. The difference is that the diameter of the voids is $36.4 \mu\text{m}$, $24.1 \mu\text{m}$, $14.4 \mu\text{m}$ and $10.1 \mu\text{m}$, respectively (all voids with the same size). Note that, to make the size of all voids the same, we have increased/decreased the size of the small/large voids of the parent microstructures (SS5Z-R1, SS5Z-R3, Ti0.5X-R3, INC1XY-R1) from which (V1, V2, V3, V4) are derived. The results of these calculations correspond to the green markers.

The evolution of the necking strain ϵ_{xx}^c with the initial averaged void volume fraction $f^0(\%)$ is shown in Figs. 12(a) and 12(b) for the two strain rates considered. The decrease of the necking strain with the initial void volume fraction has been fitted with linear functions showing that the actual porosity approach (black solid lines) predicts greater necking strains than the homogenized porosity model (red solid lines) for all microstructures and realizations considered. Notice that the fitted curves are roughly parallel, such that the relative difference between the results obtained with both approaches increases with the initial void volume fraction. For instance,

$$\text{for } \dot{\epsilon}_{xx}^0 = 10000 \text{ s}^{-1} \text{ the gap between the fitted curves is } \frac{\epsilon_{xx}^c|_{f^0=0.01\%}^{\text{actual}} - \epsilon_{xx}^c|_{f^0=0.01\%}^{\text{homogenized}}}{\epsilon_{xx}^c|_{f^0=0.01\%}^{\text{actual}}} \bigg|_{\dot{\epsilon}_{xx}^0=10000 \text{ s}^{-1}} \bigg|_{\chi=0} \approx 13\% \text{ and}$$

$$\frac{\epsilon_{xx}^c|_{f^0=0.04\%}^{\text{actual}} - \epsilon_{xx}^c|_{f^0=0.04\%}^{\text{homogenized}}}{\epsilon_{xx}^c|_{f^0=0.04\%}^{\text{actual}}} \bigg|_{\dot{\epsilon}_{xx}^0=10000 \text{ s}^{-1}} \bigg|_{\chi=0} = 17\%, \text{ for } f^0 = 0.01\% \text{ and } 0.04\%, \text{ respectively. For the same initial void volume fractions and greater loading rate } 60000 \text{ s}^{-1}, \text{ the gap increases up to } 16\% \text{ and } 20\%, \text{ respectively. These}$$

results show that the predictions of the homogenized approach are closer to the calculations with explicitly resolved pores for low strain rates and small initial void volume fractions. However, note that the mesh used in the actual voids simulation is not fully converged (see Section 3.1). It is possible that using a finer mesh (for the actual voids simulations) leads to lower differences between the two modeling approaches. The effect of the initial void volume fraction on the necking strain is slightly dependent on the loading rate. For instance, the linear functions fitted to the actual porosity results (black solid lines) predict that the drop of the necking strain with f^0 for 10000 s^{-1} and 60000 s^{-1} is $\frac{\varepsilon_{xx}^c|_{f^0=0.01\%}^{\text{actual}} - \varepsilon_{xx}^c|_{f^0=0.04\%}^{\text{actual}}}{\varepsilon_{xx}^c|_{f^0=0.01\%}^{\text{actual}}} \Big|_{\dot{\varepsilon}_{xx}^0=10000 \text{ s}^{-1}}^{\chi=0} \approx 27\%$ and $\frac{\varepsilon_{xx}^c|_{f^0=0.01\%}^{\text{actual}} - \varepsilon_{xx}^c|_{f^0=0.04\%}^{\text{actual}}}{\varepsilon_{xx}^c|_{f^0=0.01\%}^{\text{actual}}} \Big|_{\dot{\varepsilon}_{xx}^0=60000 \text{ s}^{-1}}^{\chi=0} \approx 30\%$, respectively. On the other hand, notice that the green markers, corresponding to the calculations (V1, V2, V3, V4) with all pores of the same size, are slightly above the black markers obtained with computations which contain voids of different diameters, i.e., the distribution of voids sizes affects the necking strain (for a given initial void volume fraction in the specimen). Namely, these results suggest that decreasing the size of the larger pores leads to an increase of the specimen ductility.

The evolution of the necking strain ε_{xx}^c with the maximum void diameter d_{max} is shown in Figs. 12(c) and 12(d) for $\dot{\varepsilon}_{xx}^0 = 10000 \text{ s}^{-1}$ and 60000 s^{-1} , respectively. The necking strain decreases with the maximum void size, such that the results obtained for both actual porosity model and homogenized porosity model have been fitted with linear functions (of similar slope). Quantitative analysis of the fitted curves for the strain rate 10000 s^{-1} shows that the relative difference between both modeling approaches increases with the maximum void diameter: $\frac{\varepsilon_{xx}^c|_{d_{\text{max}}=30 \text{ }\mu\text{m}}^{\text{actual}} - \varepsilon_{xx}^c|_{d_{\text{max}}=30 \text{ }\mu\text{m}}^{\text{homogenized}}}{\varepsilon_{xx}^c|_{d_{\text{max}}=30 \text{ }\mu\text{m}}^{\text{actual}}} \Big|_{\dot{\varepsilon}_{xx}^0=10000 \text{ s}^{-1}}^{\chi=0} \approx 12\%$ and $\frac{\varepsilon_{xx}^c|_{d_{\text{max}}=100 \text{ }\mu\text{m}}^{\text{actual}} - \varepsilon_{xx}^c|_{d_{\text{max}}=100 \text{ }\mu\text{m}}^{\text{homogenized}}}{\varepsilon_{xx}^c|_{d_{\text{max}}=100 \text{ }\mu\text{m}}^{\text{actual}}} \Big|_{\dot{\varepsilon}_{xx}^0=10000 \text{ s}^{-1}}^{\chi=0} \approx 17\%$. Increasing the imposed loading rate to 60000 s^{-1} raises these figures up to 15% and 20%, respectively, suggesting that the homogenized porosity model provides results closer to the calculations with actual voids as the loading rate decreases and the voids are smaller. Moreover, the drop in the necking strain with increasing d_{max} predicted by the function fitted to the actual porosity calculations is $\frac{\varepsilon_{xx}^c|_{d_{\text{max}}=30 \text{ }\mu\text{m}}^{\text{actual}} - \varepsilon_{xx}^c|_{d_{\text{max}}=100 \text{ }\mu\text{m}}^{\text{actual}}}{\varepsilon_{xx}^c|_{d_{\text{max}}=30 \text{ }\mu\text{m}}^{\text{actual}}} \Big|_{\dot{\varepsilon}_{xx}^0=10000 \text{ s}^{-1}}^{\chi=0} \approx 31\%$ for 10000 s^{-1} , and $\approx 36\%$ for 60000 s^{-1} . For the microstructures and strain rates considered, the effect of the maximum void size in the necking strain slightly increases with the loading rate. On the other hand, notice that the green markers corresponding to the virtual microstructures V1, V2, V3 and V4 also predict a *roughly* linear decrease of the necking strain with d_{max} , however the slope of the fitted curve is greater than in the case of the calculations with the parent microstructures. For instance, the result for V1 is well below the black solid line: for this virtual microstructure, the increase in the necking strain due to the decrease of the maximum voids size with respect to the parent microstructure SS5Z-R1 does not balance the relatively small necking strain caused by the large initial void volume fraction (which is the same of the parent microstructure). These results make apparent

that the necking strain is determined by the interplay between f^0 and $d_{\max}$, with the initial void volume fraction playing a critical role in the specimen ductility.

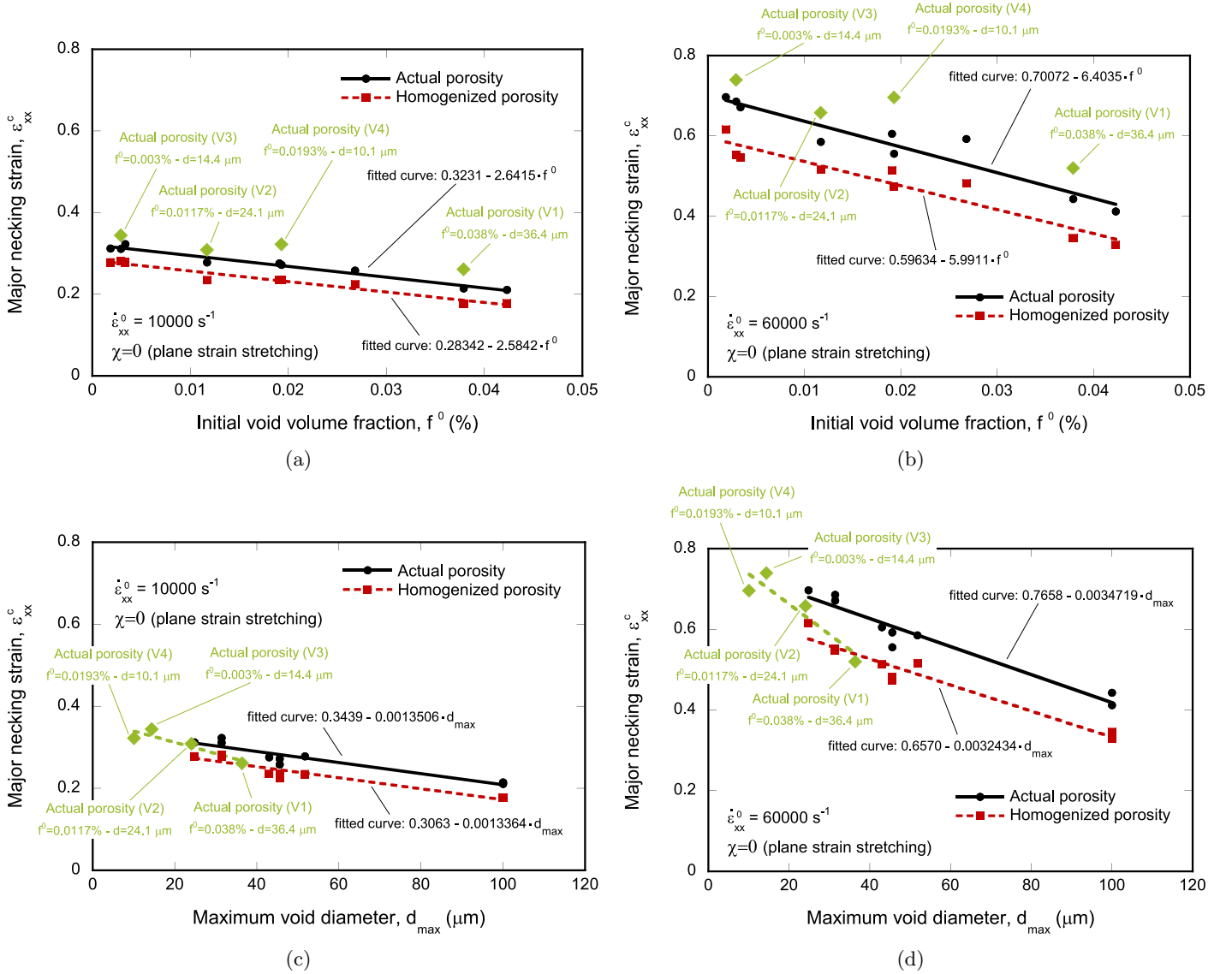


Figure 12: Comparison between actual porosity and homogenized porosity models. Results are shown for the 3 microstructures investigated in this work and the 3 realizations (R1, R2, R3) that we have generated per microstructure. Actual porosity calculations for *virtual* microstructures (V1, V2, V3, V4) with all pores of the same size are also included. Loading path $\chi = 0$ (plane strain stretching). Major necking strain ε_{xx}^c versus initial void volume fraction f^0 (%) for two different loading rates: (a) $\dot{\varepsilon}_{xx}^0 = 10000 \text{ s}^{-1}$ and (b) $\dot{\varepsilon}_{xx}^0 = 60000 \text{ s}^{-1}$. Major necking strain ε_{xx}^c versus maximum void diameter $d_{\max}$ for two different loading rates: (c) $\dot{\varepsilon}_{xx}^0 = 10000 \text{ s}^{-1}$ and (d) $\dot{\varepsilon}_{xx}^0 = 60000 \text{ s}^{-1}$. For interpretation of the references to color in this figure, the reader is referred to the web version of this article.

505

Fig. 13 displays the results obtained for a different loading path $\chi = 0.4$ (instead of $\chi = 0$). Notice the qualitative agreement between the results of Figs. 12 and 13, i.e., varying the loading path leads to different quantitative results while the general trends are the same.

508

509 The evolution of the necking strain ε_{xx}^c with the initial void volume fraction $f^0(\%)$ is shown in Figs. 13(a)
 510 and 13(b). The gap between the linear fitted curves is $\frac{\varepsilon_{xx}^c|_{f^0=0.01\%}^{\text{actual}} - \varepsilon_{xx}^c|_{f^0=0.01\%}^{\text{homogenized}}}{\varepsilon_{xx}^c|_{f^0=0.01\%}^{\text{actual}}} \Big|_{\dot{\varepsilon}_{xx}^0=10000 \text{ s}^{-1}}^{\chi=0.4} \approx 22\%$ and
 511 $\frac{\varepsilon_{xx}^c|_{f^0=0.04\%}^{\text{actual}} - \varepsilon_{xx}^c|_{f^0=0.04\%}^{\text{homogenized}}}{\varepsilon_{xx}^c|_{f^0=0.04\%}^{\text{actual}}} \Big|_{\dot{\varepsilon}_{xx}^0=10000 \text{ s}^{-1}}^{\chi=0.4} \approx 28\%$ for the lower loading rate 10000 s^{-1} , while these numbers increase
 512 up to 29% and 30%, respectively, for 60000 s^{-1} . Comparison with the results reported in Figs. 12(a) and 12(b) show
 513 that the predictions of the homogenized porosity approach are further away from the actual porosity calculations
 514 as the loading path increases from plane strain tension to $\chi = 0.4$. Moreover, the decrease in the necking strain
 515 with the initial void volume fraction predicted by the linear functions fitted to the actual porosity calculations
 516 is $\frac{\varepsilon_{xx}^c|_{f^0=0.01\%}^{\text{actual}} - \varepsilon_{xx}^c|_{f^0=0.04\%}^{\text{actual}}}{\varepsilon_{xx}^c|_{f^0=0.01\%}^{\text{actual}}} \Big|_{\dot{\varepsilon}_{xx}^0=10000 \text{ s}^{-1}}^{\chi=0.4} \approx 38\%$ for 10000 s^{-1} , and 39% for the higher strain rate 60000 s^{-1} .
 517 These figures are greater than for $\chi = 0$ (see Figs. 12(a) and 12(b)), which shows that the drop in the necking
 518 strain with the initial void volume fraction is greater as the loading path moves from plane strain towards biaxial
 519 stretching. On the other hand, the results obtained with the virtual microstructures (V1, V2, V3, V4) lie above the
 520 linear functions fitted to the actual porosity calculations, making apparent that for a given void volume fraction,
 521 decreasing the maximum diameter of the voids delays necking formation (note that the green markers are more
 522 far from the black solid line than in the case of $\chi = 0$, see Figs. 12(a) and 12(b)).

523 The evolution of the necking strain ε_{xx}^c with the maximum void diameter d_{max} is shown in Figs. 13(c) and 13(d).
 524 The relative difference between the results obtained with the homogenized porosity model and the actual porosity
 525 model increases with the maximum void diameter and the strain rate: $\frac{\varepsilon_{xx}^c|_{d_{\text{max}}=30 \text{ }\mu\text{m}}^{\text{actual}} - \varepsilon_{xx}^c|_{d_{\text{max}}=30 \text{ }\mu\text{m}}^{\text{homogenized}}}{\varepsilon_{xx}^c|_{d_{\text{max}}=30 \text{ }\mu\text{m}}^{\text{actual}}} \Big|_{\dot{\varepsilon}_{xx}^0=10000 \text{ s}^{-1}}^{\chi=0.4} \approx$
 526 21% and $\frac{\varepsilon_{xx}^c|_{d_{\text{max}}=100 \text{ }\mu\text{m}}^{\text{actual}} - \varepsilon_{xx}^c|_{d_{\text{max}}=100 \text{ }\mu\text{m}}^{\text{homogenized}}}{\varepsilon_{xx}^c|_{d_{\text{max}}=100 \text{ }\mu\text{m}}^{\text{actual}}} \Big|_{\dot{\varepsilon}_{xx}^0=10000 \text{ s}^{-1}}^{\chi=0.4} \approx 29\%$, while these figures become 29% and 31% for
 527 60000 s^{-1} . The raise with respect to the same computations performed for plane strain confirms that the dif-
 528 ferences between homogenized and actual porosity approaches are boosted with the increase of the loading path
 529 parameter. Moreover, the drop of the necking strain with the maximum diameter of the voids predicted by the
 530 linear function fitted to the actual porosity calculations is $\frac{\varepsilon_{xx}^c|_{d_{\text{max}}=30 \text{ }\mu\text{m}}^{\text{actual}} - \varepsilon_{xx}^c|_{d_{\text{max}}=100 \text{ }\mu\text{m}}^{\text{actual}}}{\varepsilon_{xx}^c|_{d_{\text{max}}=30 \text{ }\mu\text{m}}^{\text{actual}}} \Big|_{\dot{\varepsilon}_{xx}^0=10000 \text{ s}^{-1}}^{\chi=0.4} \approx 45\%$ for
 531 10000 s^{-1} , and $\frac{\varepsilon_{xx}^c|_{d_{\text{max}}=30 \text{ }\mu\text{m}}^{\text{actual}} - \varepsilon_{xx}^c|_{d_{\text{max}}=100 \text{ }\mu\text{m}}^{\text{actual}}}{\varepsilon_{xx}^c|_{d_{\text{max}}=30 \text{ }\mu\text{m}}^{\text{actual}}} \Big|_{\dot{\varepsilon}_{xx}^0=60000 \text{ s}^{-1}}^{\chi=0.4} \approx 46\%$ for 60000 s^{-1} , respectively. These figures are
 532 greater than for $\chi = 0$ (see Figs. 12(c) and 12(d)), which shows that the decrease of the necking strain with d_{max}
 533 is more for biaxial stretching than for plane strain. On the other hand, note that the effect of the maximum void
 534 diameter on the necking strain is sensitive to the initial void volume fraction. Similarly to the results presented in
 535 Figs. 12(c) and 12(d), the necking strain obtained with the virtual microstructure V1 is well below the solid black
 536 curve, i.e., for a given value of the maximum void diameter, increasing the void volume fraction leads to a drop in

537 sample ductility.

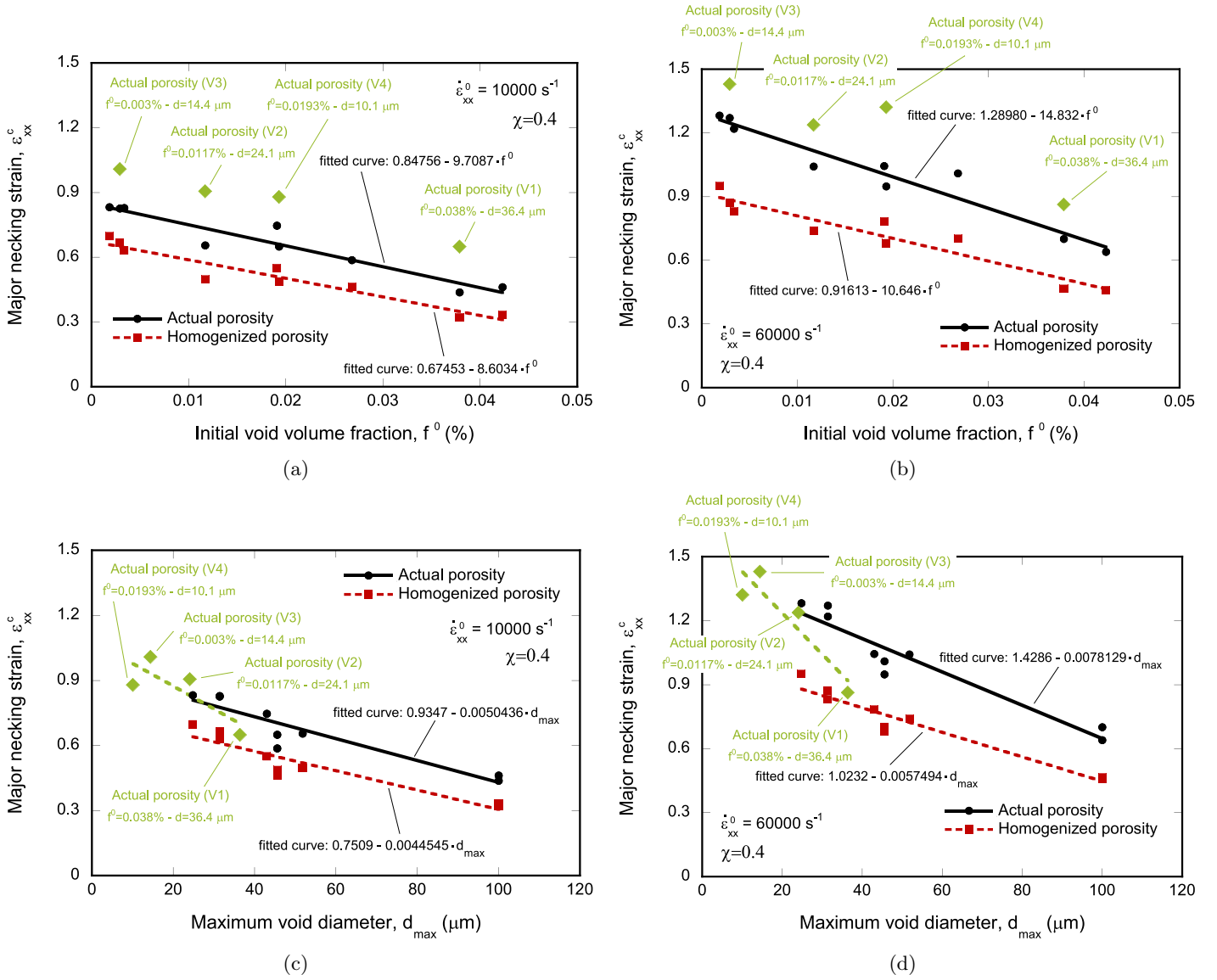


Figure 13: Comparison between actual porosity and homogenized porosity models. Results are shown for the 3 microstructures investigated in this work and the 3 realizations (R1, R2, R3) that we have generated per microstructure. Actual porosity calculations for *virtual* microstructures (V1, V2, V3, V4) with all pores of the same size are also included. Loading path $\chi = 0.4$. Major necking strain ε_{xx}^c versus initial void volume fraction f^0 (%) for two different loading rates: (a) $\dot{\varepsilon}_{xx}^0 = 10000 \text{ s}^{-1}$ and (b) $\dot{\varepsilon}_{xx}^0 = 60000 \text{ s}^{-1}$. Major necking strain ε_{xx}^c versus maximum void diameter $d_{\max}$ for two different loading rates: (c) $\dot{\varepsilon}_{xx}^0 = 10000 \text{ s}^{-1}$ and (d) $\dot{\varepsilon}_{xx}^0 = 60000 \text{ s}^{-1}$. For interpretation of the references to color in this figure, the reader is referred to the web version of this article.

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539 Moreover, we have performed selected calculations with the Gurson-Tvergaard continuum plasticity theory in
 540 which the initial porosity is taken to be homogeneous in the strip (not shown for the sake of brevity). These
 541 simulations have led to very high necking strains (much greater than the calculations with explicitly resolved
 542 pores) or no necking for some loading conditions (for high strain rates and large values of the loading path),

making apparent that the distribution of initial void volume fraction in the homogenized porosity approach is essential to capture the general trends predicted by the calculations with explicitly resolved porosity.

The effect of the heterogeneous spatial distribution of porosity on the specimen ductility is further illustrated in Fig. 14, which displays the evolution of the major necking strain ε_{xx}^c with the standard deviation of the cells porosity distribution SD for calculations performed with both the actual porosity model (black markers) and the homogenized porosity model (red markers) for the three realizations of the three microstructures investigated, see Tables 2 and 3. The standard deviation of the cells porosity distribution is a measure of the heterogeneity of the spatial allocation of initial void volume fraction along the specimen, and it is computed from the homogenized porosity model (which is divided into 16 cells with $L_c = 0.5$ mm and initial void volume fraction computed with equation (14)), using the following relation $SD = \sqrt{\frac{\sum_{i=1}^{N_c} (f_i^0 - f^0)^2}{N_c}}$, where N_c is the number of cells and f_i^0 is the initial void fraction in cell i . Results are shown for $\dot{\varepsilon}_{xx}^0 = 10000 \text{ s}^{-1}$ and $\chi = 0$ in Fig. 14(a), and for $\dot{\varepsilon}_{xx}^0 = 60000 \text{ s}^{-1}$ and $\chi = 0.4$ in Fig. 14(b). Both the calculations with explicitly resolved pores and with homogenized porosity have been fitted to logarithmic functions. The necking strain decreases nonlinearly as the heterogeneity of the spatial distribution of porosity in the specimen increases. Namely, the rate of decrease of the necking strain is large for low values of SD and becomes gradually less as SD increases. Actual porosity and homogenized porosity calculations for virtual microstructures (V1, V2, V3, V4) are also included in Fig. 14 (green markers). One can observe that the results for all the microstructures are very close to the logarithmic fitting curves, which reinforces the idea that the heterogeneity of the porosity distribution is an important factor affecting the specimen ductility, such that a small heterogeneity leads to an important drop in the necking strain (in comparison with the homogeneous material). The same conclusion has been obtained from calculations in which the material behavior corresponds to Tantalum and Copper, with parameters taken from Mercier et al. (2010), see Appendix A. Notice that Molinari and Clifton (1987) also obtained a logarithmic relationship between the critical strain leading to shear band formation and the amplitude of geometric defects in specimens subjected to simple shear loading (see Fig. 4 therein). In fact, the results presented in Fig. 14 suggest that the porous microstructure acts as a material imperfection. The voids are randomly distributed within the material. This causes spatial variations of the initial void fraction and of the macroscopic material behavior, which affect the development of necking.

4.3. The influence of loading rate

This section displays results for the three realizations of the three porous microstructures investigated, see Tables 2 and 3. Two different loading paths, $\chi = 0$ and 0.4, and several initial major strain rates ranging from $\dot{\varepsilon}_{xx}^0 = 10000 \text{ s}^{-1}$ to 60000 s^{-1} , are considered for the analysis.

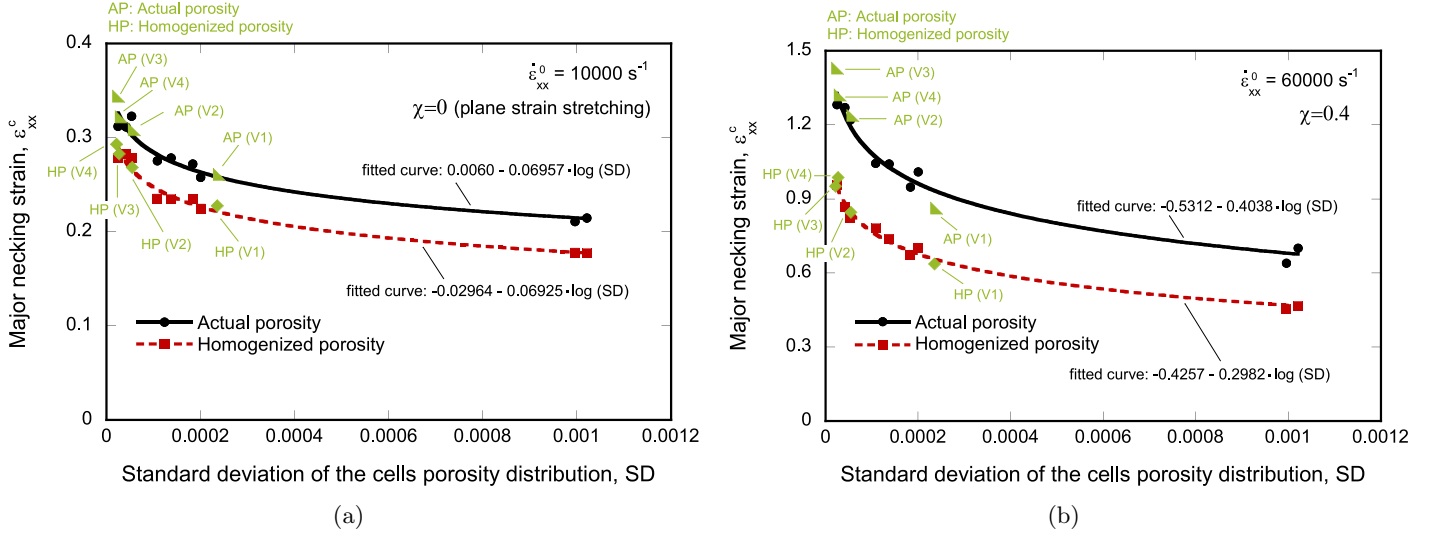


Figure 14: Comparison between actual porosity and homogenized porosity models. Results are shown for the 3 microstructures investigated in this work and the 3 realizations (R1, R2, R3) that we have generated per microstructure. Actual porosity calculations and homogenized porosity calculations for *virtual* microstructures (V1, V2, V3, V4) are also included (green markers). Major necking strain ε_{xx}^c versus standard deviation of the cells porosity distribution SD for: (a) $\dot{\varepsilon}_{xx}^0 = 10000 \text{ s}^{-1}$ and $\chi = 0$, and (b) $\dot{\varepsilon}_{xx}^0 = 60000 \text{ s}^{-1}$ and $\chi = 0.4$. For interpretation of the references to color in this figure, the reader is referred to the web version of this article.

Fig. 15 shows the evolution of the major necking strain ε_{xx}^c with the loading rate $\dot{\varepsilon}_{xx}^0$ for calculations performed with the Gurson-Tvergaard continuum plasticity theory and with explicitly resolved voids. The results are fitted to linear functions (solid red and black lines, respectively) following the same color coding used in Figs. 12, 13 and 14. The general trend for the three microstructures and both loading paths is that the necking strain increases monotonically with the loading rate (due to the stabilizing effect of inertia), and that the values of ε_{xx}^c predicted by the actual porosity calculations are greater (in agreement with the results shown in Figs. 12, 13 and 14). Consistent with the results presented in Figs. 12, 13 and 14, the differences between the models with homogenized porosity and actual voids increase as the loading rate increases and the loading path moves away from plane strain towards biaxial stretching. However, for the sake of brevity, we do not show a quantitative analysis to substantiate these trends (as we did in Section 4.2), so that the reader is referred to the fitted linear functions of Fig. 15 to compute the gap in the necking strains for different loading conditions.

Figs. 15(a) and 15(b) picture the results corresponding to the three realizations of microstructure INC1XY for $\chi = 0$ and 0.4, respectively. The linear functions fitted to the actual porosity results predict that the increase of the necking strain with loading rate is more for plane strain stretching than for $\chi = 0.4$, namely

$$\frac{\varepsilon_{xx}^c|_{\dot{\varepsilon}_{xx}^0=60000 \text{ s}^{-1}}^{\text{actual}} - \varepsilon_{xx}^c|_{\dot{\varepsilon}_{xx}^0=10000 \text{ s}^{-1}}^{\text{actual}}}{\varepsilon_{xx}^c|_{\dot{\varepsilon}_{xx}^0=10000 \text{ s}^{-1}}^{\text{actual}}} \Big|_{\text{INC1XY}} \Big|_{\chi=0} \approx 113\% \text{ and } \frac{\varepsilon_{xx}^c|_{\dot{\varepsilon}_{xx}^0=60000 \text{ s}^{-1}}^{\text{actual}} - \varepsilon_{xx}^c|_{\dot{\varepsilon}_{xx}^0=10000 \text{ s}^{-1}}^{\text{actual}}}{\varepsilon_{xx}^c|_{\dot{\varepsilon}_{xx}^0=10000 \text{ s}^{-1}}^{\text{actual}}} \Big|_{\text{INC1XY}} \Big|_{\chi=0.4} \approx 51\%,$$

in agreement with the results of Rodríguez-Martínez et al. (2017) who showed that the stabilizing effect of inertia

decreases as the loading path moves towards biaxial stretching (as we mentioned in Section 4.2). Besides delaying necking formation, inertia effects also increase the number of necks formed in the strip. For instance, Table 4 shows that the average number of necks (for the three realizations) for the actual porosity calculations with microstructure INC1XY turns from 4 at 10000 s^{-1} to 5 at 60000 s^{-1} for plane strain stretching. For loading path $\chi = 0.4$, and the same loading rates, the average number of necks is 3 and 4, respectively (we have rounded to the nearest integer as it is meaningless to give the number of necks with decimal digits). The nucleation of additional necks with the increase of the loading rate is apparent in Figs. 16(a) and 16(b) which show the evolution of the effective plastic strain and the distribution of initial void volume fraction along the normalized axial coordinate of the strip for realization R3 and loading paths $\chi = 0$ and 0.4 , respectively. Recall that the effective plastic strain is measured along the path indicated in Fig. 1, located on the free surface of the specimen. Results for two loading rates are presented, $\dot{\varepsilon}_{xx}^0 = 10000 \text{ s}^{-1}$ and 60000 s^{-1} . While three necks are nucleated for the lower strain rate at sections of the strip with large values of the initial void volume fraction, two more necks appear at 60000 s^{-1} . On the one hand, neck number 2 at low strain rate splits up into two (red numbers 2 and 3) because inertia and material straining favor the formation of smaller necks (see Figs. 14 and 5 of Rodríguez-Martínez et al. (2013) and Rodríguez-Martínez et al. (2017), respectively). The formation of these two smaller necks seems to be assisted by the porous microstructure, as the excursions of strain appear near two highs of the initial void volume fraction. On the other hand, neck number 5 at 60000 s^{-1} nucleates at a section of the strip that was neck-free at lower strain rate. This neck also seems to be assisted by the microstructure, as there is a local increase of the porosity near $\bar{X} = 1$. These results suggest that both porous microstructure and inertia play a role in the resulting necking patterns. While the microstructure seems to impose the nucleation of necks at specific locations (weak points) of the specimen for all the velocities investigated, inertia delays localization and favors the nucleation of additional necks at higher loading rates (these results are consistent with the calculations shown in Figs. 16 and 20 of Marvi-Mashhadi et al. (2021)).

Figs. 15(c) and 15(d) show the evolution of the major necking strain ε_{xx}^c with the initial major strain rate $\dot{\varepsilon}_{xx}^0$ for the calculations with microstructure SS5Z. The stabilizing effect of inertia is similar to microstructure INC1XY, the increase of the necking strain being $\frac{\varepsilon_{xx}^c|_{\dot{\varepsilon}_{xx}^0=60000 \text{ s}^{-1}}^{\text{actual}} - \varepsilon_{xx}^c|_{\dot{\varepsilon}_{xx}^0=10000 \text{ s}^{-1}}^{\text{actual}}}{\varepsilon_{xx}^c|_{\dot{\varepsilon}_{xx}^0=10000 \text{ s}^{-1}}^{\text{actual}}} \Big|_{\chi=0}^{\chi=0.4} \approx 100\%$ and $\frac{\varepsilon_{xx}^c|_{\dot{\varepsilon}_{xx}^0=60000 \text{ s}^{-1}}^{\text{actual}} - \varepsilon_{xx}^c|_{\dot{\varepsilon}_{xx}^0=10000 \text{ s}^{-1}}^{\text{actual}}}{\varepsilon_{xx}^c|_{\dot{\varepsilon}_{xx}^0=10000 \text{ s}^{-1}}^{\text{actual}}} \Big|_{\text{SS5Z}} \approx 51\%$ for loading paths $\chi = 0$ and 0.4 , respectively. For the results obtained with microstructure Ti0.5Z in Figs. 15(e) and 15(f) these figures slightly increase up to 112% and 52%, respectively. These results make apparent that the delay in the necking formation with the increase of the loading rate depends to some extent on the microstructure. This behavior, however, does not seem to be caused by

the dynamic behavior of the voids (microinertia), as it is proven to be negligible before necking localization, see Appendix D. More likely, it is due the interplay between the microstructure (which seems to act as a material imperfection) and macroscale inertia effects.

Number of necks - Microstructure: INC1XY - Actual porosity						
	Loading path: $\chi = 0$			Loading path: $\chi = 0.4$		
Initial major strain rate	R1	R2	R3	R1	R2	R3
10000 s ⁻¹	4	4	3	3	3	3
20000 s ⁻¹	4	4	3	3	3	3
40000 s ⁻¹	4	4	4	4	4	3
60000 s ⁻¹	5	6	5	4	4	5

Table 4: Number of necks obtained for microstructure INC1XY and three realizations (R1, R2, R3). Results corresponding to calculations with explicitly resolved pores for two loading paths: $\chi = 0$ (plane strain stretching) and $\chi = 0.4$. Four loading rates are considered: $\dot{\varepsilon}_{xx}^0 = 10000 \text{ s}^{-1}$, 20000 s^{-1} , 40000 s^{-1} and 60000 s^{-1} .

4.4. The influence of loading path

This section shows forming limit diagrams for the three porous microstructures considered, and two different loading rates, $\dot{\varepsilon}_{xx}^0 = 10000 \text{ s}^{-1}$ and 60000 s^{-1} (lowest and highest strain rate considered). The loading paths considered range from $\chi = 0$ to 0.75 (calculations for $\chi > 0.5$ are only carried out for microstructure SS5Z). In addition, the effect of the loading path on the shape evolution of the voids for the calculations with explicitly resolved pores is investigated.

Fig. 17 pictures the major necking strain ε_{xx}^c versus the minor necking strain ε_{yy}^c for calculations performed with both the homogenized porosity model and the actual porosity model (recall from Section 4.1 the procedure for computing the major and minor necking strains). Results for the three realizations performed per microstructure are considered. The color coding is the same used in Figs. 12, 13, 14 and 15. The data are fitted with linear functions: solid lines for 10000 s^{-1} , and dashed lines for 60000 s^{-1} . The results are qualitatively very similar for the three microstructures considered: compare Figs. 17(a), 17(b) and 17(c) for INC1XY, SS5Z and Ti0.5Z, respectively. Namely, the calculations with the explicitly resolved pores predict greater necking formability, i.e., the predictions of the homogenized porosity approach are on the conservative side. Moreover, the quantitative differences between the results obtained with both modeling strategies increase with the loading rate (as anticipated in Sections 4.2 and 4.3). Notice that the slope of the linear functions used to fit the data is greater for $\dot{\varepsilon}_{xx}^0 = 10000 \text{ s}^{-1}$ than for

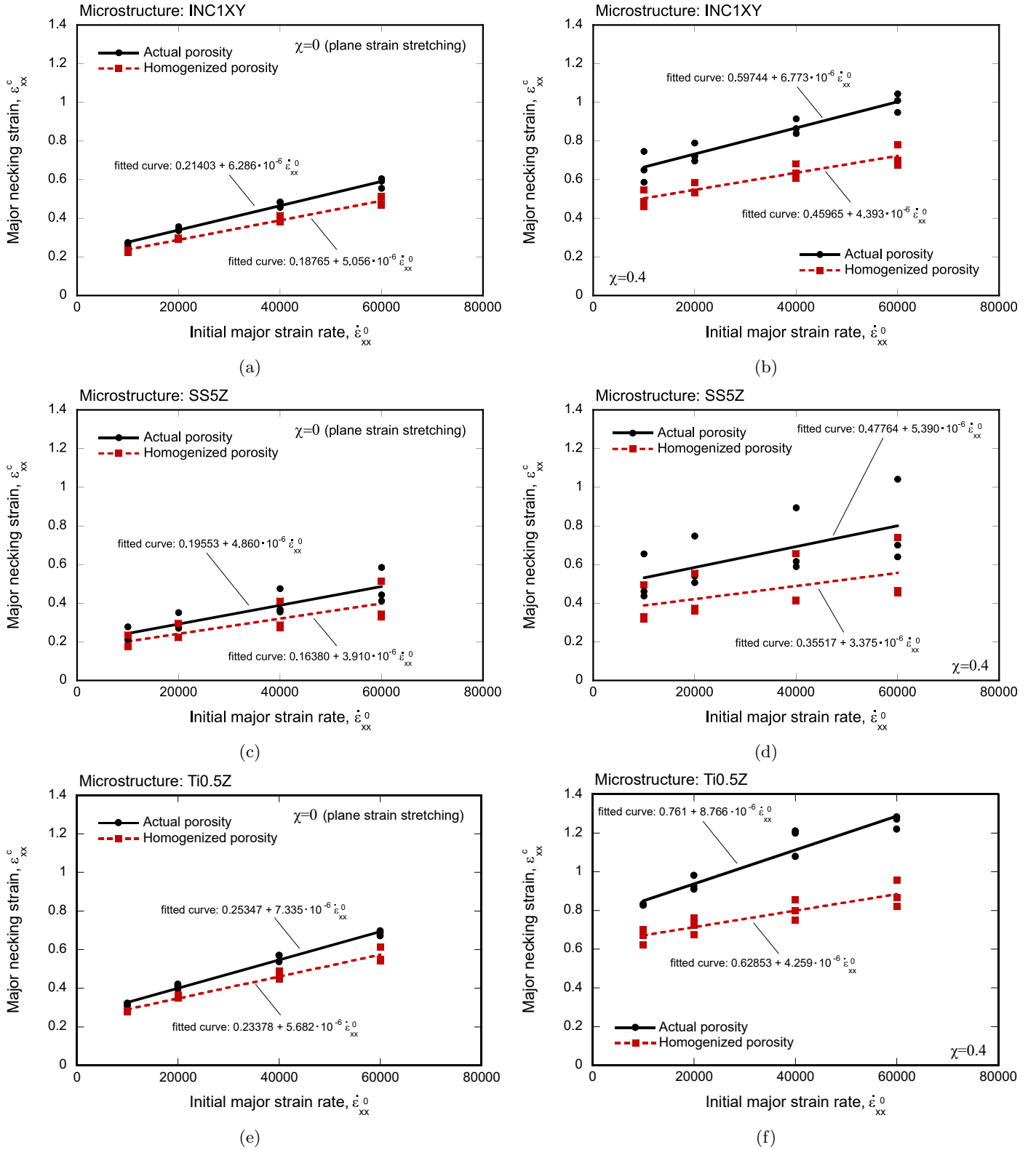


Figure 15: Major necking strain ε_{xx}^c versus initial major strain rate $\dot{\varepsilon}_{xx}^0$. Comparison between results obtained with the actual porosity model and the homogenized porosity model for the three microstructures investigated in this work. Microstructure INC1XY: (a) $\chi = 0$ and (b) $\chi = 0.4$. Microstructure SS5Z: (c) $\chi = 0$ and (d) $\chi = 0.4$. Microstructure Ti0.5Z: (e) $\chi = 0$ and (f) $\chi = 0.4$. For interpretation of the references to color in this figure, the reader is referred to the web version of this article.

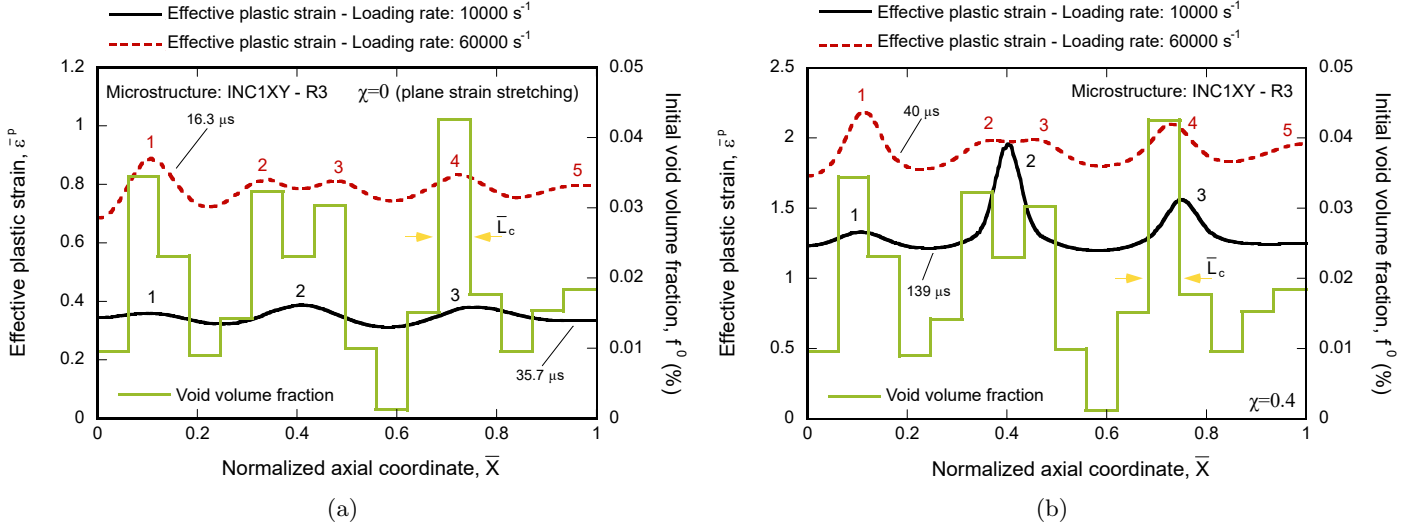


Figure 16: Microstructure INC1XY and realization R3. Evolution of effective plastic strain $\bar{\varepsilon}^p$ and initial void volume fraction f^0 (%) along the normalized axial coordinate $\bar{X} = \frac{X}{L_x^0}$ (see Fig. 1). The effective plastic strain is measured in calculations with actual porosity, for two different loading rates, $\dot{\varepsilon}_{xx}^0 = 10000 \text{ s}^{-1}$ and $\dot{\varepsilon}_{xx}^0 = 60000 \text{ s}^{-1}$, and two different loading paths: (a) $\chi = 0$ (plane strain stretching) and (b) $\chi = 0.4$. The loading time corresponds to the necking condition. The void volume fraction is computed dividing the sample into 16 cells with $L_c = 0.5 \text{ mm}$ (the strategy used in the homogenized porosity model). Note that $\bar{L}_c = \frac{L_c}{L_x^0}$ is the normalized cell size.

60000 s^{-1} . These results make apparent that the delay on necking formation as the loading path moves towards
 biaxial stretching is less at higher loading rates.

Moreover, the effect of the loading path on the voids shape evolution in the calculations with actual porosity
 is illustrated in Figs. 18 and 19 which show the 3D reconstruction of a pore corresponding to microstructure
 INC1XY-R3 for $\chi = 0$ and $\chi = 0.4$, respectively. The void geometry is reconstructed plotting the surface defined
 by the convex hull that forms the void. The loading rate is $\dot{\varepsilon}_{xx}^0 = 60000 \text{ s}^{-1}$ and the initial diameter of the void is
 43 μm . The pore is located at the normalized axial coordinate $\bar{X} = 0.72$, within the necked section 4 indicated in
 red in Fig. 16. The origin of the Cartesian coordinate system (X', Y', Z') is located at the center of mass of the
 void, so that the axes X', Y', Z' are parallel to the axes X, Y, Z shown in Fig. 1.

For the plane strain case, $\chi = 0$, Fig. 18 shows the void for $t = 0 \mu\text{s}$, 5.5 μs , 11 μs and 16 μs (necking time, see
 Fig. 16(a)). As the loading progresses, the initially spherical void turns into an ellipsoid. The pore elongates along
 the major loading direction X' , flattens in the thickness direction Z' , and slightly enlarges along the Y' direction
 despite the plane strain constraint. For the loading time $t = 16 \mu\text{s}$ (necking time), the volume of the voids has
 increased by 42%.

For the biaxial stretching case, $\chi = 0.4$, Fig. 19 shows the void for $t = 0 \mu\text{s}$, 10 μs , 20 μs and 30 μs (before
 necking time, see Fig. 16(b)). The loading times selected are greater than in the case of plane strain, because the

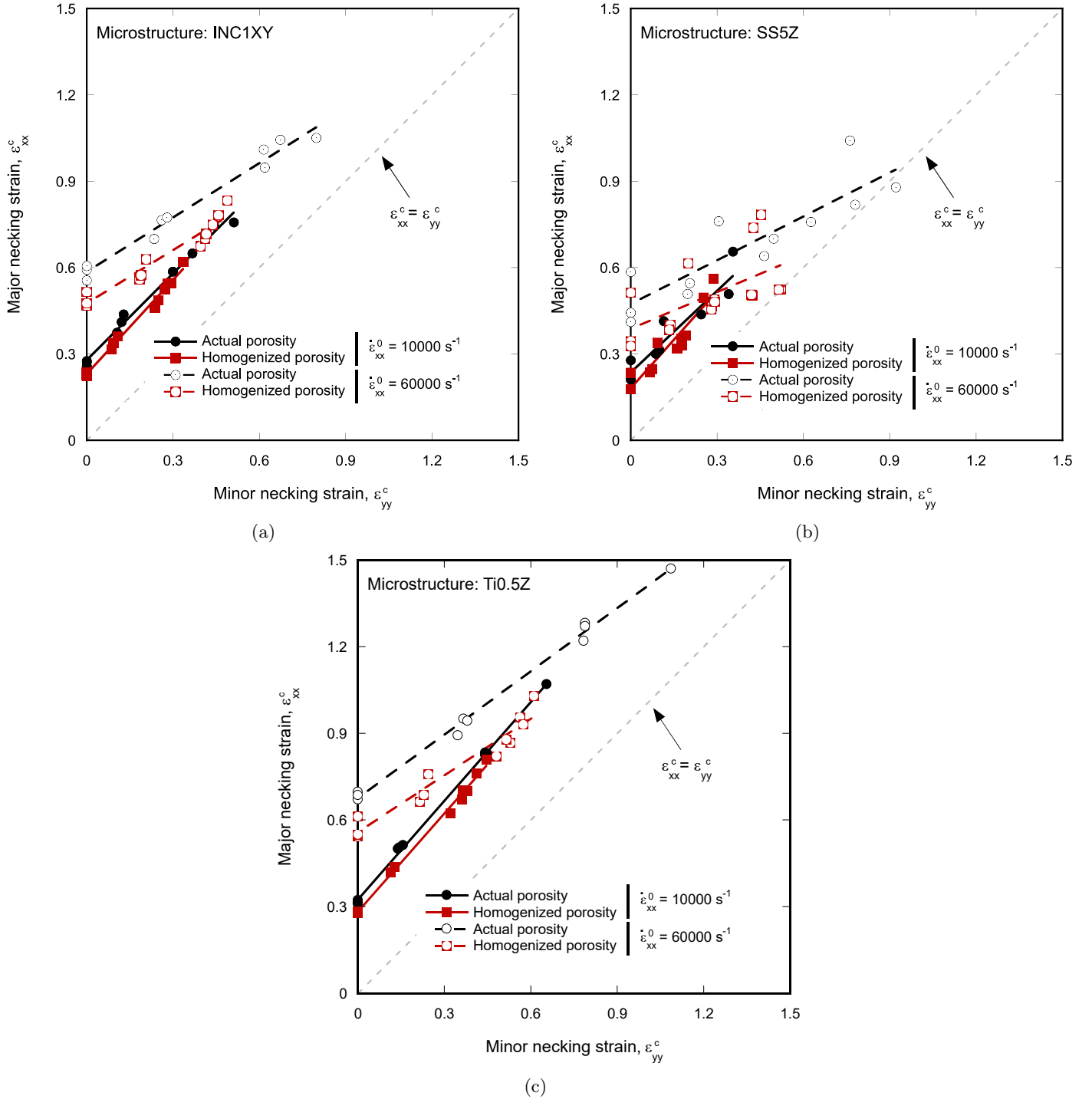


Figure 17: Forming limit diagrams, major necking strain ϵ_{xx}^c versus minor necking strain ϵ_{yy}^c . Comparison between results obtained with the actual porosity model and the homogenized porosity model for two different loading rates, $\dot{\epsilon}_{xx}^0 = 10000 \text{ s}^{-1}$ and $\dot{\epsilon}_{xx}^0 = 60000 \text{ s}^{-1}$, and the three microstructures investigated in this work: (a) INC1XY, (b) SS5Z and (c) Ti0.5Z. For interpretation of the references to color in this figure, the reader is referred to the web version of this article.

necking strain for $\chi = 0.4$ is greater ($\varepsilon_{xx}^c = 1.044$ for $\chi = 0.4$ versus $\varepsilon_{xx}^c = 0.6054$ for $\chi = 0$). The pore stretches out along X' and Y' directions, so that the initially spherical void evolves into a penny-shaped pore before necking localization (the pronounced penny-shape makes difficult to reconstruct the void for $t > 30 \mu s$). For the loading time $t = 30 \mu s$ the volume of the void has increased by 55%, i.e., the increase of the volume of the void before localization is slightly higher for $\chi = 0.4$ than for $\chi = 0$.

These results show that the severe distortion of the shape of the voids during loading does not prevent the homogenized porosity calculations from yielding necking forming limits in qualitative agreement with the actual porosity simulations, despite the Gurson-Tvergaard model assumes that the pores remain spherical during deformation. Hence, a natural continuation of this research is to consider Gurson-type constitutive models accounting for void shape change effects (Madou and Leblond, 2012a,b; Madou et al., 2013).

5. Concluding remarks

In this paper, we have performed microstructurally-informed finite element simulations to investigate the effect of porosity on the necking formability of ductile sheets subjected to in-plane biaxial stretching. The calculations have been carried out using actual porosity models which include spatial and size distributions of explicitly resolved voids, and homogenized porosity models in which the initial void volume fraction distribution is computed from the void distributions included in the simulations with actual porous microstructure. We have considered different porous microstructures with the mean void volume fraction varying from $\approx 0.003\%$ to $\approx 0.03\%$ and the size of the pores between $6 \mu m$ and $100 \mu m$. The analysis of the necking formability limits and the necking patterns predicted by the two approaches has led to the following conclusions:

- The porous microstructure has a significant influence on necking strain, most likely due to the heterogeneity of the spatial distribution of porosity in the specimen. The specimen ductility decreases nonlinearly as the heterogeneity of the spatial distribution of porosity in the specimen increases. The influence of the initial microstructure becomes more pronounced when the loading path moves from plane strain towards biaxial stretching.
- The porous microstructure imposes the nucleation of necks at specific locations of the specimen for all the velocities investigated, while inertia delays localization and favors the nucleation of additional necks at higher loading rates.
- The effect of microinertia has been shown to be small prior localized necking, so that the dynamic behavior

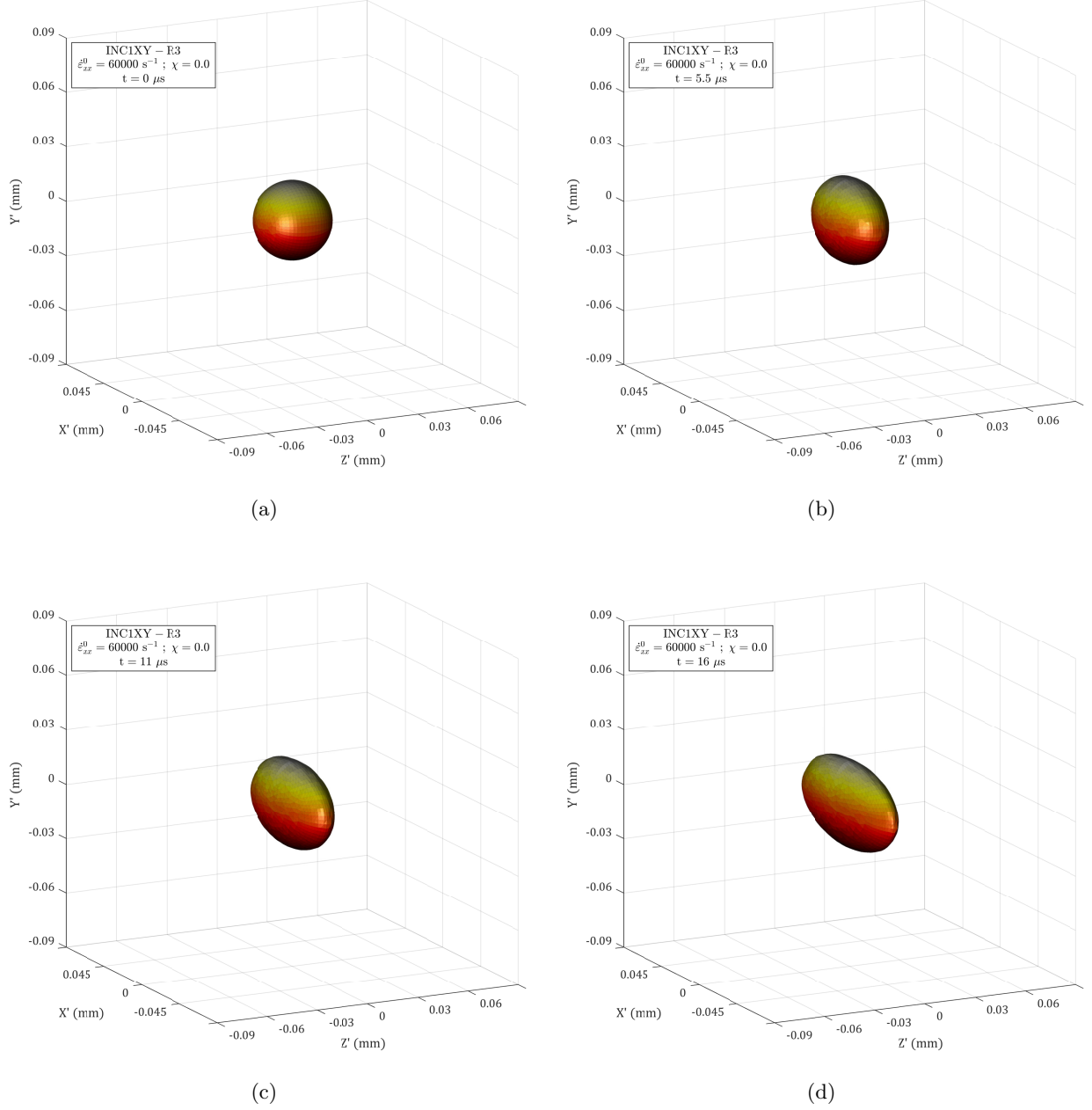


Figure 18: Microstructure INC1XY and realization R3. 3D reconstruction of the surface of the void located at coordinates $X = 5.82 \text{ mm}$, $Y = -0.23 \text{ mm}$ and $Z = 0.94 \text{ mm}$, referred to the Cartesian coordinate system (X, Y, Z) indicated in Fig. 1. Evolution of the void shape for different loading times: (a) $t = 0 \mu s$, (b) $t = 5.5 \mu s$, (c) $t = 11 \mu s$ and (d) $t = 16 \mu s$. The necking time is $16.3 \mu s$. The loading rate is $\dot{\epsilon}_{xx}^0 = 60000 \text{ s}^{-1}$ and the loading path is $\chi = 0$ (plane strain stretching). The origin of the Cartesian coordinate system (X', Y', Z') is located at the center of mass of the void, so that the axes X', Y', Z' are parallel to the axes X, Y, Z shown in Fig. 1. The initial diameter of the void is $43 \mu m$. The void lies within the necked section 4 indicated in red in Fig. 16(a).

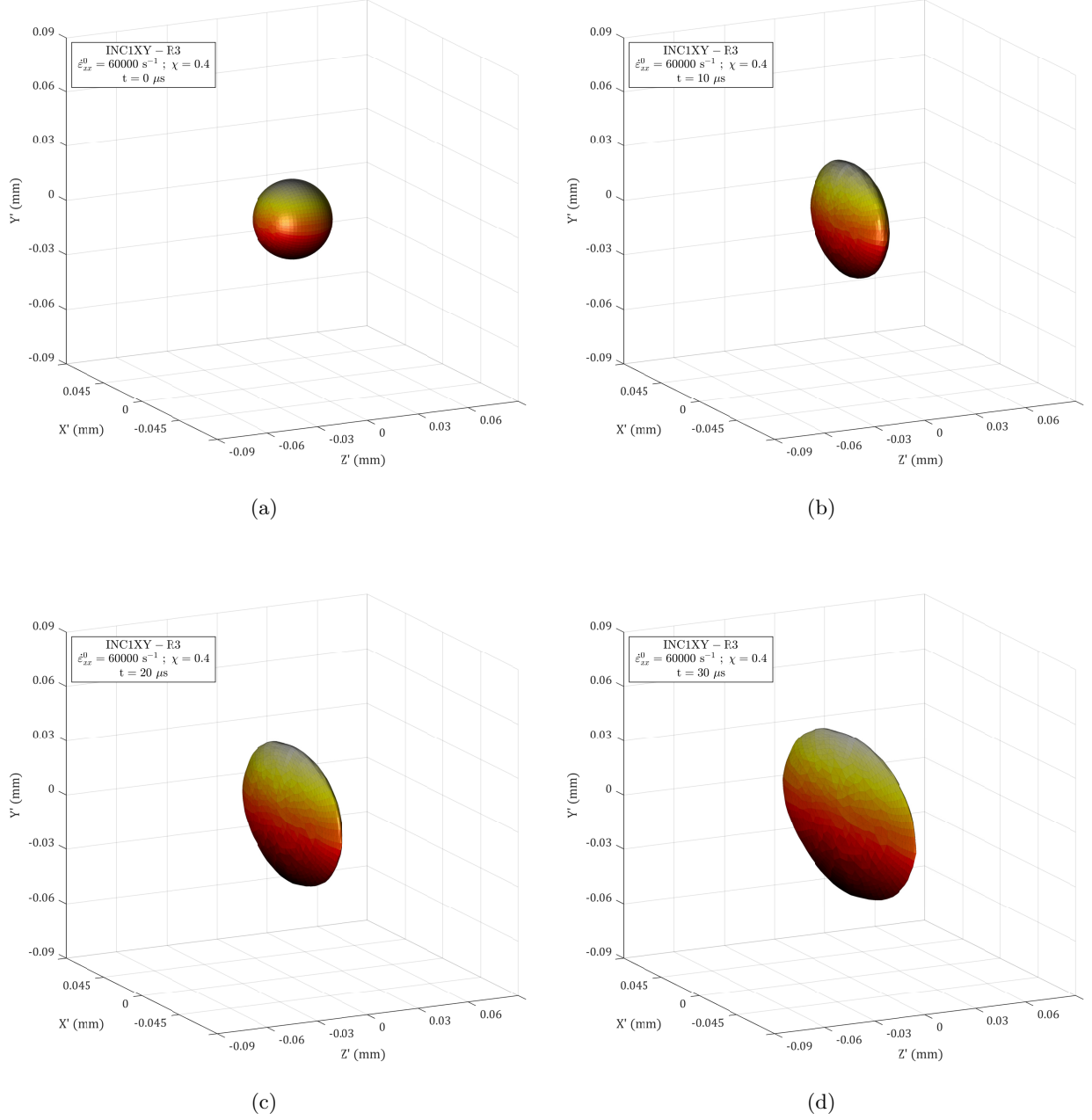


Figure 19: Microstructure INC1XY and realization R3. 3D reconstruction of the surface of the void located at coordinates $X = 5.82 \text{ mm}$, $Y = -0.23 \text{ mm}$ and $Z = 0.94 \text{ mm}$, referred to the Cartesian coordinate system (X, Y, Z) indicated in Fig. 1. Evolution of the void shape for different loading times: (a) $t = 0 \mu\text{s}$, (b) $t = 10 \mu\text{s}$, (c) $t = 20 \mu\text{s}$ and (d) $t = 30 \mu\text{s}$. The necking time is $40 \mu\text{s}$. The loading rate is $\dot{\epsilon}_{xx}^0 = 60000 \text{ s}^{-1}$ and the loading path is $\chi = 0.4$. The origin of the Cartesian coordinate system (X', Y', Z') is located at the center of mass of the void, so that the axes X', Y', Z' are parallel to the axes X, Y, Z shown in Fig. 1.. The initial diameter of the void is $43 \mu\text{m}$. The void lies within the necked section 4 indicated in red in Fig. 16(b).

of the voids does not affect the necking formability of the specimen for the range of voids sizes and initial void volume fractions investigated.

- The homogenized porosity calculations provide results for the necking strain which are in qualitative agreement with the actual porosity simulations, the quantitative differences being generally less than 25%. Moreover, the number and the location of the necks predicted by the homogenized porosity approach and the actual porosity calculations are very similar for all loading rates and loading paths investigated. Provided that the spatial and size distribution of voids in the specimen is known, the homogenized porosity model predicts the same overall results and general trends than the calculations performed with actual distribution of pores. This is a salient outcome of this paper which endorses the application of Gurson-type continuum models to compute the necking formability of porous metal sheets at high strain rates, as they provide conservative predictions for the necking strains for all the loading conditions considered and require much less computational resources than simulations in which voids are explicitly resolved.

These conclusions have been substantiated for elastic/plastic materials, with yielding defined by the von Mises criterion, and parameters corresponding to AISI 430 Steel, Copper and Tantalum.

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Conflict of interest

The authors declare that they have no conflict of interest.

Author contributions

J. C. Nieto-Fuentes: Conceptualization; Data curation; Formal analysis; Funding acquisition; Investigation; Software; Validation; Writing - original draft; Writing - review & editing. **N. Jacques:** Conceptualization; Methodology; Software; Validation; Formal analysis; Investigation; Writing - review & editing. **M. Marvi-Mashhadi:** Investigation; Methodology; Software; Validation; Writing - review & editing. **K. E. N'souglo:** Investigation; Methodology; Software; Validation; Writing - review & editing. **J. A. Rodríguez-Martínez:** Conceptualization; Formal analysis; Funding acquisition; Investigation; Methodology; Project administration; Resources; Supervision; Validation; Visualization; Roles/Writing - original draft; Writing - review & editing.

Appendix A. The influence of material behavior

The effect of the porous microstructure on dynamic necking formability is investigated for two additional sets of constitutive parameters corresponding to Copper and Tantalum, see Table A.5. The goal is to supplement the analysis of Section 4 with additional results for different material behaviors to broaden the validity of the conclusions obtained in this paper. In this regard, notice that these two materials show large differences in the values of the strain hardening, strain rate hardening and temperature sensitivity exponents. Moreover, the density of Tantalum is approximately double than Copper.

Symbol	Property and units	Copper	Tantalum
ρ^o	Initial density (kg/m ³), Eq. (10)	8930	16700
C_p	Specific heat (J/kg K), Eq. (10)	383	140
K	Bulk modulus (GPa), Eq. (3)	129	199
G	Shear modulus (GPa), Eq. (3)	46	68
σ_Y^0	Initial yield strength (MPa), Eq. (8)	0	0
σ_K	Strain hardening modulus (MPa), Eq. (8)	570.75	553.15
n	Strain hardening exponent, Eq. (8)	0.35	0.136
$\dot{\epsilon}_{ref}$	Reference strain rate (s ⁻¹), Eq. (8)	1	1
m	Strain rate sensitivity exponent, Eq. (8)	0.0098	0.0452
T_{ref}	Reference temperature, Eq. (8)	300	300
ν	Temperature sensitivity exponent, Eq. (8)	-0.111	-0.591
q_1	Material parameter, Eq. (11)	1.25	1.25
q_2	Material parameter, Eq. (11)	1	1
β	Taylor-Quinney coefficient, Eq. (10)	1	1

Table A.5: Material parameters corresponding to Copper and Tantalum used in the finite element simulations reported in Appendix A. Data after Mercier et al. (2010), except the parameters q_1 and q_2 which take standard values used in the literature.

Fig. A.20 shows the evolution of the effective plastic strain $\bar{\varepsilon}^p$ and the initial void volume fraction f^0 along the normalized axial coordinate $\bar{X}$ for calculations performed with microstructure SS5Z and realization R3. The results obtained with both actual porosity and homogenized porosity models for Copper and Tantalum are included in subfigures A.20(a) and A.20(b), respectively. The loading path is $\chi = 0$ and the loading rate 60000 s^{-1} . The loading time corresponds to the necking condition.

The calculations with actual porosity and homogenized porosity yield results which are qualitatively very similar, the $\bar{\varepsilon}^p - \bar{X}$ curves computed with the Gurson-Tvergaard theory being shifted downwards for both Copper and Tantalum because the homogenized porosity approach predicts earlier necking formation (as in the calculations with constitutive parameters of AISI 430 Steel shown in Section 4). Moreover, the necking patterns obtained with Copper and Tantalum are also qualitatively very similar to each other, such that for both materials the necks are nucleated at the same locations (in the same locations as for AISI 430 Steel, see Fig. 7(b)), showing that the effect of the porous microstructure on the localization process is large and general, as the same necks are incepted for three different sets of material parameters. The quantitative differences between the necking patterns for Copper and Tantalum appear in the rate of growth of the necks, as for the latter material the strain gradients in the specimen are more pronounced (the necks seem to grow faster) and the necking time is smaller, most likely due to the lower strain hardening and the higher thermal softening of Tantalum.

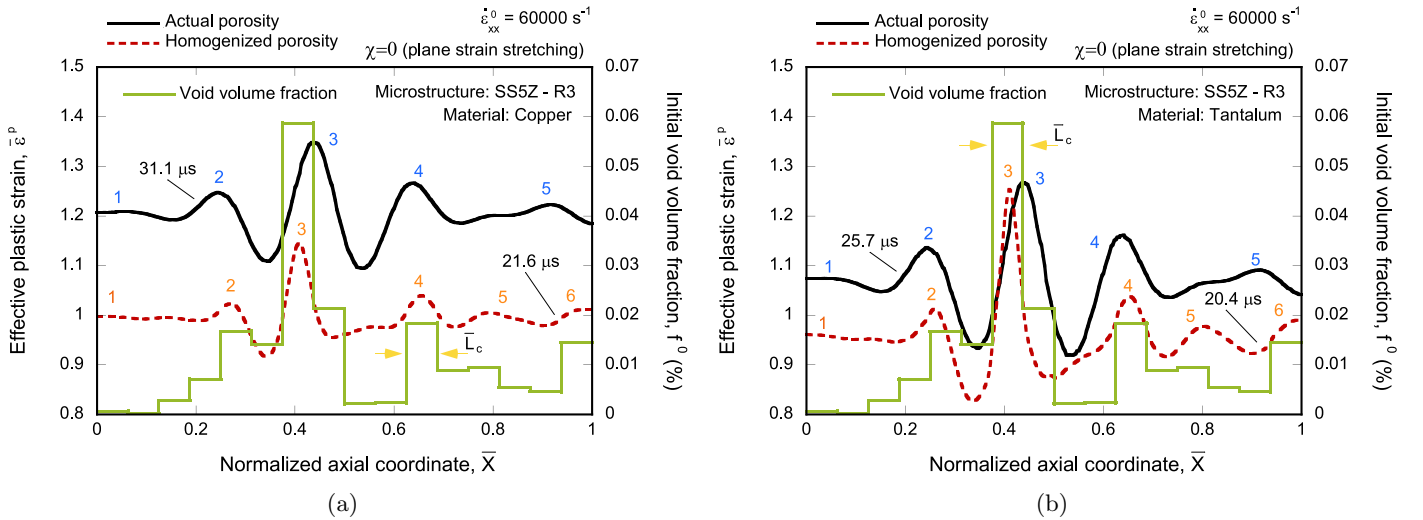


Figure A.20: Microstructure SS5Z and realization R3. Evolution of effective plastic strain $\bar{\varepsilon}^p$ and initial void volume fraction f^0 (%) along the normalized axial coordinate $\bar{X} = \frac{X}{L_0^x}$ (see Fig. 1). Comparison between results obtained with the actual porosity model and the homogenized porosity model for loading path $\chi = 0$ (plane strain stretching) and loading rate $\dot{\varepsilon}_{xx}^0 = 60000 \text{ s}^{-1}$. Material parameters taken from Mercier et al. (2010) corresponding to: (a) Copper and (b) Tantalum. The loading time corresponds to the necking condition. The void volume fraction is computed dividing the sample into 16 cells with $L_c = 0.5 \text{ mm}$ (the strategy used in the homogenized porosity model). Note that $\bar{L}_c = \frac{L_c}{L_0^x}$ is the normalized cell size.

Fig. A.21 shows the evolution of the major necking strain ε_{xx}^c with the initial void volume fraction f^0 obtained with actual porosity and homogenized porosity models for both Copper and Tantalum, see subfigures A.21(a) and A.21(b), respectively. The results are qualitatively the same obtained for AISI 430 Steel in Fig. 12(b) for the same loading path $\chi = 0$ and loading rate 60000 s^{-1} . For the microstructures investigated, the increase of the void volume fraction leads to a roughly linear decrease of the necking strain for the three sets of material parameters considered. However, notice that the slope coefficients are slightly different for the different sets of material parameters. The same qualitative results are obtained for the evolution of the major necking strain with the largest pore of the microstructure, so that ε_{xx}^c decreases roughly linearly with $d_{\max}$ for AISI 430 Steel, Copper and Tantalum, see subfigures 12(d), A.22(a) and A.22(b).

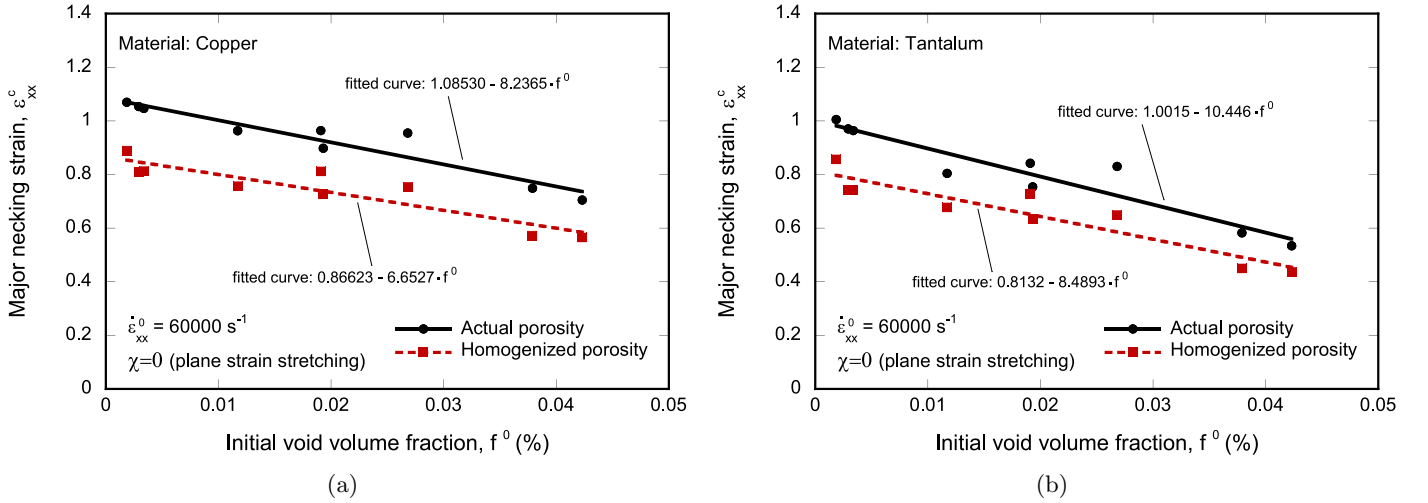


Figure A.21: Comparison between actual porosity and homogenized porosity models. Results are shown for the 3 microstructures investigated in this work and the 3 realizations (R1, R2, R3) that we have generated per microstructure. Major necking strain ε_{xx}^c versus initial void volume fraction f^0 (%) for $\dot{\varepsilon}_{xx}^0 = 60000 \text{ s}^{-1}$ and $\chi = 0$. Material parameters taken from Mercier et al. (2010) corresponding to: (a) Copper and (b) Tantalum.

The evolution of the major necking strain ε_{xx}^c with the standard deviation of the cells porosity distribution SD for Copper and Tantalum is shown in subfigures A.23(a) and A.23(b), respectively, for the same loading path $\chi = 0$ and loading rate $\dot{\varepsilon}_{xx}^0 = 60000 \text{ s}^{-1}$ used in the calculations of Figs. A.20, A.21 and A.22. Both the simulations with explicitly resolved pores and the simulations with homogenized porosity predict a nonlinear decrease of the necking strain with the standard deviation of the cells porosity distribution that has been fitted to logarithmic functions. The results for Copper and Tantalum are qualitatively the same obtained for AISI 430 Steel in Fig. 14. It becomes apparent that the heterogeneity in the porosity distribution is an important factor controlling the specimen ductility for the three material behaviors investigated in this work.

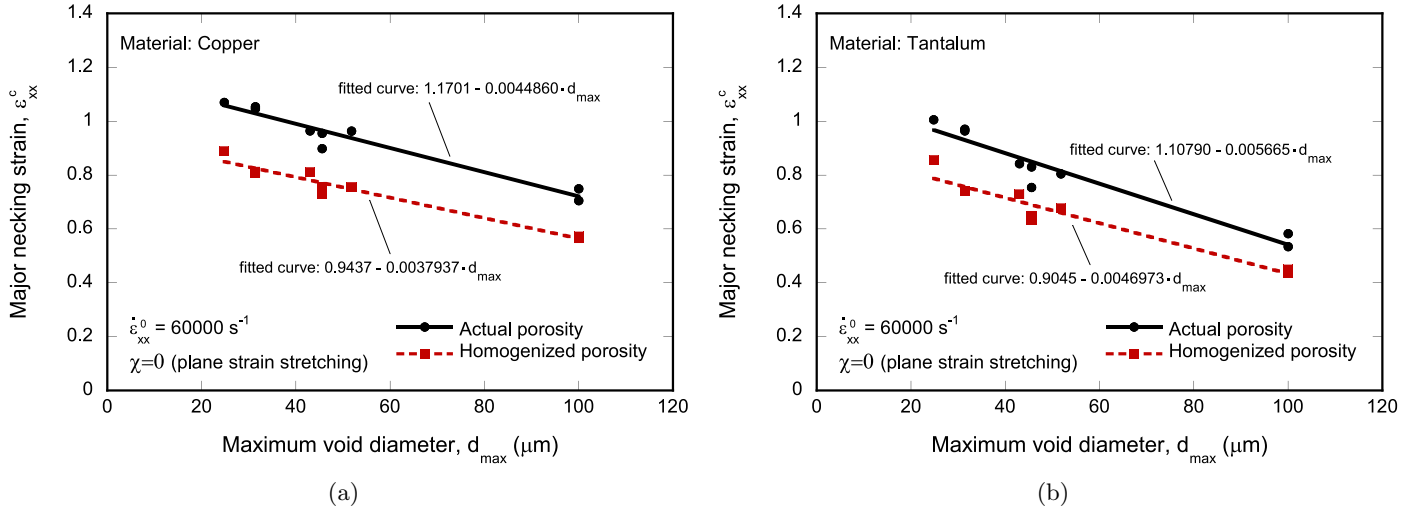


Figure A.22: Comparison between actual porosity and homogenized porosity models. Results are shown for the 3 microstructures investigated in this work and the 3 realizations (R1, R2, R3) that we have generated per microstructure. Major necking strain ε_{xx}^c versus maximum void diameter $d_{\max}$ for $\dot{\varepsilon}_{xx}^0 = 60000 \text{ s}^{-1}$ and $\chi = 0$. Material parameters taken from Mercier et al. (2010) corresponding to: (a) Copper and (b) Tantalum.

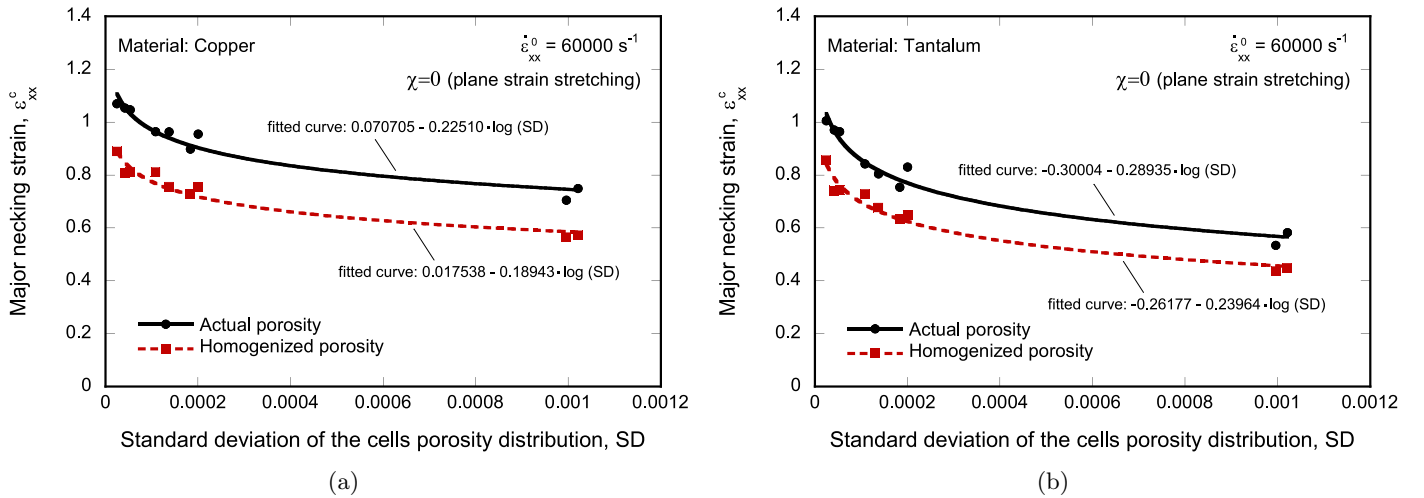


Figure A.23: Comparison between actual porosity and homogenized porosity models. Results are shown for the 3 microstructures investigated in this work and the 3 realizations (R1, R2, R3) that we have generated per microstructure. Major necking strain ε_{xx}^c versus standard deviation of the cells porosity distribution SD for $\dot{\varepsilon}_{xx}^0 = 60000 \text{ s}^{-1}$ and $\chi = 0$. Material parameters taken from Mercier et al. (2010) corresponding to: (a) Copper and (b) Tantalum.

763 Appendix B. Mesh sensitivity analysis

764 The influence of mesh on the finite element results has been investigated for both the calculations with homog-
 765 enized porosity and with explicitly resolved pores.

766

767 Fig. B.24 shows the evolution of the effective plastic strain along the normalized axial coordinate for calculations
 768 with explicitly resolved voids corresponding to microstructure SS5Z and realization R1. The loading rate is
 769 60000 s^{-1} and the loading path $\chi = 0$ (see Fig. 6). The results obtained for three different mesh discretizations
 770 are compared: Mesh 1 has been used in the simulations reported in Section 4, Mesh 2 increases the number of
 771 elements by a factor of 7, and Mesh 3 includes the same number of elements of Mesh 1, but the element type is
 772 C3D10 (10-node quadratic tetrahedron) instead of C3D4 (4-node linear tetrahedron). Note that the discretization
 773 does not alter the position and the number of necks, while the necking strain is $\approx 9\%$ and $\approx 14\%$ smaller for Mesh
 774 2 and Mesh 3, respectively, than for Mesh 1. The same trend has been obtained from the comparison performed
 775 for other loading rates and loading paths (while the results are not shown for the sake of brevity).

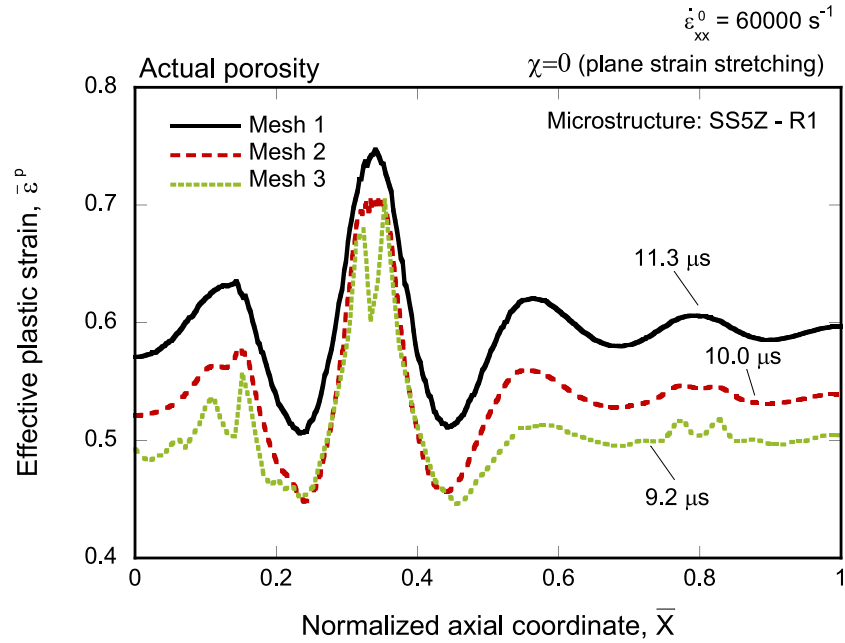


Figure B.24: Microstructure SS5Z and realization R1. Material parameters corresponding to AISI 430 Steel. Evolution of effective plastic strain $\bar{\epsilon}^p$ along the normalized axial coordinate $\bar{X} = \frac{X}{L_0}$ (see Fig. 1) for calculations with explicitly resolved voids. The loading rate is 60000 s^{-1} and the loading path $\chi = 0$ (see Fig. 6). The results obtained for three different mesh discretizations are compared: Mesh 1 has been used in the simulations reported in Section 4, Mesh 2 increases the number of elements by a factor of 7, and Mesh 3 includes the same number of elements of Mesh 1, but the element type is C3D10 instead of C3D4.

776

777 Fig. B.25 shows the evolution of the effective plastic strain along the normalized axial coordinate for calculations

with homogenized porosity. The porous microstructure, the loading rate and the loading path are the same considered in Fig. B.24. The results correspond to two different mesh discretizations: the simulations reported in Section 4 have been performed with Mesh 1, while Mesh 2 increases the number of elements by a factor of 8. Note that the $\bar{\varepsilon}^p - \bar{X}$ curves are virtually independent of the discretization.

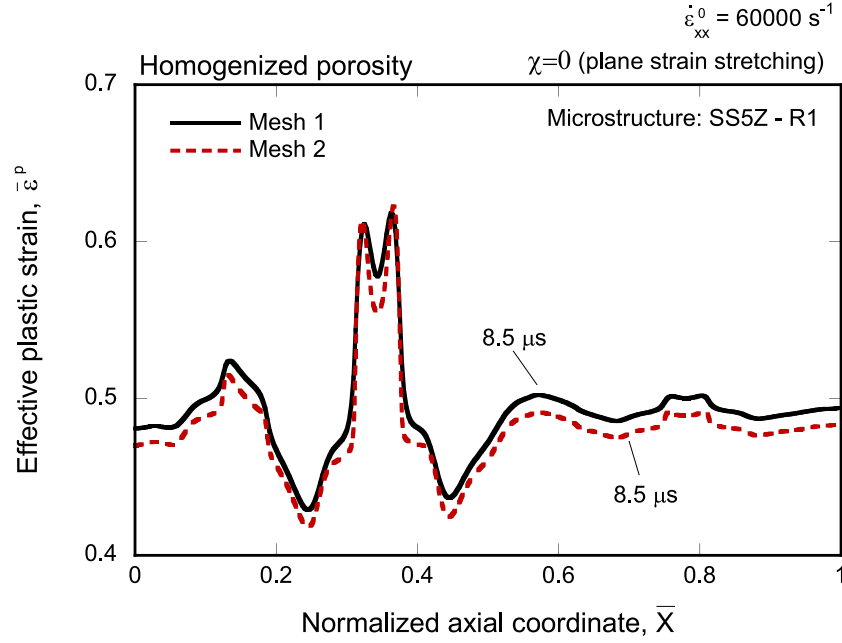


Figure B.25: Microstructure SS5Z and realization R1. Material parameters corresponding to AISI 430 Steel. Evolution of effective plastic strain $\bar{\varepsilon}^p$ along the normalized axial coordinate $\bar{X} = \frac{X}{L_0}$ (see Fig. 1) for calculations with homogenized porosity. The loading rate is 60000 s^{-1} and the loading path $\chi = 0$ (see Fig. 6). The results obtained for two different mesh discretizations are compared: Mesh 1 has been used in the simulations reported in Section 4 and Mesh 2 increases the number of elements by a factor of 8.

Appendix C. The influence of porosity cell size

The spatial distribution of initial void volume fraction in the homogenized porosity model is performed splitting up the strip into cells of size L_c , see Section 3.2. The initial void volume fraction allotted to each cell is calculated from the positions and the sizes of the voids included in the finite element models with actual porosity. Recall that the size of the cells must be taken greater than the average spacing between voids, so that the void volume fraction in the cells is different from zero. For the microstructure INC1XY, this condition is met by both $L_c = 0.5 \text{ mm}$ and $L_c = 0.25 \text{ mm}$. The results presented in Section 4 have been obtained with $L_c = 0.5 \text{ mm}$ (as this value was also used for microstructures Ti0.5Z and SS5Z). However, in order to investigate the effect of the cell size on the finite element results, calculations were also carried out for $L_c = 0.25 \text{ mm}$. Fig. C.26 shows the relative difference in necking strain $\Delta_c = \frac{\varepsilon_{xx}^c|_{L_c=0.25 \text{ mm}} - \varepsilon_{xx}^c|_{L_c=0.5 \text{ mm}}}{\varepsilon_{xx}^c|_{L_c=0.5 \text{ mm}}}$ as a function of the major necking strain $\varepsilon_{xx}^c|_{L_c=0.5 \text{ mm}}$. Note that $\varepsilon_{xx}^c|_{L_c=0.25 \text{ mm}}$ and $\varepsilon_{xx}^c|_{L_c=0.5 \text{ mm}}$ refer to the major necking strains computed with cell sizes $L_c = 0.25 \text{ mm}$

and $L_c = 0.5$ mm, respectively. The results correspond to the three realizations (R1, R2, R3), three loading paths $\chi = 0, 0.2$ and 0.4 , and four loading rates $\dot{\varepsilon}_{xx}^0 = 10000 \text{ s}^{-1}, 20000 \text{ s}^{-1}, 40000 \text{ s}^{-1}$ and 60000 s^{-1} (each marker corresponds to a given combination of loading path, loading rate and microstructural realization). **The constitutive parameters correspond to AISI 430 Steel.** The average value of Δ_c is -2.5% and the minimum value (corresponding of the maximum absolute value) is -8.7% . While Δ_c is generally negative (i.e., the major necking strain obtained with $L_c = 0.25$ mm is generally slightly lower), for specific loading conditions, and for realizations R2 and R3, Δ_c turns to be positive. Fig. C.26 also shows that the effect of the cells size is more important for loading conditions leading to large major necking strains (i.e., high strain rate and/or large values of the loading path).

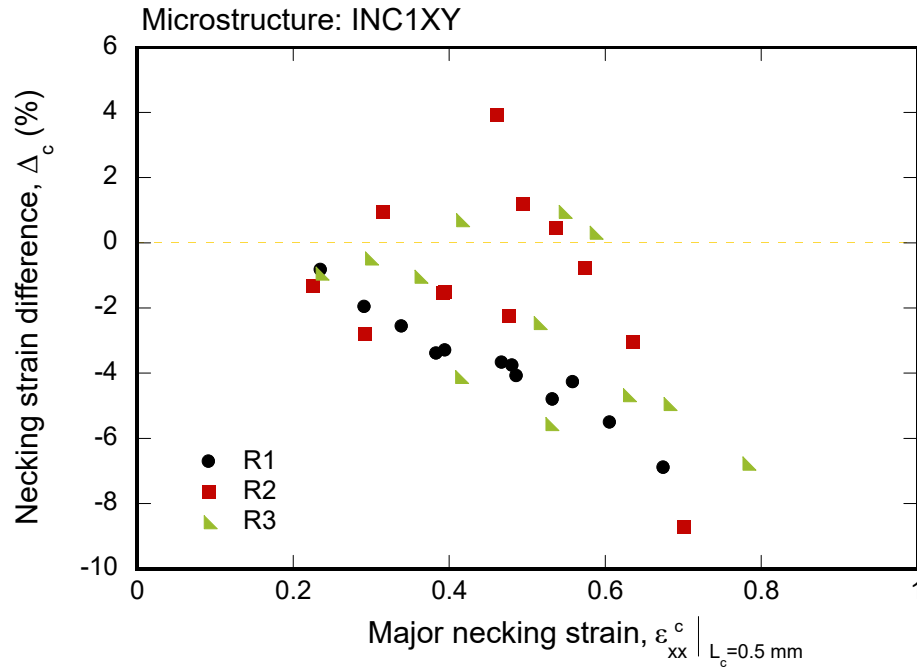


Figure C.26: Influence of the cell size on the major necking strain computed with the homogenized porosity model. Relative difference in major necking strain $\Delta_c = \frac{\varepsilon_{xx}^c|_{L_c=0.25 \text{ mm}} - \varepsilon_{xx}^c|_{L_c=0.5 \text{ mm}}}{\varepsilon_{xx}^c|_{L_c=0.5 \text{ mm}}}$ versus major necking strain $\varepsilon_{xx}^c|_{L_c=0.5 \text{ mm}}$. Results corresponding to microstructure INC1XY and AISI 430 Steel parameters, for three realizations (R1, R2, R3), three loading paths $\chi = 0, 0.2$ and 0.4 , and four loading rates $\dot{\varepsilon}_{xx}^0 = 10000 \text{ s}^{-1}, 20000 \text{ s}^{-1}, 40000 \text{ s}^{-1}$ and 60000 s^{-1} .

Appendix D. The influence of microinertia

The effect of microinertia on the macroscopic material response was found to be important in several problems, such as dynamic cracking (Jacques et al., 2012, 2015), spall fracture (Czarnota et al., 2008; Jacques et al., 2010) and shock propagation in porous and cellular solids (Czarnota et al., 2017, 2020; Barthélémy et al., 2016), see Section 1. While microinertia effects are accounted for in the finite element simulations with actual voids because the porous microstructure is explicitly represented (Section 3.1), this is not the case in the calculations performed

with the Gurson-Tvergaard yield criterion (Section 3.2), as this constitutive model was derived from a static homogenization approach in which the representative volume element of the porous material is assumed to be in static equilibrium (note that all the calculations reported in Section 4 take into account inertia effects that arise at the macroscale via the momentum balance equation). In order to assess the specific effect of microinertia on the dynamic formability of porous materials, we have carried out homogenized porosity simulations using the constitutive model for porous solids proposed by Jacques et al. (2012), which extends the Gurson-Tvergaard yield criterion to consider microinertia effects using a dynamic homogenization approach, see Section 1.

Fig. D.27 shows the temporal evolution of the axial logarithmic strain outside the neck (material point A in Fig. 6(b)), and of the porosity at the neck center (material point B in Fig. 6(b)), for calculations performed with the microinertia-based constitutive model developed by Jacques et al. (2012) and with the Gurson-Tvergaard yield criterion (which does not take microinertia into account). The results correspond to microstructure SS5Z-R1, imposed loading rate $\dot{\varepsilon}_{xx}^0 = 60000 \text{ s}^{-1}$ and loading path $\chi = 0$ (plane strain stretching), see Fig. 6. The strain outside the neck is virtually the same for the microinertia-based constitutive model and for the Gurson-Tvergaard yield criterion, so that microinertia does not affect the formability of the specimen (the saturation of the strain at $t \approx 8 \mu\text{s}$ corresponds to the necking strain). Moreover, the stabilizing effect of microinertia on the porosity growth becomes noticeable only after necking localization, for $t > 10 \mu\text{s}$. The same overall results and trends have been obtained from finite element simulations performed with microstructures Ti0.5Z and INC1XY. On the other hand, note that the initial porosity is relatively low for the three microstructures investigated in this paper, less than 0.05%. It is possible that the stabilizing effect of microinertia is more important for materials with larger initial void volume fraction, for which the onset of fracture may precede necking (Zheng et al., 2020).

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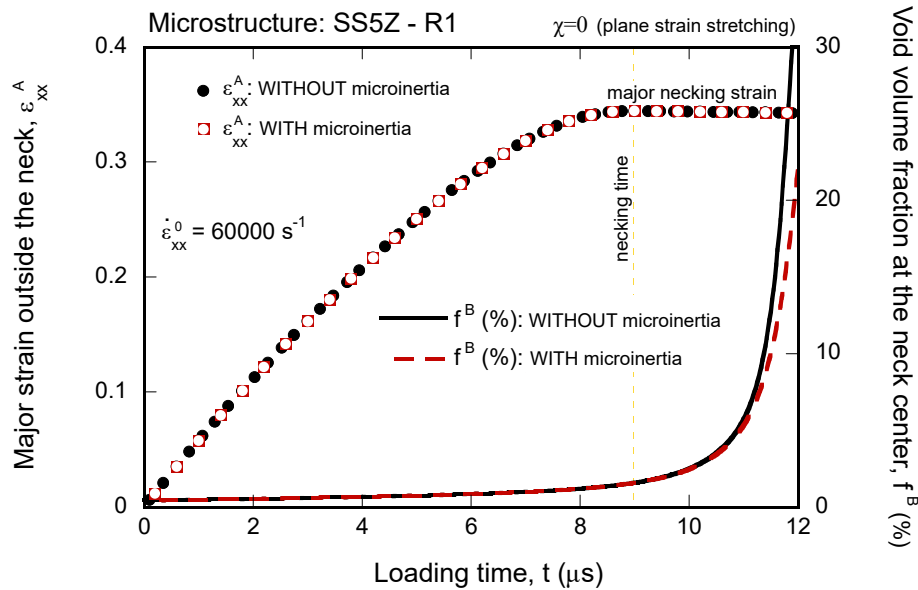


Figure D.27: Temporal evolution of the axial logarithmic strain outside the neck (A in Fig. 6(b)), and of the porosity at the neck center (B in Fig. 6(b)). Calculations performed with the microinertia-based constitutive model developed by Jacques et al. (2012) and with the Gurson-Tvergaard yield criterion (which does not take microinertia into account) are compared. Results correspond to microstructure SS5Z-R1, AISI 430 Steel parameters, loading rate $\dot{\varepsilon}_{xx}^0 = 60000 \text{ s}^{-1}$ and loading path $\chi = 0$ (plane strain stretching).

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