MODELLING CRASHES AS SELF-EXCITING POINT PROCESS: A STUDY IN DHAKA METROPOLITAN AREA

by

Md. Fahimur Rahman Shuvo

BACHELOR OF SCIENCE IN CIVIL ENGINEERING

DEPARTMENT OF CIVIL ENGINEERING
BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY
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Md. Fahimur Rahman Shuvo

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1. Name of the Supervisor — Chairman
   Designation
   Address

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2. Name of the Internal Member — Member
   Designation
   Address

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3. Name of the Internal Member — Member
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DECLARATION

I hereby declare that this thesis is my own work and to the best of my knowledge it contains no materials previously published or written by another person, or have been accepted for the award of any other degree or diploma at Bangladesh University of Engineering and Technology (BUET) or any other educational institution, except where due acknowledgment is made. I also declare that the intellectual content of this thesis is the product of my own work and any contribution made to the research by others, with whom I have worked at BUET or elsewhere, is explicitly acknowledged.

Signature with date

Name

Dedicated

to

My beloved and respected parents

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ABSTRACT

The thesis pertains to the investigation of whether the crashes taking place within the Dhaka Metropolitan Area exhibit properties of self-excitement and clustering. The motivation behind the study stems from the fact that a crash incites congestion in the traffic network, which in turn increases the odds of more crashes in the system. A univariate exponentially decaying model following Hawkes Process is developed for the study of the crashes with four parameters: background intensity μ, parameter α denoting an increase in crash intensity once a crash takes place, intensity decay rate β, and the time of influence of a crash on the system τ. Using RStudio, the Most Likely Estimates of the parameters are determined which are then used to compute the Goodness of Fit of the model along with the Stationary Condition which gives an idea about the safety of the traffic network in the long run in terms of crash intensity. The result of the study shows that the crashes in a macroscopic level are not self-exciting but there is evidence to prove that the crashes in a smaller scale indeed tend to be self-exciting. The values of the parameters obtained are large in magnitude, hinting that the road network becomes extremely unsafe once a crash takes place, making the traffic system prone to more crashes. It also implies that the road network tends to become safe again very quickly with a large crash intensity decay rate. The effect of a crash in the crash intensity of the system tends to last longer which might prove that the crashes are indeed self-exciting but the combined effect of too many crashes in the influential timeframe τ tend to make the occurrence of crashes appear as randomly distributed.

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LIST OF ABBREVIATIONS

ADU = Accident Data Unit

AIC = Akaike Information Criterion

ARF = Accident Report Form

ARI = Accident Research Institute

BFGS = Broyden–Fletcher–Goldfarb–Shanno

BUET = Bangladesh University of Engineering and Technology

DMA = Dhaka Metropolitan Area

KS = Komlogorov-Smirnov

MAAP = Microcomputer Accident Analysis Package

MLE = Most Likely Estimate

CHAPTER 1
INTRODUCTION

1.1 Background and Motivations

Over the last several years, the number of automobiles in Bangladesh has increased by leaps and bounds. This is particularly noticeable in the big cities and towns like the capital Dhaka and other divisional centers. The increase in the number of vehicles has improved the standard of living of the people and allowed for a more widespread transportation network. However, all of those are achieved at the price of increased risk of traffic crashes. People's lives are endangered and significant financial losses are incurred due to these traffic crashes. According to the Crash Research Institute (ARI) of Bangladesh University of Engineering and Technology (BUET), 64 people lose their lives and 150 people face injury of any kind on a daily average over the last several years in the country. Furthermore, the economic losses due to these crashes can amount to more than Tk. 30,000 crore per year, according to a news report published in Dhaka Tribune. Under such circumstances, studying the safety and dependability of the country's large metropolitans' traffic network is a matter of both theoretical and practical importance.

There are various approaches to studying the safety and reliability of traffic networks. Plenty of necessary steps is taken every year for improving the safety conditions of the traffic networks of the country. A probabilistic approach for forecasting the number of crashes most likely to occur in a given time period is an intriguing concept that could be investigated further. Using probability theories and models, many events like earthquakes, financial crises, and even crimes are being handled efficiently, which otherwise would have been unforeseen and therefore catastrophic. Similar strategies could be taken to reduce traffic crashes and which is the topic of this study.

This study concerning the crashes taking place within the traffic networks of Dhaka Metropolitan Area (DMA) treats the crashes as a self-exciting process, that is, if a crash takes place within the network, it is assumed that the probability of another crash taking place within the network increases. Although traffic crashes are not known for directly exhibiting the property of self-excitement like earthquake or financing, it is worth investigating the safety of traffic networks considering the crashes as self-exciting. In most cases, traffic crashes required a period of waiting time to be dealt with. When incidents are being investigated, traffic congestion might occur, increasing the risk of traffic crashes. In a nutshell, traffic crashes can lead to traffic congestion, and traffic congestion can lead to more traffic crashes. Under such a phenomenon, traffic crashes can indirectly exhibit self-excitation.

Traffic crashes require human and material resources to be dealt with after the crash. As such, congestion is likely to occur while investigation and post-crash countermeasures are being taken. These include emergency medical services for the people, salvage of vehicles and restoration of road and traffic devices.

A study by Retallack and Ostendorf (2019) suggests that congestion can influence the occurrence of traffic crashes in unexpected ways. Crashes are influenced by both number and velocity of the vehicles in a road network. Number of crashes increases when the number of vehicles increases in a road network, almost in the similar fashion to how number of crashes increases with the increase in speed. Occasionally, the highest rate of crashes, mostly fatal, are observed in roads with the highest and lowest volume-to-capacity ratios, implying that crash occurrence is a function of not only speed but also the quantity of automobiles on the road. That is, crashes are more common in low traffic volumes due to high speeds, whereas crash rates are higher in large traffic volumes due to more cars on the road (Retallack and Ostendorf, 2019).

A reflection of the findings of the aforementioned by Retallack and Ostendorf (2019) can be vividly seen in the traffic network of DMA. Upon plotting the crashes within DMA traffic network taking place from 2017 up to 2020 against the time of the day, it is seen that a huge number of crashes take place between 9 AM and 12 noon. Out of the 991 crashes in the four years, 241 of them took place between 9 AM and 12 noon, amounting to 24.32% of the crashes. The number of trips destined towards workplaces, educational institutes and the like are highest during this time of the day, indicative of one of the daily peak traffic flows of DMA’s traffic network. As such, the traffic flow is greater than the saturation capacity of the road networks during these hours and hence, a large amount of congestion is observed. Ergo, there is an indirect link between the crashes and congestion in case of DMA’s traffic network.

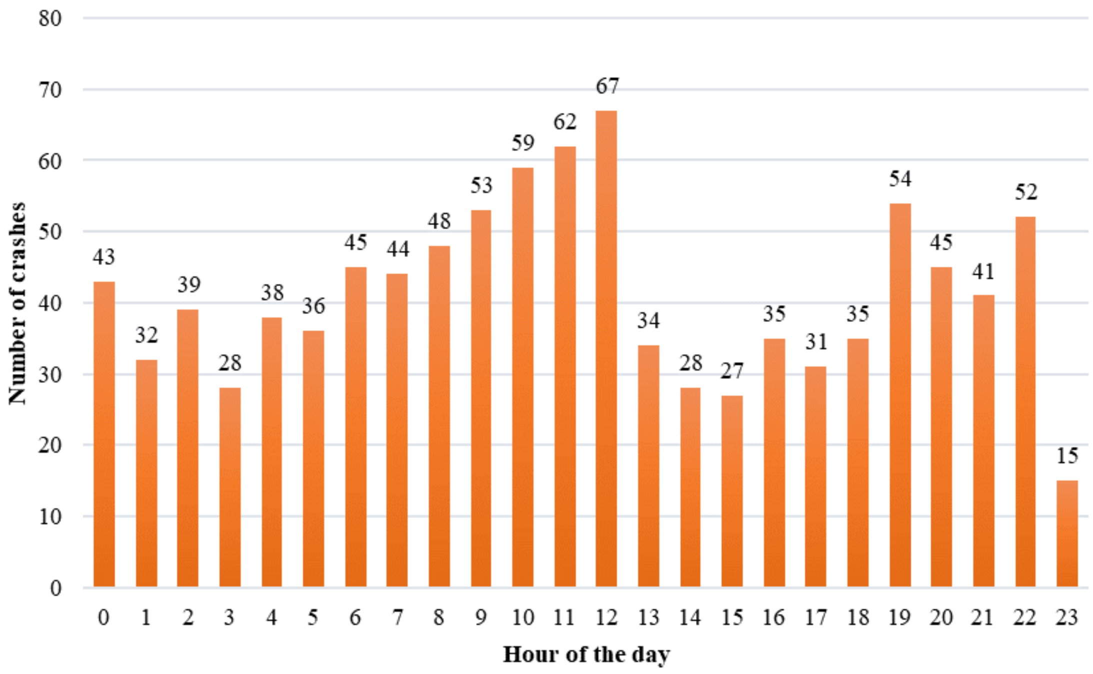

Figure 1-1. Hourly crashes within the Dhaka Metropolitan Area, obtained from the crash data available at ARI in BUET

Since Dhaka Metropolitan Area is the largest administrative and business hub in the country, the city's road network is critical, and it is expected to be considerably safer and more reliable than any other road networks in the country. As a result, probabilistic accident analysis is crucial for DMA and hence the study is being carried out to see if the process traffic crashes within this traffic network of interest is a self-exciting point process.

An approach could have been taken to analyze the crashes using Poisson Distribution, which is often done for processes without self-excitement properties. However, other studies and researches provide evidence that analysis of systems using the Hawkes Process yields a better analysis outcome compared to the same done using Poisson Distribution. Furthermore, this study also aims to see how much the crashes within the DMA network exhibit the property of self-excitement – whether the crashes are self-exciting or self-regulating or there is no dependency on the past events within the system.

The crashes are considered as a Hawkes Process in this study, which was extensively developed and studied by Alan G. Hawkes in 1974. There are many different point processes which are referred to as Hawkes Process. However, the processes for which Hawkes did the most work himself were univariate self-exciting temporal point processes. Such processes had a conditional intensity function λ(t) of the following form:

$$\lambda(t) = \mu(t) + \sum_{t_i<t} h(t - t_i)$$

Here, $\lambda(t)$ is the background rate of the process of interest, and $h(t)$ is the excitement kernel function, accounting for the self-excitement of the process. It is to be noted that depending on the nature of $h(t)$, a process might be self-regulating instead of self-exciting, that is, the likelihood of another event within the process or systems decreases with every new event. In a Poisson Process, there is no such kernel function as the events in the process are considered self-independent.

Originally, Hawkes Process and its relevant studies were developed for the analysis of the self-exciting properties of earthquakes. Soon afterward, the analysis of various systems in many other fields such as criminology, finance, computer networking, etc. considering them Hawkes Processes became famous.

1.2 Research Objectives and Overview

The general objective of this study is to analyze the safety and reliability of DMA's traffic network considering the crashes taking place within the system as a Hawkes Process. For this purpose, a univariate model with exponentially decaying kernel function is developed with crash data for the years from 2017 up to 2020 inclusive.

The specific objectives of the study are as follows:

1. To determine the Most Likely Estimates (MLEs) of the parameters of the model;
2. To assess the goodness of fit of the model using Chi-Square test; and
3. To analyze the reliability and safety of DMA's traffic network using the MLE of parameters in the stationary condition.

1.3 Scope of the Research

The study aims to investigate the crashes within the DMA traffic network to see if they are self-exciting or not. The multivariate non-linear models used for similar studies were outside the scope of this study. Also, no spatial analysis has been done and crashes occurring before January, 2017 were not taken into account.

1.4 Organization of the thesis

The thesis has been divided into five chapters.

Chapter 1: Introduction provides the background and motivations of the research. The overall objectives, expected outcomes and the scope of the studies are also described in this chapter.

Chapter 2: Literature Review explores the implications of Hawkes Process along with its context in Traffic Crash System. The review of related studies is also done in this chapter.

Chapter 3: Methodology describes the methodology adopted to carry out the parameter estimation, goodness of fit test and checking of stationary condition required for this research.

Chapter 4: Results and Discussion describes the results of the investigation done as per the processes mentioned in the Methodology chapter, required for the expected outcomes mentioned in the Introduction chapter.

Chapter 5: Conclusions summarizes the major contributions of this study along with the limitations and recommendations for future studies and road safety engineering.

CHAPTER 2
LITERATURE REVIEW

2.1 Introduction

Crashes in general don't exhibit any self-exciting or self-regulating characteristics. As such, there is less than a handful of research done on traffic crash analyses considering crash processes as the Hawkes Processes. However, there is immense work done in many other disciplines such as criminology, computer networking, environmental studies, finance, seismology, and the like. The underlying mathematics and analyses at the core of all of these studies are essentially the same. Therefore, it is possible to develop relevant models and criteria for the analysis of traffic crashes as a self-exciting process.

In this chapter, the general definitions for Hawkes Process and its related concepts are established first, followed by elucidation of the themes from the standpoint of a traffic crash system. The chapter then reviews the relevant literature available in this particular field of research.

2.2 Hawkes Process in general

2.2.1 Point process

A point process is a probabilistic model for random scatterings of points on an X-dimensional space. Often, point processes are used to represent the occurrence of random events across time, with each occurrence being revealed one by one as time passes. Considering times of realizations $t_1, t_2, \dots, t_n$ where $t_1 < t_2 < \cdots < t_n$, the point process can be expressed as the set:

$$\{t_1, t_2, \dots, t_d\}$$

Point processes are occasionally dubbed counting processes as both terms are synonymous.

2.2.2 Self-Exciting Point Process

A point process is considered self-exciting if the occurrence of an event incites an increase in the likelihood of future events. Mathematically, a process N will be considered self-exciting if it satisfies the following condition:

$$\mathrm{cov}\big(N(t_1, t_2), N(t_2, t_3)\big) > 0$$

Here, $t_1 < t_2 < t_3$ and the notation cov represents the covariance between two processes. As covariance is the measure of the correlation between two or more sets of random variables, it can be intuitively concluded that a self-exciting point process is a point process where there exist strong positive correlations among the times of realizations of the events within the process.

2.2.3 Temporal Point Process

A point process whose realizations take place along an arbitrary time axis can be defined as a temporal point process. Such a point process is expressed as $\{t_j\}$ for values of j from an index set J.

Schoenberg *et al.*, 2002 expressed the values of t_j as arbitrary real numbers corresponding to the time of realization of the j-th event within the process while the index set J is similar to the set of integers $\mathbb{Z}$. This viewpoint, popular among certain pieces of literature, corresponds to looking at temporal point processes to see how long events last when the events are spaced out as per a discrete set containing countable time parameters.

On the other hand, other authors represent the temporal point processes as binary events (Paninski, 2013). In such a viewpoint, t_j is a set containing two elements for each value of j coming from a finite index set J. It considers temporal point processes as indicators of whether a finite number of events has taken place or not.

Considering an indicator function I associated with the point process $N(t)$ of any particular sequence of events:

$$I = \begin{cases} 1 & \text{if the event takes place} \\ 0 & \text{otherwise} \end{cases}$$

For a sequence of time of events $T = \{t_j : j \geq 1\}$, the temporal point process $N(t)$ can be established as such:

$$N(t) := \sum_{t_j < t} I_T$$

Such a temporal point process would have the following properties.

- $N(0) = 0$ i.e., the number of events at the start of the counting process is zero.
- The function $N(t)$ is right-continuous. It follows the condition $\lim_{s \to t^+} N(s) = N(t)$
- The function $N(t)$ is a step function with an increment of $+1$ for each of the times an event takes place.

2.2.4 Conditional Intensity Function

The conditional intensity function of a point process is the infinitesimal expected rate at which the events are expected to occur. For a temporal point process $N(t)$, the conditional intensity function is defined as:

$$\lambda(t) = \lim_{\Delta t \to 0} \frac{E[N(t + \Delta t) - N(t)]}{\Delta t}$$

2.2.5 Hawkes process

A self-exciting temporal point process $N(t)$ is considered to be a Hawkes Process when its conditional intensity function takes the form:

$$\lambda(t) = \mu + \sum_{t_i < t} h(t - t_i)$$

Here, μ is the background intensity rate of the process and $h(t)$ is the kernel function. As the process is considered self-exciting, the kernel function is also referred to as the excitement function or response function.

The background intensity rate μ accounts for the randomness of the process based purely on the number of events and time of occurrences. It is also synonymous to the rate of events or the mean as found in a Poisson Distribution. In some cases, it is represented in the form of a function of time as $\mu(t)$. However, under such consideration, the initial background rate at $t = 0$ needs to be defined by $\mu(0) = \mu_0$.

The excitement kernel function $h(t)$ introduces the influence of past events' realization on the probability of a future event's realization, thereby making the point process different from a Poisson Distribution. By summing the values of $h(t - t_i)$ for all $t_i < t$ $\{i = 1,2, \ldots\}$, the influences of all events prior to time t are accounted for.

In actuality, Hawkes processes are referred to a number of different point processes by different authors. Hawkes made the most progress for univariate self-exciting temporal point processes with a linear conditional intensity function (Hawkes, 1974). As a result, such processes have been coined as Hawkes processes by several authors. In general, however, such processes are referred to as linear Hawkes processes, as opposed to their non-linear counterparts with non-linear conditional intensity functions.

2.3 Hawkes Process in Traffic Crash System

Hawkes Process is used for the analysis of processes with cluster and self-exciting qualities, which is why there are almost no studies in analyzing the safety of a traffic network using Hawkes Process. But in reality, there might be underlying self-excitement within the crash patterns not visible on the surface. Under such case, analysis using Hawkes Process can accurately capture the actual occurrences of traffic accidents, contrary to analysis using Poisson Process. Such studies of traffic crashes within a network as Hawkes Process would serve as a valuable tool for assessing the reliability and safety of the traffic network. The findings and outcomes of the studies would lead to proper preventive measures such as choice of route and renovative strategies such as fixing road geometry.

However, contrary to the lack of studies in traffic crash sector, several researches have been conducted in the railway and marine sectors of transportation safety analysis using self-exciting point processes. As safety and reliability of several processes within the railway and marine sectors have been studied, it provides hope that the same could be done for traffic crash studies.

2.3.1 Self-exciting Model for Traffic Crash System

If the times of realizations of traffic crashes are denoted as $\{t_1, t_2, \dots, t_n\}$ over a continuous time axis, the crash counting process can be established as:

$$N(t) := \sum_{t_i<t} I_C$$

Here, I_C is the crash indicator function which equals 1 if a crash takes place and 0 otherwise. The crash counting process $N(t)$ itself gives a preliminary idea about the safety of the traffic network in regards to crashes. The higher the value of $N(t)$ is, the more the number of crashes within the timeframe $[0, t)$ is.

It is to be noted that the crash counting process is of interest only for observing the times of crashes and not the number of crashes taking place in a particular instant of time. In reality, there might be more than one crash taking place in time $t_i$but all such crashes are mapped under one instance of time.

To rephrase, the crash counting process counts only the times of crashes taking place and assumes that the probability of two or more crashes taking place within a very short period of time is negligible. Mathematically,

$$\lim_{\Delta t \to 0} \mathbb{P}(N(t + \Delta t) - N(t) \geq 2) = 0$$

However, the most important point in developing a self-exciting model for crashes is the selection of kernel excitement function. In most studies, the kernel function is taken in the following exponential form, the form on which Hawkes originally developed the Hawkes Process on (Hawkes, 1974):

$$h(t) = \alpha e^{-\beta t}$$

It makes intuitive sense to pick such a form, as each event in the process would boost the intensity of future events by α, and then the intensity would gradually decrease over time at a rate β.

The extra parameter that needs to be considered for self-exciting crashes is the delay parameter τ which incites self-excitement within the traffic network's crashes. Thus, the kernel function of interest would take the following form:

$$h(t) = \begin{cases} \alpha e^{-\beta(t-\tau)} & t \geq \tau \\ \alpha & 0 < t < \tau \\ 0 & t < 0 \end{cases}$$

Therefore, the conditional intensity function $\lambda(t)$ of the crash counting process $N(t)$ can be defined as:

$$\lambda(t) := \mu + \sum_{t_i < t} h(t - t_i) = \mu + \sum_{t_i < t} \left(\alpha I_{t - t_i < \tau} + \alpha e^{-\beta(t - t_i - \tau)} I_{t - t_i \geq \tau} \right)$$

$$= \mu + \sum_{t - t_i < \tau} \alpha n_1 + \sum_{t - t_i \geq \tau} \alpha e^{-\beta(t - t_i - \tau)}$$

The bounds on the parameters are such that $\mu > 0, \alpha > 0, \beta > 0, \tau \geq 0$. The background intensity rate μ indicates the probability of crashes without any self-excitement, which can also be identified as the probability of crashes per unit time at the start of the timeline when $t = 0$. When a crash takes place, the probability of another crash taking place is boosted by α and it remains so until a time equal to τ has elapsed. Afterwards, the probability decreases in the form of exponential decay at a rate β.

2.3.2 Stationary Condition

The stationary condition of a self-exciting crash process illustrates the stability of the traffic network in the long run. For a point process to be stationary, it has to satisfy the following criterion:

$$\lim_{t\to\infty} E\big(\lambda(t)\big) = \lambda_s < \infty$$

In the long-term scenario, the expected number of crashes must tend towards some finite constant λ_s. A traffic network where this condition is not met would be deemed unsafe, as it would suggest that the probability, and hence the expected number of crashes, would keep on increasing as time goes on, making the network more and more unreliable with passage of time.

The conditional intensity function for Hawkes Process can be written in an integral form as below (Hawkes, 1974):

$$\lambda(t) = \mu + \int_{-\infty}^{t} h(t-u)dN(u)$$

$$\therefore E[\lambda(t)] = E\left[\mu + \int_{-\infty}^{t} h(t-u)dN(u)\right] = \mu + \int_{-\infty}^{t} h(t-u)E[dN(u)]$$

From the definition of conditional intensity function in Chapter 2.2.4:

$$\lambda(u) = \lim_{du\to 0} \frac{E[N(u+du) - N(u)]}{du} = \frac{E[dN(u)]}{du}$$

$$\therefore E[dN(u)] = \lambda(u)du$$

Substituting the value of $E[dN(u)]$ from this equation to the previously obtained value of $E[\lambda(t)]$, it follows:

$$E[\lambda(t)] = \mu + \int_{-\infty}^{t} h(t-u)\lambda(u)du$$

Taking the limit $t \to \infty$, it is obtained:

$$\lim_{t\to\infty} E[\lambda(t)] = \lim_{t\to\infty}\left[\mu + \int_{-\infty}^{t} h(t-u)\lambda(u)du\right] = \mu + \int_{-\infty}^{\infty} h(t-u)\left(\lim_{u\to\infty}\lambda(u)\right)du$$

In order to conclude that the traffic network of interest is safe and stable, $\lim_{t\to\infty} E\big(\lambda(t)\big) = \lambda_s < \infty$ must be true. Ergo,

$$\lambda_s = \mu + \int_{-\infty}^{\infty} h(t-u)\lambda_s du = \mu + \lambda_s \int_{-\infty}^{\infty} h(t-u)du$$

$$\therefore \lambda_s = \frac{\mu}{1 - \int_{-\infty}^{\infty} h(t-u)du} < \infty$$

$$\rightarrow 1 - \int_{-\infty}^{\infty} h(t-u)du > 0$$

$$\rightarrow \int_{-\infty}^{\infty} h(t-u)du < 1$$

Substituting $y = t - u$, the integration can be rewritten as:

$$\int_{-\infty}^{\infty} h(t-u)du = -\int_{\infty}^{-\infty} h(y)dy = \int_{-\infty}^{\infty} h(y)dy$$

Thus, the condition that is needed to be checked for the stability of the traffic network's crash process can be established as:

$$\int_{-\infty}^{\infty} h(y)dy < 1$$

$$\rightarrow \int_{-\infty}^{\infty} \left[0 \cdot I_{y<0} + \alpha \cdot I_{0<y<\tau} + \alpha e^{-\beta(y-\tau)} \cdot I_{y\geq\tau}\right] dy < 1$$

$$\rightarrow \int_{-\infty}^{0} 0 \cdot 1dy + \int_{0}^{\tau} \alpha \cdot 1dy + \int_{\tau}^{\infty} \alpha e^{-\beta(y-\tau)} \cdot 1dy < 1$$

$$\rightarrow 0 + \alpha\tau + \alpha\left(-\frac{1}{\beta}\right)\left(e^{-\beta\cdot\infty} - e^{0}\right) < 1$$

$$\rightarrow \alpha\tau + \frac{\alpha}{\beta} < 1$$

Therefore, if the value of $\alpha\tau + \frac{\alpha}{\beta}$ for a traffic network is obtained to be less than unity, it can be concluded that the network is safe and stable in the long-run in terms of crashes.

2.4 Relevant Studies

Traffic crashes are matters of serious issue as they disrupt national growth with the incurring losses of human lives and country's valuable economy. For this reason, chief among others, traffic crashes are studied extensively all year round. A fair share of these studies is done using probabilistic analyses over the traffic networks. However, almost no studies have been conducted considering crashes as Hawkes Process, since traffic crashes do not show self-exciting properties, at the surface level at the very least.

However, traffic crashes taking place in a road network often led to congestion within the network, which in turn increases the likeliness of another crash to take place within the same traffic network. Therefore, it is very crucial to investigate traffic networks considering crashes as Hawkes Process with self-excitation rather than as Poisson Process with no probabilistic dependency between two or among more crashes, which is often done in most studies.

Despite the lack of abundance of previous studies regarding traffic crash analysis using Hawkes Process, there is a traffic crash model developed by Li *et al.* (2018) which considers crashes as Hawkes Process in order to examine the safety and reliability of a city's transportation network based on the crashes taking place within the network. In their study, the likelihood functions for the parameters' MLEs for such a model along with the algorithms to calculate the statistical characteristics of the model have been provided as well.

Despite the dearth of study in the traffic crash sector, the underlying ideas in analyzing any system using Hawkes Process is the same at the roots. With the basic definition of the conditional intensity function at its core, different disciplines of studies branch out from it based on their systems and intuitive parameters. As the fundamental basics of other studies using the Hawkes Process are aligned with those of this study, many such pieces of research in the fields of social networking, criminology, etc. have been studied.

Tench *et al.* (2016) used Hawkes Processes to analyze and model the improvised explosive device attacks in Northern Ireland. The study has used both univariate and multivariate models of the Hawkes Process as suitable over different phases of time along with comparing the cases analyzed using a normal Poisson Distribution. Models developed by Hawkes Processes were statistically better fitting than those developed based on Poisson Distribution as the inclusion of future events' dependencies on past events portrayed a more accurate scenario.

Fox *et al.* (2016) has done a similar study but on Email Networking. They have come to the same conclusion stating models based on self-exciting point processes perform better than those based on stationary Poisson Distribution.

The main motivation behind using Hawkes Process over Poisson Process for traffic crashes in this study originates from the aforementioned studies. As the consideration of dependency on past events leads to better modeling of criminology and email networking, the same might be true for analyzing traffic crashes from a probabilistic viewpoint.

To develop the likelihood equations of the parameters for the traffic crash process, the procedure developed by Ozaki (1979) in his paper was studied. To further complement the development of equations, studies by Laub *et al.* (2015) were reviewed. The likelihood functions and MLEs of the parameters are further discussed in the upcoming chapter.

Another study by Li *et al.* (2020) used Laplace Transformation and Inverse Laplace Transformation to find numerical solutions of a linear Hawkes Process. To demonstrate the application of the numerical method, a WeChat network model was developed in that study. While Laplace and Inverse Laplace Transformations were not used for this study, the fundamentals of the WeChat model were observed for the stationary condition of the model.

A study by Brown et al. (2002) was found particularly useful for testing the goodness of fit for models developed for Hawkes Process. In the study, the Time-Rescaling Theorem and its application for point processes were discussed, which stated that the residuals of a good fit for any point process are independent and has a stationary Poisson Distribution with unit rate. The theorem has been used in the research by Tench *et al.* (2016) on the improvised explosive device attacks in Northern Ireland.

This study does not encompass the entirety of analysis of traffic crashes as Hawkes Process but rather leaves insights aiming to inspire future studies to be conducted in a similar vein. Much more development in transport safety analysis sector using ideas of self-exciting point processes can be accomplished as the number of studies in this vein continues to expand.

2.5 Summary

A lot of work has been done in the analysis of systems following Hawkes Process in different disciplines, despite lack of work in the transportation engineering sector. The works done in other fields can be easily modified and applied for the traffic crashes, which would serve vital roles in road safety engineering.

CHAPTER 3
METHODOLOGY

3.1 Introduction

In order to work with the traffic crashes as Hawkes Process, it is necessary to position the crashes within a continuous temporal axis with the times of realizations of the crashes as discrete points in the timeline. The crashes within the DMA traffic network can be modeled with the conditional intensity function stated in the previous chapters upon estimating the parameters α, β, μ and τ using the crash data for the network. Once the values of these parameters are determined, the study on other properties to analyze the reliability and safety of the DMA traffic network can be proceeded.

In this chapter, the theory, procedure and calculations required for determining the Most Likely Estimates (MLEs) of the parameters α, β, μ and τ are explained along with all the necessary programming codes used in the process. Afterwards, the methodology of conducting a chi-square test to determine the goodness of fit i.e., whether the crashes follow self-exciting trends, is described. Finally, the chapter is concluded stating the limitations, assumptions, and range of validity of the analyses used for this study.

3.2 Methodology Overview

The study can be broadly divided into two major segments:

- Determining the MLEs of the parameters of interest for the proposed Hawkes Process model;
- Testing for the Goodness of the Fit of the models for the calculated parameter.

The flow of work is summarized in Figure 3-1.

Crash data for the crashes taking place in the DMA traffic network is obtained from ARI in BUET. The crashes are recorded in timestamps of year, month and date. Considering the midnight of January 1, 2017 as $t = 0$, the times of realization of each crash, marked as $t_i\{i = 1,2,3, \dots\}$ for the i-th crash, are converted to countable points on the time axis. Microsoft Excel is used for the process.

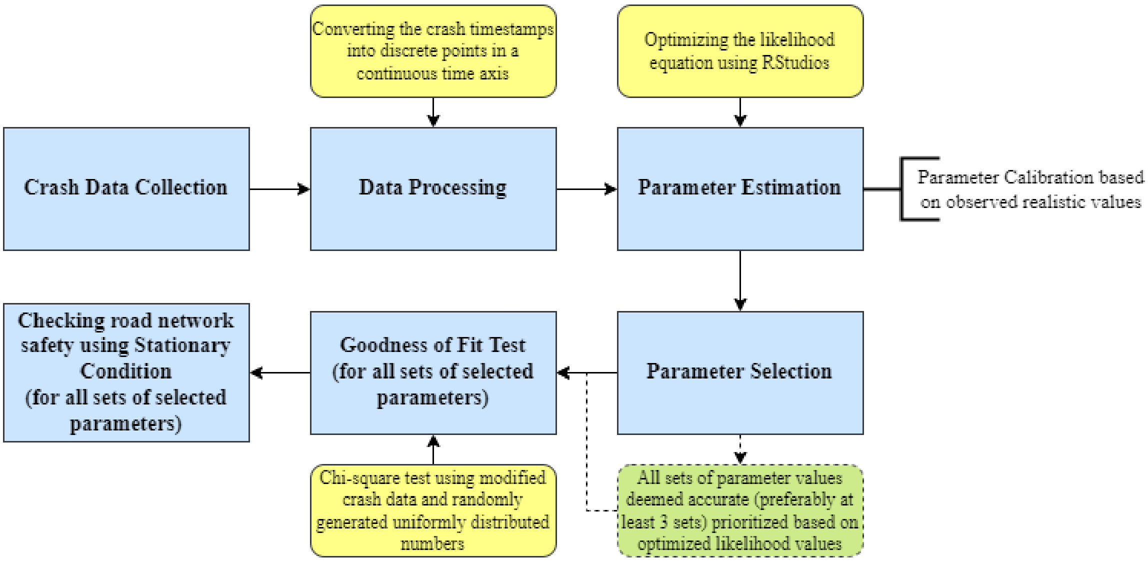

Figure 3-1. Overview of the methodology

The conditional intensity function and kernel excitation function for the crash counting process for DMA traffic is selected as described in Chapter 2.3.1.:

$$\lambda(t) = \mu + \sum_{t_i<t} h(t - t_i) = \sum_{t_i<t} \left(\alpha I_{t-t_i<\tau} + \alpha e^{-\beta(t-t_i-\tau)} I_{t-t_i\geq\tau}\right)$$

The optimum likelihood of the parameters $\Theta = \{\alpha, \beta, \mu, \tau\}$ is done by maximizing the value of likelihood function:

$$L(\Theta) = \prod_{j=1}^{n} \lambda(t_j)\, e^{-\int_0^t \lambda(t)dt}$$

For the optimization of the likelihood function, the `optim` function in R is used. As the most difficult step of analysis of Hawkes Processes is the selection of parameters replicating the in-field scenario, more than one set of parameters are selected for all subsequent analyses.

In order to test for the goodness of fit, the residuals of the crash process are calculated which are then manipulated to form a new variable with an expected exponential distribution. Ultimately, by checking if the new variable is exponentially distributed or not, the goodness of fit can be judged. This process is based on the Time-Rescaling Theorem by Brown *et al.* (2002).

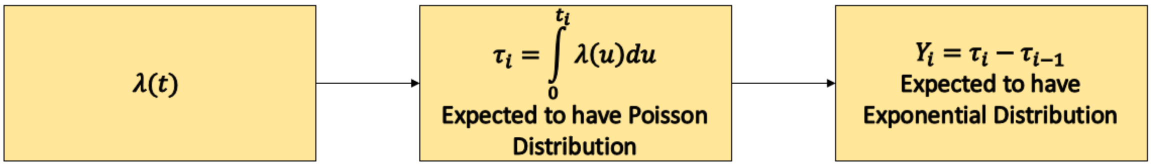

Figure 3-2. Generation of new variables in order to test the goodness of fit

The calculation for stationary condition of the traffic network in the long term is straightforward. Using the expression derived in Chapter 2.3.2, it can be checked whether the condition $\alpha\tau + \frac{\alpha}{\beta} < 1$ condition is fulfilled for the obtained for the determined sets of parameters. For all the sets of parameters accepted, the condition is checked individually.

3.3 Study Area

The study is conducted in the Dhaka Metropolitan Area (DMA), located in the District of Dhaka under the Dhaka Division. Dhaka being the capital city of the country, DMA is the administrative and business hub for every kind of economy and administration in the national level.

With the rapid growth of economy and urbanization, the urban size of DMA continues to expand with the passage of time, both in terms of geographical area and population. As of 2017, the city spanned an urban area of 305.47 square kilometers (117.94 square miles) and a metropolitan area of 2161.17 square kilometers (834.432 square miles). There are almost 9 million people living within this small area, making the city of the densest urban settings in the entire world.

The study area being densely population in comparison to the available land and road infrastructures result in an exceedingly high number of social activities resulting in flow saturation for the road networks. With more trips than available road spaces, there are a significant level of causalities every year within the city's network. In the four years from 2017 to 2020, 1494 crashes have been reported to the Dhaka Metropolitan Police (Source: ARI, BUET).

Figure 3-3. The administrative map of the Dhaka Metropolitan Area, the area of interest of this study.

Since the city is the largest center of administrative and business activities, the road network in the city is of utmost importance and is expected to be much safe and reliable in the context of traffic crashes. Hence, the study is conducted in DMA's traffic network to observe if the traffic crashes represent a self-exciting point process.

3.4 Data Collection

The crash data used for this study are secondary data obtained in soft copies from the ARI under BUET. Four Microsoft Excel Worksheets are provided by Dr. Asif Raihan from ARI, each worksheet containing the crash reports for each of the years from 2017 to 2020, inclusive.

When a traffic crash takes place within the DMA traffic network, a sub-inspector of the Dhaka Metropolitan Police completes an Accident Report Form (ARF) after visiting the accident scene and verifying the information. The ARF is subsequently transmitted to the DMP database, which is the Accident Data Units (ADU) for the DMA zone. There are ten such regional ADUs, established in 1998 in order to process and analyze the crash data under respective jurisdictions. The contents of the ARF are shared with the ARI of BUET in order to incorporate these data with the Microcomputer Accident Analysis Package (MAAP) software. However, the contents are also available in .xlsx file format for different purposes.

The ARF contains all the information related to the crashes necessary to examine the crashes, including but not limited to the following:

- Location,
- Accident ID,
- Day of week,
- Date,
- Time,
- Severity,
- Road Condition,
- Traffic Control,
- Road User Movement, etc.

Out these data, the data relevant to the study of traffic crashes as Hawkes Process are Day of week, Date and Time. The model selected is a univariate model with time as the variable and hence, the other entries of the ARF are not of importance in particular. A separate Microsoft Excel worksheet is created combining the contents of the four provided worksheets for further processing of the data.

3.5 Data Processing

The analysis in the study focuses on the times of events instead of the events themselves. As a result, the raw data directly from the ARI is needed to be modified for the use in this study.

Firstly, any entries in the worksheet with missing date or time or both of crash is discarded. Often times, it is not possible to investigate and record all the information in the ARF, due to reasons like involvement of criminal activities with the crash, lack of clarity for the description of the crash and the like. As a result, it is expected to have a few entries in the crash worksheet to have empty cells in the date and time columns. Such entries were omitted.

Secondly, for multiple crashes with the exact same timestamps are merged into one entry. Otherwise, the counting process would violate the properties of a temporal point process as stated in Chapter 2.2.3. After these two modifications, there are 991 entries in the new worksheet remaining out of the original 1494 crash entries across the original four excel worksheets.

Finally, considering the midnight of January 1, 2017 as the origin of the temporal axis $t = 0$, the times of realizations of the crashes are calculated, taking one day as the unit of time. Mathematically, For the i-th crash in the D-th day of the M-th month in year Y taking place in H hours and m minutes, the time of crash t_i is calculated as:

$$t_i = \sum_{j=0}^{M-1} d_j + D + \frac{H}{24} + \frac{m}{1440} - 1$$

Here, d_j is the total number of days in the j-th month, with $d_0 = 0$. For example, $d_1 = 31$ for January, $d_2 = 28$ for February on a non-leap year and the like.

The table in the Microsoft Excel worksheet where all the modifications are done has been provided in the Appendix A. The processing of data done in Excel along with the formula used to do so are stated below.

The ‘Hour’ and the ‘Minute’ column are created to find the hours and minutes of each of the crashes from the ‘Time’ column. The following Excel formula are used for the calculation of ‘Hour’ and ‘Minute’ values respectively:

```
[@Hour] =([@Time]-[@Minute])/100
[@Minute] =MOD([@Time],100)
```

Since the time is in the HHMM format in both the original .xlsx files and the new worksheet, the above formula is used to find the hour from the first two digits and the minute from the last two digits.

The number of days between the temporal origin and the time of crash is calculated in the ‘di’ column with the following formula:

```
[@[di]] =DAYS(DATE(2000+[@Year],[@Month],[@[Date in month]])
,DATE(2017,1,1))
```

The function `DAYS()` returns the number of days between two days taking the end date and start date as the parameters. The function `DATE()` returns the number that represents the date in Microsoft Excel date-time code taking three numeric parameters: a year, a month and a day in the month respectively.

Finally in the column ‘ti’ of the new worksheet, the timestamps of each of the accidents are calculated which is used as t_i for the model and subsequent calculation and analysis. The following formula is used for calculation of ‘ti’:

```
[@[ti]] =[@[di]]+[@Hour]/24+[@Minute]/(60*24)
```

Thus, the times of realizations of the crashes are obtained in a continuous temporal axis.

3.6 Most Likely Estimates of the Parameters:

3.6.1 Likelihood Equation

The conditional intensity function of the DMA crash counting process has the following form:

$$\lambda(t) = \mu + \sum_{t_i<t} \left(\alpha I_{t-t_i<\tau} + \alpha e^{-\beta(t-t_i-\tau)} I_{t-t_i\geq\tau}\right)$$

The intensity function $\lambda(t)$ denotes the expected rate of crashes per day at time t, calculating upon the following four parameters:

- The background intensity rate μ indicates the element of randomness of the crash process i.e., the intensity of crash occurrence without accounting for self-excitement;
- The parameter α indicates the increase in intensity of crash occurrence as soon as a crash takes place;
- The parameter β indicates the rate at which the intensity gradually decreases over time as crashes do not occur;
- The parameter τ indicates how long the influence of one crash on the intensity remains within the crash process, also accounting for the time required for taking necessary steps for a crash to be dealt with.

In order to estimate the values of these parameters, the optimum value of the likelihood function is calculated. The likelihood function is the joint probability of the observed crashes as a function of the parameters stated above, and has the following form:

$$L = \prod_{j=1}^{n} \lambda(t_j)\, e^{-\int_0^{t_n} \lambda(t)dt}$$

Due to the nature of the likelihood function L, it is often difficult to find its global maximum. Firstly, the calculation of optimization is often done in iterations and based on the converge of successive iterations, the result can end up in a location maximum instead of a global maximum. Secondly, it is complicated to determine both analytical and empirical solutions for the product of the intensity functions along with the integral exponent.

However, the expression can be simplified by taking the ln of the terms on both sides. Logarithm is a monotonically increasing function, and hence maximizing the logarithmic value of a function is analogous to maximizing the function itself. Thus, the log-likelihood function for the crash counting process becomes:

$$l = \ln L = \sum_{j=1}^{n} \ln\left[\lambda(t_j)\right] - \int_0^{t_n} \lambda(t)dt$$

Evaluating the log-likelihood function by substituting the value of the conditional intensity function, the following expression is obtained:

$$\begin{aligned} l = \sum_{j=1}^{n} \ln\left[\mu + \sum_{i=1}^{j-1}\left\{\alpha I_{t_j - t_i < \tau} + \alpha e^{-\beta(t_j - t_i - \tau)} I_{t_j - t_i \geq \tau}\right\}\right] - \mu t_n \\ - \alpha \sum_{j=1}^{n-1}\sum_{i=1}^{j}\left[\left\{t_{j+1} - t_j\right\} I_{t_{j+1} - t_i < \tau} - \frac{1}{\beta}\left\{e^{-\beta(t_{j+1} - t_i - \tau)} - e^{-\beta(t_j - t_i - \tau)}\right\} I_{t_j - t_i \geq \tau} \right. \\ \left. + \left\{\left(\tau - t_j + t_i\right) - \frac{1}{\beta}\left(e^{-\beta(t_{j+1} - t_i - \tau)} - 1\right)\right\} I_{t_j - t_i < \tau < t_{j+1} - t_i}\right] \end{aligned}$$

3.6.2 Computational Methodology

In order to maximize the log-likelihood function, the `optim` function in R is used. However, the function `optim` minimizes the function of interest whereas the study aims to maximize the log-likelihood function. Therefore, the argument function used for the `optim` function is the negative log-likelihood function since maximizing the log-likelihood function is analogous to minimizing the negative log-likelihood function. The negative log-likelihood function in R is shown in Appendix B.

The arguments required as the input for the `optim` function along with the inputs for this study are explained below:

'`par`': The initial values or guesses for the parameters. For the study, multiple different initial values are entered and the optimum value of negative log-likelihood is obtained for each of these initial values in multiple iterations. The ranges for the initial values are provided in the table.

Table 3-1. Ranges of initial values for the parameter before optimization

Parameter	Range of initial values	Motivation
$\boldsymbol{\mu}$	From 0.1 upto 1.0 with an increment of 0.1 in each iteration	The arithmetic mean of crashes per day is $\frac{991}{4\times365} = 0.679$ ... and the background intensity rate should not deviate far from this value
$\boldsymbol{\alpha}$	Random number uniformly distributed between 0 and 1.0	
$\boldsymbol{\beta}$	Random number uniformly distributed between 0 and 2.0	
$\boldsymbol{\tau}$	From 1.0 upto 10.0 with an increment of 1.0 in each iteration	The largest gap between consecutive crashes is $1270.02 - 1258.76 = 11.26$ days (between 930^{th} and 931^{st} crashes) and the probabilistic influence of the crashes in the network should not last more than this duration

From the values in the table, it is seen that 100 iterations of optimization are to be done for different combinations of the initial guesses of the parameters.

'`fn`': The function to be minimized. For this study's case, the function is the negative log-likelihood function as shown in Appendix B.

'`data`': The argument for the crash timestamps in the negative log-likelihood. The table presented in Appendix A is imported to R as a data frame and then the column 'ti' is separated as a vector for input as the data parameter.

'`method`': The optimization algorithm that the `optim` function will use for the process. In this study, the L-BFGS-B method, short for Limited-memory Broyden–Fletcher–Goldfarb–Shanno (BFGS) algorithm, is used.

'`lower`': The lower bounds for the parameters. As stated in the previous chapters, all the parameters for the crash process are non-negative numbers. However, entering zero as the lower bound creates computational issues. Therefore, the lower bound is entered as 1e-10 in R corresponding to 10^{-10} which can be approximated to 0.

'`upper`': The upper bounds for the parameters. In actuality, the parameters are not supposed to have an upper bound. But due to the lack of machine with large computation powers and the way the initial guesses are set up, an upper bound of 26.271 is placed.

A function `generate_mle_table` is created in R which runs a for-loop where the initial values of the parameters are taken for each iteration and the optimized parameters are calculated and tabulated. The RStudio code for the function is presented in Appendix C while the table with the values of the optimized parameters is provided in Appendix D.

3.7 Checking the Goodness of Fit

Once the parameters are obtained, the model can be checked if the crashes truly exhibit properties of a Hawkes Process by the use of Time-Rescaling Theorem (Brown *et al.*, 2002). The following integration can be performed to transform the times of realizations of crashes t_j to a new variable T_j:

$$\mathrm{T}_j = \int_0^{t_j} \lambda(t)dt$$

The integration can be evaluated as follows for the crash counting process established in this study:

$$\mathrm{T}_j = \int_0^{t_j} \left[\mu + \sum_{t_i<t} \left(\alpha I_{t-t_i<\tau} + \alpha e^{-\beta(t-t_i-\tau)} I_{t-t_i\geq\tau}\right)\right] dt$$

$$= \mu t_j + \sum_{t_i<t_j} \left(\alpha t_j I_{t_j-t_i<\tau} - \frac{\alpha}{\beta} e^{-\beta(t_j-t_i-\tau)} I_{t_j-t_i\geq\tau}\right)$$

According to Papangelou (1972), the variables T_j are independent and follows a stationary Poisson Process, if $\lambda(t)$ exhibits a good fit for a Hawkes Process. The inter-arrival times of such a process $Y_j = \mathrm{T}_j - \mathrm{T}_{j-1}$ is expected to be exponentially distributed.

A simple Chi-square test can be checked whether the variable Y_j follows exponential distribution or not. The null hypothesis shall be H_0: Y_j comes from a uniform distribution. The distribution shall be divided into 10 class intervals for the test with a level of significance of 0.05. After calculation of Y_j, the data can be sorted into 10 classes i.e., $k = 10$. Chi-Square test is conducted for each set of parameters accepted from parameter estimation. The procedure is same for all of the tests and a general format is presented here.

For each test, the λ_{ex} parameter of the exponential distribution is calculated as the inverse of average of the Y_j values i.e., $\lambda_{ex} = \frac{1}{\bar{Y}}$. It is to be noted that the λ_{ex} is not related to the notation λ used from conditional intensity function. The bounds for the i-th class interval is bounded by a_{i-1} and a_i where a_i is calculated as:

$$a_i = -\frac{1}{\lambda_{ex}} \ln(1 - ip)$$

Here, $p = \frac{1}{k} = \frac{1}{10} = 0.1$ for this study, for all the tests. Once the bounds for the class intervals are determined, the observed frequency O_i for the i-th class interval is calculated as number of observations of Y between greater than a_{i-1} and smaller than a_i. If an observation is found exactly equal to a_i, it has been placed in the i-th class interval. The expected frequency E_i is calculated as $\frac{N}{k}$. For this study, $N = 991$ from the number of crashes and $k = 10$ as decided above. Therefore, the expected frequency $E_i = 99.1$ for all the class intervals.

Finally, the χ^2 statistic is calculated as $\sum \frac{(O_I - E_i)^2}{E_i}$ and is compared with a critical χ^2 value corresponding to 0.05 level of significance and $10 - 1 = 9$ degree of freedom. If the condition, $\chi^2_{observed} < \chi^2_{critical}$ is fulfilled, the model is a good fit for that particular set of parameters for which the test is conducted.

3.8 Summary

Mostly numerical works are needed to be done for this study. Microsoft Excel is used for the processing of data before they are exported to RStudio for bulk of the work including parameter estimation and goodness of fit check. The results and findings are easily obtained through the processes and calculations described as above.

CHAPTER 4
RESULTS AND DISCUSSION

4.1 Introduction

From the conducted calculations, the crashes appear to be not self-exciting, at least on the surface viewpoint. There is a lack of evidence to support that the crashes exhibit self-excitement nature yet some interesting observations are found while conducting the investigation.

The chapter deals with the results obtained from the calculations as per the methodology described. The findings for each of the specific objectives are discussed in separate chapters followed by discussions about the results and findings.

4.2 Parameter Estimation

4.2.1 Crash Counting Process

The plot of the crash counting process $N(t)$ against time t is presented in Figure 4-1. It can be observed that the number of crashes tend to increase linearly with time.

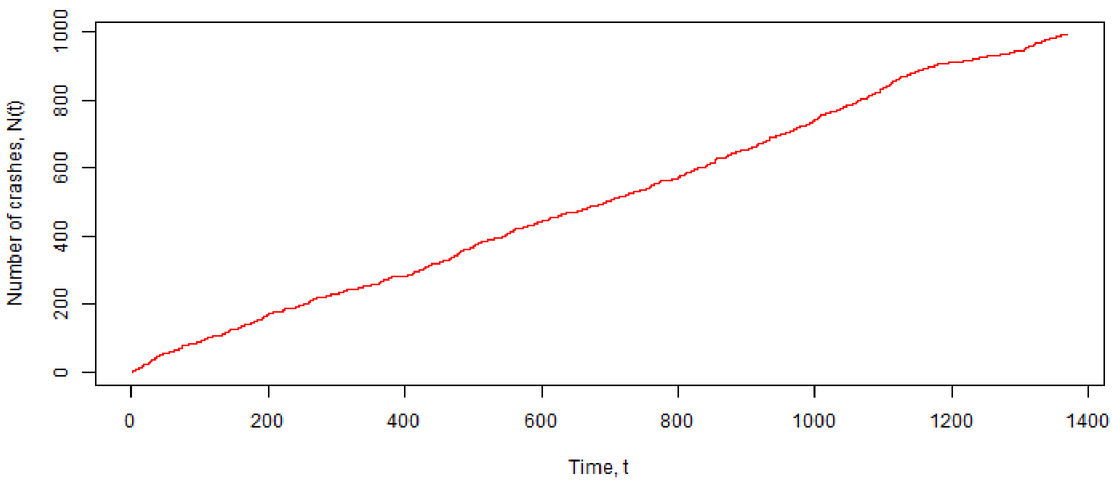

Figure 4-1. Total number of crashes within the system plotted against time.

This is a preliminary idea that the crashes are more dependent on time, rather than on the occurrence of the crashes themselves, which ultimately proves that the crashes are a random process. In fact, it appears that the crash counting variable is much strongly correlated to the passage of time, so much so that a crash counting dependent variable $\hat{N}$ can be correlated with an independent time variable t using a linear regression equation:

$$\hat{N}(t) = -7.49 + 1.36t$$

Such an equation would have a R^2 value of 0.9976, implying a very strong correlation.

4.2.2 MLE of Parameters

From the table in Appendix D, it is seen that the parameters tend to diverge out of the bounds set for the calculation i.e., $(0, 26.271]$ and there are seven sets unique parameter sets obtained from different initial guesses. These parameter sets are summarized in Table 4-1.

Table 4-1. Summary of the determined sets of parameters

Set	μ	α	β	τ	Negative Log-Likelihood	Remarks
1	10^{-10}	26.271	10^{-10}	10^{-10}	-5.80×10^{9}	α diverges
2	10^{-10}	26.271	26.271	26.271	-5.81×10^{303}	α, β, τ diverge
3	0.1	26.271	26.271	26.271	-5.81×10^{303}	α, β, τ diverge
4	0.2	26.271	26.271	26.271	-5.81×10^{303}	α, β, τ diverge
5	0.3	26.271	26.271	26.271	-5.81×10^{303}	α, β, τ diverge
6	1	26.271	26.271	26.271	-5.81×10^{303}	α, β, τ diverge
7	26.271	26.271	26.271	26.271	-5.81×10^{303}	All parameters diverge

It is to be noted that although the values for the parameters are obtained to be 26.271 in certain scenarios, this number can not be their conclusive value. This number is the upper bound fixed by the `upper` parameter in the `optim` function. The fact that the parameters value was found to be 26.271 implies that the likelihood function's maximum value was not determined in the bounds set for our initial condition.

The `optim` function works by making iterative calculations by calculating the function to be optimized at multiple points starting from the initial guess provided. Based on several optimization algorithms, the `optim` function locates the optimum value within the provided bounds. If the function to be optimized has divergent slope for the limits of parameters' initial guesses and given upper and lower bounds, then `optim` fails to find a local extremum within the boundary conditions and constraints, let alone an absolute extremum. In such a case, the iterations tend towards the extremum values found near the boundary conditions which is ultimately returned by the `optim` function. This is the case that has happened for this study.

However, the interesting observation is the likelihood value. Likelihood value was found large for diverging parameters (Set 2-7) with contrast to non-divergent parameters (Set 1). It strongly implied that the likelihood function is indeed devoid of a local or absolute maximum in the boundary conditions and initial guesses set for this study.

4.2.3 Conditional Intensity Function

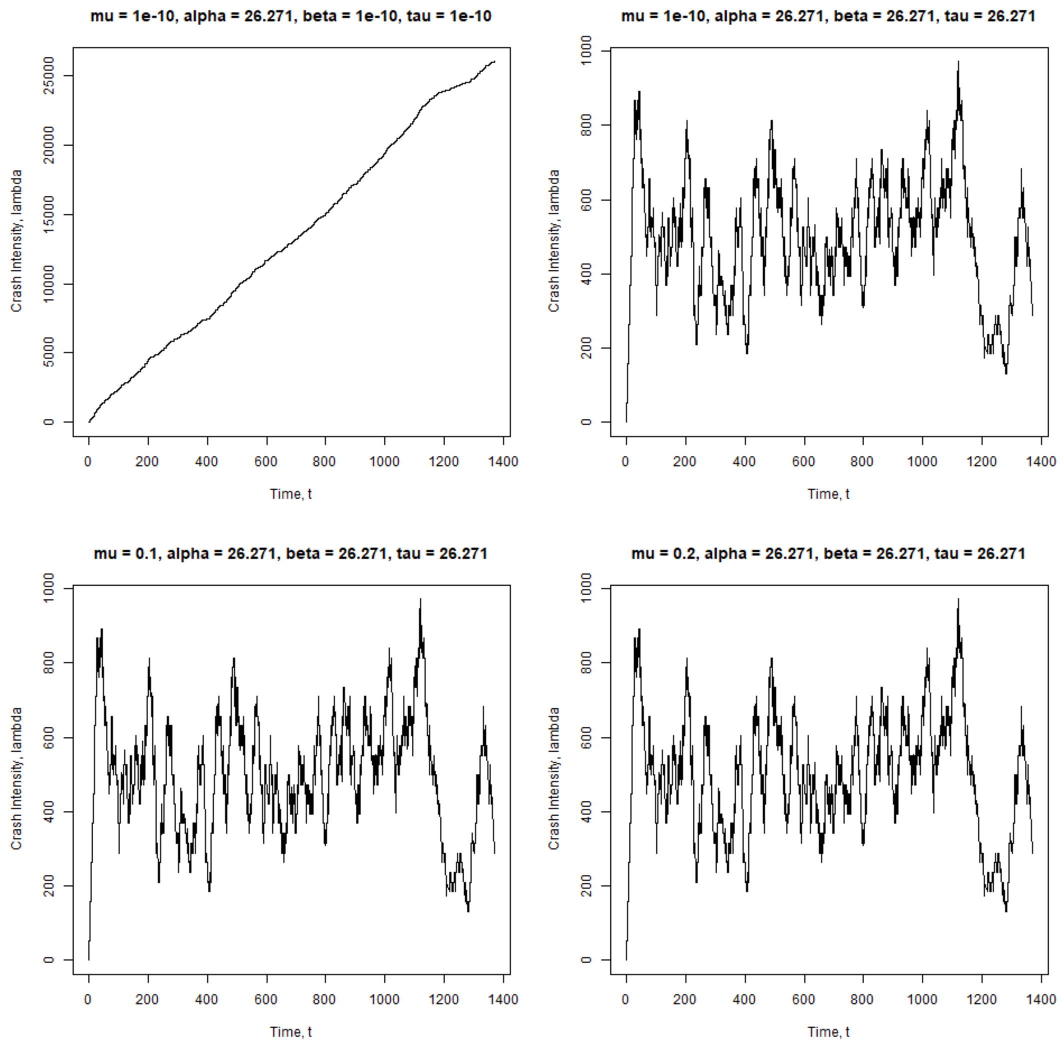

Figure 4-2. Conditional Intensity Function for parameter sets 1-4

The conditional intensity function's graphs are provided for the parameter sets in figures 4-2 and 4-3. From the structure of the kernel excitement function selected, it is obvious that the conditional intensity function's graphical shape is dictated by α, β and

τ. The value of μ shifts the graph along the vertical axis. Such results are seen from the figures.

Despite the values being offshoot from realistic scenarios, the graph obtained for Set 1 represents a scenario far more unrealistic than the other sets of parameters.

In set of parameters Set 1, the values of β and τ were not divergent but the overall likelihood of the set of parameters was found relatively smaller. But the conditional intensity function, as observed from the graph, continues to increase as time goes on implying that the road network becomes more and more prone to crashes as time continues and crashes take place.

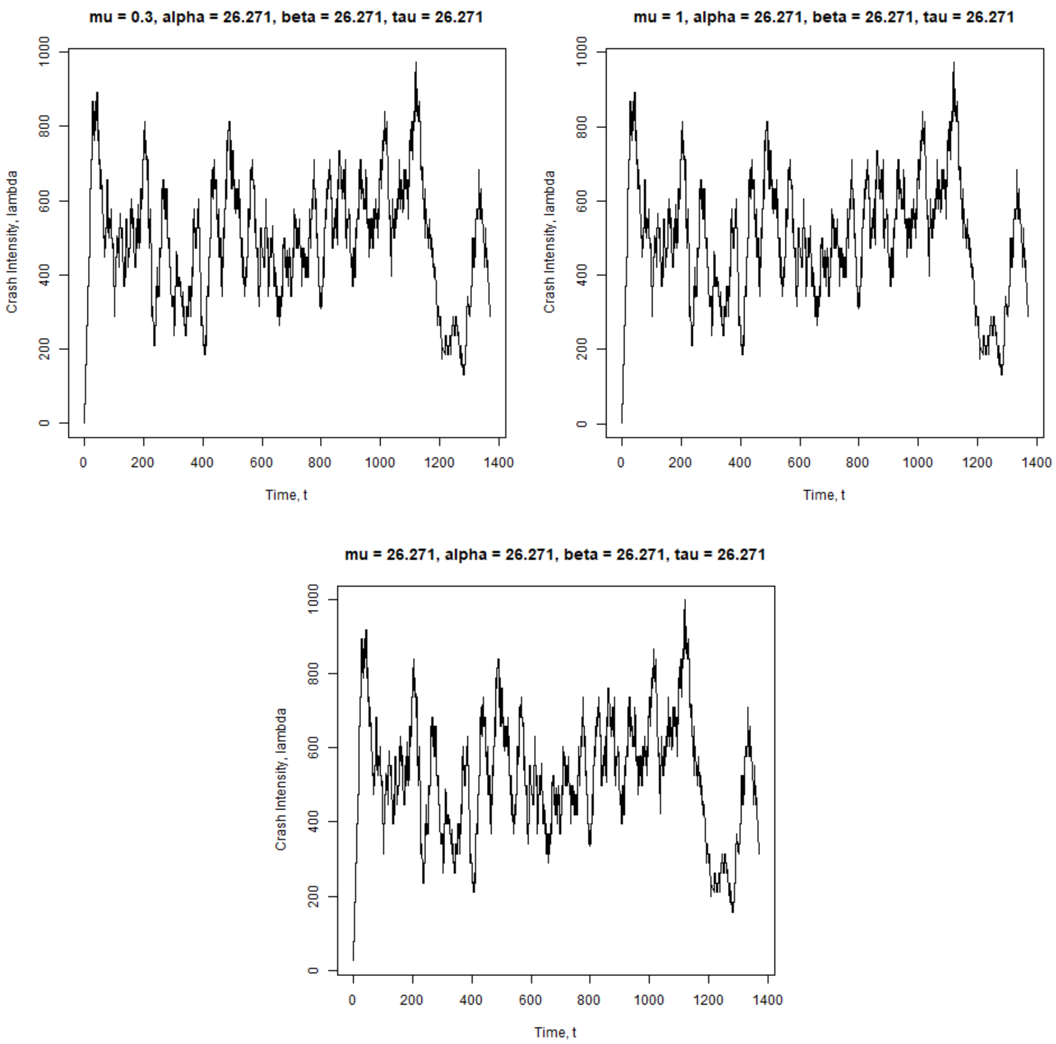

Figure 4-3. Conditional Intensity Function for parameter sets 5-7

However, in Sets 2-7, the values of α, β and τ are divergent and the overall likelihoods of the parameters were much higher. The intensity of the crashes for these sets fluctuated with time, increased in some instances while decreased in other. Such a scenario in real traffic network would imply that the network is unsafe at times but not entirely unreliable in terms of crashes and safety.

4.3 Goodness of Fit

The summarized result from the Chi-Square tests is presented in Table 4-2. It is seen that none of the parameter sets for the proposed model are a good fit for crashes in DMA as a Hawkes Process. However, the higher the values went, the lower the value of χ^2 statistic. Therefore, it is implied that the model with a set of higher values of parameter might result in a good fit.

Table 4-2. Summary of Chi-Square test for Goodness of Fit

Parameter Set	$\chi^2_{observed}$	$\chi^2_{\alpha=0.05,dof=9}$	Conclusion
1	8901.01	16.919	Not a good fit
2	643.24	16.919	Not a good fit
3	643.24	16.919	Not a good fit
4	643.24	16.919	Not a good fit
5	643.24	16.919	Not a good fit
6	642.69	16.919	Not a good fit
7	630.64	16.919	Not a good fit

On the contrary, too high values of the parameters do not make intuitive sense for the crashes depicted in the practical scenarios. The result might even change based on a different class interval size. The number of class intervals k can be increased to get a better χ^2 statistic.

4.4 Stationary Condition

The safety and reliability of the traffic network can be determined by computing the expression $\alpha\tau + \frac{\alpha}{\beta}$ as discussed in Chapter 2.3.2. The summarized calculation for the safety and reliability check of the traffic network in stationary condition for each set of the parameters is presented in Table 4-3.

Table 4-3. Stationary Condition Check for Safety

Parameter Set(s)	α	β	τ	$\alpha\tau + \frac{\alpha}{\beta}$	Conclusion
1	26.271	10^{-10}	10^{-10}	2.627×10^{11}	Unsafe
2-7	26.271	26.271	26.271	691.165	Unsafe

It is seen that for all sets of parameters, the traffic network appears unsafe. As time goes on after a crash takes place, the crash intensity keeps on increasing and the road becomes more and more unsafe in terms of crashes.

The findings for the stationary condition for the parameters in Set 1 are in conjunction to the computed conditional intensity function in Chapter 4.2.3. The value of $\alpha\tau + \frac{\alpha}{\beta}$ for Set 1 is higher than that for the other sets by several orders of magnitudes, implying that the road network would be more unsafe for Set 1 by a lot. And it is the exact conclusion derived during computation of the conditional intensity function, where it was seen that the crash intensity increased to extreme values with time for Set 1 in comparison to other sets.

4.5 Discussions

4.5.1 Parameter Estimation

The parameters' optimum values found was sensitive to the provided boundary condition and initial values. While the values obtained in this study was not found to be convergent towards a maximum value of likelihood function, the process could be repeated for a different boundary condition and set of initial values. Replicating the study with different condition, a different scenario is very likely to arise.

From a mathematical point of view, the model appears to be much dictated by the values of β and τ. If a convergent criterion is determined for these two parameters, it is believed that the analysis of crashes as Hawkes Process could be more fruitful.

As seen from the values of the MLE of the parameters, they tend to be really high. Each of their significances are described below.

- The value of μ fluctuated across different sets. Therefore, it is safe to say that the background rate of DMA's crash process is hard to predict following Hawkes Process.
- A high value of α indicates that the roads become very unsafe as soon as a crash takes place. The crash intensity immediately boosts up within the network and another crash is likely to take place.
- High values of τ contributes to the unreliability of the network along with the high value of α. The influence of a crash in the crash intensity of the network tends to remain for a very long time.

- Counterbalancing the high values of α, a high value of β implies the network's crash intensity also decreases very swiftly with time.

Combining the effects of having a high α and a high β, it can be said that the traffic network is 'fickle'. The network becomes very unsafe in terms of crash intensity whenever a crash takes place and reverts back to safe condition very swiftly as well.

Although most of the evidences aim at the crashes not exhibiting self-excitement, a counter argument could be made using the obtained high value of τ. It implies that the probabilistic effects of a crash remain for a long time within the transportation network. As more and more crashes take place within this timeframe, the combined effect of all these crashes obscure the self-excitement and illustrates a rather random pattern among the crashes. Hence, it is too early to discard the possibility of self-exciting properties within the traffic network of Dhaka Metropolitan Area.

4.5.2 Goodness of Fit Test

The Goodness of Fit test also suggests that the crashes are not self-exciting in nature. But it was observed that the χ^2 statistic decreased as the values of the parameters increased. Therefore, the observed and expected data would keep getting closer as the parameters' value is increased. But in practical scenarios, obtaining a large value for a parameter would not make intuitive sense especially for μ and α.

However, there is a major takeaway from the Chi-Square test. The χ^2 statistic for Set 1 of the parameters was extremely high compared to the χ^2 statistic for Sets 2-7. It implies that the parameters in Sets 2-7 are better fitting than the parameters in Set 1. But as discussed previously, the optimized value of the parameters could not be found in the boundary condition and initial guess provided. If the boundary condition and constraints were changed, a properly optimized set of parameters could have been obtained representing a good fit.

4.5.3 Stationary Condition

The check for stationary condition implied that the number of crashes would keep on increasing by leaps and bounds in the DMA traffic network as time passed. It would make the network gradually more and more unsafe with each crash occurring in time.

It is dictated by the high values of the parameters α and τ. The influence of a crash on the overall traffic system is immense for DMA. Firstly, a crash boosts the crash intensity in the traffic network by a huge order of magnitude because of the high value of α. Secondly, the influence of the crash remains within the network for large time as seen from the high values of τ.

In order to have a traffic network safe from crashes in the long run, it requires having a large value of β and small values for α and τ. Although β was found large in this study, it was not large enough to compensate for the large values of α and τ together.

It is also observed that the expression for checking the stationary condition in the long run is independent of μ. The randomness of the system does not affect the system in the long run and it is the eventual likeliness of crashes to take place even after all crash preventive measures are in place.

4.6 Summary

For some obstructions and limitations to the studies, some predicted and some unforeseen, the results obtained might not seem functional. Although the crashes did not exhibit properties of self-excitement, there were some interesting observations otherwise. These observations provided new insights about not only the nature of traffic crashes Dhaka Metropolitan Area but also the process of analysis of systems following Hawkes Process.

CHAPTER 5
CONCLUSION

5.1 Introduction

This study investigated the crashes taking place within the Dhaka Metropolitan Area traffic network to see if they followed Hawkes Process i.e., exhibit properties of self-excitement. The results and findings hint towards a lack of self-excitement in the crash system, though it is too early to dismiss the possibility of crashes being self-exciting at face value.

In this chapter, the major findings and contribution of this study is mentioned, followed by the limitations of the study. The possible scopes of improvement in this vein of analysis and future works that might be based on this study are also discussed afterwards.

5.2 Major Findings of the Study

The results and findings provide more insights once they are analyzed, as compared to interpreting for the first time. Although there was a lack of evidence for self-excitement in a holistic sense, the following major observations were made.

- ❖ In a holistic scale, the crashes were extremely correlated to the flow of time. They were random events appearing as very less dependent on their occurrences i.e., they appeared to be a Poisson Process.
- ❖ On the contrary, the crashes did exhibit self-exciting nature indirectly in a smaller scale. Crashes correlates to the formation of congestion which in turn correlates to occurrences of more crashes as seen in Figure 1-1.
- ❖ Alternately, it may also be concluded that the crash process is extremely self-exciting in a holistic level, with high values of α, β and τ. The effect of self-excitement of a crash remains for a long time within the traffic network, obscuring the self-exciting nature and depicting a pattern of randomness.

- ❖ The background randomness of the traffic crashes is very hard to pinpoint under the assumption of them being a Hawkes Process. But it should be extremely easy to measure their randomness as a Poisson Process.
- ❖ The DMA traffic network becomes very risky and extremely prone to crashes whenever a crash takes place. The crash intensity also decays quickly over time, making the network relatively safer again. There is a huge amount of fluctuation in the crash intensity (as number of crashes per day) in the traffic network.
- ❖ Due to lack of computational powers, the exact optimized values for the parameters could not be identified in this study. However, it is evident that an optimized solution exists and can be determined by different initial guesses and boundary constraints.
- ❖ The calculation for the optimized values of the parameters is strongly dictated by the values of β and τ.
- ❖ The model used in this study was not a good fit for the crash process in DMA. However, as the values of the parameters increased, the differences between observed values and expected values decreased.
- ❖ In terms of the stationary condition of the model, the traffic network is found unsafe in the long run with number of crashes continuing to increase as time progresses.
- ❖ In order to make the traffic network safe in the long run, it is necessary to decrease the value of α and τ and increase the value of β.
- ❖ The randomness of the system does not affect the safety of the system in the long run. Crashes are bound to take place even after all crash preventive measures are in place.

5.3 Limitations of the Study

The study had several limitations and constraints, some unforeseen and some predicted. These are discussed as below.

- The model deals with the time of realization of the events rather than the events themselves. Most of the calculations either break or become extremely convoluted if there are more than one event in the exact same moment. The crash intensity in this study is calculated assuming not more than one crashes taking place at any instant of time whereas there might be more than one crash at a time in reality.

– Hawkes Process is better modeled with data from the infinite past. It is not possible to collect crash data from infinite past.
– There was an invisible constraint in terms of computational capacity of the devices used for this study. With better devices, the MLE of the parameters can be calculated to a better degree and the model can be a better fit of the real scenario.

5.4 Recommendations for Future Works

The findings of this study can be utilized for future works and researches. The insights and observations can be helpful for road safety engineering, whereas the limitations can be overcome in future studies aiming to analyze crashes as Hawkes Process.

5.4.1 Future Studies

There are ample opportunities for future studies of crashes to be improved based on this study. Some of the key pointers are mentioned below.

- There is scope of using Machine Learning for the analysis of Hawkes Processes. The use of Machine Learning in the analysis of crashes as Hawkes Process can yield better results.
- Using better algorithms and equations, the calculations can be simplified and properly approximated. It is analogous to using computational devices of higher capacities and getting better estimates for the parameters and fitting the data better into the model.
- There are many other possible tests that can be done for the Goodness of Fit test. Some popular alternatives to the Chi-Square test done in this study are the Kolmogorov-Smirnov (KS) test and the Akaike Information Criterion (AIC).
- The basic assumptions of this study can be used to calculate the statistical characteristics of the DMA traffic network, such as the expected values and variances of number of crashes $N(t)$ and crash intensity $\lambda(t)$.

- The study can include spatial factors and investigate the crashes in smaller zones. A spatial-temporal analysis on a zonal level might depict the crash condition even better than the aggregated study. It is very plausible that crashes in certain areas of Dhaka Metropolitan Area exhibit self-excitement while other areas don’t.

5.4.2 Road Safety Engineering

Road safety engineers can detect whether the crash prevention measures to be taken should be selected for the entire DMA or for spatial or temporal zones. The prevention measures can include improvement of road designs, speed management, roadside hazards management and the like.

- If further study proves that the crashes in DMA are truly random in nature, then crash prevention measures should be comprehensive of the entire network.
- If the crashes are self-exciting throughout the network, the preventive measures should be strongly enforced right after a crash takes place.
- If the crashes are self-exciting in certain zones, then preventive measures are to be deployed to those areas with more priority than the other areas.

5.5 Concluding Remarks

Crash analysis is a vital sector for analysis and research. If it is possible to predict and prevent crashes ahead of time, it can save countless valuable lives and properties. Many agencies are already using concepts of Hawkes Process to bring about remarkable results in their disciplines, such as PredPol in predicting crimes and seismologists in predicting earthquakes and volcanic eruptions. Similar feats can be accomplished in the sector of transportation, preventing crashes and improving people’s safety and quality of life.

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APPENDIX

APPENDIX
Appendix A

Table A-1. Table in Microsoft Excel Worksheet containing crash timestamps and times of realizations of the traffic crashes.

i	Year	Month	Date in month	Time	Hour	Minute	di	ti
1	17	1	2	1110	11	10	1	1.47
2	17	1	3	615	6	15	2	2.26
3	17	1	4	40	0	40	3	3.03
4	17	1	4	1800	18	0	3	3.75
5	17	1	5	1530	15	30	4	4.65
6	17	1	6	1000	10	0	5	5.42
7	17	1	7	930	9	30	6	6.40
8	17	1	9	1130	11	30	8	8.48
9	17	1	9	2240	22	40	8	8.94
10	17	1	10	1230	12	30	9	9.52
11	17	1	11	845	8	45	10	10.36
12	17	1	12	900	9	0	11	11.38
13	17	1	13	1210	12	10	12	12.51
14	17	1	14	1125	11	25	13	13.48
15	17	1	15	1030	10	30	14	14.44
16	17	1	16	910	9	10	15	15.38
17	17	1	16	1500	15	0	15	15.63
18	17	1	18	2047	20	47	17	17.87
19	17	1	19	400	4	0	18	18.17
20	17	1	19	1930	19	30	18	18.81
21	17	1	20	156	1	56	19	19.08
22	17	1	20	2130	21	30	19	19.90
23	17	1	20	2210	22	10	19	19.92
24	17	1	21	1100	11	0	20	20.46
25	17	1	24	1200	12	0	23	23.50
26	17	1	24	2330	23	30	23	23.98
27	17	1	25	915	9	15	24	24.39
28	17	1	26	1330	13	30	25	25.56
29	17	1	26	2340	23	40	25	25.99
30	17	1	28	730	7	30	27	27.31
31	17	1	28	940	9	40	27	27.40
32	17	1	28	2145	21	45	27	27.91
33	17	1	29	830	8	30	28	28.35
34	17	1	29	1010	10	10	28	28.42
35	17	1	30	1835	18	35	29	29.77

36	17	1	30	2205	22	5	29	29.92
37	17	2	3	130	1	30	33	33.06
38	17	2	4	12	0	12	34	34.01
39	17	2	4	1530	15	30	34	34.65
40	17	2	5	630	6	30	35	35.27
41	17	2	6	1230	12	30	36	36.52
42	17	2	6	2000	20	0	36	36.83
43	17	2	7	800	8	0	37	37.33
44	17	2	7	910	9	10	37	37.38
45	17	2	8	330	3	30	38	38.15
46	17	2	8	2310	23	10	38	38.97
47	17	2	10	1515	15	15	40	40.64
48	17	2	10	1640	16	40	40	40.69
49	17	2	12	40	0	40	42	42.03
50	17	2	12	1230	12	30	42	42.52
51	17	2	12	1745	17	45	42	42.74
52	17	2	15	2230	22	30	45	45.94
53	17	2	16	1115	11	15	46	46.47
54	17	2	17	140	1	40	47	47.07
55	17	2	22	1230	12	30	52	52.52
56	17	2	22	1300	13	0	52	52.54
57	17	2	23	2230	22	30	53	53.94
58	17	2	24	400	4	0	54	54.17
59	17	2	25	30	0	30	55	55.02
60	17	2	25	635	6	35	55	55.27
61	17	2	28	245	2	45	58	58.11
62	17	3	2	1400	14	0	60	60.58
63	17	3	5	330	3	30	63	63.15
64	17	3	5	2215	22	15	63	63.93
65	17	3	6	2005	20	5	64	64.84
66	17	3	6	2030	20	30	64	64.85
67	17	3	9	1330	13	30	67	67.56
68	17	3	10	1345	13	45	68	68.57
69	17	3	11	1130	11	30	69	69.48
70	17	3	12	1400	14	0	70	70.58
71	17	3	12	2045	20	45	70	70.86
72	17	3	14	830	8	30	72	72.35
73	17	3	15	1210	12	10	73	73.51
74	17	3	16	200	2	0	74	74.08
75	17	3	16	900	9	0	74	74.38
76	17	3	16	915	9	15	74	74.39
77	17	3	17	735	7	35	75	75.32
78	17	3	18	550	5	50	76	76.24
79	17	3	18	1730	17	30	76	76.73
80	17	3	24	1930	19	30	82	82.81
81	17	3	25	305	3	5	83	83.13

82	17	3	26	1145	11	45	84	84.49
83	17	3	30	1135	11	35	88	88.48
84	17	3	31	1205	12	5	89	89.50
85	17	3	31	2230	22	30	89	89.94
86	17	4	5	30	0	30	94	94.02
87	17	4	6	110	1	10	95	95.05
88	17	4	7	800	8	0	96	96.33
89	17	4	10	1045	10	45	99	99.45
90	17	4	13	2305	23	5	102	102.96
91	17	4	14	130	1	30	103	103.06
92	17	4	14	1255	12	55	103	103.54
93	17	4	15	815	8	15	104	104.34
94	17	4	15	2100	21	0	104	104.88
95	17	4	16	2140	21	40	105	105.90
96	17	4	17	930	9	30	106	106.40
97	17	4	19	2120	21	20	108	108.89
98	17	4	20	45	0	45	109	109.03
99	17	4	20	330	3	30	109	109.15
100	17	4	20	2040	20	40	109	109.86
101	17	4	22	1130	11	30	111	111.48
102	17	4	27	1130	11	30	116	116.48
103	17	4	27	1330	13	30	116	116.56
104	17	4	29	630	6	30	118	118.27
105	17	4	30	530	5	30	119	119.23
106	17	4	30	1430	14	30	119	119.60
107	17	5	1	730	7	30	120	120.31
108	17	5	2	1130	11	30	121	121.48
109	17	5	9	400	4	0	128	128.17
110	17	5	12	1640	16	40	131	131.69
111	17	5	13	1915	19	15	132	132.80
112	17	5	15	900	9	0	134	134.38
113	17	5	16	245	2	45	135	135.11
114	17	5	16	1640	16	40	135	135.69
115	17	5	17	1200	12	0	136	136.50
116	17	5	19	830	8	30	138	138.35
117	17	5	20	1015	10	15	139	139.43
118	17	5	20	1800	18	0	139	139.75
119	17	5	22	300	3	0	141	141.13
120	17	5	22	905	9	5	141	141.38
121	17	5	22	1730	17	30	141	141.73
122	17	5	23	520	5	20	142	142.22
123	17	5	24	2200	22	0	143	143.92
124	17	5	26	1230	12	30	145	145.52
125	17	5	30	1530	15	30	149	149.65
126	17	6	1	1945	19	45	151	151.82
127	17	6	4	715	7	15	154	154.30

128	17	6	6	730	7	30	156	156.31
129	17	6	6	1320	13	20	156	156.56
130	17	6	6	1945	19	45	156	156.82
131	17	6	6	2045	20	45	156	156.86
132	17	6	7	1030	10	30	157	157.44
133	17	6	10	1330	13	30	160	160.56
134	17	6	11	1845	18	45	161	161.78
135	17	6	11	2230	22	30	161	161.94
136	17	6	14	240	2	40	164	164.11
137	17	6	15	1400	14	0	165	165.58
138	17	6	15	1845	18	45	165	165.78
139	17	6	17	515	5	15	167	167.22
140	17	6	21	1745	17	45	171	171.74
141	17	6	24	1930	19	30	174	174.81
142	17	6	24	2000	20	0	174	174.83
143	17	6	25	400	4	0	175	175.17
144	17	6	25	1030	10	30	175	175.44
145	17	6	29	615	6	15	179	179.26
146	17	6	29	700	7	0	179	179.29
147	17	6	29	1445	14	45	179	179.61
148	17	6	30	1655	16	55	180	180.70
149	17	7	1	1230	12	30	181	181.52
150	17	7	2	1440	14	40	182	182.61
151	17	7	4	35	0	35	184	184.02
152	17	7	4	1900	19	0	184	184.79
153	17	7	5	500	5	0	185	185.21
154	17	7	8	540	5	40	188	188.24
155	17	7	9	1530	15	30	189	189.65
156	17	7	10	630	6	30	190	190.27
157	17	7	10	745	7	45	190	190.32
158	17	7	10	930	9	30	190	190.40
159	17	7	10	1635	16	35	190	190.69
160	17	7	10	1930	19	30	190	190.81
161	17	7	12	1330	13	30	192	192.56
162	17	7	12	1900	19	0	192	192.79
163	17	7	15	1230	12	30	195	195.52
164	17	7	15	1945	19	45	195	195.82
165	17	7	16	1620	16	20	196	196.68
166	17	7	18	1000	10	0	198	198.42
167	17	7	18	2130	21	30	198	198.90
168	17	7	19	800	8	0	199	199.33
169	17	7	20	700	7	0	200	200.29
170	17	7	21	15	0	15	201	201.01
171	17	7	21	830	8	30	201	201.35
172	17	7	21	1000	10	0	201	201.42
173	17	7	21	1305	13	5	201	201.55

174	17	7	21	2030	20	30	201	201.85
175	17	7	24	630	6	30	204	204.27
176	17	7	27	5	0	5	207	207.00
177	17	7	28	1400	14	0	208	208.58
178	17	7	28	1610	16	10	208	208.67
179	17	8	1	500	5	0	212	212.21
180	17	8	1	1930	19	30	212	212.81
181	17	8	10	10	0	10	221	221.01
182	17	8	10	30	0	30	221	221.02
183	17	8	10	755	7	55	221	221.33
184	17	8	10	1735	17	35	221	221.73
185	17	8	13	915	9	15	224	224.39
186	17	8	19	830	8	30	230	230.35
187	17	8	24	530	5	30	235	235.23
188	17	8	26	1400	14	0	237	237.58
189	17	8	28	1445	14	45	239	239.61
190	17	8	29	1020	10	20	240	240.43
191	17	8	30	1900	19	0	241	241.79
192	17	9	1	330	3	30	243	243.15
193	17	9	1	400	4	0	243	243.17
194	17	9	1	1230	12	30	243	243.52
195	17	9	3	920	9	20	245	245.39
196	17	9	4	345	3	45	246	246.16
197	17	9	6	10	0	10	248	248.01
198	17	9	7	520	5	20	249	249.22
199	17	9	10	815	8	15	252	252.34
200	17	9	11	1200	12	0	253	253.50
201	17	9	11	1315	13	15	253	253.55
202	17	9	12	1245	12	45	254	254.53
203	17	9	12	2330	23	30	254	254.98
204	17	9	16	2030	20	30	258	258.85
205	17	9	17	2100	21	0	259	259.88
206	17	9	18	530	5	30	260	260.23
207	17	9	18	710	7	10	260	260.30
208	17	9	18	930	9	30	260	260.40
209	17	9	19	2000	20	0	261	261.83
210	17	9	20	1030	10	30	262	262.44
211	17	9	20	2100	21	0	262	262.88
212	17	9	22	2030	20	30	264	264.85
213	17	9	23	700	7	0	265	265.29
214	17	9	24	1305	13	5	266	266.55
215	17	9	25	430	4	30	267	267.19
216	17	9	26	2230	22	30	268	268.94
217	17	9	28	2030	20	30	270	270.85
218	17	9	29	930	9	30	271	271.40
219	17	10	3	700	7	0	275	275.29

220	17	10	3	1035	10	35	275	275.44
221	17	10	4	630	6	30	276	276.27
222	17	10	4	935	9	35	276	276.40
223	17	10	13	2130	21	30	285	285.90
224	17	10	14	700	7	0	286	286.29
225	17	10	15	815	8	15	287	287.34
226	17	10	16	200	2	0	288	288.08
227	17	10	16	1000	10	0	288	288.42
228	17	10	19	2010	20	10	291	291.84
229	17	10	23	1025	10	25	295	295.43
230	17	10	25	1545	15	45	297	297.66
231	17	10	27	630	6	30	299	299.27
232	17	11	1	1230	12	30	304	304.52
233	17	11	2	345	3	45	305	305.16
234	17	11	2	2000	20	0	305	305.83
235	17	11	4	2240	22	40	307	307.94
236	17	11	5	400	4	0	308	308.17
237	17	11	7	450	4	50	310	310.20
238	17	11	8	700	7	0	311	311.29
239	17	11	8	1000	10	0	311	311.42
240	17	11	9	430	4	30	312	312.19
241	17	11	10	430	4	30	313	313.19
242	17	11	12	1235	12	35	315	315.52
243	17	11	17	1630	16	30	320	320.69
244	17	11	20	1330	13	30	323	323.56
245	17	11	28	340	3	40	331	331.15
246	17	11	28	1100	11	0	331	331.46
247	17	11	29	1530	15	30	332	332.65
248	17	12	3	1600	16	0	336	336.67
249	17	12	4	130	1	30	337	337.06
250	17	12	5	605	6	5	338	338.25
251	17	12	6	1530	15	30	339	339.65
252	17	12	11	2000	20	0	344	344.83
253	17	12	13	45	0	45	346	346.03
254	17	12	13	1040	10	40	346	346.44
255	17	12	14	2000	20	0	347	347.83
256	17	12	18	1000	10	0	351	351.42
257	17	12	18	1010	10	10	351	351.42
258	17	12	19	930	9	30	352	352.40
259	17	12	27	220	2	20	360	360.10
260	17	12	27	650	6	50	360	360.28
261	17	12	28	500	5	0	361	361.21
262	17	12	29	545	5	45	362	362.24
263	17	12	30	700	7	0	363	363.29
264	18	1	1	10	0	10	365	365.01
265	18	1	1	230	2	30	365	365.10

266	18	1	1	800	8	0	365	365.33
267	18	1	2	10	0	10	366	366.01
268	18	1	3	1415	14	15	367	367.59
269	18	1	4	5	0	5	368	368.00
270	18	1	4	1730	17	30	368	368.73
271	18	1	4	2000	20	0	368	368.83
272	18	1	5	220	2	20	369	369.10
273	18	1	6	645	6	45	370	370.28
274	18	1	10	1600	16	0	374	374.67
275	18	1	12	900	9	0	376	376.38
276	18	1	14	1400	14	0	378	378.58
277	18	1	15	1155	11	55	379	379.50
278	18	1	16	210	2	10	380	380.09
279	18	1	18	200	2	0	382	382.08
280	18	1	18	445	4	45	382	382.20
281	18	1	21	1720	17	20	385	385.72
282	18	1	28	900	9	0	392	392.38
283	18	2	1	1200	12	0	396	396.50
284	18	2	9	2230	22	30	404	404.94
285	18	2	11	645	6	45	406	406.28
286	18	2	12	115	1	15	407	407.05
287	18	2	12	430	4	30	407	407.19
288	18	2	14	1815	18	15	409	409.76
289	18	2	15	1730	17	30	410	410.73
290	18	2	17	1210	12	10	412	412.51
291	18	2	18	705	7	5	413	413.30
292	18	2	18	925	9	25	413	413.39
293	18	2	19	1045	10	45	414	414.45
294	18	2	19	1330	13	30	414	414.56
295	18	2	23	230	2	30	418	418.10
296	18	2	24	1915	19	15	419	419.80
297	18	2	24	1930	19	30	419	419.81
298	18	2	25	125	1	25	420	420.06
299	18	2	25	830	8	30	420	420.35
300	18	2	27	1100	11	0	422	422.46
301	18	3	1	35	0	35	424	424.02
302	18	3	1	2200	22	0	424	424.92
303	18	3	4	200	2	0	427	427.08
304	18	3	4	315	3	15	427	427.14
305	18	3	5	100	1	0	428	428.04
306	18	3	5	1130	11	30	428	428.48
307	18	3	6	200	2	0	429	429.08
308	18	3	6	645	6	45	429	429.28
309	18	3	7	2030	20	30	430	430.85
310	18	3	9	630	6	30	432	432.27
311	18	3	10	2230	22	30	433	433.94

312	18	3	11	500	5	0	434	434.21
313	18	3	13	130	1	30	436	436.06
314	18	3	13	200	2	0	436	436.08
315	18	3	14	620	6	20	437	437.26
316	18	3	14	915	9	15	437	437.39
317	18	3	16	230	2	30	439	439.10
318	18	3	17	420	4	20	440	440.18
319	18	3	18	2030	20	30	441	441.85
320	18	3	21	1515	15	15	444	444.64
321	18	3	24	1045	10	45	447	447.45
322	18	3	28	700	7	0	451	451.29
323	18	3	29	1030	10	30	452	452.44
324	18	3	29	1320	13	20	452	452.56
325	18	3	30	2100	21	0	453	453.88
326	18	4	1	2200	22	0	455	455.92
327	18	4	3	1100	11	0	457	457.46
328	18	4	3	1630	16	30	457	457.69
329	18	4	5	1000	10	0	459	459.42
330	18	4	10	930	9	30	464	464.40
331	18	4	10	1115	11	15	464	464.47
332	18	4	11	830	8	30	465	465.35
333	18	4	12	430	4	30	466	466.19
334	18	4	12	755	7	55	466	466.33
335	18	4	13	110	1	10	467	467.05
336	18	4	14	710	7	10	468	468.30
337	18	4	14	2110	21	10	468	468.88
338	18	4	16	915	9	15	470	470.39
339	18	4	16	2100	21	0	470	470.88
340	18	4	18	20	0	20	472	472.01
341	18	4	18	240	2	40	472	472.11
342	18	4	18	1000	10	0	472	472.42
343	18	4	20	2100	21	0	474	474.88
344	18	4	21	1930	19	30	475	475.81
345	18	4	23	115	1	15	477	477.05
346	18	4	23	530	5	30	477	477.23
347	18	4	24	1300	13	0	478	478.54
348	18	4	24	2330	23	30	478	478.98
349	18	4	25	220	2	20	479	479.10
350	18	4	25	230	2	30	479	479.10
351	18	4	25	500	5	0	479	479.21
352	18	4	26	2100	21	0	480	480.88
353	18	4	27	950	9	50	481	481.41
354	18	4	27	1230	12	30	481	481.52
355	18	4	28	1450	14	50	482	482.62
356	18	4	29	1340	13	40	483	483.57
357	18	4	30	1200	12	0	484	484.50

358	18	5	1	1900	19	0	485	485.79
359	18	5	1	2115	21	15	485	485.89
360	18	5	4	400	4	0	488	488.17
361	18	5	6	1900	19	0	490	490.79
362	18	5	11	800	8	0	495	495.33
363	18	5	11	1630	16	30	495	495.69
364	18	5	11	2120	21	20	495	495.89
365	18	5	12	445	4	45	496	496.20
366	18	5	12	1000	10	0	496	496.42
367	18	5	14	2230	22	30	498	498.94
368	18	5	15	2115	21	15	499	499.89
369	18	5	16	1200	12	0	500	500.50
370	18	5	16	1930	19	30	500	500.81
371	18	5	17	1000	10	0	501	501.42
372	18	5	19	330	3	30	503	503.15
373	18	5	19	2030	20	30	503	503.85
374	18	5	22	930	9	30	506	506.40
375	18	5	22	2020	20	20	506	506.85
376	18	5	22	2145	21	45	506	506.91
377	18	5	23	1800	18	0	507	507.75
378	18	5	24	100	1	0	508	508.04
379	18	5	24	400	4	0	508	508.17
380	18	5	27	1430	14	30	511	511.60
381	18	5	28	430	4	30	512	512.19
382	18	5	29	120	1	20	513	513.06
383	18	5	29	445	4	45	513	513.20
384	18	6	3	910	9	10	518	518.38
385	18	6	4	1230	12	30	519	519.52
386	18	6	6	530	5	30	521	521.23
387	18	6	6	2110	21	10	521	521.88
388	18	6	7	1930	19	30	522	522.81
389	18	6	8	1015	10	15	523	523.43
390	18	6	10	1715	17	15	525	525.72
391	18	6	15	1230	12	30	530	530.52
392	18	6	16	245	2	45	531	531.11
393	18	6	18	1230	12	30	533	533.52
394	18	6	18	1500	15	0	533	533.63
395	18	6	21	450	4	50	536	536.20
396	18	6	21	2240	22	40	536	536.94
397	18	6	27	1845	18	45	542	542.78
398	18	6	28	200	2	0	543	543.08
399	18	6	29	230	2	30	544	544.10
400	18	7	1	930	9	30	546	546.40
401	18	7	1	2250	22	50	546	546.95
402	18	7	2	900	9	0	547	547.38
403	18	7	4	45	0	45	549	549.03

404	18	7	4	100	1	0	549	549.04
405	18	7	4	920	9	20	549	549.39
406	18	7	5	950	9	50	550	550.41
407	18	7	5	1815	18	15	550	550.76
408	18	7	6	1745	17	45	551	551.74
409	18	7	7	1810	18	10	552	552.76
410	18	7	8	325	3	25	553	553.14
411	18	7	8	1645	16	45	553	553.70
412	18	7	9	325	3	25	554	554.14
413	18	7	12	1100	11	0	557	557.46
414	18	7	14	630	6	30	559	559.27
415	18	7	14	1245	12	45	559	559.53
416	18	7	15	141	1	41	560	560.07
417	18	7	15	800	8	0	560	560.33
418	18	7	15	2045	20	45	560	560.86
419	18	7	16	600	6	0	561	561.25
420	18	7	16	1130	11	30	561	561.48
421	18	7	17	1230	12	30	562	562.52
422	18	7	18	1035	10	35	563	563.44
423	18	7	23	430	4	30	568	568.19
424	18	7	26	400	4	0	571	571.17
425	18	7	28	230	2	30	573	573.10
426	18	7	29	1230	12	30	574	574.52
427	18	8	1	730	7	30	577	577.31
428	18	8	1	1230	12	30	577	577.52
429	18	8	2	905	9	5	578	578.38
430	18	8	3	1215	12	15	579	579.51
431	18	8	9	645	6	45	585	585.28
432	18	8	11	100	1	0	587	587.04
433	18	8	11	300	3	0	587	587.13
434	18	8	12	1525	15	25	588	588.64
435	18	8	13	2230	22	30	589	589.94
436	18	8	15	2045	20	45	591	591.86
437	18	8	17	1045	10	45	593	593.45
438	18	8	17	1445	14	45	593	593.61
439	18	8	18	200	2	0	594	594.08
440	18	8	18	830	8	30	594	594.35
441	18	8	18	1205	12	5	594	594.50
442	18	8	19	1550	15	50	595	595.66
443	18	8	19	1800	18	0	595	595.75
444	18	8	25	2230	22	30	601	601.94
445	18	8	28	730	7	30	604	604.31
446	18	8	28	2025	20	25	604	604.85
447	18	9	2	210	2	10	609	609.09
448	18	9	2	500	5	0	609	609.21
449	18	9	2	1545	15	45	609	609.66

450	18	9	2	2210	22	10	609	609.92
451	18	9	4	645	6	45	611	611.28
452	18	9	4	2045	20	45	611	611.86
453	18	9	5	1945	19	45	612	612.82
454	18	9	5	2305	23	5	612	612.96
455	18	9	11	1930	19	30	618	618.81
456	18	9	14	1930	19	30	621	621.81
457	18	9	15	1030	10	30	622	622.44
458	18	9	16	2040	20	40	623	623.86
459	18	9	17	1600	16	0	624	624.67
460	18	9	17	2245	22	45	624	624.95
461	18	9	19	1830	18	30	626	626.77
462	18	9	20	2015	20	15	627	627.84
463	18	9	22	1030	10	30	629	629.44
464	18	9	24	1021	10	21	631	631.43
465	18	9	24	1100	11	0	631	631.46
466	18	9	27	845	8	45	634	634.36
467	18	9	28	1000	10	0	635	635.42
468	18	9	28	1750	17	50	635	635.74
469	18	10	4	1730	17	30	641	641.73
470	18	10	7	1630	16	30	644	644.69
471	18	10	12	1930	19	30	649	649.81
472	18	10	14	2205	22	5	651	651.92
473	18	10	15	1400	14	0	652	652.58
474	18	10	18	1630	16	30	655	655.69
475	18	10	19	735	7	35	656	656.32
476	18	10	22	1310	13	10	659	659.55
477	18	10	22	2200	22	0	659	659.92
478	18	10	22	2335	23	35	659	659.98
479	18	10	23	2230	22	30	660	660.94
480	18	10	26	1500	15	0	663	663.63
481	18	10	29	1630	16	30	666	666.69
482	18	10	30	400	4	0	667	667.17
483	18	10	31	600	6	0	668	668.25
484	18	11	1	1500	15	0	669	669.63
485	18	11	1	2130	21	30	669	669.90
486	18	11	2	645	6	45	670	670.28
487	18	11	2	1810	18	10	670	670.76
488	18	11	3	12	0	12	671	671.01
489	18	11	5	730	7	30	673	673.31
490	18	11	11	1100	11	0	679	679.46
491	18	11	15	630	6	30	683	683.27
492	18	11	16	1945	19	45	684	684.82
493	18	11	17	2000	20	0	685	685.83
494	18	11	20	400	4	0	688	688.17
495	18	11	20	2100	21	0	688	688.88

496	18	11	22	15	0	15	690	690.01
497	18	11	22	745	7	45	690	690.32
498	18	11	23	2240	22	40	691	691.94
499	18	11	26	730	7	30	694	694.31
500	18	11	27	1830	18	30	695	695.77
501	18	11	28	1330	13	30	696	696.56
502	18	11	29	2215	22	15	697	697.93
503	18	12	3	700	7	0	701	701.29
504	18	12	3	1800	18	0	701	701.75
505	18	12	4	500	5	0	702	702.21
506	18	12	4	1130	11	30	702	702.48
507	18	12	4	1300	13	0	702	702.54
508	18	12	8	930	9	30	706	706.40
509	18	12	8	1800	18	0	706	706.75
510	18	12	9	150	1	50	707	707.08
511	18	12	9	500	5	0	707	707.21
512	18	12	11	500	5	0	709	709.21
513	18	12	13	1115	11	15	711	711.47
514	18	12	16	540	5	40	714	714.24
515	18	12	17	430	4	30	715	715.19
516	18	12	19	1315	13	15	717	717.55
517	18	12	20	415	4	15	718	718.18
518	18	12	20	2020	20	20	718	718.85
519	18	12	23	1800	18	0	721	721.75
520	18	12	25	130	1	30	723	723.06
521	18	12	26	820	8	20	724	724.35
522	18	12	26	1700	17	0	724	724.71
523	18	12	29	1230	12	30	727	727.52
524	18	12	29	1930	19	30	727	727.81
525	19	1	1	1400	14	0	730	730.58
526	19	1	5	1300	13	0	734	734.54
527	19	1	5	2130	21	30	734	734.90
528	19	1	6	210	2	10	735	735.09
529	19	1	8	845	8	45	737	737.36
530	19	1	9	315	3	15	738	738.14
531	19	1	9	1210	12	10	738	738.51
532	19	1	13	1445	14	45	742	742.61
533	19	1	14	1210	12	10	743	743.51
534	19	1	17	755	7	55	746	746.33
535	19	1	18	2115	21	15	747	747.89
536	19	1	21	430	4	30	750	750.19
537	19	1	21	1745	17	45	750	750.74
538	19	1	23	2250	22	50	752	752.95
539	19	1	24	1715	17	15	753	753.72
540	19	1	26	1915	19	15	755	755.80
541	19	1	26	2253	22	53	755	755.95

542	19	1	27	1215	12	15	756	756.51
543	19	1	28	45	0	45	757	757.03
544	19	1	28	215	2	15	757	757.09
545	19	1	29	2330	23	30	758	758.98
546	19	1	30	2215	22	15	759	759.93
547	19	1	31	500	5	0	760	760.21
548	19	2	1	2200	22	0	761	761.92
549	19	2	3	1200	12	0	763	763.50
550	19	2	5	330	3	30	765	765.15
551	19	2	5	750	7	50	765	765.33
552	19	2	5	1640	16	40	765	765.69
553	19	2	7	1130	11	30	767	767.48
554	19	2	8	57	0	57	768	768.04
555	19	2	9	55	0	55	769	769.04
556	19	2	10	2140	21	40	770	770.90
557	19	2	11	1410	14	10	771	771.59
558	19	2	12	1840	18	40	772	772.78
559	19	2	13	530	5	30	773	773.23
560	19	2	13	2230	22	30	773	773.94
561	19	2	14	112	1	12	774	774.05
562	19	2	21	1125	11	25	781	781.48
563	19	2	22	515	5	15	782	782.22
564	19	2	23	1315	13	15	783	783.55
565	19	2	27	1840	18	40	787	787.78
566	19	3	2	2230	22	30	790	790.94
567	19	3	6	430	4	30	794	794.19
568	19	3	8	1430	14	30	796	796.60
569	19	3	9	630	6	30	797	797.27
570	19	3	11	130	1	30	799	799.06
571	19	3	11	330	3	30	799	799.15
572	19	3	11	1530	15	30	799	799.65
573	19	3	12	600	6	0	800	800.25
574	19	3	14	1930	19	30	802	802.81
575	19	3	15	1030	10	30	803	803.44
576	19	3	16	600	6	0	804	804.25
577	19	3	17	1945	19	45	805	805.82
578	19	3	19	720	7	20	807	807.31
579	19	3	20	2115	21	15	808	808.89
580	19	3	22	730	7	30	810	810.31
581	19	3	22	930	9	30	810	810.40
582	19	3	22	1900	19	0	810	810.79
583	19	3	22	2200	22	0	810	810.92
584	19	3	23	1230	12	30	811	811.52
585	19	3	24	1915	19	15	812	812.80
586	19	3	25	1200	12	0	813	813.50
587	19	3	27	2100	21	0	815	815.88

588	19	3	29	130	1	30	817	817.06
589	19	3	29	1100	11	0	817	817.46
590	19	3	30	1130	11	30	818	818.48
591	19	4	1	1640	16	40	820	820.69
592	19	4	3	1330	13	30	822	822.56
593	19	4	4	10	0	10	823	823.01
594	19	4	5	1140	11	40	824	824.49
595	19	4	5	1830	18	30	824	824.77
596	19	4	6	1115	11	15	825	825.47
597	19	4	7	600	6	0	826	826.25
598	19	4	7	800	8	0	826	826.33
599	19	4	8	1130	11	30	827	827.48
600	19	4	10	410	4	10	829	829.17
601	19	4	10	1050	10	50	829	829.45
602	19	4	16	545	5	45	835	835.24
603	19	4	19	515	5	15	838	838.22
604	19	4	19	1800	18	0	838	838.75
605	19	4	21	800	8	0	840	840.33
606	19	4	21	2115	21	15	840	840.89
607	19	4	23	700	7	0	842	842.29
608	19	4	23	720	7	20	842	842.31
609	19	4	25	910	9	10	844	844.38
610	19	4	25	1045	10	45	844	844.45
611	19	4	27	1930	19	30	846	846.81
612	19	4	28	1200	12	0	847	847.50
613	19	4	28	1750	17	50	847	847.74
614	19	4	30	530	5	30	849	849.23
615	19	5	1	230	2	30	850	850.10
616	19	5	1	1930	19	30	850	850.81
617	19	5	1	2245	22	45	850	850.95
618	19	5	4	1910	19	10	853	853.80
619	19	5	5	30	0	30	854	854.02
620	19	5	5	645	6	45	854	854.28
621	19	5	5	800	8	0	854	854.33
622	19	5	5	1030	10	30	854	854.44
623	19	5	6	815	8	15	855	855.34
624	19	5	6	835	8	35	855	855.36
625	19	5	7	1745	17	45	856	856.74
626	19	5	8	803	8	3	857	857.34
627	19	5	8	830	8	30	857	857.35
628	19	5	10	2240	22	40	859	859.94
629	19	5	11	1835	18	35	860	860.77
630	19	5	17	1930	19	30	866	866.81
631	19	5	19	1100	11	0	868	868.46
632	19	5	20	515	5	15	869	869.22
633	19	5	20	1730	17	30	869	869.73

634	19	5	22	230	2	30	871	871.10
635	19	5	22	1730	17	30	871	871.73
636	19	5	23	1000	10	0	872	872.42
637	19	5	24	1500	15	0	873	873.63
638	19	5	25	1630	16	30	874	874.69
639	19	5	27	1030	10	30	876	876.44
640	19	5	27	1210	12	10	876	876.51
641	19	5	28	1000	10	0	877	877.42
642	19	5	30	500	5	0	879	879.21
643	19	5	30	1030	10	30	879	879.44
644	19	5	30	1930	19	30	879	879.81
645	19	6	1	800	8	0	881	881.33
646	19	6	4	620	6	20	884	884.26
647	19	6	5	400	4	0	885	885.17
648	19	6	5	800	8	0	885	885.33
649	19	6	6	1245	12	45	886	886.53
650	19	6	11	230	2	30	891	891.10
651	19	6	11	630	6	30	891	891.27
652	19	6	13	1630	16	30	893	893.69
653	19	6	17	930	9	30	897	897.40
654	19	6	20	1500	15	0	900	900.63
655	19	6	22	1215	12	15	902	902.51
656	19	6	25	935	9	35	905	905.40
657	19	6	25	1650	16	50	905	905.70
658	19	6	25	2335	23	35	905	905.98
659	19	6	26	715	7	15	906	906.30
660	19	6	26	1430	14	30	906	906.60
661	19	6	27	1150	11	50	907	907.49
662	19	6	28	1130	11	30	908	908.48
663	19	6	29	2030	20	30	909	909.85
664	19	7	2	600	6	0	912	912.25
665	19	7	3	2340	23	40	913	913.99
666	19	7	4	1030	10	30	914	914.44
667	19	7	4	1630	16	30	914	914.69
668	19	7	6	205	2	5	916	916.09
669	19	7	6	1040	10	40	916	916.44
670	19	7	7	1130	11	30	917	917.48
671	19	7	7	2000	20	0	917	917.83
672	19	7	8	2130	21	30	918	918.90
673	19	7	10	1600	16	0	920	920.67
674	19	7	13	615	6	15	923	923.26
675	19	7	15	510	5	10	925	925.22
676	19	7	15	2320	23	20	925	925.97
677	19	7	16	1330	13	30	926	926.56
678	19	7	16	2130	21	30	926	926.90
679	19	7	17	1900	19	0	927	927.79

680	19	7	18	845	8	45	928	928.36
681	19	7	19	300	3	0	929	929.13
682	19	7	21	630	6	30	931	931.27
683	19	7	22	1235	12	35	932	932.52
684	19	7	23	630	6	30	933	933.27
685	19	7	24	115	1	15	934	934.05
686	19	7	24	615	6	15	934	934.26
687	19	7	24	900	9	0	934	934.38
688	19	7	24	1600	16	0	934	934.67
689	19	7	30	905	9	5	940	940.38
690	19	8	1	335	3	35	942	942.15
691	19	8	1	930	9	30	942	942.40
692	19	8	1	2040	20	40	942	942.86
693	19	8	2	2200	22	0	943	943.92
694	19	8	3	830	8	30	944	944.35
695	19	8	4	1630	16	30	945	945.69
696	19	8	7	1710	17	10	948	948.72
697	19	8	9	1430	14	30	950	950.60
698	19	8	9	1500	15	0	950	950.63
699	19	8	9	1730	17	30	950	950.73
700	19	8	9	1900	19	0	950	950.79
701	19	8	12	900	9	0	953	953.38
702	19	8	15	1630	16	30	956	956.69
703	19	8	17	950	9	50	958	958.41
704	19	8	18	20	0	20	959	959.01
705	19	8	19	2140	21	40	960	960.90
706	19	8	21	123	1	23	962	962.06
707	19	8	22	1	0	1	963	963.00
708	19	8	23	1030	10	30	964	964.44
709	19	8	24	1645	16	45	965	965.70
710	19	8	27	1130	11	30	968	968.48
711	19	8	27	1345	13	45	968	968.57
712	19	8	28	1000	10	0	969	969.42
713	19	8	29	1230	12	30	970	970.52
714	19	9	1	530	5	30	973	973.23
715	19	9	1	1730	17	30	973	973.73
716	19	9	1	1915	19	15	973	973.80
717	19	9	2	2130	21	30	974	974.90
718	19	9	4	1925	19	25	976	976.81
719	19	9	5	900	9	0	977	977.38
720	19	9	5	1100	11	0	977	977.46
721	19	9	6	2255	22	55	978	978.95
722	19	9	7	2145	21	45	979	979.91
723	19	9	9	430	4	30	981	981.19
724	19	9	10	520	5	20	982	982.22
725	19	9	12	415	4	15	984	984.18

726	19	9	15	1130	11	30	987	987.48
727	19	9	16	830	8	30	988	988.35
728	19	9	19	715	7	15	991	991.30
729	19	9	19	1515	15	15	991	991.64
730	19	9	19	1900	19	0	991	991.79
731	19	9	20	430	4	30	992	992.19
732	19	9	21	2250	22	50	993	993.95
733	19	9	22	740	7	40	994	994.32
734	19	9	23	2110	21	10	995	995.88
735	19	9	24	630	6	30	996	996.27
736	19	9	24	1245	12	45	996	996.53
737	19	9	24	1630	16	30	996	996.69
738	19	9	24	2050	20	50	996	996.87
739	19	9	25	945	9	45	997	997.41
740	19	9	27	1245	12	45	999	999.53
741	19	9	27	1930	19	30	999	999.81
742	19	9	30	1045	10	45	1002	1002.45
743	19	9	30	1430	14	30	1002	1002.60
744	19	9	30	1715	17	15	1002	1002.72
745	19	10	1	820	8	20	1003	1003.35
746	19	10	2	1152	11	52	1004	1004.49
747	19	10	2	2035	20	35	1004	1004.86
748	19	10	3	1700	17	0	1005	1005.71
749	19	10	4	1035	10	35	1006	1006.44
750	19	10	4	1145	11	45	1006	1006.49
751	19	10	5	830	8	30	1007	1007.35
752	19	10	6	1035	10	35	1008	1008.44
753	19	10	6	2100	21	0	1008	1008.88
754	19	10	7	445	4	45	1009	1009.20
755	19	10	11	1330	13	30	1013	1013.56
756	19	10	11	1930	19	30	1013	1013.81
757	19	10	13	605	6	5	1015	1015.25
758	19	10	13	850	8	50	1015	1015.37
759	19	10	13	1900	19	0	1015	1015.79
760	19	10	16	1130	11	30	1018	1018.48
761	19	10	17	1145	11	45	1019	1019.49
762	19	10	18	1040	10	40	1020	1020.44
763	19	10	19	1030	10	30	1021	1021.44
764	19	10	19	1645	16	45	1021	1021.70
765	19	10	24	125	1	25	1026	1026.06
766	19	10	27	200	2	0	1029	1029.08
767	19	10	28	330	3	30	1030	1030.15
768	19	10	29	415	4	15	1031	1031.18
769	19	10	30	1945	19	45	1032	1032.82
770	19	11	2	1330	13	30	1035	1035.56
771	19	11	3	1845	18	45	1036	1036.78

772	19	11	4	1100	11	0	1037	1037.46
773	19	11	4	1115	11	15	1037	1037.47
774	19	11	4	1845	18	45	1037	1037.78
775	19	11	6	835	8	35	1039	1039.36
776	19	11	6	1830	18	30	1039	1039.77
777	19	11	7	1745	17	45	1040	1040.74
778	19	11	8	350	3	50	1041	1041.16
779	19	11	8	1000	10	0	1041	1041.42
780	19	11	12	850	8	50	1045	1045.37
781	19	11	13	810	8	10	1046	1046.34
782	19	11	13	1240	12	40	1046	1046.53
783	19	11	13	1450	14	50	1046	1046.62
784	19	11	16	830	8	30	1049	1049.35
785	19	11	17	1500	15	0	1050	1050.63
786	19	11	18	700	7	0	1051	1051.29
787	19	11	21	900	9	0	1054	1054.38
788	19	11	21	1130	11	30	1054	1054.48
789	19	11	23	330	3	30	1056	1056.15
790	19	11	23	2015	20	15	1056	1056.84
791	19	11	25	1815	18	15	1058	1058.76
792	19	11	26	1345	13	45	1059	1059.57
793	19	11	27	210	2	10	1060	1060.09
794	19	11	28	2230	22	30	1061	1061.94
795	19	11	29	20	0	20	1062	1062.01
796	19	11	29	1810	18	10	1062	1062.76
797	19	11	29	2200	22	0	1062	1062.92
798	19	12	2	730	7	30	1065	1065.31
799	19	12	2	1010	10	10	1065	1065.42
800	19	12	2	2030	20	30	1065	1065.85
801	19	12	4	845	8	45	1067	1067.36
802	19	12	5	500	5	0	1068	1068.21
803	19	12	6	145	1	45	1069	1069.07
804	19	12	7	555	5	55	1070	1070.25
805	19	12	9	1915	19	15	1072	1072.80
806	19	12	12	1530	15	30	1075	1075.65
807	19	12	13	605	6	5	1076	1076.25
808	19	12	13	1630	16	30	1076	1076.69
809	19	12	15	1100	11	0	1078	1078.46
810	19	12	15	1450	14	50	1078	1078.62
811	19	12	15	1800	18	0	1078	1078.75
812	19	12	17	25	0	25	1080	1080.02
813	19	12	18	230	2	30	1081	1081.10
814	19	12	19	530	5	30	1082	1082.23
815	19	12	20	5	0	5	1083	1083.00
816	19	12	21	2200	22	0	1084	1084.92
817	19	12	22	1930	19	30	1085	1085.81

818	19	12	24	1115	11	15	1087	1087.47
819	19	12	24	1215	12	15	1087	1087.51
820	19	12	25	1255	12	55	1088	1088.54
821	19	12	26	2210	22	10	1089	1089.92
822	19	12	28	2155	21	55	1091	1091.91
823	19	12	30	2000	20	0	1093	1093.83
824	19	12	31	745	7	45	1094	1094.32
825	19	12	31	1115	11	15	1094	1094.47
826	20	1	1	1	0	1	1095	1095.00
827	20	1	1	5	0	5	1095	1095.00
828	20	1	1	930	9	30	1095	1095.40
829	20	1	2	420	4	20	1096	1096.18
830	20	1	2	1330	13	30	1096	1096.56
831	20	1	2	1725	17	25	1096	1096.73
832	20	1	3	730	7	30	1097	1097.31
833	20	1	3	2230	22	30	1097	1097.94
834	20	1	5	830	8	30	1099	1099.35
835	20	1	5	1330	13	30	1099	1099.56
836	20	1	8	3	0	3	1102	1102.00
837	20	1	9	1030	10	30	1103	1103.44
838	20	1	9	1945	19	45	1103	1103.82
839	20	1	10	1655	16	55	1104	1104.70
840	20	1	12	1815	18	15	1106	1106.76
841	20	1	13	1100	11	0	1107	1107.46
842	20	1	13	1445	14	45	1107	1107.61
843	20	1	14	1200	12	0	1108	1108.50
844	20	1	14	1230	12	30	1108	1108.52
845	20	1	14	1700	17	0	1108	1108.71
846	20	1	16	2240	22	40	1110	1110.94
847	20	1	17	1340	13	40	1111	1111.57
848	20	1	17	2030	20	30	1111	1111.85
849	20	1	18	910	9	10	1112	1112.38
850	20	1	20	550	5	50	1114	1114.24
851	20	1	20	1130	11	30	1114	1114.48
852	20	1	20	1830	18	30	1114	1114.77
853	20	1	20	2045	20	45	1114	1114.86
854	20	1	21	2200	22	0	1115	1115.92
855	20	1	23	235	2	35	1117	1117.11
856	20	1	23	1500	15	0	1117	1117.63
857	20	1	24	1	0	1	1118	1118.00
858	20	1	24	830	8	30	1118	1118.35
859	20	1	26	3	0	3	1120	1120.00
860	20	1	27	1235	12	35	1121	1121.52
861	20	1	28	1130	11	30	1122	1122.48
862	20	1	28	1200	12	0	1122	1122.50
863	20	1	28	1900	19	0	1122	1122.79

864	20	1	29	305	3	5	1123	1123.13
865	20	1	29	1130	11	30	1123	1123.48
866	20	1	29	1500	15	0	1123	1123.63
867	20	2	1	300	3	0	1126	1126.13
868	20	2	4	730	7	30	1129	1129.31
869	20	2	4	950	9	50	1129	1129.41
870	20	2	5	930	9	30	1130	1130.40
871	20	2	8	1020	10	20	1133	1133.43
872	20	2	10	845	8	45	1135	1135.36
873	20	2	10	1535	15	35	1135	1135.65
874	20	2	13	1200	12	0	1138	1138.50
875	20	2	14	225	2	25	1139	1139.10
876	20	2	15	1330	13	30	1140	1140.56
877	20	2	15	1930	19	30	1140	1140.81
878	20	2	17	1155	11	55	1142	1142.50
879	20	2	18	145	1	45	1143	1143.07
880	20	2	19	2100	21	0	1144	1144.88
881	20	2	21	1600	16	0	1146	1146.67
882	20	2	22	25	0	25	1147	1147.02
883	20	2	23	1145	11	45	1148	1148.49
884	20	2	23	1230	12	30	1148	1148.52
885	20	2	24	1930	19	30	1149	1149.81
886	20	2	26	15	0	15	1151	1151.01
887	20	2	26	1700	17	0	1151	1151.71
888	20	2	26	1930	19	30	1151	1151.81
889	20	2	28	1000	10	0	1153	1153.42
890	20	3	3	830	8	30	1157	1157.35
891	20	3	7	730	7	30	1161	1161.31
892	20	3	8	730	7	30	1162	1162.31
893	20	3	9	1130	11	30	1163	1163.48
894	20	3	9	1200	12	0	1163	1163.50
895	20	3	11	5	0	5	1165	1165.00
896	20	3	12	630	6	30	1166	1166.27
897	20	3	13	320	3	20	1167	1167.14
898	20	3	18	1700	17	0	1172	1172.71
899	20	3	18	2025	20	25	1172	1172.85
900	20	3	20	630	6	30	1174	1174.27
901	20	3	21	2120	21	20	1175	1175.89
902	20	3	22	905	9	5	1176	1176.38
903	20	3	23	310	3	10	1177	1177.13
904	20	3	24	1850	18	50	1178	1178.78
905	20	3	26	445	4	45	1180	1180.20
906	20	3	31	1045	10	45	1185	1185.45
907	20	4	7	730	7	30	1192	1192.31
908	20	4	8	1630	16	30	1193	1193.69
909	20	4	11	600	6	0	1196	1196.25

910	20	4	14	1215	12	15	1199	1199.51
911	20	4	16	1100	11	0	1201	1201.46
912	20	4	21	1145	11	45	1206	1206.49
913	20	4	23	1230	12	30	1208	1208.52
914	20	5	2	2020	20	20	1217	1217.85
915	20	5	3	930	9	30	1218	1218.40
916	20	5	4	1700	17	0	1219	1219.71
917	20	5	5	2045	20	45	1220	1220.86
918	20	5	12	115	1	15	1227	1227.05
919	20	5	15	1300	13	0	1230	1230.54
920	20	5	16	2230	22	30	1231	1231.94
921	20	5	20	1020	10	20	1235	1235.43
922	20	5	22	615	6	15	1237	1237.26
923	20	5	24	2130	21	30	1239	1239.90
924	20	5	28	1100	11	0	1243	1243.46
925	20	5	29	1250	12	50	1244	1244.53
926	20	5	31	2100	21	0	1246	1246.88
927	20	6	4	1135	11	35	1250	1250.48
928	20	6	4	2145	21	45	1250	1250.91
929	20	6	9	130	1	30	1255	1255.06
930	20	6	12	1815	18	15	1258	1258.76
931	20	6	24	30	0	30	1270	1270.02
932	20	6	25	115	1	15	1271	1271.05
933	20	6	25	1200	12	0	1271	1271.50
934	20	7	2	10	0	10	1278	1278.01
935	20	7	7	805	8	5	1283	1283.34
936	20	7	8	1230	12	30	1284	1284.52
937	20	7	8	1410	14	10	1284	1284.59
938	20	7	11	2200	22	0	1287	1287.92
939	20	7	12	1130	11	30	1288	1288.48
940	20	7	14	100	1	0	1290	1290.04
941	20	7	14	1115	11	15	1290	1290.47
942	20	7	14	2015	20	15	1290	1290.84
943	20	7	17	630	6	30	1293	1293.27
944	20	7	21	915	9	15	1297	1297.39
945	20	7	23	2250	22	50	1299	1299.95
946	20	7	28	200	2	0	1304	1304.08
947	20	7	30	2145	21	45	1306	1306.91
948	20	7	31	330	3	30	1307	1307.15
949	20	7	31	1500	15	0	1307	1307.63
950	20	7	31	2330	23	30	1307	1307.98
951	20	8	1	1200	12	0	1308	1308.50
952	20	8	2	15	0	15	1309	1309.01
953	20	8	2	200	2	0	1309	1309.08
954	20	8	4	1700	17	0	1311	1311.71
955	20	8	6	1230	12	30	1313	1313.52

956	20	8	6	1420	14	20	1313	1313.60
957	20	8	7	850	8	50	1314	1314.37
958	20	8	8	915	9	15	1315	1315.39
959	20	8	10	600	6	0	1317	1317.25
960	20	8	11	1800	18	0	1318	1318.75
961	20	8	12	1120	11	20	1319	1319.47
962	20	8	14	1030	10	30	1321	1321.44
963	20	8	14	1215	12	15	1321	1321.51
964	20	8	14	1430	14	30	1321	1321.60
965	20	8	16	1	0	1	1323	1323.00
966	20	8	16	2010	20	10	1323	1323.84
967	20	8	21	1230	12	30	1328	1328.52
968	20	8	22	2230	22	30	1329	1329.94
969	20	8	23	1900	19	0	1330	1330.79
970	20	8	24	315	3	15	1331	1331.14
971	20	8	24	1800	18	0	1331	1331.75
972	20	8	25	2330	23	30	1332	1332.98
973	20	8	27	1930	19	30	1334	1334.81
974	20	8	28	140	1	40	1335	1335.07
975	20	8	29	1300	13	0	1336	1336.54
976	20	8	30	1030	10	30	1337	1337.44
977	20	8	30	1630	16	30	1337	1337.69
978	20	9	1	1140	11	40	1339	1339.49
979	20	9	3	400	4	0	1341	1341.17
980	20	9	5	200	2	0	1343	1343.08
981	20	9	7	2245	22	45	1345	1345.95
982	20	9	11	600	6	0	1349	1349.25
983	20	9	12	1200	12	0	1350	1350.50
984	20	9	15	830	8	30	1353	1353.35
985	20	9	15	1900	19	0	1353	1353.79
986	20	9	16	2215	22	15	1354	1354.93
987	20	9	17	1000	10	0	1355	1355.42
988	20	9	20	2300	23	0	1358	1358.96
989	20	9	21	720	7	20	1359	1359.31
990	20	9	22	230	2	30	1360	1360.10
991	20	9	30	1830	18	30	1368	1368.77

Appendix B

Negative log-likelihood function written in R.

```
function(params, data) {
  mu = params[1]
  alpha = params[2]
  beta = params[3]
  tau = params[4]
  t = sort(data)
  n = length(t)
  sum_j1 = 0
  for (j in 1:n) {
    sum_i1 = 0
    for (i in 1:(j-1)) {
      if (t[j]-t[i]<tau) {
        sum_i1 = sum_i1 + alpha
      }
      if (t[j]-t[i]>=tau) {
        sum_i1 = sum_i1 + alpha*exp(-beta*(t[j]-t[i]-tau))
      }
    }
    sum_j1 = sum_j1 + log(mu + sum_i1)
  }
  sum_j2 = 0
  for (j in 1:(n-1)) {
    sum_i2 =0
    for (i in 1:j) {
      if (t[j+1]-t[i]<tau) {
        sum_i2 = sum_i2 + (t[j+1]-t[j])
      }
      if (t[j]-t[i]>=tau) {
        sum_i2 = sum_i2 + (-1/beta)*(exp(-beta*(t[j+1]-t[j]-
tau))-exp(-beta*(t[j]-t[i]-tau)))
      }
      if (t[j]-t[i]<tau && tau<t[j+1]-t[i]) {
        sum_i2 = sum_i2 + (tau-t[j]-t[i]-(1/beta)*(exp(-
beta*(t[j+1]-t[i]-tau))-1))
      }
    }
    sum_j2 = sum_j2 + sum_i2
  }
  loglik = sum_j1 - mu*t[n] - alpha*sum_j2
  return(-loglik)
}
```

Appendix C

Function generate_mle_table() written in R.

```
function(mu_cap, alpha_cap, beta_cap) {
  mle_table = data.frame(0, 0, 0, 0, 0, 0, 0, 0, 0)
  # generating the table
  names(mle_table) = c('mu_in','alpha_in', 'beta_in',
'tau_in', 'mu_opt', 'tau_opt', 'beta_opt', 'tau_opt',
'neg_loglik')
  for (mu in 0.1*(1:10)) {
    for (tau in 1:10) {
      params = c(mu, runif(1, 0, 1), runif(1, 0, 2), tau)
      opt = optim(par = params, fn = neg_loglik, data =
timestamps, method = "L-BFGS-B", lower = 1e-10, upper =
26.271)
      mle_table[nrow(mle_table)+1,] = c(params, opt$par,
opt$value)
    }
  }
  write_xlsx(mle_table, "mle_table.xlsx")
  # library(writexl) is required
}
```

Appendix D

Table D-1. Table containing the MLE of parameters generated by the `generate_mle_table` function.

mu_in	alpha_in	beta_in	tau_in	mu_opt	alpha_opt	beta_opt	tau_opt	neg_loglik
0.10	0.50	0.44	1.00	1.00E-10	26.27	1.00E-10	1.00E-10	-5.80E+09
0.10	0.92	1.31	2.00	1.00E-10	26.27	26.27	26.27	-5.81E+303
0.10	0.21	1.63	3.00	1.00E-10	26.27	26.27	26.27	-5.81E+303
0.10	0.78	1.32	4.00	1.00E-10	26.27	26.27	26.27	-5.81E+303
0.10	0.23	0.27	5.00	1.00E-10	26.27	26.27	26.27	-5.81E+303
0.10	0.73	0.97	6.00	1.00E-10	26.27	26.27	26.27	-5.81E+303
0.10	0.71	1.52	7.00	1.00E-10	26.27	26.27	26.27	-5.81E+303
0.10	0.07	1.21	8.00	26.27	26.27	26.27	26.27	-5.81E+303
0.10	0.11	1.65	9.00	1.00E-10	26.27	26.27	26.27	-5.81E+303
0.10	0.29	1.66	10.00	0.10	26.27	26.27	26.27	-5.81E+303
0.20	0.02	0.42	1.00	1.00E-10	26.27	1.00E-10	1.00E-10	-5.80E+09
0.20	0.31	0.22	2.00	1.00E-10	26.27	1.00E-10	1.00E-10	-5.80E+09
0.20	0.25	1.05	3.00	1.00E-10	26.27	26.27	26.27	-5.81E+303
0.20	0.10	0.93	4.00	26.27	26.27	26.27	26.27	-5.81E+303
0.20	0.82	1.95	5.00	1.00E-10	26.27	26.27	26.27	-5.81E+303
0.20	0.73	1.28	6.00	1.00E-10	26.27	26.27	26.27	-5.81E+303
0.20	0.61	0.51	7.00	1.00E-10	26.27	26.27	26.27	-5.81E+303
0.20	0.17	1.80	8.00	0.20	26.27	26.27	26.27	-5.81E+303
0.20	0.85	0.92	9.00	1.00E-10	26.27	26.27	26.27	-5.81E+303
0.20	0.69	0.30	10.00	1.00E-10	26.27	26.27	26.27	-5.81E+303
0.30	0.37	0.04	1.00	1.00E-10	26.27	1.00E-10	1.00E-10	-5.80E+09
0.30	0.34	1.85	2.00	1.00E-10	26.27	26.27	26.27	-5.81E+303
0.30	0.56	1.53	3.00	1.00E-10	26.27	26.27	26.27	-5.81E+303
0.30	0.13	1.63	4.00	1.00E-10	26.27	26.27	26.27	-5.81E+303
0.30	0.88	0.08	5.00	1.00E-10	26.27	1.00E-10	1.00E-10	-5.80E+09
0.30	0.77	0.40	6.00	1.00E-10	26.27	26.27	26.27	-5.81E+303
0.30	0.29	0.01	7.00	1.00E-10	26.27	1.00E-10	1.00E-10	-5.80E+09
0.30	0.75	1.11	8.00	1.00E-10	26.27	26.27	26.27	-5.81E+303
0.30	0.95	1.30	9.00	1.00E-10	26.27	26.27	26.27	-5.81E+303
0.30	0.23	1.92	10.00	0.30	26.27	26.27	26.27	-5.81E+303
0.40	0.38	0.87	1.00	1.00E-10	26.27	1.00E-10	1.00E-10	-5.80E+09
0.40	0.50	1.16	2.00	1.00E-10	26.27	26.27	26.27	-5.81E+303
0.40	0.16	0.81	3.00	1.00E-10	26.27	26.27	26.27	-5.81E+303
0.40	0.44	1.19	4.00	1.00E-10	26.27	26.27	26.27	-5.81E+303
0.40	0.34	0.24	5.00	1.00E-10	26.27	1.00E-10	1.00E-10	-5.80E+09
0.40	0.73	0.82	6.00	1.00E-10	26.27	26.27	26.27	-5.81E+303
0.40	0.38	0.20	7.00	1.00E-10	26.27	26.27	26.27	-5.81E+303
0.40	0.19	0.53	8.00	1.00E-10	26.27	26.27	26.27	-5.81E+303
0.40	0.06	1.55	9.00	1.00E-10	26.27	26.27	26.27	-5.81E+303
0.40	0.01	0.21	10.00	26.27	26.27	26.27	26.27	-5.81E+303
0.50	0.91	0.33	1.00	1.00E-10	26.27	1.00E-10	1.00E-10	-5.80E+09

0.50	0.95	0.10	2.00	1.00E-10	26.27	1.00E-10	1.00E-10	-5.80E+09
0.50	0.53	0.01	3.00	1.00E-10	26.27	1.00E-10	1.00E-10	-5.80E+09
0.50	0.44	0.45	4.00	1.00E-10	26.27	26.27	26.27	-5.81E+303
0.50	0.06	0.55	5.00	1.00E-10	26.27	26.27	26.27	-5.81E+303
0.50	0.51	0.89	6.00	1.00E-10	26.27	26.27	26.27	-5.81E+303
0.50	0.50	0.84	7.00	1.00E-10	26.27	26.27	26.27	-5.81E+303
0.50	0.50	0.79	8.00	1.00E-10	26.27	26.27	26.27	-5.81E+303
0.50	0.64	0.21	9.00	1.00E-10	26.27	26.27	26.27	-5.81E+303
0.50	0.42	0.75	10.00	1.00E-10	26.27	26.27	26.27	-5.81E+303
0.60	0.88	0.75	1.00	1.00E-10	26.27	1.00E-10	1.00E-10	-5.80E+09
0.60	0.15	1.08	2.00	1.00E-10	26.27	26.27	26.27	-5.81E+303
0.60	0.33	1.06	3.00	1.00E-10	26.27	26.27	26.27	-5.81E+303
0.60	0.17	0.18	4.00	1.00E-10	26.27	1.00E-10	1.00E-10	-5.80E+09
0.60	0.25	1.36	5.00	1.00E-10	26.27	26.27	26.27	-5.81E+303
0.60	0.23	0.63	6.00	1.00E-10	26.27	26.27	26.27	-5.81E+303
0.60	0.89	1.18	7.00	1.00E-10	26.27	26.27	26.27	-5.81E+303
0.60	0.60	0.20	8.00	1.00E-10	26.27	26.27	26.27	-5.81E+303
0.60	0.97	1.09	9.00	1.00E-10	26.27	26.27	26.27	-5.81E+303
0.60	0.33	1.28	10.00	1.00E-10	26.27	26.27	26.27	-5.81E+303
0.70	0.97	1.11	1.00	1.00E-10	26.27	1.00E-10	1.00E-10	-5.80E+09
0.70	0.66	1.80	2.00	1.00E-10	26.27	26.27	26.27	-5.81E+303
0.70	0.02	0.99	3.00	1.00E-10	26.27	26.27	26.27	-5.81E+303
0.70	0.83	0.71	4.00	1.00E-10	26.27	26.27	26.27	-5.81E+303
0.70	0.25	0.64	5.00	1.00E-10	26.27	26.27	26.27	-5.81E+303
0.70	0.01	0.17	6.00	1.00E-10	26.27	1.00E-10	1.00E-10	-5.80E+09
0.70	0.36	1.99	7.00	1.00E-10	26.27	26.27	26.27	-5.81E+303
0.70	0.56	0.95	8.00	1.00E-10	26.27	26.27	26.27	-5.81E+303
0.70	0.14	0.95	9.00	1.00E-10	26.27	26.27	26.27	-5.81E+303
0.70	0.85	0.75	10.00	1.00E-10	26.27	26.27	26.27	-5.81E+303
0.80	0.22	1.39	1.00	1.00E-10	26.27	1.00E-10	1.00E-10	-5.80E+09
0.80	0.98	0.60	2.00	1.00E-10	26.27	1.00E-10	1.00E-10	-5.80E+09
0.80	0.68	0.11	3.00	1.00E-10	26.27	1.00E-10	1.00E-10	-5.80E+09
0.80	0.70	1.49	4.00	1.00E-10	26.27	26.27	26.27	-5.81E+303
0.80	0.15	0.40	5.00	1.00E-10	26.27	26.27	26.27	-5.81E+303
0.80	0.67	1.35	6.00	1.00E-10	26.27	26.27	26.27	-5.81E+303
0.80	0.75	0.10	7.00	1.00E-10	26.27	1.00E-10	1.00E-10	-5.80E+09
0.80	0.19	0.45	8.00	1.00E-10	26.27	26.27	26.27	-5.81E+303
0.80	0.58	1.89	9.00	1.00E-10	26.27	26.27	26.27	-5.81E+303
0.80	0.36	0.02	10.00	1.00E-10	26.27	1.00E-10	1.00E-10	-5.80E+09
0.90	0.72	0.48	1.00	1.00E-10	26.27	1.00E-10	1.00E-10	-5.80E+09
0.90	0.87	1.84	2.00	1.00E-10	26.27	26.27	26.27	-5.81E+303
0.90	0.22	1.34	3.00	1.00E-10	26.27	26.27	26.27	-5.81E+303
0.90	0.89	0.63	4.00	1.00E-10	26.27	26.27	26.27	-5.81E+303
0.90	0.14	1.75	5.00	1.00E-10	26.27	26.27	26.27	-5.81E+303
0.90	0.21	1.95	6.00	1.00E-10	26.27	26.27	26.27	-5.81E+303
0.90	0.58	1.33	7.00	1.00E-10	26.27	26.27	26.27	-5.81E+303

0.90	0.10	0.20	8.00	1.00E-10	26.27	26.27	26.27	-5.81E+303
0.90	0.39	1.62	9.00	1.00E-10	26.27	26.27	26.27	-5.81E+303
0.90	0.22	0.33	10.00	1.00E-10	26.27	26.27	26.27	-5.81E+303
1.00	0.96	0.84	1.00	1.00E-10	26.27	1.00E-10	1.00E-10	-5.80E+09
1.00	0.57	0.00	2.00	1.00E-10	26.27	1.00E-10	1.00E-10	-5.80E+09
1.00	0.12	0.42	3.00	1.00E-10	26.27	1.00E-10	1.00E-10	-5.80E+09
1.00	0.20	0.66	4.00	1.00E-10	26.27	26.27	26.27	-5.81E+303
1.00	0.19	1.82	5.00	1.00E-10	26.27	26.27	26.27	-5.81E+303
1.00	0.50	1.60	6.00	1.00E-10	26.27	26.27	26.27	-5.81E+303
1.00	0.87	0.94	7.00	1.00E-10	26.27	26.27	26.27	-5.81E+303
1.00	0.14	1.51	8.00	1.00E-10	26.27	26.27	26.27	-5.81E+303
1.00	0.22	1.15	9.00	1.00E-10	26.27	26.27	26.27	-5.81E+303
1.00	0.62	1.52	10.00	1.00	26.27	26.27	26.27	-5.81E+303